Pre calculus
Chapter 1

Assessments

Weekly quizzes, weekly assignments, midterm examinations, and final examination.

Grading policy

Grading policy:

- Weekly Quizzes	(10%)
- Assignments	(20%)
- Midterm Exam	(30%)
- Final Exam	(40%)
Total	(100%)

Use the syllabus.

Not everything from the chapter.

Chapters arranged for teaching with worked examples at the end.

Pearsons my Maths will be available from next week

1, 2 & 3	**Numbers and Algebraic Operations, Equations**	• Order of operations • Scientific Notation • SI Units and unit conversions • Exponents, roots and radicals and its properties • Manipulations of algebraic expressions and Solving Equations • Applied word problems	1.4 Exponents 1.5 Scientific Notation 1.6 Roots and Radicals 1.7 Adding and subtracting algebraic expressions 1.8 Multiplication of algebraic expressions 1.9 Division of algebraic expressions 1.10 Solving Equations 1.11 Formulas and literal equations 1.12 Applied word problems	**HW1 Quiz1**

Ch. 1.4: Exponents

- We use exponents to demonstrate when a number is multiplied by itself ***n*** times.

$$\text{base} \longrightarrow a^n \longleftarrow \text{exponent}$$

- Only exponents of the same base may be combined.

Laws of Exponents

- Product Law: $$a^m \times a^n = a^{m+n}$$

- Quotient Law: $$\frac{a^m}{a^n} = a^{m-n}$$ **$m > n$, $a \neq 0$**

$$\frac{a^m}{a^n} = \frac{1}{a^{n-m}}$$ **$m < n$, $a \neq 0$**

Laws of Exponents (*continued*)

- Power Law:

$$\boxed{(a^m)^n = a^{mn}}$$

$$\boxed{(ab)^n = a^n b^n}$$

$$\boxed{\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}}$$

$$b \neq 0$$

Zero & Negative Exponents

- Any number or variable raised to a zero exponent **0** is equal to **1**.
    - That is: $a^0 = 1$, $a \neq 0$.
- A negative exponent is defined by:

$$\boxed{a^{-n} = \frac{1}{a^n}}$$

Order of Operations

1. Operations within specific groupings
2. Powers
3. Multiplications and divisions (from left to right)
4. Additions and divisions (from left to right)

Evaluating Algebraic Expressions

- An algebraic expression is evaluated by substituting given values of the **literal numbers (x, y etc)** in the expression and calculating the result.

Example

- Evaluate the following algebraic expression when $x = -1$.

$$5x^3 + 7x^2 - 2x + 1$$

$$= 5(-1)^3 + 7(-1)^2 - 2(-1) + 1$$

$$= -5 + 7 + 2 + 1$$

$$= 5$$

Examples

$\sqrt{ab} = \sqrt{a}\sqrt{b}$

Product Law: $a^m \times a^n = a^{m+n}$

Quotient Law: $\frac{a^m}{a^n} = a^{m-n}$ $\quad m > n,$

Power Law: $(a^m)^n = a^{mn}$

$(ab)^n = a^n b^n$

5. $x^3 \cdot x^4$

7. $2b^4b^2$

8. $3k^5(k)$

10. $\frac{x^6}{x} =$

11. $\frac{n^5}{7n^9}$

12. $\frac{3s}{s^4} =$

5. $x^3 \cdot x^4 = x^{3+4} = x^7$

6. $y^2 y^7 = y^{2+7} = y^9$

7. $2b^4 b^2 = 2b^{4+2} = 2b^6$

8. $3k^5(k) = 3k^{5+1} = 3k^6$

9. $\dfrac{m^5}{m^3} = m^{5-3} = m^2$

11. $\dfrac{n^5}{7n^9} = \dfrac{n^{5-9}}{7} = \dfrac{n^{-4}}{7} = \dfrac{1}{7n^4}$

12. $\dfrac{3s}{s^4} = 3s^{1-4} = 3s^{-3} = \dfrac{3}{s^3}$

Ch. 1.5: Scientific Notation

- How is a very large or a very small number expressed?
- We express the number in ***scientific notation.***
- We use $P \times 10^{k}$
- The exponent of **10** tells us how many decimal places are in the number.

Examples

1. $6.873 \times 10^{11} = 687\ 300\ 000\ 000$
2. $5.67 \times 10^{-6} = .000\ 005\ 67$

- Check to see how you can use your calculator to express numbers in scientific notation and perform multiplication & division with numbers that are expressed in scientific notation.

Examples

29. $1280(865{,}000)(43.8) = 4.85\times10^{10}$

30. $0.0000569(3{,}190{,}000) = 1.82\times10^{2}$

31. $$\frac{0.0732(6710)}{0.00134(0.0231)} = \frac{7.32\times10^{-2}\times6.71\times10^{3}}{1.34\times10^{-3}\times2.31\times10^{-2}}$$
$$= 1.59\times10^{7}$$

32. $$\frac{0.00452}{2430(97{,}100)} = 1.92\times10^{-11}$$

33. $(3.642\times10^{-8})(2.736\times10^{5}) = 9.965\times10^{-3}$

34. $$\frac{(7.309\times10^{-1})^{2}}{5.9843(2.5036\times10^{-20})} = 3.566\times10^{18}$$

35. $$\frac{(3.69\times10^{-7})(4.61\times10^{21})}{0.0504} = 3.40\times10^{16}$$

36. $$\frac{(9.907\times10^{7})(1.08\times10^{12})^{2}}{(3.603\times10^{-5})(2054)} = 1.56\times10^{33}$$

Ch. 1.6: Roots & Radicals

- The **square** root of a number x is one of **two** equal factors whose product is x.

$$\sqrt{144} = \sqrt{2 \times 2 \times 2 \times 2 \times 3 \times 3}$$
$$= \sqrt{4} \times \sqrt{4} \times \sqrt{9}$$
$$= 2 \times 2 \times 3 = \mathbf{12}$$

Because 12 x 12 = 144

Roots & Radicals

- The **cube** root of a number x is one of **three** equal factors whose product is x.

$$\sqrt[3]{216} = \sqrt{2 \times 2 \times 2 \times 3 \times 3 \times 3}$$
$$= \sqrt[3]{8} \times \sqrt[3]{27}$$
$$= 2 \times 3 = \mathbf{6}$$

Because 6 x 6 x 6 = 216

We usually use prime numbers to give us our simplest answer.

A **prime number** (or a prime) is a natural number greater than 1 that has no positive divisors other than 1 and itself.

n Roots

- The n^{th} root of a number a is one of n equal factors whose product is a. This is denoted by:

$$\sqrt[n]{a}$$

radicand

index

radical sign

Properties of Roots & Radicals

1. We define the principal ***n*** th root of ***a*** to be positive if ***a*** is positive & to be negative if ***a*** is negative and ***n*** is odd.
2. The square root of a product of positive numbers is the product of their square roots.

$$\boxed{\sqrt{ab} = \sqrt{a}\sqrt{b}}$$

Examples

eg $\sqrt[5]{32} = 2$

$\sqrt[5]{n \times n \times n \times n \times n}$

Eg 2 $\sqrt[5]{2 \times 2 \times 2 \times 2 \times 2}$

Examples

16. $\sqrt[5]{-32} = -2$

17. $\left(\sqrt{5}\right)^2 = 5$

18. $\left(\sqrt[3]{31}\right)^3 = \sqrt[3]{31}\sqrt[3]{31}\sqrt[3]{31} = 31$

19. $\left(-\sqrt[3]{-47}\right)^3 = (-1)^3\left(\sqrt[3]{-47}\right)^3 = -1(-47) = 47$

20. $\left(\sqrt[5]{-23}\right)^5 = -23$

21. $\left(-\sqrt[4]{53}\right)^4 = 53$

22. $-\sqrt{32} = -\sqrt{16 \cdot 2} = -4\sqrt{2}$

23. $\sqrt{1200} = \sqrt{400(3)} = 20\sqrt{3}$

24. $\sqrt{50} = \sqrt{25 \cdot 2} = 5\sqrt{2}$

Ch. 1.7: Addition & Subtraction of Algebraic Expressions

- We can work with algebraic expressions as we would with any real numbers.
- Be sure to follow the order of operations.
- Collect like terms watching for any negative terms.

Terminology used with Algebraic Expressions

- Monomial:
  - an algebraic expression containing only one term.
  - ***Examples***: $3\boldsymbol{x}^5$, 7, $-15\boldsymbol{x}^2$
- Binomial:
  - an algebraic expression containing two terms.
  - ***Examples*** : $3\boldsymbol{x}^5 - 4\boldsymbol{x}$, $2\boldsymbol{x} + 7$, $-15\boldsymbol{x}^2 - 20$

Terminology used with Algebraic Expressions (*continued*)

- Trinomial:
  - an algebraic expression containing three terms.
  - ***Examples***: $3x^5 - 4x + 1$, $x^3 - 2x + 7$
- Multinomial:
  - Any expression containing two or more terms.
  - ***Examples*** : $3x^5 - 8x^2 + 4x + 1$

In mathematics, a **coefficient** is a multiplicative factor in some term of a polynomial, a series or any expression; it is usually a number, but in any case does not involve any variables of the expression. For instance in

$$7x^2 - 3xy + 1.5 + y$$

the first two terms respectively have the coefficients 7 and −3. The third term 1.5 is a constant. The final term does not have any explicitly written coefficient, but is considered to have coefficient 1,

Terminology used with Algebraic Expressions (*continued*)

- Coefficient:
  - The numbers & literal symbols multiplying any given factor in an algebraic expression.
- Numerical coefficient:
  - The product of all the numbers in explicit form.
- Similar or like terms:
  - All terms that differ at most in their numerical coefficients.

Addition & Subtraction of Algebraic Expressions

- In adding and subtracting algebraic expressions, we combine similar (or like) terms into a single term.
- The final simplified expression will contain only terms that are not similar.

Example

Notice how the sign of each term in the second algebraic expression is reversed.

- Simplify by collecting like terms:

$$(-6x^4 + 3x^2 + 6) - (2x^4 + 5x^3 - 5x^2 + 7)$$

$$= -6x^4 + 3x^2 + 6 - 2x^4 - 5x^3 + 5x^2 - 7$$

$$= (-6x^4 - 2x^4) - 5x^3 + (3x^2 + 5x^2) + (6 - 7)$$

$$= -8x^4 - 5x^3 + 8x^2 - 1$$

Examples

$$a^2b - a^2b^2 - 2a^2b$$

Examples

5. $5x + 7x - 4x = 12x - 4x = 8x$

6. $6t - 3t - 4t = -t$

7. $2y - y + 4x = y(2-1) + 4x = y + 4x$

8. $4C + L - 6C = -2C + L$

9. $2F - 2T - 2 + 3F - T = 5F - 3T - 2$

10. $x - 2y + 3x - y + z = 4x - 3y + z$

11. $a^2b - a^2b^2 - 2a^2b = a^2b - 2a^2b - a^2b^2 = -a^2b - a^2b^2$

12. $xy^2 - 3x^2y^2 + 2xy^2 = 3xy^2 - 3x^2y^2$

Ch. 1.8: Multiplication of Algebraic Expressions

- To find the products of two or more monomials, we use the laws of exponents & the laws for multiplying signed numbers.
  1. Multiply the numerical coefficients.
  2. Multiply the literal numbers.
  3. Combine any exponents when bases are the same.

Example

- Multiply:

$$5x^3(9x^2 - 7x)$$

1. Multiply the numerical coefficients.

$$= 45x^5 - 35x^4$$

2. Multiply the literal numbers.

3. Combine any exponents when bases are the same.

Multiplication of Algebraic Expressions

- When working with multinomials, be sure to use the distributive property over each term.
- ***Example***:

$$\begin{aligned}(x-3)^2 &= (x-3)(x-3)\\ &= x^2-3x-3x+9\\ &\boxed{= x^2-6x+9}\end{aligned}$$

General factoring method.

$$(a+b)(c+d)$$

Last

Inner

$$(a+b)(c+d) = ac + ad + bc + bd$$

F O I L

First

Outer

Examples

Examples

20. $-2\left(-3st^3\right)(3s-4t)=6st^3(3s-4t)=18s^2t^3-24st^4$

21. $(x-3)(x+5)=x^2+5x-3x-15=x^2+2x-15$

22. $(a+7)(a+1)=a^2+8a+7$

55.
$$\begin{aligned}(x+y)^3&=(x+y)(x+y)(x+y)\\&=\left(x^2+2xy+y^2\right)(x+y)\\&=x^3+x^2y+2x^2y+2xy^2+xy^2+y^3\\&=x^3+3x^2y+3xy^2+y^3\neq x^3+y^3\end{aligned}$$

56.
$$\begin{aligned}&(x+y)\left(x^2-xy+y^2\right)\\&=x^3-x^2y+xy^2+x^2y-xy^2+y^3\\&=x^3+y^3\end{aligned}$$

Ch. 1.9: Division of Algebraic Expressions

- When dividing algebraic expressions once again use the laws of exponents and the laws for dividing signed numbers.
- Combine the exponents if the bases are the same.

Division of Algebraic Expressions

- The quotient of a multinomial divided by a monomial is found by dividing each term of the multinomial by the monomial and adding the results.
- That is:

$$\boxed{\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}}$$

Polynomials in $\boldsymbol{x}$

- If each term in an algebraic sum is a number or is of the form ax^n, where n is a nonnegative integer.
- ***Degree of the polynomial***:
  - The greatest value of the exponent n that appears.

Division of One Polynomial by Another

This is long division using polynomials

First watch a video on long division

http://www.youtube.com/watch?v=FXgV9ySNusc

Then we will look at a procedure.

Division of One Polynomial by Another

- Arrange the dividend & the divisor in descending powers of the variable.
- We divide similarly as we would with long division.

Example

- We are asked to solve

$$\left(6x^2 - 13x + 7\right) \div \left(x + 1\right)$$

- Note that each polynomial is arranged in descending order of powers.

Solution

$$
\begin{array}{r|l}
 & 6x - 19 \\
\hline
x + 1 & 6x^2 - 13x + 7 \\
 & 6x^2 + 6x \\
\hline
 & \quad\quad -19x + 7 \\
 & \quad\quad -19x - 19 \\
\hline
 & \quad\quad\quad\quad 26
\end{array}
$$

remainder $\longrightarrow$ 26

1. Arrange the dividend & the divisor in descending powers of the variable. We divide similarly as we would with long division.
2. Divide the first term of the dividend by the first term of the divisor. This gives us our first term in the quotient.
3. Multiply the entire divisor by the first term of the quotient. Subtract this product from the dividend.
4. Divide the first term of the difference by the first term of the divisor. This gives us the second term.
5. Multiply the entire divisor by this term of the quotient. Subtract this product from the difference.
6. This gives the remainder (repeat if needed)

Final Answer

- Therefore, the quotient for:

$$\left(6x^2 - 13x + 7\right) \div (x + 1)$$

$$= 6x - 19 + \frac{26}{x+1}$$

remainder

① make sure in order $x^2 \to x^1 \to x^0$

RECAP

Solution

Quotient

$$
\begin{array}{r}
6x \; -19 \\
x+1 \overline{\smash{\big)}\, 6x^2 - 13x + 7} \\
\underline{6x^2 + 6x} \\
\boxed{-19x + 7} \\
\underline{-19x - 19} \\
26
\end{array}
$$

← Divident

Devisor

First term

② $\frac{6x^2}{x} = 6x$

First term

③ multiply divisor by 1st term quotien
$6x(x+1)$
subtract this from above

④ Next term is $\frac{-19x}{x} = -19$

remainder → 26

⑤ multiply devisor by this term $-19(x+1)$ then subtract

ans $= 6x - 19 + \frac{26}{x+1}$

⑥ this gives remainder
[If no more x terms stop]

Examples

$$3x-4\overline{\smash{)}6x^2-5x-9}$$

Examples

7. $\dfrac{-16r^3t^5}{-4r^5t} = \dfrac{4t^{5-1}}{r^{5-3}} = \dfrac{4t^4}{r^2}$

8. $\dfrac{51mn^5}{17m^2n^2} = \dfrac{3n^3}{m}$

9. $\dfrac{(15x^2)(4bx)(2y)}{30bxy} = 4x^2$

32. $$\begin{array}{r} 2x+1 \\ 3x-4\overline{)6x^2-5x-9} \\ \underline{6x^2-8x} \\ 3x-9 \\ \underline{3x-4} \\ -5 \end{array}$$

$$(6x^2-5x-9)\div(3x-4) = 2x+1+\frac{-5}{3x-4}$$

Ch. 1.10: Solving Equations

- An equation is an algebraic statement that two algebraic expressions are equal.
- Any value of the literal numbers representing the ***unknown*** that produces equality when ***substituted*** in the equation is said to ***satisfy*** the equation.
- An equation valid only for certain values of the unknown is a ***conditional equation***.

Solving Equations

- To solve an equation we find the values of the unknown that satisfy it.
- ***Key Rule*** when solving equations.

Perform the same operation

on both sides of the equation.

Procedure for Solving Equations

1. Remove grouping symbols (distributive law).
2. Combine any like terms of each side (also after *step 3*).
3. Perform the same operations on both sides, until ***x* = *result*** is obtained.
4. Check the solution in the original equation.

Example

- Solve this linear equation:

$$3(x-4)-6(1-3x)=20$$

$$3x-12-6+18x=20$$

$$21x=38$$

$$x=\frac{38}{21}\cong 1.81$$

1. Remove grouping symbols (distributive law).
2. Combine any like terms of each side (also after *step 3*).
3. Perform the same operations on both sides, until ***x = result*** is obtained.

Check

- Solve this linear equation:

$$3\left(\boxed{\frac{38}{21}}-4\right)-6\left(1-3\times\boxed{\frac{38}{21}}\right)\overset{?}{=}20$$

$$20 = 20$$

Examples

Examples

29. $\dfrac{4x-2(x-4)}{3}=8$

$4x-2(x-4)=24$

$4x-2x+8=24$

$2x=16$

$x=8$

30. $2x=\dfrac{3-5(7-3x)}{4}$

$8x=3-5(7-3x)$

$8x=3-35+15x$

$-7x=-32$

$x=\dfrac{32}{7}$

31. $|x|-1=8$

$|x|=9$

$x=-9$ or $x=9$

32. $2-|x|=4$

$-|x|=2$

$|x|=-2$, no solution

50. $210(3x)=55.3x+38.5(8.25-3x)$

$630x=55.3x+317.625-115.5x$

$690.2x=317.625$

$x=0.46$ m

Ch. 1.11: Formulas & Literal Equations

- A ***formula*** is an equation that expresses the relationship between two or more related quantities.
- We can isolate the desired symbol by using algebraic operations on the literal numbers.
- When expected to substitute a given value into the formula, we should first isolate the given variable.

Example

- In the given formula, isolate for ***e***.

$$C = \frac{2eAk_1k_2}{d\left(k_1 + k_2\right)}$$

- ***Solution***:

1. We multiply both sides by $\boldsymbol{d(k_1 + k_2)}$.
2. Divide both sides by $\boldsymbol{2Ak_1k_2}$.

Solution

1. We multiply both sides by $\boldsymbol{d(k_1 + k_2)}$
2. Divide both sides by $\boldsymbol{2Ak_1k_2}$.

$$C = \frac{2eAk_1k_2}{d(k_1+k_2)} \Rightarrow Cd(k_1+k_2) = 2eAk_1k_2$$

$$\boxed{\frac{Cd(k_1+k_2)}{2Ak_1k_2} = e}$$

Examples

Examples

5. $E = IR$

$$\frac{E}{I} = \frac{IR}{I}$$

$$R = \frac{E}{I}$$

6. $PV = nRT$

$$T = \frac{PV}{nR}$$

35. $$P = \frac{V_1(V_2 - V_1)}{gJ}$$

$$gJP = V_1V_2 - V_1^2$$

$$gJP + V_1^2 = V_1V_2$$

$$V_2 = \frac{gJP + V_1^2}{V_1}$$

7. $rL = g_2 - g_1$

$$g_1 = g_2 - rL$$

8. $W = S_dT - Q$

$$Q = S_dT - W$$

Ch. 1.12: Applied Word Problems

- Mathematical questions in science, biology, etc. do not present themselves in neat, tidy equations.
- They are often presented as word problems which must be solved.
- The following is a step-by-step approach that you can use to solve word problems.

Procedure for Solving Word Problems

1. Read the statement of the problem. First, read it for a general overview. Then read it slowly & carefully, listing the information provided.
2. Clearly identify the unknown quantities & then assign an appropriate letter to represent one of them, stating this choice clearly.
3. Specify the other unknown quantities in terms of the one in step 2.

Procedure for Solving Word Problems (*continued*)

4. If possible, make a sketch using the known & unknown quantities.
5. Analyze the statement of the problem & write the necessary equation.
6. Solve the equation, clearly stating the solution.
7. Check the solution with the original statement of the problem.

Example

- The sum of 3 electric currents that come together at a point in an integrated circuit is zero. If the second current is double the first & the third current is 9.2 μA more than the first, what are the currents?
- (The sign of a current indicates the direction of flow.)

Solution

1. Read the statement of the problem. First, read it for a general overview. Then read it again slowly & carefully, listing the information provided.

- ***Information provided***:
  - ✓ 3 separate electric currents.
  - ✓ Their sum total is zero.
  - ✓ The currents can be positive or negative.

Solution *(continued)*

2. Clearly identify the unknown quantities & then assign an appropriate letter to represent one of them, stating this choice clearly.

- ***Unknown quantities***:
  - ✓ First current: x
  - ✓ Each current listed is in terms of the first electric current.

Solution *(continued)*

3. Specify the other unknown quantities in terms of the one in step 2.

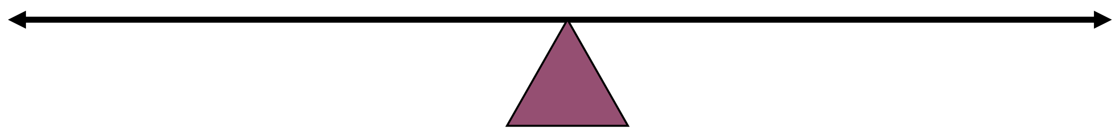
- ***Unknown quantities***:
  - ✓ First current: x
  - ✓ Second current: $2x$
  - ✓ Third current: $x + 9.2$

Solution *(continued)*

4. If possible, make a sketch using the known & unknown quantities.

- ***Sketch***:

First current + Second current + Third current = 0



Solution *(continued)*

5. Analyze the statement of the problem & write the necessary equation.

- ***The equation***:

✓ $$x + 2x + (x + 9.2) = 0$$

Solution *(continued)*

6. Solve the equation, clearly stating the solution.

- ***The equation***:

$$x + 2x + (x + 9.2) = 0$$

$$x = -2.3$$

  - ***Therefore***,
    - ✓ First current: $x = -2.3$
    - ✓ Second current: $2(-2.3) = -4.6$
    - ✓ Third current: $(-2.3 + 9.2) = 6.9$

Solution *(continued)*

7. Check the solution with the original statement of the problem.

- ***Check***:

$$(-2.3) + (-4.6) + (-6.9) \overset{?}{=} 0$$

$$0 \overset{\checkmark}{=} 0$$

- ***Conclusion***: The solution is correct.

Examples

Examples

6. Let x = flow rate of first stream in ft^3/s;

$x + 1700$ = flow rate of second stream in ft^3/s

$$(x + x + 1700) \cdot 3600 = 1.98 \times 10^7$$

$$x = 1900\ \mathrm{ft}^3/\mathrm{s}$$

$$x + 1700 = 3600\ \mathrm{ft}^3/\mathrm{s}$$

18.

$$G_1 + G_2 = 750 \Rightarrow G_2 = 750 - G_1$$

$$0.65G_1 + 0.75G_2 = 530$$

$$0.65G_1 + 0.75(750 - G_1) = 530$$

$$0.65G_1 + 562.2 - 0.75G_1 = 530$$

$$-0.10G_1 = -32.5$$

$$G_1 = 325\ MW$$

$$G_2 = 750 - G_1 = 425\ MW$$

SUMMARY BELOW

ORDER of operations

1. Operations within specific groupings
2. Powers
3. Multiplications and divisions (from left to right)
4. Additions and divisions (from left to right)

- commutative $a + b = b + a$
- associative $(a + b) + c = a + (b + c)$
- distributive $a\,(b + c) = ab + ac$

Division by zero is undefined

index $\rightarrow \sqrt[n]{a} \leftarrow$ radicand

radical sign

Mon = 1
Bi = 2
Tri = 3

$$\boxed{\sqrt{ab} = \sqrt{a}\sqrt{b}}$$

Product Law: $\boxed{a^m \times a^n = a^{m+n}}$

Quotient Law: $\boxed{\dfrac{a^m}{a^n} = a^{m-n}}$ $m > n,$

Power Law: $\boxed{(a^m)^n = a^{mn}}$

$\boxed{(ab)^n = a^n b^n}$

Accuracy: number of significant digits

Precision: decimal position of the last significant digit.

base $\rightarrow a^n \leftarrow$ exponent

Approx = measure
Exact = count

Scientific notation $P \times 10^k$

- Multiply:

1. Multiply the numerical coefficients.

$$5x^3(9x^2 - 7x)$$

2. Multiply the literal numbers.

$$= 45x^5 - 35x^4$$

3. Combine any exponents when bases are the same.

$$\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$$

$$
\begin{array}{r|l}
 & 6x \quad -19 \\
\hline
x+1 & 6x^2 - 13x + 7 \\
 & 6x^2 + 6x \\
\hline
 & \quad -19x + 7 \\
 & \quad -19x - 19 \\
\hline
 & \qquad\quad 26
\end{array}
$$

remainder $\rightarrow 26$

Perform the same operation

on both sides of the equation.

1. Remove grouping symbols (distributive law).
2. Combine any like terms of each side (also after *step 3*).
3. Perform the same operations on both sides, until $x = result$ is obtained.
4. Check the solution in the original equation.

Chapter 1 Review Exercises (p56) From mymath (Pearsons)

4	50
16	64
23	82
28	93
	99

worked examples

Make sure you understand These worked examples

1.8 1, 2, 3, 4, 5, 6

1

(a) $3c^5(-4c^2) = -12c^7$ multiply numerical coefficients and add exponents of c

(b) $(-2b^2y^3)(-9aby^5) = 18ab^3y^8$ add exponents of same base

(c) $2xy(-6cx^2)(3xcy^2) = -36c^2x^4y^3$

EXAMPLE 2 Product containing power of a monomial

(a) $-3a(2a^2x)^3 = -3a(8a^6x^3) = -24a^7x^3$

(b) $2s^3(-st^4)^2(4s^2t) = 2s^3(s^2t^8)(4s^2t) = 8s^7t^9$

EXAMPLE 3 Product of monomial and polynomial

(a) $2ax(3ax^2 - 4yz) = 2ax(3ax^2) + (2ax)(-4yz) = 6a^2x^3 - 8axyz$

(b) $5cy^2(-7cx - ac) = (5cy^2)(-7cx) + (5cy^2)(-ac) = -35c^2xy^2 - 5ac^2y^2$ ■

EXAMPLE 4 Product of polynomials

$$(x - 2)(x + 3) = x(x) + x(3) + (-2)(x) + (-2)(3)$$

$$= x^2 + 3x - 2x - 6 = x^2 + x - 6$$

EXAMPLE 5 Power of a polynomial

two factors

(a) $(x + 5)^2 = (x + 5)(x + 5) = x^2 + 5x + 5x + 25$

$$= x^2 + 10x + 25$$

(b) $(2a - b)^3 = (2a - b)(2a - b)(2a - b)$ the exponent 3 indicates three factors

$$\begin{aligned}
&= (2a - b)(4a^2 - 2ab - 2ab + b^2) \\
&= (2a - b)(4a^2 - 4ab + b^2) \\
&= 8a^3 - 8a^2b + 2ab^2 - 4a^2b + 4ab^2 - b^3 \\
&= 8a^3 - 12a^2b + 6ab^2 - b^3
\end{aligned}$$

We should note in illustration (a) that

$(x + 5)^2$ ***is <u>not</u> equal to*** $x^2 + 25$

because the term $10x$ is not included. We must follow the proper procedure and not simply square each of the terms within the parentheses.

EXAMPLE 6 Products in an application

An expression used with a lens of a certain telescope is simplified as shown.

$$\begin{aligned}
&a(a+b)^2 + a^3 - (a+b)(2a^2 - s^2) \\
&\quad = a(a+b)(a+b) + a^3 - (2a^3 - as^2 + 2a^2b - bs^2) \\
&\quad = a(a^2 + ab + ab + b^2) + a^3 - 2a^3 + as^2 - 2a^2b + bs^2 \\
&\quad = a^3 + a^2b + a^2b + ab^2 - a^3 + as^2 - 2a^2b + bs^2 \\
&\quad = ab^2 + as^2 + bs^2
\end{aligned}$$

1.9 1, 2, 3, 4, 5

EXAMPLE 1 Dividing monomials

(a) $\dfrac{3c^7}{c^2} = 3c^{7-2} = 3c^5$

(b) $\dfrac{16x^3y^5}{4xy^2} = \dfrac{16}{4}(x^{3-1})(y^{5-2}) = 4x^2y^3$

divide coefficients

subtract exponents

(c) $\dfrac{-6a^2xy^2}{2axy^4} = -\left(\dfrac{6}{2}\right)\dfrac{a^{2-1}x^{1-1}}{y^{4-2}} = -\dfrac{3a}{y^2}$

EXAMPLE 2 Dividing by a monomial

(a) $$\frac{4a^2 + 8a}{2a} = \frac{4a^2}{2a} + \frac{8a}{2a} = 2a + 4$$

each term of numerator divided by denominator

(b) $$\frac{4x^3y - 8x^3y^2 + 2x^2y}{2x^2y} = \frac{4x^3y}{2x^2y} - \frac{8x^3y^2}{2x^2y} + \frac{2x^2y}{2x^2y}$$

$$= 2x - 4xy + 1$$

EXAMPLE 3 Application of dividing by a monomial

The expression $\dfrac{2p + v^2d + 2ydg}{2dg}$ is used when analyzing the operation of an irrigation pump. Performing the indicated division, we have

$$\frac{2p + v^2d + 2ydg}{2dg} = \frac{p}{dg} + \frac{v^2}{2g} + y$$

EXAMPLE 4 Dividing one polynomial by another

Perform the division $(6x^2 + x - 2) \div (2x - 1)$.

$\left(\text{This division can also be indicated in the fractional form } \dfrac{6x^2 + x - 2}{2x - 1}.\right)$

We set up the division as we would for long division in arithmetic. Then, following the procedure outlined above, we have the following:

$$
\begin{array}{r|l}
 & \;\;3x + 2 \\ \hline
2x - 1 & 6x^2 + x - 2 \\
 & 6x^2 - 3x \\ \hline
 & \qquad\;\; 4x - 2 \\
 & \qquad\;\; 4x - 2 \\ \hline
 & \qquad\qquad\;\; 0
\end{array}
$$

$\dfrac{6x^2}{2x}$ divide first term of dividend by first term of divisor

$3x(2x - 1)$

subtract

$6x^2 - 6x^2 = 0$

$x - (-3x) = 4x$

subtract

EXAMPLE 5 Quotient with a remainder

Perform the division $\dfrac{8x^3 + 4x^2 + 3}{4x^2 - 1}$. Because there is no x-term in the dividend, we should leave space for any x-terms that might arise (which we will show as $0x$).

$$
\begin{array}{lrl|rrrrl}
 & & & & & 2x & +\ 1 & \\
\cline{4-7}
\text{divisor} & 4x^2 - 1 & & 8x^3 & +\ 4x^2 & +\ 0x & +\ 3 & \text{dividend} \qquad \dfrac{8x^3}{4x^2} = 2x \\
 & & & 8x^3 & & -\ 2x & & \text{subtract} \\
\cline{4-6}
 & & & & 4x^2 & +\ 2x & +\ 3 & \qquad\qquad \dfrac{4x^2}{4x^2} = 1 \\
 & 0x - (-2x) = 2x & & & & & & \\
 & & & & 4x^2 & & -\ 1 & \text{subtract} \\
\cline{5-7}
 & & & & & 2x & +\ 4 & \text{remainder}
\end{array}
$$

Because the degree of the remainder $2x + 4$ is less than that of the divisor, we now show the quotient in this case as $2x + 1 + \dfrac{2x + 4}{4x^2 - 1}$. ■

EXAMPLE 1 Valid values for equations

The equation $3x - 5 = x + 1$ is true only if $x = 3$. Substituting 3 for x in the equation, we have $3(3) - 5 = 3 + 1$, or $4 = 4$; substituting $x = 2$, we have $1 = 3$, which is not correct.

This equation is valid for only one value of the unknown. *An equation valid only for certain values of the unknown is a* **conditional equation.** In this section, nearly all equations we solve will be conditional equations that are satisfied by only one value of the unknown.

EXAMPLE 2 Identity and contradiction

(a) The equation $x^2 - 4 = (x - 2)(x + 2)$ is true for all values of x. For example, substituting $x = 3$ in the equation, we have $3^2 - 4 = (3 - 2)(3 + 2)$, or $5 = 5$. Substituting $x = -1$, we have $(-1)^2 - 4 = (-1 - 2)(-1 + 2)$, or $-3 = -3$. *An equation valid for all values of the unknown is an* **identity.**

(b) The equation $x + 5 = x + 1$ is not true for any value of x. For any value of x we try, we find that the left side is 4 greater than the right side. *Such an equation is called a* **contradiction.**

EXAMPLE 3 Basic operations used in solving

In each of the following equations, we may isolate x, and thereby solve the equation, by performing the indicated operation.

$x - 3 = 12$	$x + 3 = 12$	$\frac{x}{3} = 12$	$3x = 12$
add 3 to both sides	subtract 3 from both sides	multiply both sides by 3	divide both sides by 3
$x - 3 + 3 = 12 + 3$	$x + 3 - 3 = 12 - 3$	$3\left(\frac{x}{3}\right) = 3(12)$	$\frac{3x}{3} = \frac{12}{3}$
$x = 15$	$x = 9$	$x = 36$	$x = 4$

Each solution should be checked by substitution in the original equation.

EXAMPLE 4 Operations used for solution; checking

Solve the equation $2t - 7 = 9$.

We are to perform basic operations to both sides of the equation to finally isolate t on one side. The steps to be followed are suggested by the form of the equation.

$$
\begin{aligned}
2t - 7 &= 9 && \text{original equation} \\
2t - 7 + 7 &= 9 + 7 && \text{add 7 to both sides} \\
2t &= 16 && \text{combine like terms} \\
\frac{2t}{2} &= \frac{16}{2} && \text{divide both sides by 2} \\
t &= 8 && \text{simplify}
\end{aligned}
$$

Therefore, we conclude that $t = 8$. Checking *in the original equation*, we have

$$2(8) - 7 \stackrel{?}{=} 9, \qquad 16 - 7 \stackrel{?}{=} 9, \qquad 9 = 9$$

The solution checks. ■

EXAMPLE 5 First remove parentheses

Solve the equation $x - 7 = 3x - (6x - 8)$.

$x - 7 = 3x - 6x + 8$	parentheses removed	
$x - 7 = -3x + 8$	x-terms combined on right	
$4x - 7 = 8$	$3x$ added to both sides	—by inspection
$4x = 15$	7 added to both sides	—by inspection
$x = \frac{15}{4}$	both sides divided by 4	—by inspection

Checking in the original equation, we obtain (after simplifying) $-\frac{13}{4} = -\frac{13}{4}$.

EXAMPLE 7 Ratio

If the ratio of x to 8 equals the ratio of 3 to 4, we have the proportion

$$\frac{x}{8} = \frac{3}{4}$$

We can solve this equation by multiplying both sides by 8. This gives

$$8\left(\frac{x}{8}\right) = 8\left(\frac{3}{4}\right), \quad \text{or} \quad x = 6$$

Substituting $x = 6$ into the original proportion gives the proportion $\frac{6}{8} = \frac{3}{4}$. Because these ratios are equal, the solution checks.

EXAMPLE 1 Solving for symbol in formula

In Einstein's formula $E = mc^2$, solve for m.

$$\frac{E}{c^2} = m \qquad \text{divide both sides by } c^2$$

$$m = \frac{E}{c^2} \qquad \text{switch sides to place } m \text{ at left}$$

The required symbol is usually placed on the left, as shown. ■

EXAMPLE 2 Symbol with subscript in formula

A formula relating acceleration a, velocity v, initial velocity v_0, and time is $v = v_0 + at$. Solve for t.

$$v - v_0 = at \qquad v_0 \text{ subtracted from both sides}$$

$$t = \frac{v - v_0}{a} \qquad \text{both sides divided by } a \text{ and then sides switched}$$ ■

EXAMPLE 3 Symbol in capital and in lowercase

In the study of the forces on a certain beam, the equation $W = \frac{L(wL + 2P)}{8}$ is used. Solve for P.

$$8W = \frac{8L(wL + 2P)}{8} \quad \text{multiply both sides by 8}$$

$$8W = L(wL + 2P) \quad \text{simplify right side}$$

$$8W = wL^2 + 2LP \quad \text{remove parentheses}$$

$$8W - wL^2 = 2LP \quad \text{subtract } wL^2 \text{ from both sides}$$

$$P = \frac{8W - wL^2}{2L} \quad \text{divide both sides by } 2L \text{ and switch sides}$$

■

EXAMPLE 4 Formula with groupings

The effect of temperature on measurements is important when measurements must be made with great accuracy. The volume V of a special precision container at temperature T in terms of the volume V_0 at temperature T_0 is given by $V = V_0[1 + b(T - T_0)]$, where b depends on the material of which the container is made. Solve for T.

Because we are to solve for T, we must isolate the term containing T. This can be done by first removing the grouping symbols.

$$V = V_0[1 + b(T - T_0)] \quad \text{original equation}$$

$$V = V_0[1 + bT - bT_0] \quad \text{remove parentheses}$$

$$V = V_0 + bTV_0 - bT_0V_0 \quad \text{remove brackets}$$

$$V - V_0 + bT_0V_0 = bTV_0 \quad \text{subtract } V_0 \text{ and add } bT_0V_0 \text{ to both sides}$$

$$T = \frac{V - V_0 + bT_0V_0}{bV_0} \quad \text{divide both sides by } bV_0 \text{ and switch sides}$$

■

EXAMPLE 1 Sum of forces on a beam

A 17-lb beam is supported at each end. The supporting force at one end is 3 lb more than at the other end. Find the forces.

Since the force at each end is required, we write

$$\text{let } F = \text{the smaller force (in lb)} \qquad \text{step 2}$$

as a way of establishing the unknown for the equation. Any appropriate letter could be used, and we could have let it represent the larger force.

Also, since the other force is 3 lb more, we write

$$F + 3 = \text{the larger force (in lb)} \qquad \text{step 3}$$

We now draw the sketch in Fig. 18. step 4

Since the forces at each end of the beam support the weight of the beam, we have the equation

$$F + (F + 3) = 17 \qquad \text{step 5}$$

This equation can now be solved: $2F = 14$

$$F = 7 \text{ lb} \qquad \text{step 6}$$

Thus, the smaller force is 7 lb, and the larger force is 10 lb. This checks with the original statement of the problem. step 7

■ Be sure to carefully identify your choice for the unknown. In most problems, there is really a choice. Using the word *let* clearly shows that a specific choice has been made.

■ The statement after "let x (or some other appropriate letter) =" should be clear. It should completely define the chosen unknown.

Fig. 18