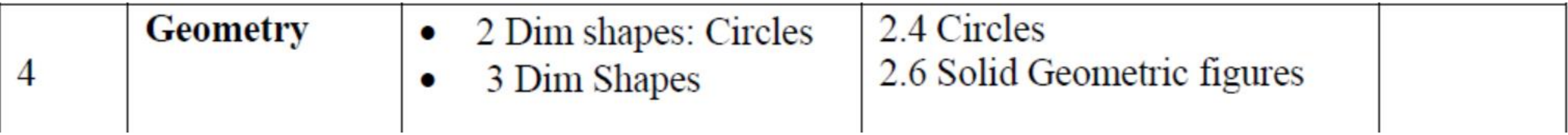
Chapter 2.

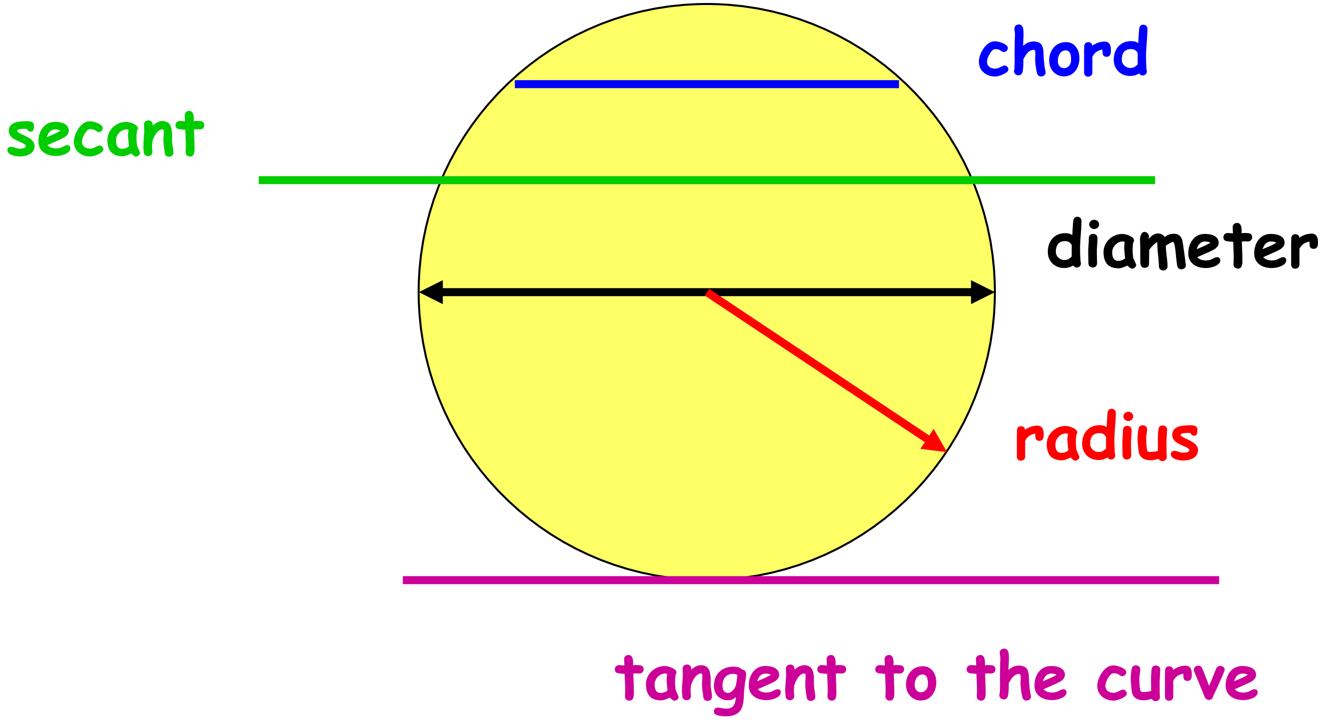
Geometry



4	**Geometry**	• 2 Dim shapes: Circles • 3 Dim Shapes	2.4 Circles 2.6 Solid Geometric figures	

Ch. 2.4: Circles

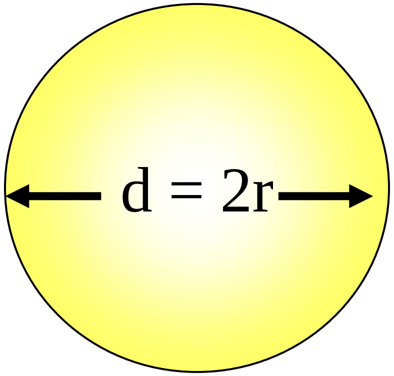
- Terminology related to the circle.



Circumference & Area of a Circle

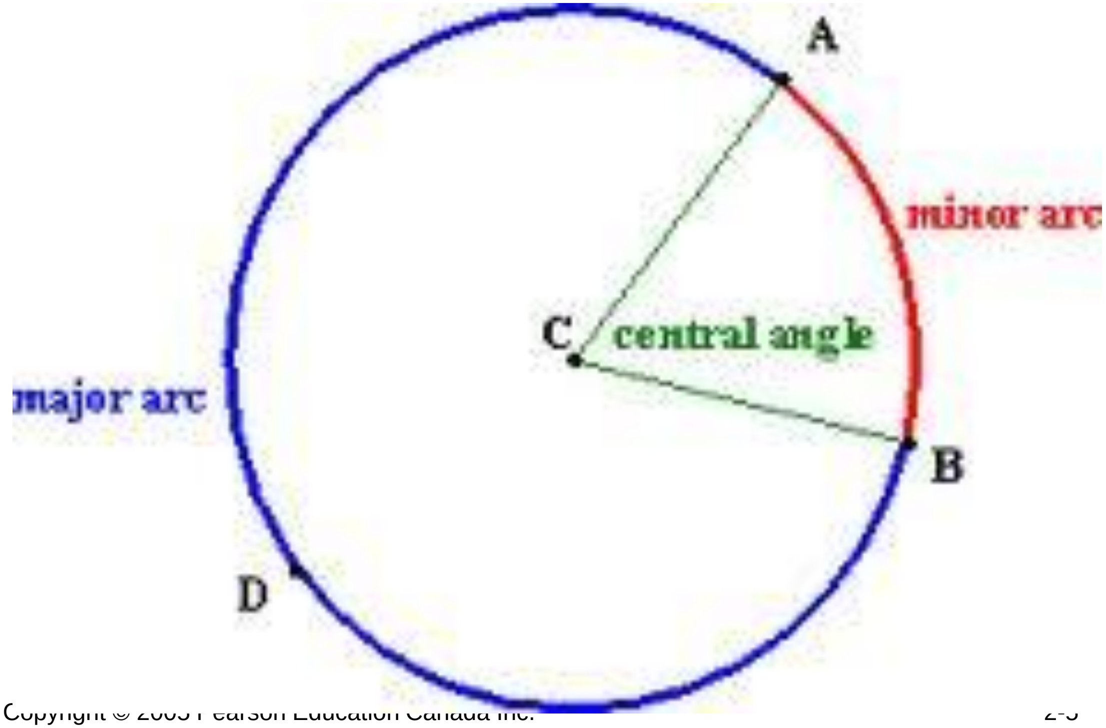
- Circumference is a perimeter of a circle and is given by $C = 2\pi r$

- $A_{circle} = \pi r^2$



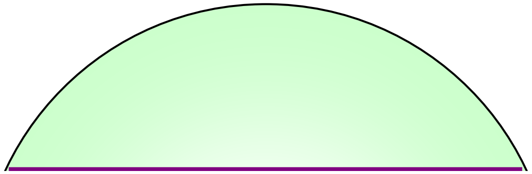
Circular Arcs & Angles

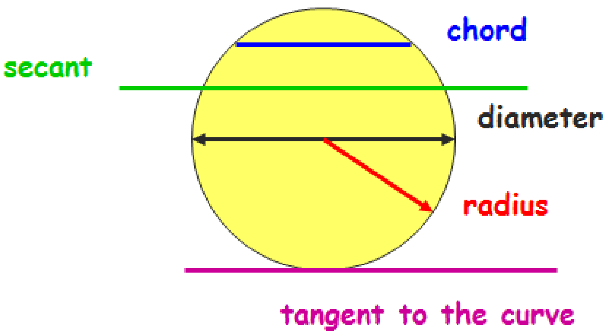
- An ***arc*** is part of a circle.
- A ***central angle*** is an angle formed at the centre by two radii.



Segment of a Circle

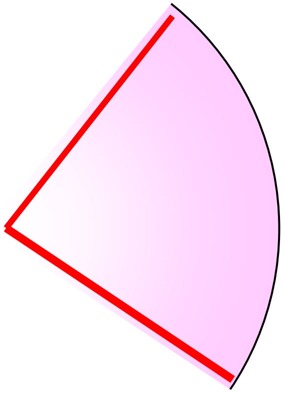
- The region bounded by a chord and its arc.





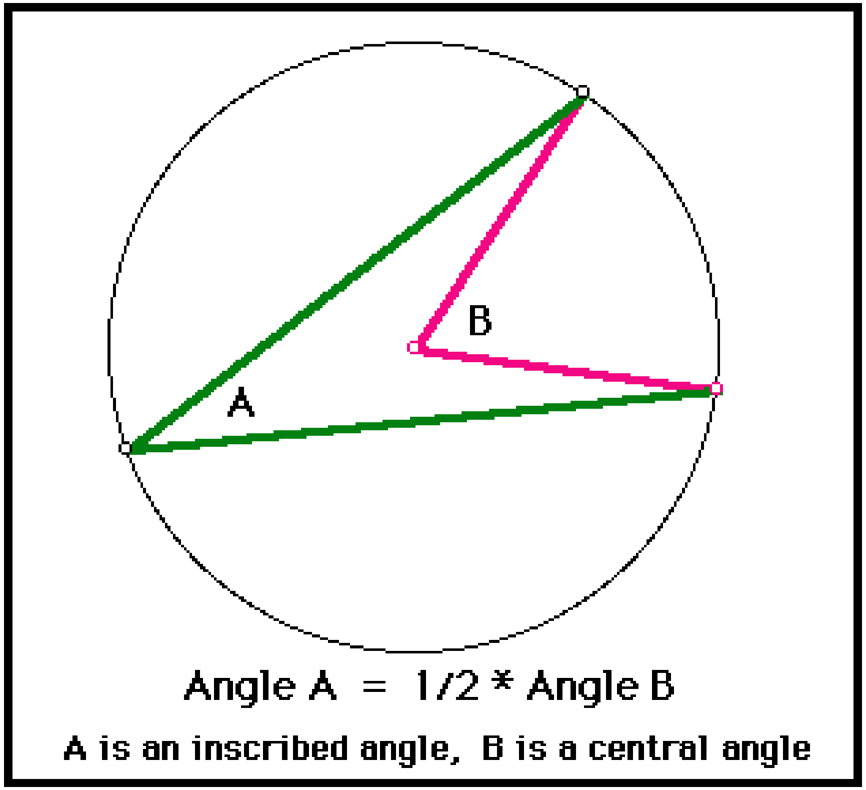
The Sector of a Circle

- The region bounded by 2 radii and the arc they intercept.



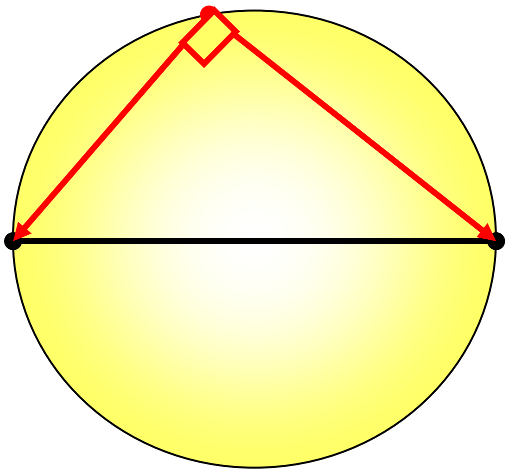
An Inscribed Angle

- An ***inscribed angle*** of an arc is one for which the endpoints of the arc are points on the sides of the angle for which the vertex is a point (not an endpoint) of the arc.



An Inscribed Angle

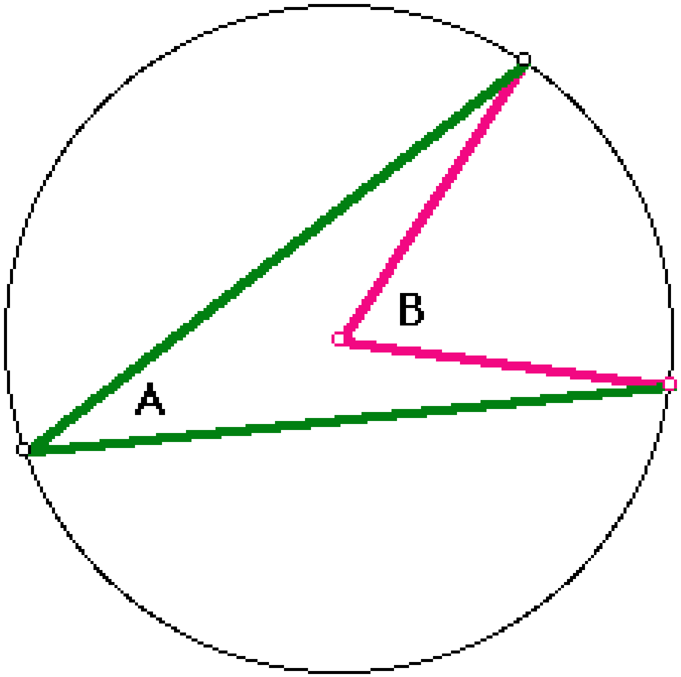
- An angle inscribed in a semi-circle is a right angle.



The measure of an inscribed angle in a circle equals half the measure of its intercepted arc.

Measure of an Inscribed Angle Theorem

If an angle is inscribed in a circle, then its measure is half the measure of its intercepted arc.



Angle A = 1/2 * Angle B

A is an inscribed angle, B is a central angle

- **Inscribed angle:** In a circle, this is an angle formed by two chords with the vertex on the circle.
- **Intercepted arc:** Corresponding to an angle, this is the portion of the circle that lies in the interior of the angle together with the endpoints of the arc.

In Exercises 17–20, refer to Fig. 2.81, where AB is a diameter, TB is a tangent line at B, and ∠ABC = 65°. Determine the indicated angles.

17. ∠*CBT*

18. ∠*BCT*

19. ∠*CAB*

20. ∠*BTC*

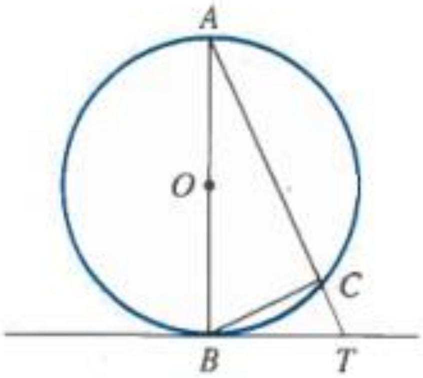


Fig. 2.81

? Ex 2.4
p66

43. A window designed between semicircular regions is shown in Fig. 2.86. Find the area of the window.

- $A_{circle} = \pi r^2$

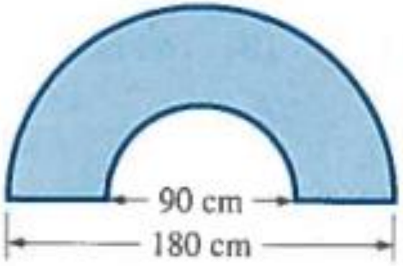


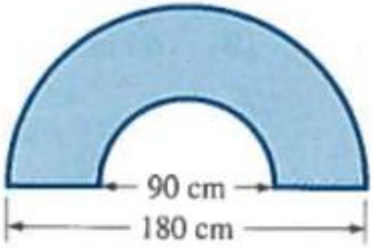
Fig. 2.86

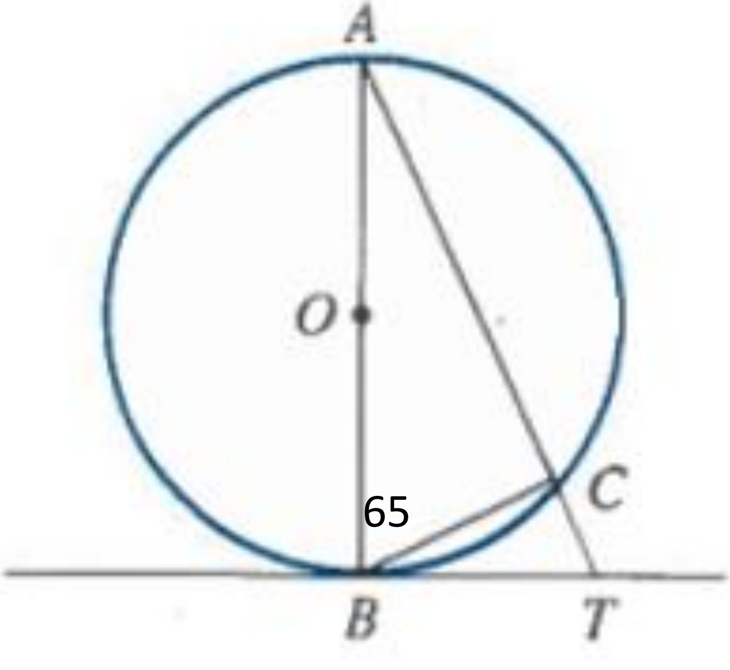
Fig. 2.87

Area of small / Area of large

44. The cross section of a large circular conduit has seven smaller equal circular conduits within it. The conduits are tangent to each other as shown in Fig. 2.87. What fraction of the large conduit is occupied by the seven smaller conduits?

$$A_{circle} = \pi r^2$$





Examples

17. $\angle CBT = 90^\circ - \angle ABC = 90^\circ - 65^\circ = 25^\circ$

18. $\angle BCT = 90^\circ$, any angle such as $\angle BCA$ inscribed in a semicircle is a right angle and $\angle BCT$ is supplementary to $\angle BCA$.

19. A tangent to a circle is perpendicular to the radius drawn to the point of contact. Therefore,

$\angle ABT = 90^\circ$

$\angle CBT = \angle ABT - \angle ABC = 90^\circ - 65^\circ = 25^\circ$;

$\angle CAB = 25^\circ$

20. $\angle BTC = 65^\circ$; $\angle CBT = 35^\circ$ since it is complementary to $\angle ABC = 65^\circ$.

43. $A = \dfrac{\pi}{2}\left(90^2 - 45^2\right)$

$A = 9500 \text{ cm}^2$

44. Let D = diameter of large conduit, then

$D = 3d$ where d = diameter of smaller conduit

$$F = \frac{\pi}{4}D^2 = 7 \cdot \frac{\pi}{4} \cdot d^2$$

$$F = \frac{7d^2}{D^2} = \frac{7d^2}{(3d)^2} = \frac{7d^2}{9d^2}$$

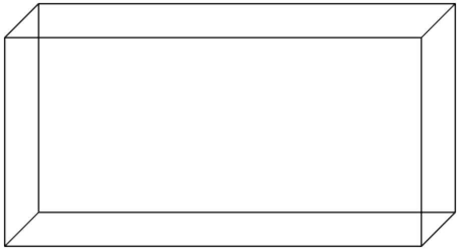
$$F = \frac{7}{9}$$

The smaller conduits occupy $\frac{7}{9}$ of the larger conduits.

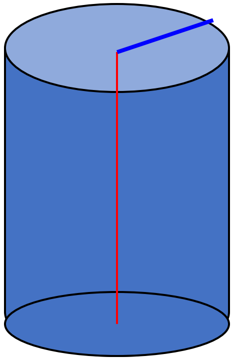
Ch. 2.6: Solid Geometric Figures

- Solid geometric figures are three dimensional figures.
- Their **volume** and **surface area** can be calculated.
- The sides of a solid figure made up of planes are known as **faces**.

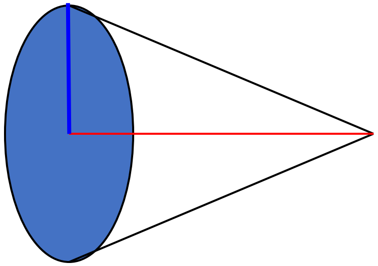
Volumes of Various Solids



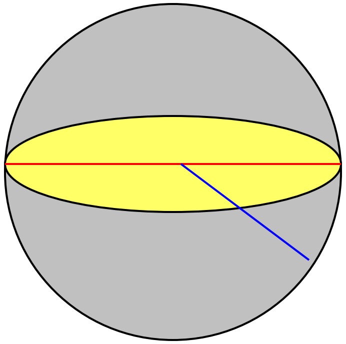
$$V = LWH$$



$$V = \pi r^2 h$$

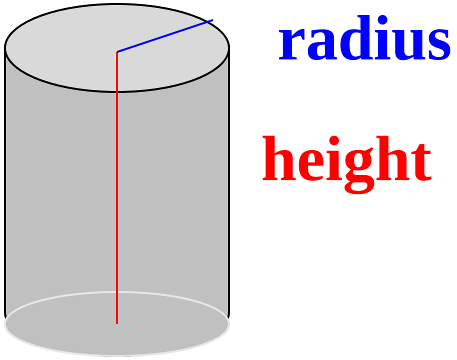


$$V = \frac{1}{3} \pi r^2 h$$

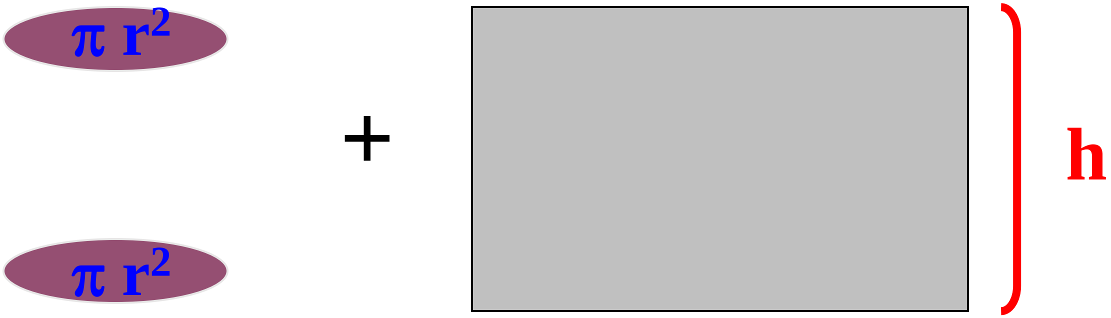


$$V = \frac{4}{3} \pi r^3$$

Surface Area of a Cylinder



area of circle = πr^2



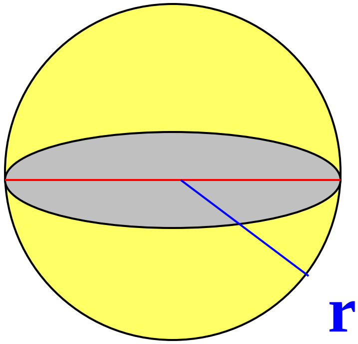
Circumference of the circle top

Surface area for cylinder = $(2)\pi r^2 + 2\pi r h$

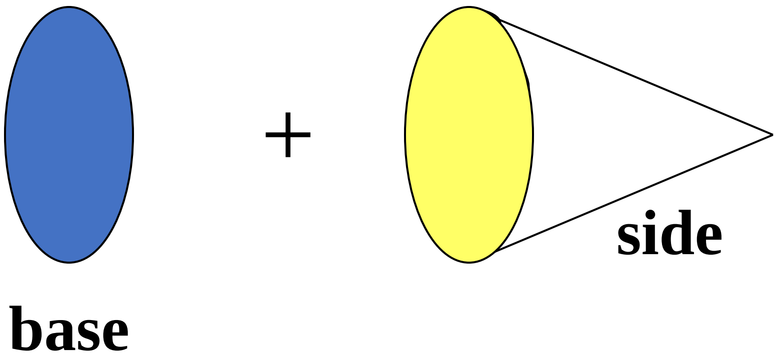
Surface Area & Volume of a Sphere

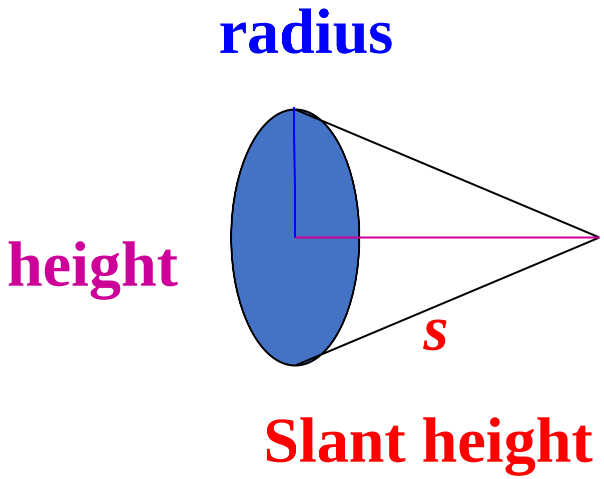
Surface area = $4 \pi r^{2}$

Volume = $\frac{4}{3} \pi r^{3}$



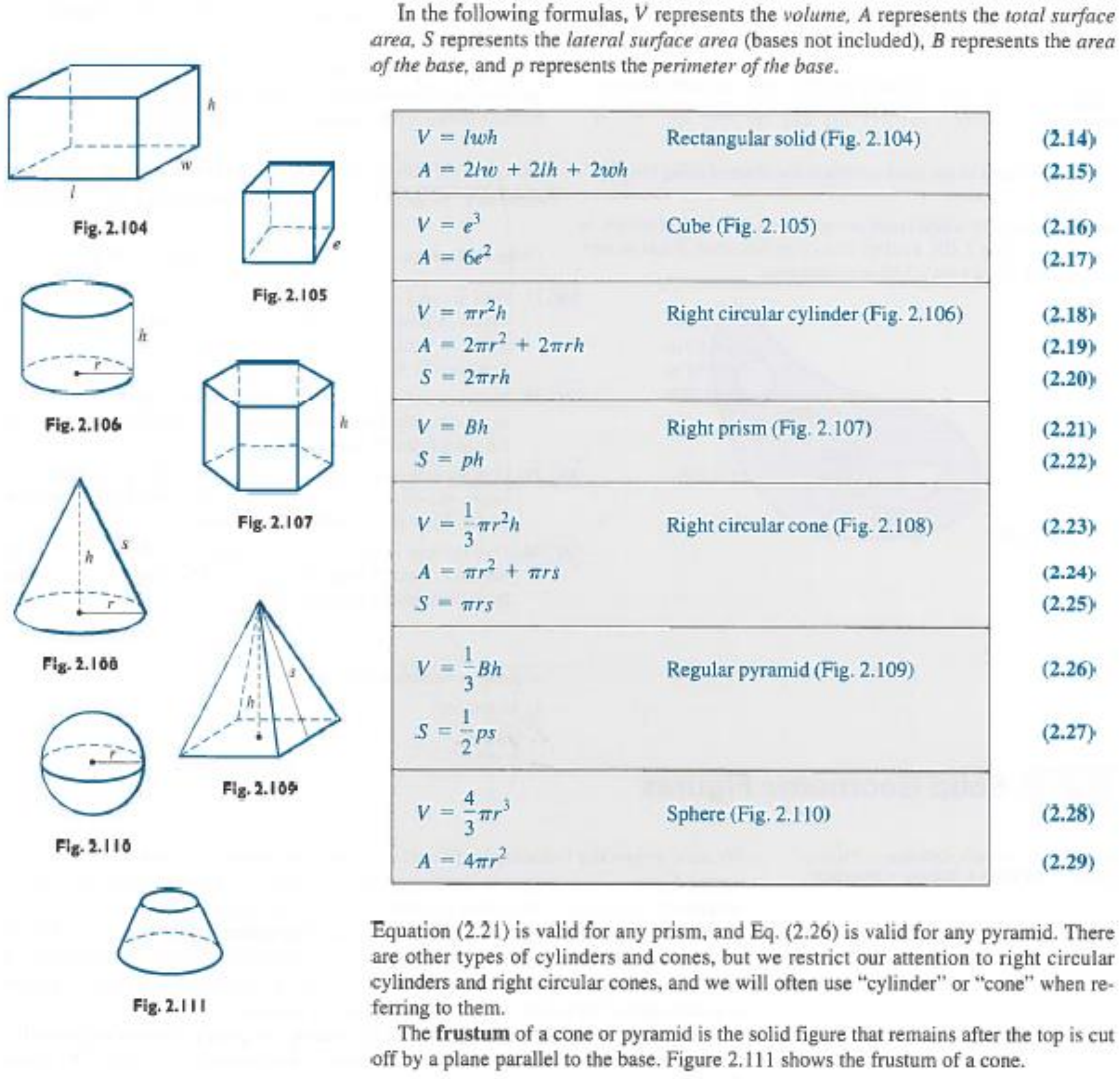
Surface Area of a Cone





$$\text{Surface area}_{\text{cone}} = \pi r^2 + \pi r s$$

Summary of Formula Used in Geometry



In the following formulas, V represents the *volume*, A represents the *total surface area*, S represents the *lateral surface area* (bases not included), B represents the *area of the base*, and p represents the *perimeter of the base*.

$V = lwh$ $A = 2lw + 2lh + 2wh$	Rectangular solid (Fig. 2.104)	(2.14) (2.15)
$V = e^3$ $A = 6e^2$	Cube (Fig. 2.105)	(2.16) (2.17)
$V = \pi r^2 h$ $A = 2\pi r^2 + 2\pi rh$ $S = 2\pi rh$	Right circular cylinder (Fig. 2.106)	(2.18) (2.19) (2.20)
$V = Bh$ $S = ph$	Right prism (Fig. 2.107)	(2.21) (2.22)
$V = \frac{1}{3}\pi r^2 h$ $A = \pi r^2 + \pi rs$ $S = \pi rs$	Right circular cone (Fig. 2.108)	(2.23) (2.24) (2.25)
$V = \frac{1}{3}Bh$ $S = \frac{1}{2}ps$	Regular pyramid (Fig. 2.109)	(2.26) (2.27)
$V = \frac{4}{3}\pi r^3$ $A = 4\pi r^2$	Sphere (Fig. 2.110)	(2.28) (2.29)

Fig. 2.104

Fig. 2.105

Fig. 2.106

Fig. 2.107

Fig. 2.108

Fig. 2.109

Fig. 2.110

Fig. 2.111

Equation (2.21) is valid for any prism, and Eq. (2.26) is valid for any pyramid. There are other types of cylinders and cones, but we restrict our attention to right circular cylinders and right circular cones, and we will often use "cylinder" or "cone" when referring to them.

The **frustum** of a cone or pyramid is the solid figure that remains after the top is cut off by a plane parallel to the base. Figure 2.111 shows the frustum of a cone.

? Ex2.6 q16-18, 31-33

15. Lateral area of regular prism: equilateral triangle base of side 1.092 m, $h = 1.025$ m

16. Lateral area of right circular cylinder: diameter = 250 ft, $h = 347$ ft

17. Volume of hemisphere: diameter = 0.83 yd

18. Volume of regular pyramid: square base of side 22.4 m, $s = 14.2$ m

$V = lwh$ $A = 2lw + 2lh + 2wh$	Rectangular solid (Fig. 2.101)	(2.14) (2.15)
$V = e^3$ $A = 6e^2$	Cube (Fig. 2.102)	(2.16) (2.17)
$V = \pi r^2 h$ $A = 2\pi r^2 + 2\pi rh$ $S = 2\pi rh$	Right circular cylinder (Fig. 2.103)	(2.18) (2.19) (2.20)
$V = Bh$ $S = ph$	Right prism (Fig. 2.104)	(2.21) (2.22)
$V = \frac{1}{3}\pi r^2 h$ $A = \pi r^2 + \pi rs$ $S = \pi rs$	Right circular cone (Fig. 2.105)	(2.23) (2.24) (2.25)
$V = \frac{1}{3}Bh$ $S = \frac{1}{2}ps$	Regular pyramid (Fig. 2.106)	(2.26) (2.27)
$V = \frac{4}{3}\pi r^3$ $A = 4\pi r^2$	Sphere (Fig. 2.107)	(2.28) (2.29)

31. The Great Pyramid of Egypt has a square base approximately 250 yd on a side. The height of the pyramid is about 160 yd. What is its volume? See Fig. 2.117.

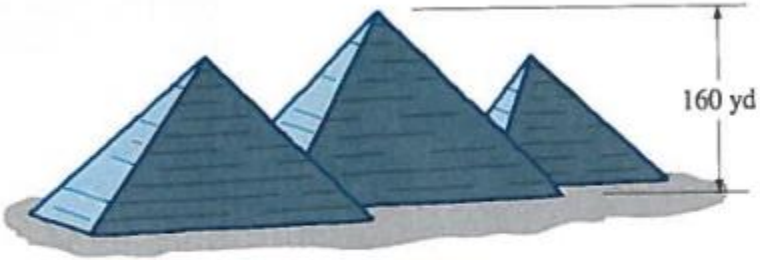


Fig. 2.117

32. A paper cup is in the shape of a cone as shown in Fig. 2.118. What is the surface area of the cup?

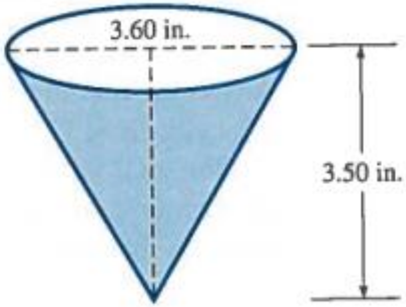


Fig. 2.118

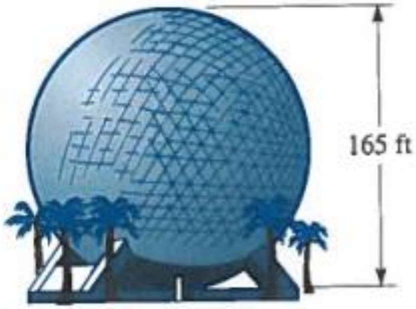


Fig. 2.119

33. *Spaceship Earth* (shown in Fig. 2.119) at Epcot Center in Florida is a sphere of 165 ft in diameter. What is the volume of *Spaceship Earth*?

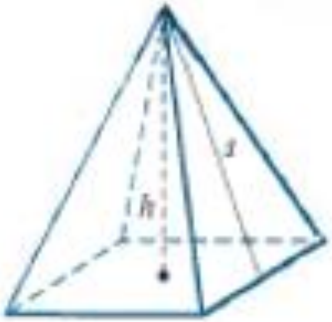
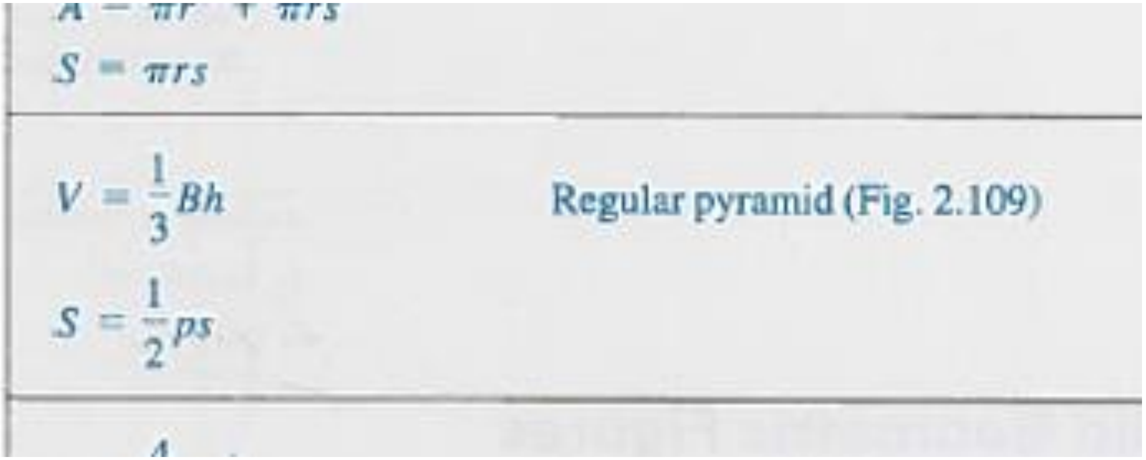


Fig. 2.109



$A = \pi r \; + \pi rs$	
$S = \pi rs$	
$V = \frac{1}{3}Bh$ $S = \frac{1}{2}ps$	Regular pyramid (Fig. 2.109)

31. The Great Pyramid of Egypt has a square base approximately 250 yd on a side. The height of the pyramid is about 160 yd. What is its volume? See Fig. 2.117.

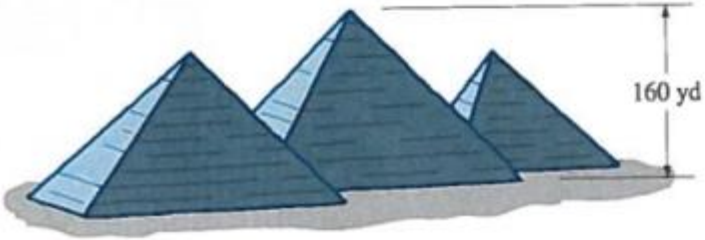


Fig. 2.117

15. $S = \frac{1}{2}ps = \frac{1}{2}(3\times 1.092)(1.025) = 3.358\text{ m}^2$

16. $S = 2\pi rh = 2\pi(d/2)h = 2\pi(250/2)(347)$

$= 273{,}000\text{ ft}^2$

17. $V = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right) = \frac{2}{3}\pi\left(\frac{0.83}{2}\right)^3 = 0.15\text{ yd}^3$

18. $b = \frac{22.4}{2} = 11.2;\ h = \sqrt{s^2 - b^2} = \sqrt{14.2^2 - 11.2^2}$

$= 8.73\text{ m}$

$V = \frac{1}{3}Bh = \frac{1}{3}(22.4)^2(8.73) = 1460\text{ m}^3$

31. $V = \frac{1}{3}BH = \frac{1}{3}(250^2)(160) = 3{,}300{,}000\text{ yd}^3$

32. $s = \sqrt{h^2 + r^2} = \sqrt{3.50^2 + 1.80^2} = 3.94\text{ in.}$

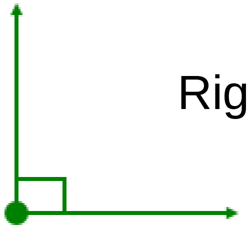
$S = \pi rs = \pi(1.80)(3.94) = 22.3\text{ in.}^2$

33. $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(d/2)^3$

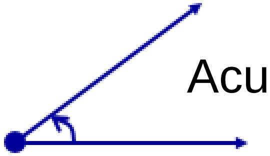
$= \frac{4}{3}\pi(165/2)^3$

$= 2.35\times 10^6\text{ ft}^3$

Examples



Right angle

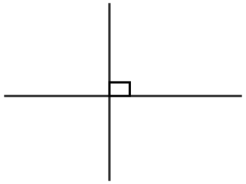


Acute angle

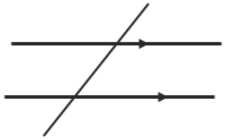
Obtuse angle

Straight angle

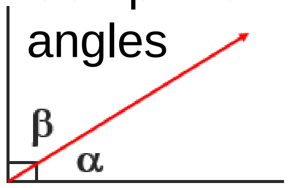
Perpendicular Lines



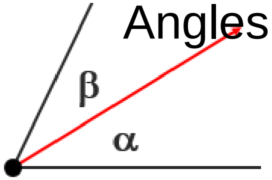
Parallel Lines



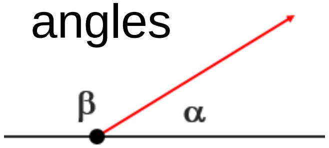
Complimentary angles



Adjacent Angles

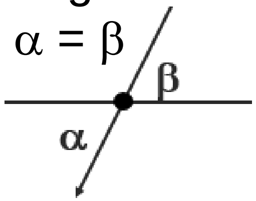


Supplementary angles

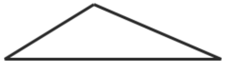


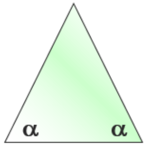
Vertical angles

$\alpha = \beta$

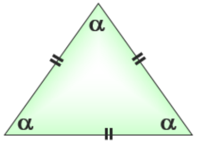


- **Scalene Triangle**:
  - No 2 sides are equal in length





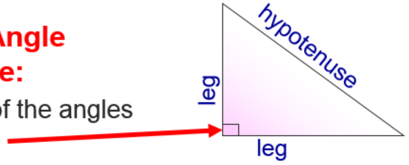
- **Isosceles Triangle**:
  - Two sides are equal in length
  - The sides leading to the base are equal; the base angles of the 2 equal sides are equal.



- **Equilateral Triangle:**
  - All sides & angles are equal.
  - $\alpha = 60°$

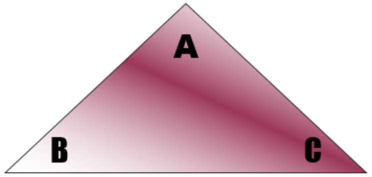
Right Angle Triangle:

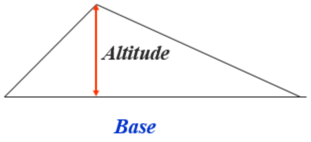
- One of the angles is 90˚

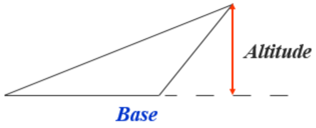


The sum of the three angles of any triangle is **180º**

$$\angle A + \angle B + \angle C = 180^{o}$$







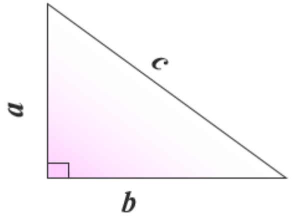
Perimeter:

- The sum of the lengths of the 3 sides.

Area: $A = \frac{bh}{2}$

Extra

The Pythagorean Theorem



$$c^2 = a^2 + b^2$$

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

Hero's formula

Where, $s = \frac{a+b+c}{2}$

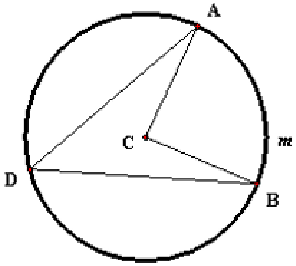
Circumference is a perimeter of a circle and is given by $C = 2\pi r$

$A_{circle} = \pi r^2$

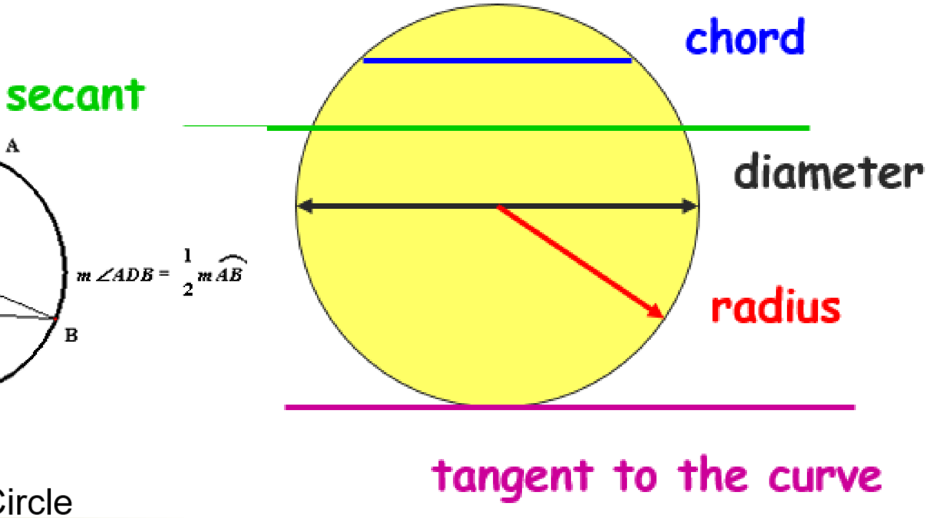
An *arc* is part of a circle.

A *central angle* is an angle formed at the centre by two radii.

Terminology related to the circle.



$m\angle ADB = \frac{1}{2} m\overset{\frown}{AB}$

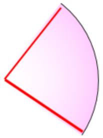


Segment of a Circle

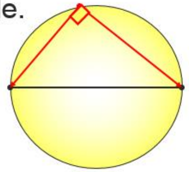
The region bounded by a chord and its arc.

The Sector of a Circle

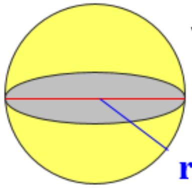
The region bounded by 2 radii and the arc they intercept.



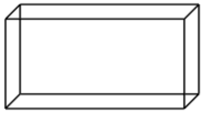
An angle inscribed in a semi-circle is a right angle.



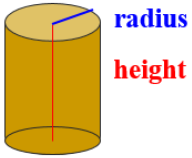
Surface area = $4 \pi r^2$



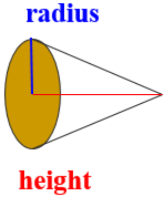
Volume = $\frac{4}{3} \pi r^3$



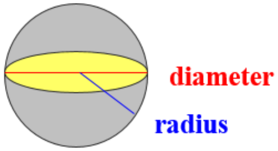
$V = LWH$



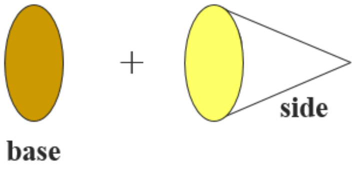
$V = \pi r^2 h$

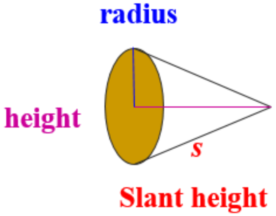


$V = \frac{1}{3} \pi r^2 h$



$V = \frac{4}{3} \pi r^3$





Surface area $_{cone} = \pi r^2 + \pi r s$

- **Parallelogram**:
  - Opposite sides are parallel

- **Rectangle**:
  - A parallelogram in which intersecting sides are perpendicular

- **Rhombus**:
  - A parallelogram with 4 equal sides

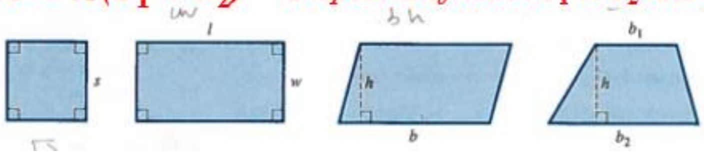
- **Square**:
  - A rectangle with four equal sides

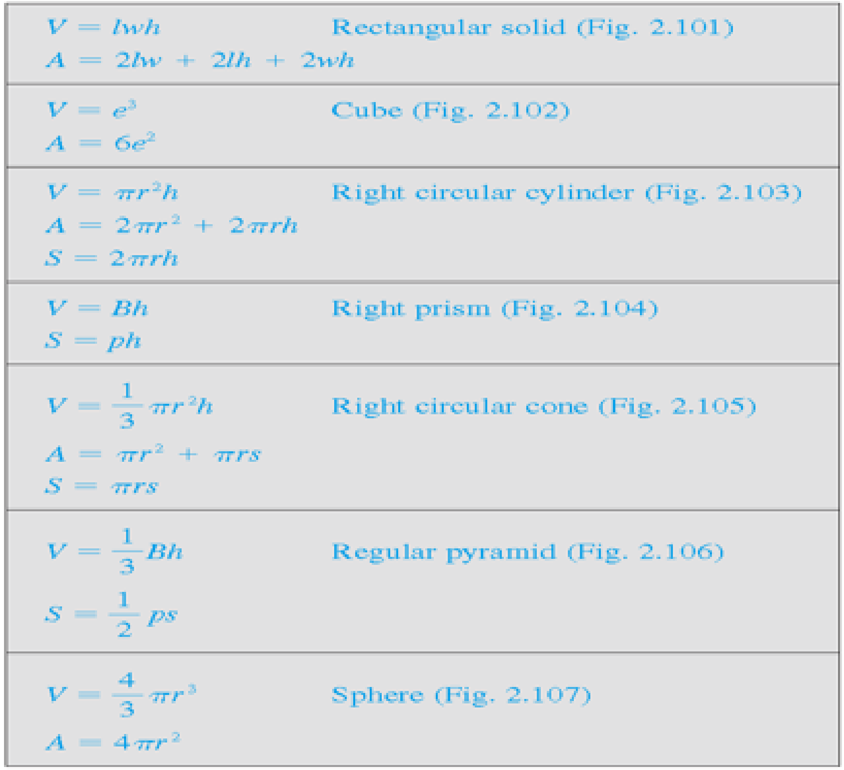
Perimeter:

- The sum of the lengths of the four sides.

Area:

- $A = x^2$ Square of side *x*
- $A = lw$ *Rectangle of length **l** and width **w***
- $A = bh$ *Parallelogram of base **b** & height **h***
- $A = ½(b_1 + b_2)$ *Trapezoid of bases b_1 & b_2 and height **h***





$V = lwh$ $A = 2lw + 2lh + 2wh$	Rectangular solid (Fig. 2.101)
$V = e^3$ $A = 6e^2$	Cube (Fig. 2.102)
$V = \pi r^2 h$ $A = 2\pi r^2 + 2\pi rh$ $S = 2\pi rh$	Right circular cylinder (Fig. 2.103)
$V = Bh$ $S = ph$	Right prism (Fig. 2.104)
$V = \frac{1}{3}\pi r^2 h$ $A = \pi r^2 + \pi rs$ $S = \pi rs$	Right circular cone (Fig. 2.105)
$V = \frac{1}{3}Bh$ $S = \frac{1}{2}ps$	Regular pyramid (Fig. 2.106)
$V = \frac{4}{3}\pi r^3$ $A = 4\pi r^2$	Sphere (Fig. 2.107)

Chapter 2 exercises

Ex 2.4 (p78)
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Ex 2.6 (p87)
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Worked examples

2.4 1, 3, 4, 5

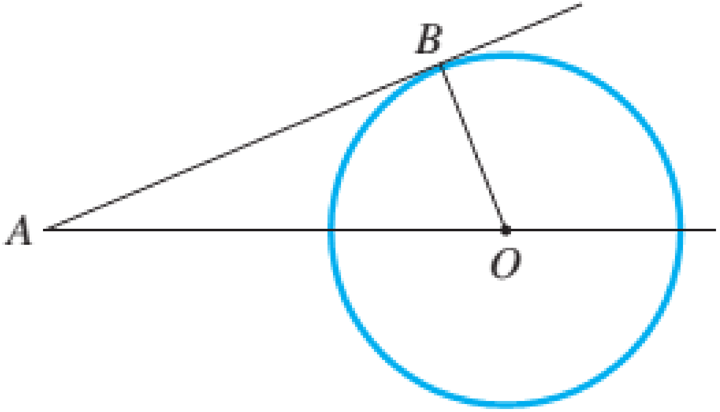
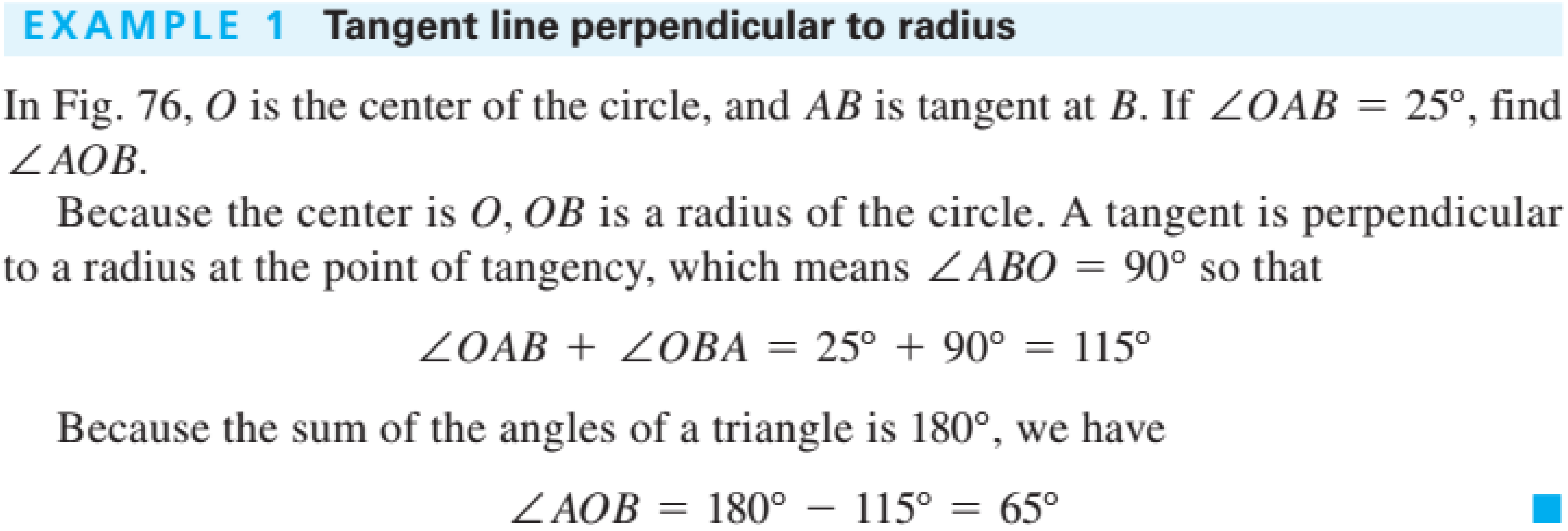


Fig. 76



EXAMPLE 1 Tangent line perpendicular to radius

In Fig. 76, O is the center of the circle, and AB is tangent at B. If $\angle OAB = 25°$, find $\angle AOB$.

Because the center is O, OB is a radius of the circle. A tangent is perpendicular to a radius at the point of tangency, which means $\angle ABO = 90°$ so that

$$\angle OAB + \angle OBA = 25° + 90° = 115°$$

Because the sum of the angles of a triangle is 180°, we have

$$\angle AOB = 180° - 115° = 65°$$

■

EXAMPLE 3 Application of perimeter and area

A machine part is a square of side 3.25 in. with a quarter-circle removed (see Fig. 78). Find the perimeter and the area of one side of the part.

Setting up a formula for the perimeter, we add the two sides of length s to *one-fourth of the circumference of a circle with radius s*. For the area, we *subtract the area of one-fourth of a circle from the area of the square*. This gives

$$p = \overset{\text{bottom and left}}{2s} + \overset{\text{circular section}}{\frac{2\pi s}{4}} = 2s + \frac{\pi s}{2} \qquad A = \overset{\text{square}}{s^2} - \overset{\text{quarter circle}}{\frac{\pi s^2}{4}}$$

where s is the side of the square and the radius of the circle. Evaluating, we have

$$p = 2(3.25) + \frac{\pi(3.25)}{2} = 11.6 \text{ in.}$$

$$A = 3.25^2 - \frac{\pi(3.25)^2}{4} = 2.27 \text{ in.}^2$$

The calculator solution for p and A is shown in Fig. 79. Note that s is stored (as x) in order to shorten the solution. ■

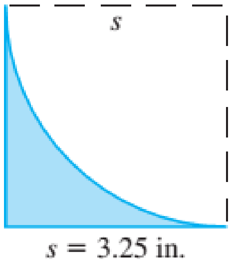
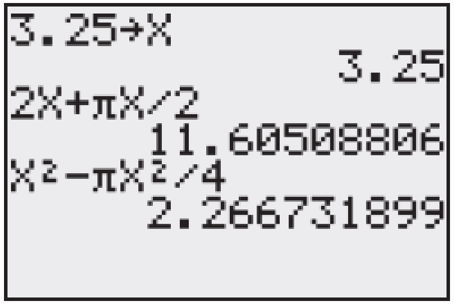


Fig. 78



```
3.25→X
                3.25
2X+πX/2
         11.60508806
X²-πX²/4
         2.266731899
```

Fig. 79

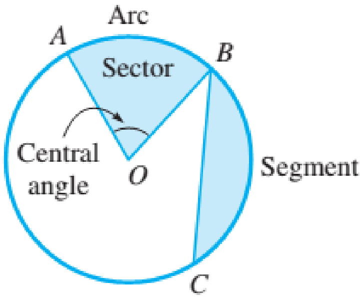


Fig. 80

EXAMPLE 4 Sector and segment

In Fig. 80, a sector of the circle is between radii OA and OB and arc AB (denoted as $\overset{\frown}{AB}$). If the measure of the central angle at O between the radii is 70°, the measure of $\overset{\frown}{AB}$ is 70°.

A segment of the circle is the region between chord BC and arc BC $(\overset{\frown}{BC})$. ■

An **inscribed angle** *of an arc is one for which the endpoints of the arc are points on the sides of the angle and for which the vertex is a point (not an endpoint) of the arc.* An important property of a circle is that *the measure of an inscribed angle is one-half of its intercepted arc.*

EXAMPLE 5 Inscribed angle

(a) In the circle shown in Fig. 81, $\angle ABC$ is inscribed in $\overset{\frown}{ABC}$, and it intercepts $\overset{\frown}{AC}$. If $\overset{\frown}{AC} = 60°$, then $\angle ABC = 30°$.

(b) In the circle shown in Fig. 82, PQ is a diameter, and $\angle PRQ$ is inscribed in the semicircular $\overset{\frown}{PRQ}$. Since $\overset{\frown}{PSQ} = 180°$, $\angle PRQ = 90°$. From this we conclude that *an angle inscribed in a semicircle is a right angle.* ■

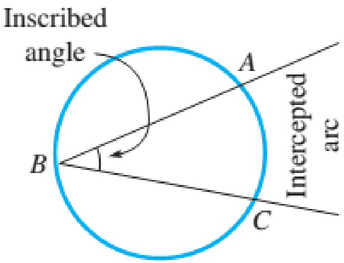


Fig. 81

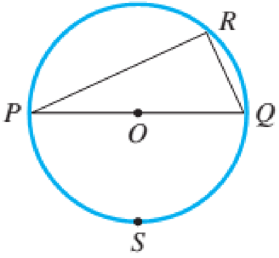
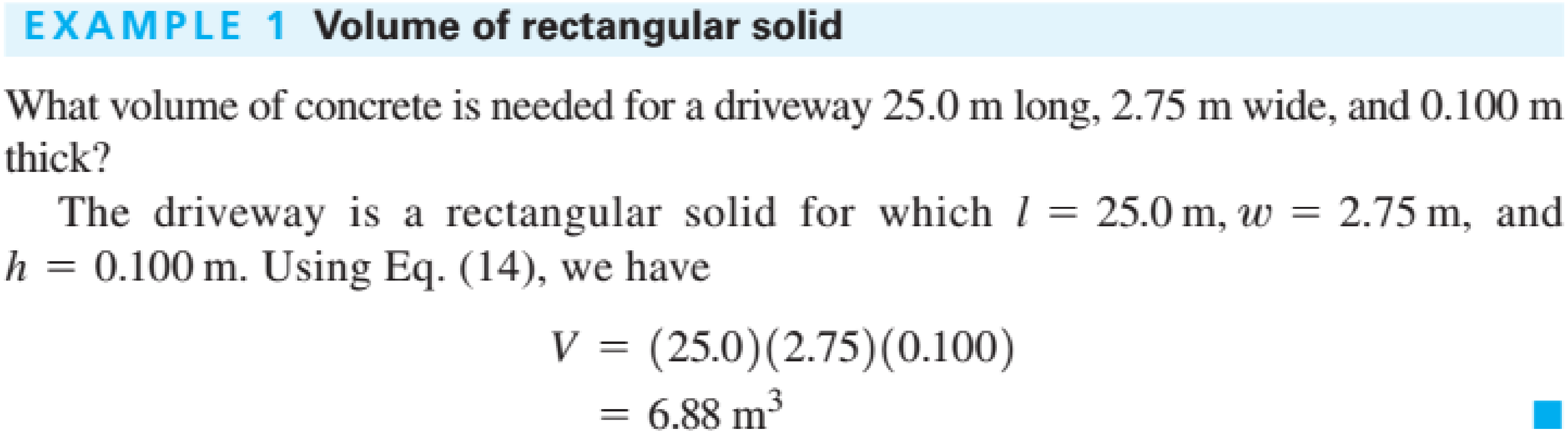


Fig. 82

2.6 1, 3



EXAMPLE 1 Volume of rectangular solid

What volume of concrete is needed for a driveway 25.0 m long, 2.75 m wide, and 0.100 m thick?

The driveway is a rectangular solid for which $l = 25.0$ m, $w = 2.75$ m, and $h = 0.100$ m. Using Eq. (14), we have

$$\begin{aligned} V &= (25.0)(2.75)(0.100) \\ &= 6.88 \text{ m}^3 \end{aligned}$$

EXAMPLE 3 Volume of combination of solids

A grain storage building is in the shape of a cylinder surmounted by a hemisphere (*half a sphere*). See Fig. 119. Find the volume of grain that can be stored if the height of the cylinder is 122 ft and its radius is 40.0 ft.

The total volume of the structure is the volume of the cylinder plus the volume of the hemisphere. By the construction we see that the radius of the hemisphere is the same as the radius of the cylinder. Therefore,

$$V = \overset{\text{cylinder}}{\pi r^2 h} + \overset{\text{hemisphere}}{\frac{1}{2}\left(\frac{4}{3}\pi r^3\right)} = \pi r^2 h + \frac{2}{3}\pi r^3$$

$$= \pi(40.0)^2(122) + \frac{2}{3}\pi(40.0)^3$$

$$= 747{,}000 \text{ ft}^3$$

■

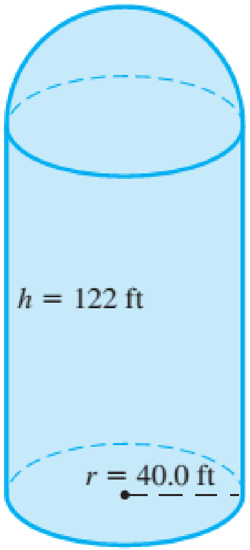


Fig. 119