Chapter 3

Functions and Graphs

• Definition of function • Cartesian coordinates • Examples of functions (Linear, second order polynomial) • Concept of inverse function	3.1. Introduction to functions 3.2. More about functions 3.3. Rectangular coordinates 3.4. Graph of functions

Ch. 3.1: Introduction to Functions

- A function is a rule that shows the relation of how one quantity depends on another.
- We can express this relationship using formulas, tables, charts and graphs.

There are two types of **variables**-**independent and dependent**.

An **independent variable** is exactly what it sounds like. It is a **variable** that stands alone and isn't changed by the other **variables** you are trying to measure.
For **example**, someone's age might be an **independent variable**.

Definition of a Function:

- Whenever a relationship exists between two variables – such that for every value of the first, there is only one corresponding value of the second – we say that the second variable is a ***function*** of the first variable.

Functional Notation

- We use the notation:

Function of x $\longrightarrow$ $f\left(x\right)$

y is a function of x

- Here, f denotes dependence on the independent variable x.

Example

- The formula for the volume of a sphere is:

$$V_{sphere} = \frac{4\pi r^3}{3} \qquad V = f(r)$$

- Its volume is **dependent** on how large or how small the sphere is which is defined by its radius.

Example *(continued)*

- If we wanted to know how the radius of the sphere changes with respect to the volume, we would rearrange the formula:

$$r = \sqrt[3]{\frac{3V_{sphere}}{4\pi}}$$

- Here the radius is dependent on the changing volume.

Example *(continued)*

- In functional notation, the equation for volume can be represented as:

$$V_{sphere} = \frac{4\pi r^3}{3} \quad \Rightarrow \quad f(r) = \frac{4\pi r^3}{3}$$

- Whatever value ***r*** represents, to find ***f(r)***, we substitute ***r*** for ***x*** in ***f(x)***.

Example *(continued)*

- Find the volume of a spherical balloon that has a **14.0** cm radius.
- Solution:

$$f(r) = \frac{4\pi r^3}{3} \Rightarrow f(14.0) = \frac{4\pi(14.0)^3}{3}$$

$$f(14.0) = 11500\ cm^3$$

Examples

Ch. 3.2: More about Functions

- ***Domain***:
  - the complete set of possible values of the independent variable (x)
- ***Range***:
  - the complete set of all possible resulting values of the dependent variable. f(x)

- Values that lead to ***division by zero*** or to ***imaginary numbers may not be included*** in the domain or the range.

Examples

- Given: $f(x) = 3x + 7$

Domain & range are defined for all real numbers.

- But given: $f(x) = \dfrac{1}{1-x}$

Domain cannot equal 1.

Range is all real numbers except 0.

Giving Meaning to Domain

- ***Example***:
  - We are to find the length of the edges of a square with an area of **272.0 cm**^{2}.

- ***Solution***:
  - $f(e) = \sqrt{272.0}$ where $\boldsymbol{e}$ is the edge of the square.
  - $f(e) = \pm 16.5$ cm
  - Since length is positive, the domain of the values is $\boldsymbol{e} \geq 0$

Negative values generally have no real meaning in applications.

Domain (x)

$$f(x) = x^2$$

dependent → $\boxed{\text{range}}$

independent → $\boxed{\text{Domain}}$

(NOT $\frac{a}{0}$, $\sqrt{-1}$)

x	$f(x)=x^2$
0	0
1	1
2	4
3	9

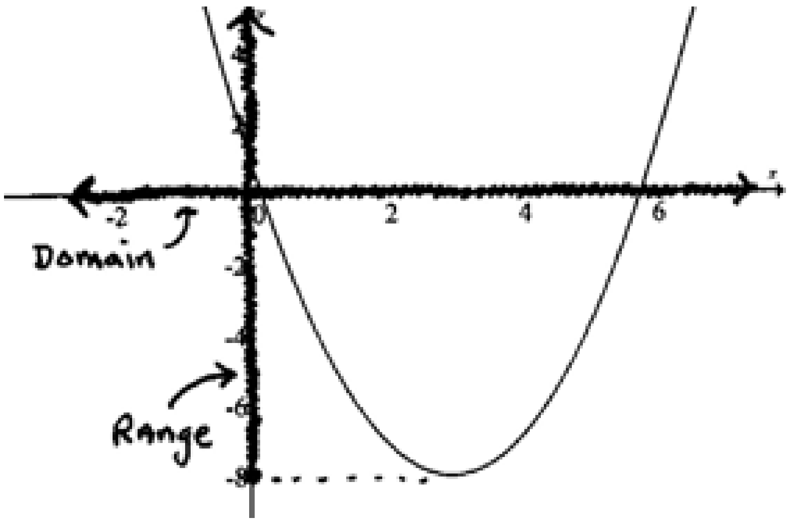
etc

$f(x)$ depends on x

(start with the independent variable and calculate the dependent)

Find the Domain THEN the corresponding range

State the domain and range.



Domain: All Real numbers $(-\infty, \infty)$

Range: All real numbers greater than or equal to -8.

Interval notation $[-8, \infty)$

In Exercises 5–14, find the domain and range of the given functions. In Exercises 11 and 12, explain your answers.

5. $f(x) = x + 5$

6. $g(u) = 3 - u^2$

7. $G(R) = \dfrac{3.2}{R}$

8. $F(r) = \sqrt{r + 4}$

Domain (x)

Examples

5. The domain and range of $f(x) = x + 5$ is all real numbers.

6. $g(u) = 3 - u^2$; since $3 - u^2$ is defined for all real numbers, the domain is the set of all real numbers. However, the range is all real numbers $g(u) \leq 3$, since u^2 is never negative.

7. $G(R) = \dfrac{3.2}{R}$ is not defined for $R = 0$.

Domain: all real numbers except 0.

Range: all real numbers except 0.

8. $F(r) = \sqrt{r+4}$ is not defined for real numbers less than -4.

Domain: all real numbers $r \geq -4$ and the range cannot be negative due to the principal square root of $r + 4$.

Range: all real numbers $F(r) \geq 0$.

Relations

- If the value of the independent variable yields one or more values of the dependent variable, the relationship is called a ***relation***.
- A function is a special type of relation.

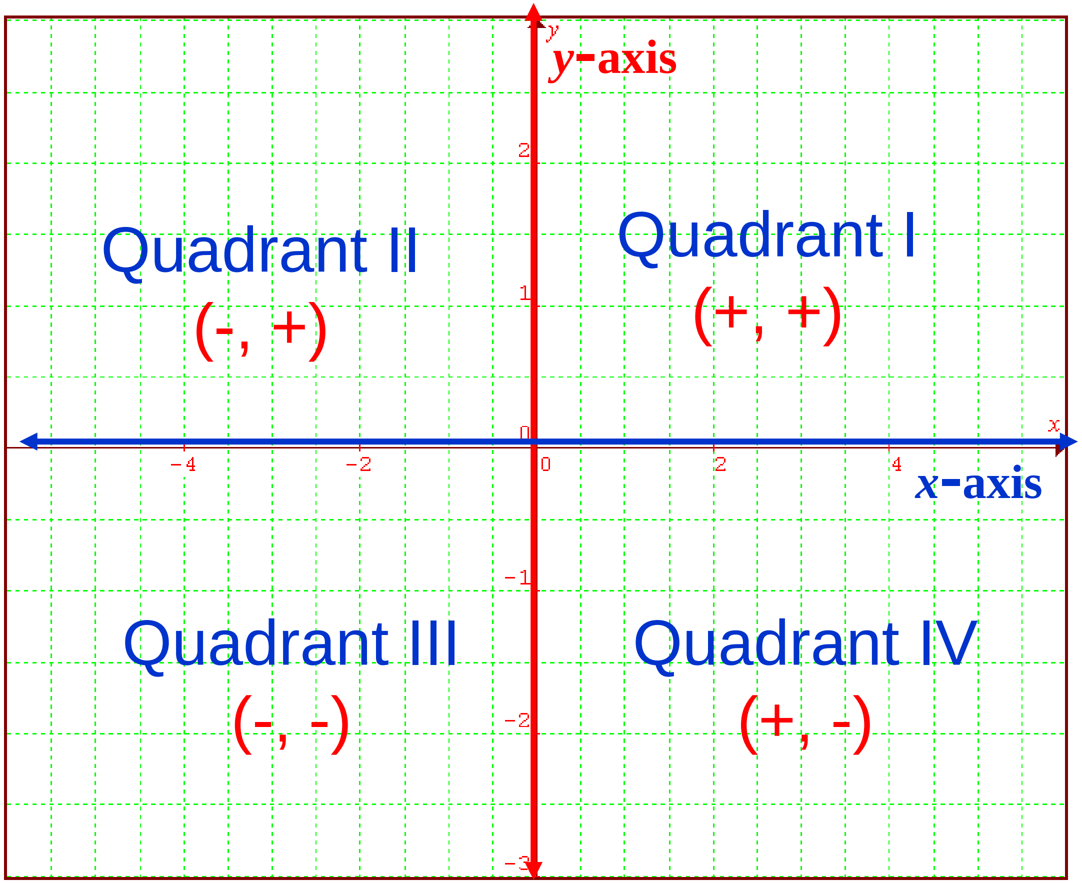
EXAMPLE 9 Relation

For $y^2 = 4x^2$, if $x = 2$, then y can be either 4 or -4. Since a value of x yields more than one value for y, we see that $y^2 = 4x^2$ is a relation, not a function. ■

Ch. 3.3: Rectangular Coordinates

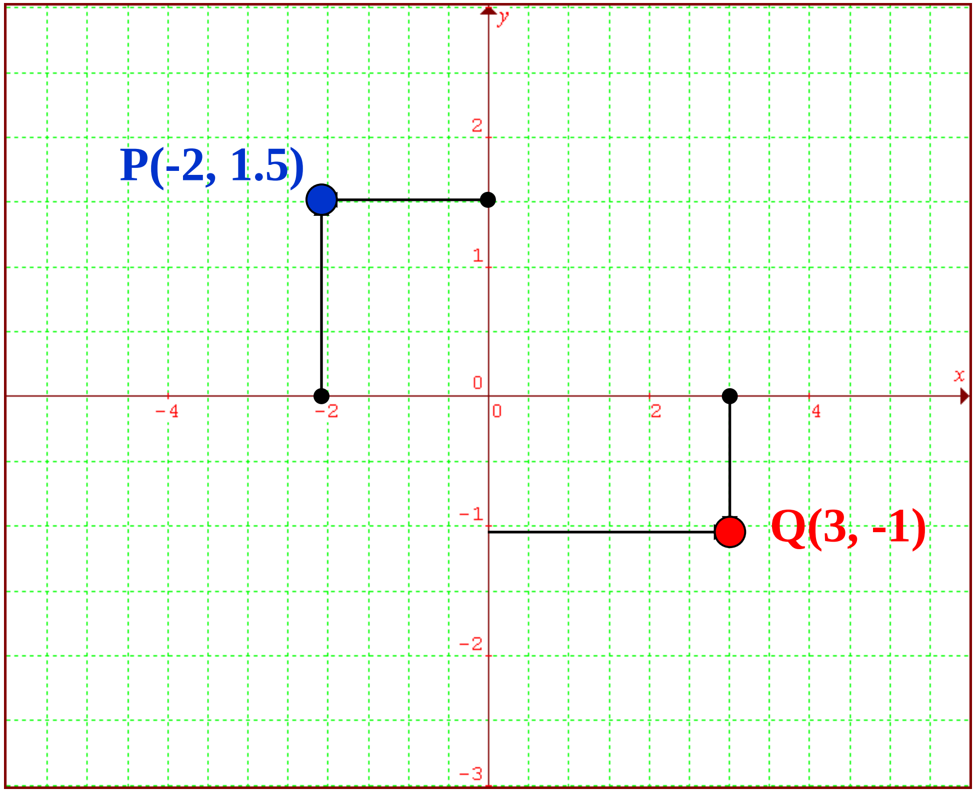
- To make a graphical representation of a function we use the Cartesian or ***xy***-plane.
- The ***x***-axis represents the independent variable.
- The ***y***-axis represents the dependent variable.

Mapping the Cartesian (*xy*) plane.



Examples

- Plot the points:
  - P(-2, 1.5)
  - Q(3, -1)



In Exercises 3 and 4, determine (at least approximately) the coordinates of the points shown in Fig. 3.13.

3. A, B, C **4.** D, E, F

In Exercises 5 and 6, plot the given points.

5. $A(2, 7), B(-1, -2), C(-4, 2)$

6. $A\left(3, \frac{1}{2}\right), B(-6, 0), C\left(-\frac{5}{2}, -5\right)$

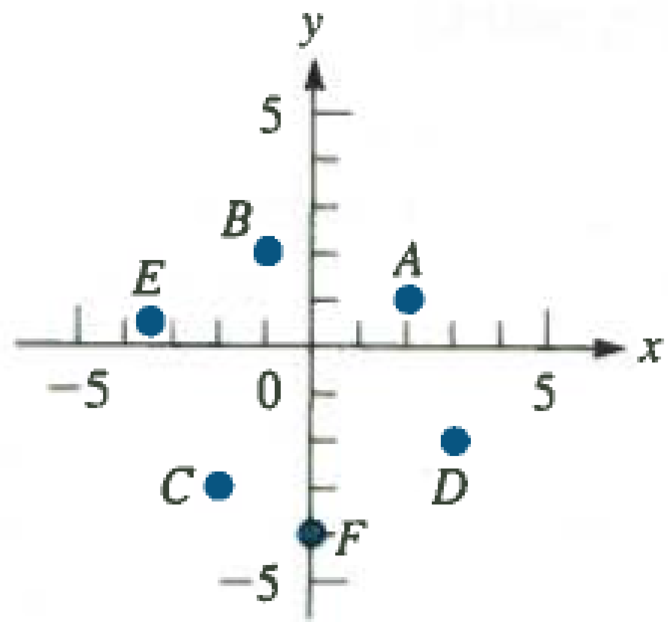


Fig. 3.13

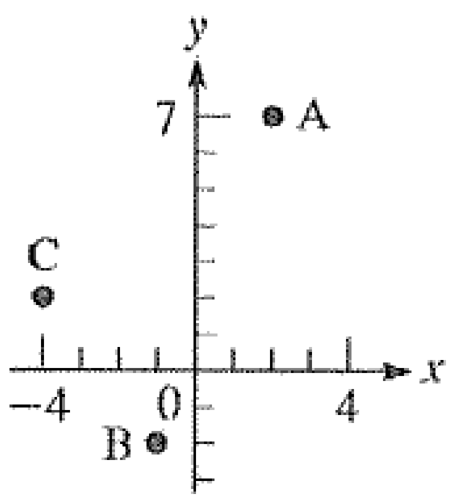
Examples

3. $A(2, 1), (-1, 2), C(-2, -3)$

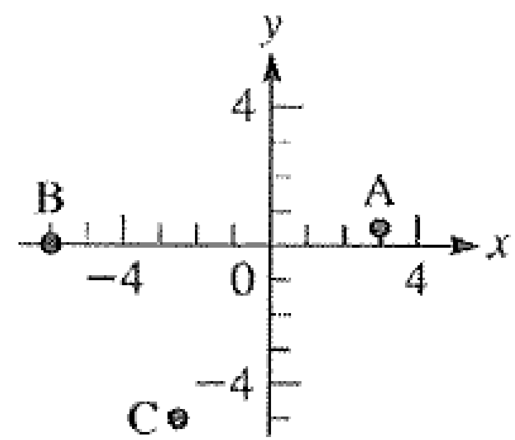
4. $D = (3, -2)$; $E = (-3.5, 0.5) = \left(-\frac{7}{2}, \frac{1}{2}\right)$;

$F = (0, -4)$

5.



6.



Ch. 3.4: The Graph of a Function

- The graph of a function is the set of all points whose coordinates (x, y) satisfy the functional relationship $y = f(x)$.
- In functional notation, we can write points as: (x, y) $\Rightarrow$ $(x, f(x))$.

Procedure for Plotting the Graph of a Function

1. Let x take on several values and calculate the corresponding values of y.
2. Tabulate these values arranging the table so that ***values of x are increasing***.
3. Plot the points and join them from left to right by a ***smooth curve*** (not short straight line segments).

Plotting Functions

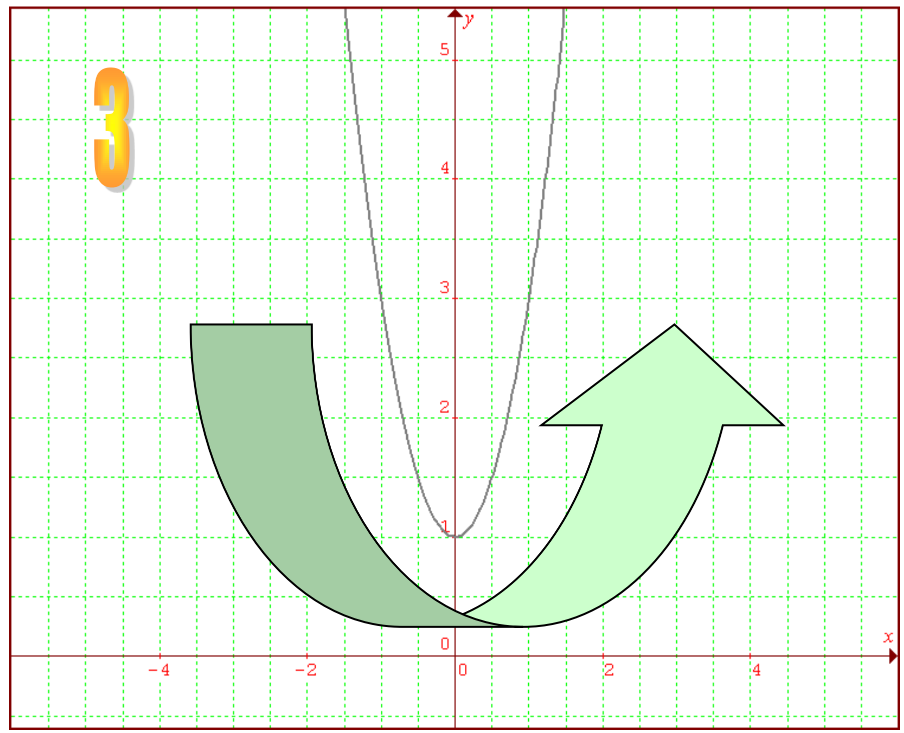
y

- Plot the function: $f(x) = 2x^2 + 1$

1

2

x	$f(x)$
-4	33
-3	19
-2	9
-1	3
0	1
1	3
2	9
3	19
4	33



Special Notes on Graphing

1. Any place that the graph changes in a way that is not expected should be checked with care.
2. The domain of the function may not include all values of x.
3. In applications, we must use only values of the variables that have meaning.

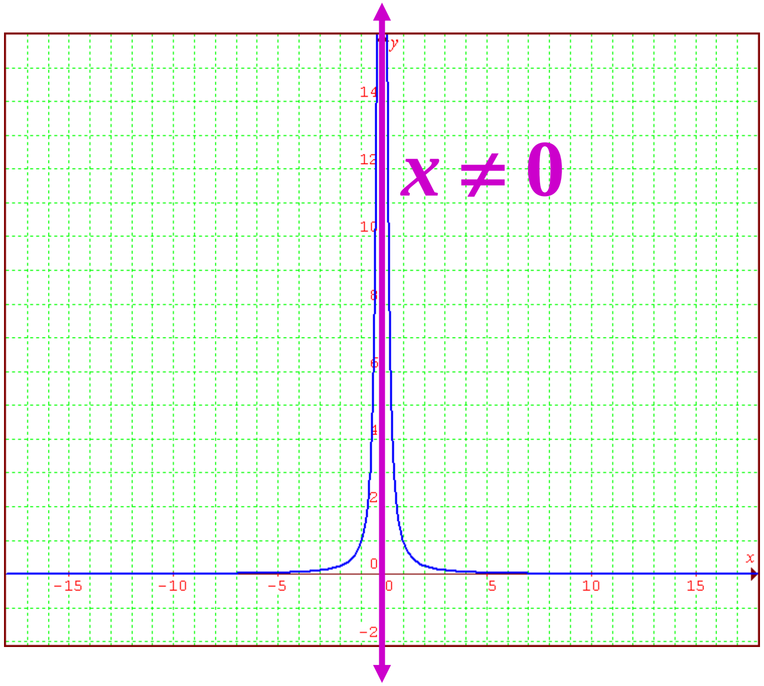
Example

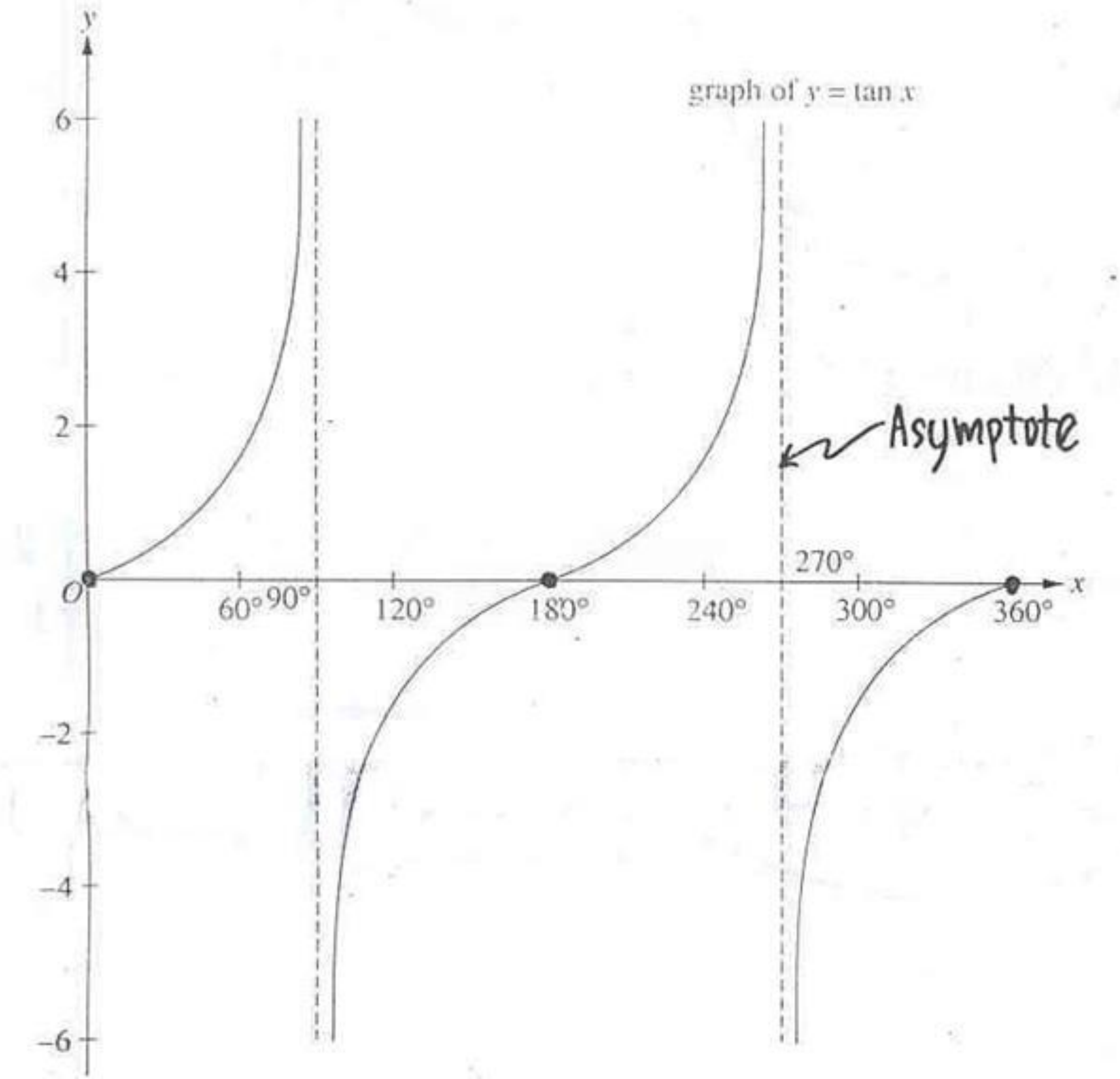
- Recall situations where the domain of a function cannot equal $\mathbf{0}$.
- In those situations, the graph may produce an **asymptote** where the values of the function never touch.

Example

- There is a ***vertical*** asymptote in the graph of the given function:

$$f(x) = \frac{1}{x^2}$$



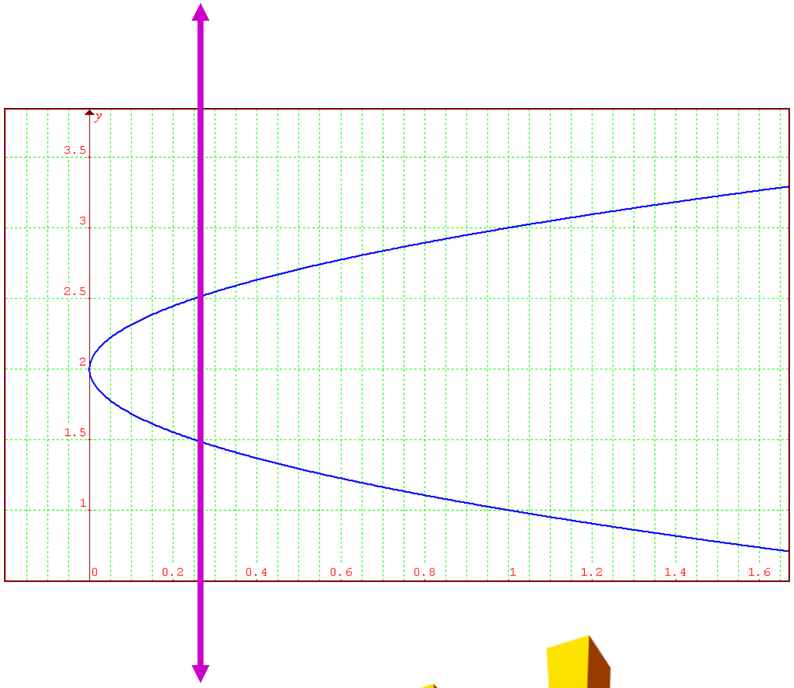

y
graph of y = tan x
6
4
2
−2
−4
−6
O
60°
90°
120°
180°
240°
270°
300°
360°
x
Asymptote

Vertical Line Test

- The ***vertical line test*** determines whether a graph as a relation is also a function.
- If any vertical line that crosses the x-axis in the domain intersects the graph in ***more than one point***, if the graph of a relation that is **not** a function.
- (A function is defined as only having one dependent value. A relation may have more than one.)

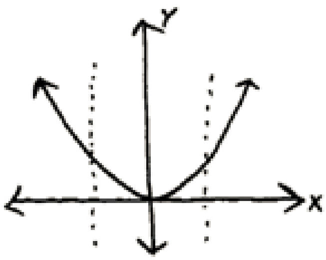
Example

- By the vertical line test, is this relation also a function?

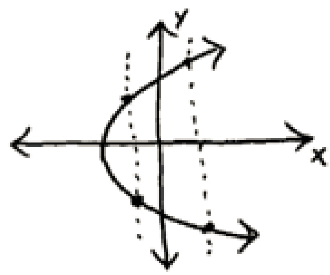


- Since the vertical line crosses the curve at 2 different points, it is **not** a function.

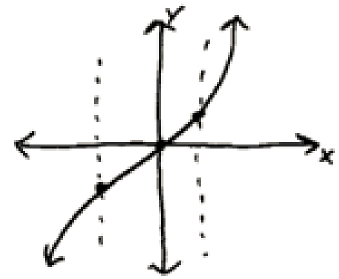
Vertical Line Test – If any vertical line intersects the graph more than once then the graph does not represent a function.



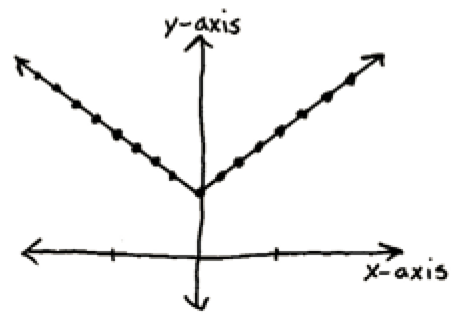
Function



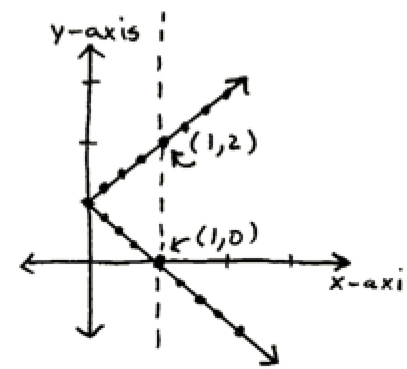
Not a Function



Function



Function



Not a Function

Examples

Domain (x)

Text book exercises.

Ex 3.1 (p98) 3,5,9,11

Ex 3.2 (p100) 1,5,13,18,23

Ex 3.3 (p104) 5,13,15

Ex 3.4 (p111) 5,19,56

Worked examples

3.1 1,3,5,6

EXAMPLE 1 Examples of functions

(a) In the equation $y = 2x$, y is a function of x, because for each value of x there is only one value of y. For example, if $x = 3$, then $y = 6$ and no other value. By arbitrarily assigning values to x, and substituting, we see that the values of y we obtain *depend* on the values of x. Therefore, x is the independent variable, and y is the dependent variable.

(b) Figure 1 shows a cube of edge e. We know that the volume V in terms of the edge is $V = e^3$. Here, V is a function of e. The dependent variable is V and the independent variable is e. ■

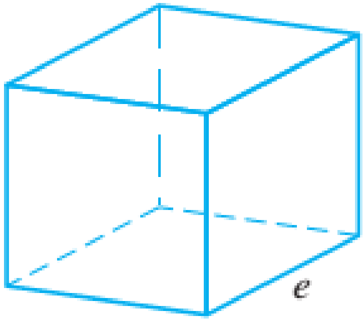


Fig. 1

EXAMPLE 3 Functional notation $f(x)$

If $y = 6x^3 - 5x$, we say that y is a function of x. This function is $6x^3 - 5x$. It is also common to write such a function as $f(x) = 6x^3 - 5x$. However, y and $f(x)$ represent the same expression, $6x^3 - 5x$. Using y, the quantities are shown, and using $f(x)$, the functional dependence is shown. ■

One of the most important uses of functional notation is to designate the value of the function for a particular value of the independent variable. That is,

the value of the function $f(x)$ *when* $x = a$ *is written as* $f(a)$.

EXAMPLE 5 Meaning of $f(a)$

If $g(t) = \dfrac{t^2}{2t + 1}$, then $g(a^3) = \dfrac{(a^3)^2}{2(a^3) + 1} = \dfrac{a^6}{2a^3 + 1}$

If $F(y) = 5 - 4y^2$, then $F(a + 1) = 5 - 4(a + 1)^2$

$$= 5 - 4(a^2 + 2a + 1) = -4a^2 - 8a + 1$$

but $F(a) + 1 = (5 - 4y^2) + 1 = 6 - 4y^2$

Carefully note the difference between $F(a + 1)$ and $F(a) + 1$.

EXAMPLE 6 Application: $f(T)$

The electric resistance R of a certain resistor as a function of the temperature T (in °C) is given by $R = 10.0 + 0.01T + 0.001T^2$. If a given temperature T is increased by 10°C, what is the value of R for the increased temperature as a function of T?

We are to determine R for a temperature of $T + 10$. Because

$$f(T) = 10.0 + 0.10T + 0.001T^2$$

then

$$\begin{aligned} f(T+10) &= 10.0 + 0.10(T+10) + 0.001(T+10)^2 && \text{substitute } T + 10 \text{ for } T \\ &= 10.0 + 0.10T + 1.0 + 0.001T^2 + 0.02T + 0.1 \\ &= 11.1 + 0.12T + 0.001T^2 \end{aligned}$$

EXAMPLE 8 Function as a set of instructions

The function $f(x) = x^2 - 3x$ tells us to "square the value of the independent variable, multiply the value of the independent variable by 3, and subtract the second result from the first." An analogy would be a computer that was programmed so that when a number was entered into the program, it would square the number, then multiply the number by 3, and finally subtract the second result from the first. This is diagramed in Fig. 3.3.

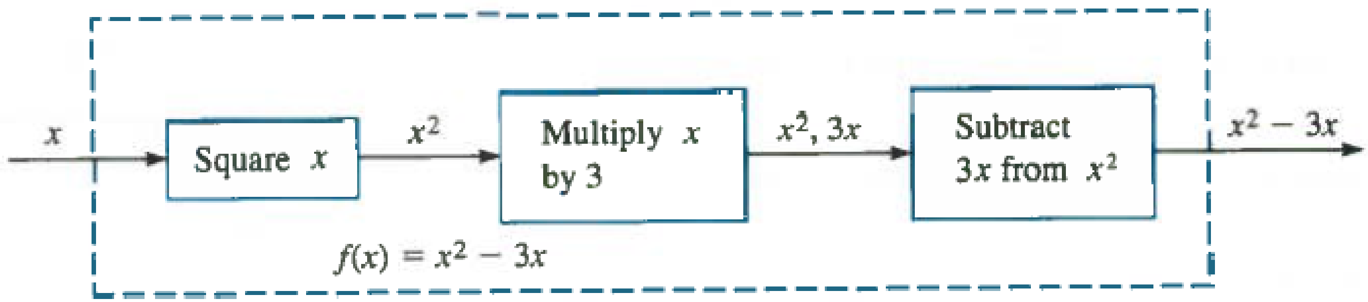


Fig. 3.3

The functions $f(t) = t^2 - 3t$ and $f(n) = n^2 - 3n$ are the same as the function $f(x) = x^2 - 3x$, since the operations performed on the independent variable are the same. ***Although different literal symbols appear, this does not change the function.***

NOTE ▸

In Exercises 5–12, find the indicated functions.

5. Express the area A of a circle as a function of (a) its radius r and (b) its diameter d.

6. Express the circumference c of a circle as a function of (a) its radius r and (b) its diameter d.

5. (a) $A(r) = \pi r^2$

(b) $A(d) = \pi\left(\frac{d}{2}\right)^2 = \frac{1}{4}\pi d^2$

6. From geometry, $c = 2\pi r$ (b) $c = \pi d$

39. The area A of the Bering Glacier in Alaska, given that its present area is $8430\ \text{km}^2$ and that it is melting at the rate of $140t\ \text{km}^2$, where t is the time in centuries.

39.
$$A = 8430 - 140t$$
$$f(t) = 8430 - 140t$$

3.2 1,2,3,5

EXAMPLE 1 Domain and range

The function $f(x) = x^2 + 2$ is defined for all real values of x. This means its domain is written as *all real numbers*. However, because x^2 is never negative, $x^2 + 2$ is never less than 2. We then write the range as *all real numbers* $f(x) \geq 2$, where the symbol $\geq$ means "is greater than or equal to."

The function $f(t) = \frac{1}{t + 2}$ is not defined for $t = -2$, for this value would require division by zero. Also, no matter how large t becomes, $f(t)$ will never exactly equal zero. Therefore, the domain of this function is *all real numbers except* -2, and the range is *all real numbers except* 0. ■

EXAMPLE 2 Domain and range

The function $g(s) = \sqrt{3 - s}$ is not defined for real numbers greater than 3, because such values make $3 - s$ negative and would result in imaginary values for $g(s)$. This means that the domain of this function is *all real numbers* $s \leq 3$, where the symbol $\leq$ means "is less than or equal to."

Also, because $\sqrt{3 - s}$ means the principal square root of $3 - s$, we know that $g(s)$ cannot be negative. This tells us that the range of the function is *all real numbers* $g(s) \geq 0$. ■

EXAMPLE 3 Find domain only

Find the domain of the function $f(x) = 16\sqrt{x} + \frac{1}{x}$.

From the term $16\sqrt{x}$, we see that x must be greater than or equal to zero in order to have real values. The term $\frac{1}{x}$ indicates that x cannot be zero, because of division by zero. Thus, putting these together, the domain is *all real numbers* $x > 0$.

As for the range, it is *all real numbers* $f(x) \geq 12$. More advanced methods are needed to determine this. ■

We have seen that the domain may be restricted because we do not use imaginary numbers or divide by zero. The domain may also be restricted by the definition of the function, or by practical considerations in an application.

EXAMPLE 4 Restricted domain

A function defined as

$$f(x) = x^2 + 4 \qquad (\text{for } x > 2)$$

has a domain restricted to real numbers greater than 2 by definition. Thus, $f(5) = 29$, but $f(1)$ is not defined, since 1 is not in the domain. Also, the range is all real numbers greater than 8.

The height h (in m) of a certain projectile as a function of the time t (in s) is

$$h = 20t - 4.9t^2$$

Negative values of time have no real meaning in this case. This is generally true in applications. Therefore, the domain is $t \geq 0$. Also, since we know the projectile will not continue in flight indefinitely, there is some upper limit on the value of t. These restrictions are not usually stated unless it affects the solution.

EXAMPLE 6 Function from verbal statement

The fixed cost for a company to operate a certain plant is \$3000 per day. It also costs \$4 for each unit produced in the plant. Express the daily cost C of operating the plant as a function of the number n of units produced.

The daily total cost C equals the fixed cost of \$3000 plus the cost of producing n units. Because the cost of producing one unit is \$4, the cost of producing n units is $4n$. Thus, the total cost C, where $C = f(n)$, is

$$C = 3000 + 4n$$

Here, we know that the domain is all values of $n \geq 0$, with some upper limit on n based on the production capacity of the plant.

EXAMPLE 7 Function from verbal statement

A metallurgist melts and mixes m grams (g) of solder that is 40% tin with n grams of another solder that is 20% tin to get a final solder mixture that contains 200 g of tin. Express n as a function of m. See Fig. 4.

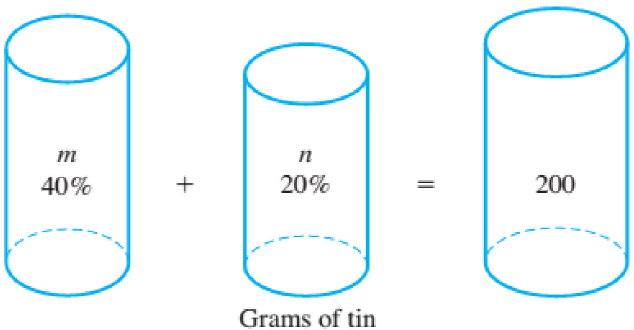


Fig. 4

The statement leads to the following equation:

$$\overset{\text{tin in first solder}}{0.40m} + \overset{\text{tin in second solder}}{0.20n} = \overset{\text{total amount of tin}}{200}$$

Because we want $n = f(m)$, we now solve for n:

$$0.20n = 200 - 0.40m$$
$$n = 1000 - 2m$$

This is the required function. Because neither m nor n can be negative, the domain is all values $0 \leq m \leq 500$ g, which means that m is greater than or equal to 0 g and less than or equal to 500 g. The range is all values $0 \leq n \leq 1000$ g. ■

EXAMPLE 8 Function from verbal statement

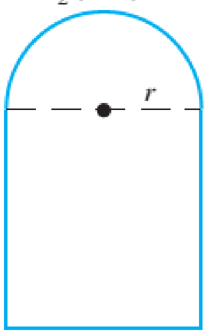


Fig. 5

An architect designs a window in the shape of a rectangle with a semicircle on top, as shown in Fig. 5. The base of the window is 10 cm less than the height of the rectangular part. Express the perimeter p of the window as a function of the radius r of the circular part.

The perimeter is the distance around the window. Because the top part is a semicircle, the length of this top circular part is $\frac{1}{2}(2\pi r)$, and the base of the window is $2r$ because it is equal in length to the dashed line (the diameter of the circle). Finally, the base being 10 cm less than the height of the rectangular part tells us that each vertical side of the rectangle is $2r + 10$. Therefore, the perimeter p, where $p = f(r)$, is

$$\begin{aligned} p &= \tfrac{1}{2}(2\pi r) + 2r + 2(2r + 10) \\ &= \pi r + 2r + 4r + 20 \\ &= \pi r + 6r + 20 \end{aligned}$$

We see that the required function is $p = \pi r + 6r + 20$. Because the radius cannot be negative and there would be no window if $r = 0$, the domain of the function is all values $0 < r \leq R$, where R is a maximum possible value of r determined by design considerations.

3.3 1,2

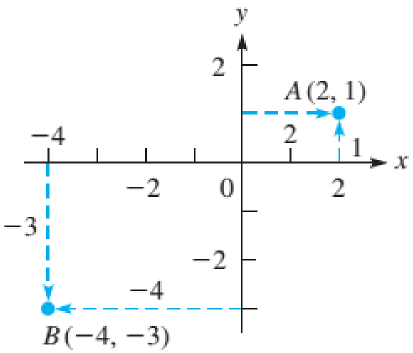


Fig. 9

EXAMPLE 1 Locating points

(a) Locate the points $A(2, 1)$ and $B(-4, -3)$ on the rectangular coordinate system.

The coordinates $(2, 1)$ for A mean that the point is 2 units to the *right* of the y-axis and 1 unit *above* the x-axis, as shown in Fig. 9. The coordinates $(-4, -3)$ for B mean that the point is 4 units to the *left* of the y-axis and 3 units *below* the x-axis, as shown. The x-coordinate of A is 2, and the y-coordinate of A is 1. For point B, the x-coordinate is -4, and the y-coordinate is -3.

(b) The positions of points $P(4, 5)$, $Q(-2, 3)$, $R(-1, -5)$, $S(4, -2)$, and $T(0, 3)$ are shown in Fig. 8. We see that this representation allows for *one point for any pair of values* (x, y). Also note that the point $T(0, 3)$ is on the y-axis. Any such point that is on either axis is not *in* any of the four quadrants. ■

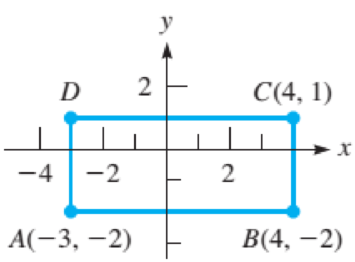


Fig. 10

EXAMPLE 2 Coordinates of vertices of rectangle

Three vertices of the rectangle in Fig. 10 are $A(-3, -2)$, $B(4, -2)$, and $C(4, 1)$. What is the fourth vertex?

We use the fact that opposite sides of a rectangle are equal and parallel to find the solution. Because both vertices of the base AB of the rectangle have a y-coordinate of -2, the base is parallel to the x-axis. Therefore, the top of the rectangle must also be parallel to the x-axis. Thus, the vertices of the top must both have a y-coordinate of 1, because one of them has a y-coordinate of 1. In the same way, the x-coordinates of the left side must both be -3. Therefore, the fourth vertex is $D(-3, 1)$. ■

EXAMPLE 1 Graphing a function by plotting points

Graph the function $f(x) = 3x - 5$.

For purposes of graphing, let $y = f(x)$, or $y = 3x - 5$. Then, let x take on various values and determine the corresponding values of y. Note that once we choose a given value of x, we have no choice about the corresponding y-value, as it is determined by evaluating the function. If $x = 0$, we find that $y = -5$. This means that the point $(0, -5)$ is on the graph of the function $3x - 5$. Choosing another value of x—for example, 1—we find that $y = -2$. This means that the point $(1, -2)$ is on the graph of the function $3x - 5$. Continuing to choose a few other values of x, we tabulate the results, as shown in Fig. 14. It is best to arrange the table so that the values of x increase; then there is no doubt how they are to be connected, for they are then connected in the order shown. Finally, we connect the points in Fig. 14 and see that the graph of the function $3x - 5$ is a straight line.

3.4

1,2, 4,7

Select values of x and calculate corresponding values of y

$f(x) = 3x - 5$

$f(-1) = 3(-1) - 5 = -8$

$f(0) = 3(0) - 5 = -5$

$f(1) = 3(1) - 5 = -2$

$f(2) = 3(2) - 5 = 1$

$f(3) = 3(3) - 5 = 4$

Tabulate with values of x increasing

x	y
-1	-8
0	-5
1	-2
2	1
3	4

Plot points and join from left to right with smooth curve

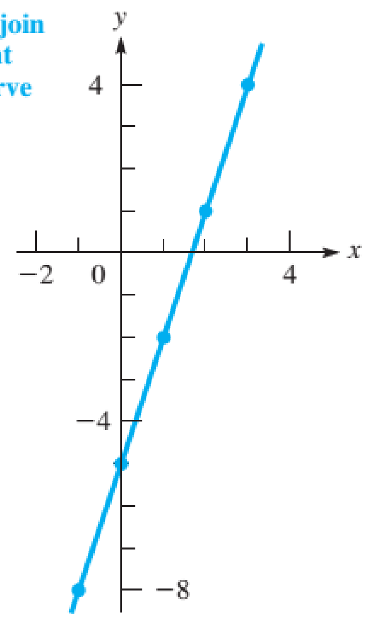


Fig. 14

EXAMPLE 2 Be careful: negative numbers

Graph the function $f(x) = 2x^2 - 4$.

First, let $y = 2x^2 - 4$ and tabulate the values as shown in Fig. 16. In determining the values in the table, take particular care to obtain the correct values of y for negative values of x. ***Mistakes are relatively common when dealing with negative numbers.*** We must carefully use the laws for signed numbers. For example, if $x = -2$, we have $y = 2(-2)^2 - 4 = 2(4) - 4 = 8 - 4 = 4$. Once the values are obtained, plot and connect the points with a smooth curve, as shown.

Select values of x and calculate corresponding values of y

$f(x) = 2x^2 - 4$

$f(-2) = 2(-2)^2 - 4 = 4$

$f(-1) = 2(-1)^2 - 4 = -2$

$f(0) = 2(0)^2 - 4 = -4$

$f(1) = 2(1)^2 - 4 = -2$

$f(2) = 2(2)^2 - 4 = 4$

Tabulate with values of x increasing

x	y
−2	4
−1	−2
0	−4
1	−2
2	4

Plot points and join from left to right with smooth curve

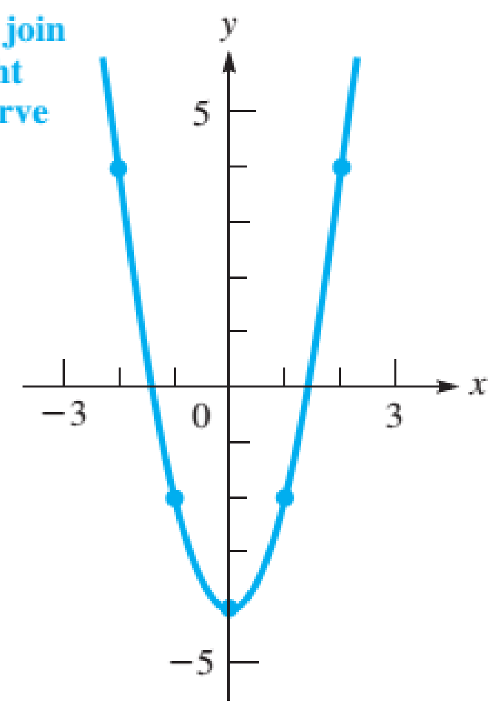


Fig. 16

EXAMPLE 4 Be careful: division by zero

x	y
-4	$3/4$
-3	$2/3$
-2	$1/2$
-1	0
$-1/2$	-1
$-1/3$	-2

x	y
$1/3$	4
$1/2$	3
1	2
2	$3/2$
3	$4/3$
4	$5/4$

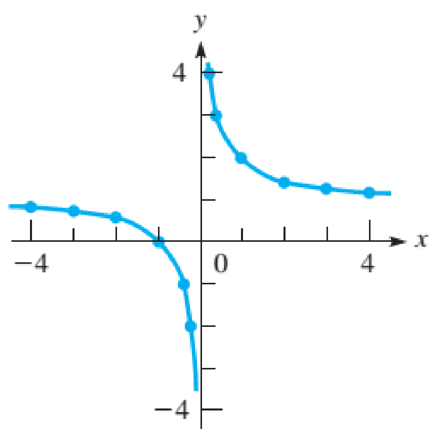


Fig. 18

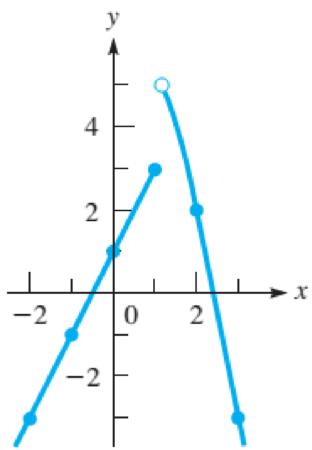
Graph the function $y = 1 + \dfrac{1}{x}$.

In finding the points on this graph, as shown in Fig. 18, note that y is not defined for $x = 0$, due to division by zero. Thus, $x = 0$ is not in the domain, and ***we must be careful not to have any part of the curve cross the y-axis*** $(x = 0)$. Although we cannot let $x = 0$, we can choose other values for x between -1 and 1 that are close to zero. In doing so, we find that as x gets closer to zero, the points get closer to the y-axis, although they do not reach or touch it. In this case, the y-axis is called an **asymptote** of the curve.

We see that the curve is smooth, except when $x = 0$, where it is not defined. ■

	x	y
$f(-2) = 2(-2) + 1 = -3$	-2	-3
$f(-1) = 2(-1) + 1 = -1$	-1	-1
$f(0) = 2(0) + 1 = 1$	0	1
$f(1) = 2(1) + 1 = 3$	1	3
$f(2) = 6 - 2^2 = 2$	2	2
$f(3) = 6 - 3^2 = -3$	3	-3

EXAMPLE 7 Function defined for intervals of domain



Graph the function $f(x) = \begin{cases} 2x + 1 & (\text{for } x \leq 1) \\ 6 - x^2 & (\text{for } x > 1) \end{cases}$.

First, let $y = f(x)$ and then tabulate the necessary values. In evaluating $f(x)$, we must be careful to use the proper part of the definition. To see where to start the curve for $x > 1$, we evaluate $6 - x^2$ for $x = 1$, but we must realize that ***the curve does not include this point*** $(1, 5)$ and starts immediately to its right. To show that it is not part of the curve, draw it as an open circle. See Fig. 21.

A function such as this one, with a "break" in it, is called *discontinuous*. ■

EXAMPLE 6 See the chapter introduction

The electric power P (in W) delivered by a certain fuel cell as a function of the resistance R (in Ω) in the circuit is given by $P = \frac{100R}{(0.50 + R)^2}$. Plot P as a function of R.

■ The unit of power, the watt (W), is named for James Watt (1736–1819), a British engineer.

Since negative values for the resistance have no physical significance, we should not plot any values of P for negative values of R. The following table is obtained:

$R(\Omega)$	0	0.25	0.50	1.0	2.0	3.0	4.0	5.0	10.0
$P(W)$	0.0	44.4	50.0	44.4	32.0	24.5	19.8	16.5	9.1

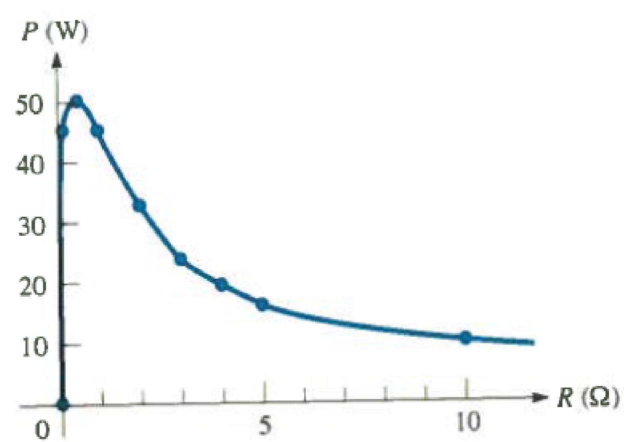


Fig. 3.20

The R values of 0.25 and 0.50 are used since P is less for $R = 2\ \Omega$ than for $R = 1\ \Omega$. The sharp change in direction should be checked by using these additional points to get a smoother curve near $R = 1\ \Omega$. See Fig. 3.20.

Note that the scale on the P-axis differs from that on the R-axis. Different scales are normally used when the variables differ in magnitudes and ranges.

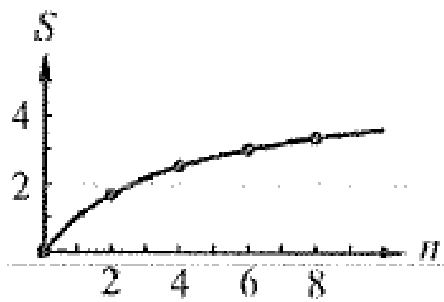
The graph gives us information about P and R. For example, the maximum power of 50 W occurs for $R = 0.50\ \Omega$. Also, P decreases as R increases beyond $0.50\ \Omega$. Reading information from a graph is discussed in the next section. ■

53. The number of times S that a certain computer can perform a computation faster with a multiprocessor than with a uniprocessor is given by $S = \dfrac{5n}{4 + n}$, where n is the number of processors. Plot S as a function of n.

54. The voltage V across a capacitor in a certain electric circuit for a 2-s interval is $V = 2t$ during the first second and $V = 4 - 2t$ during the second second. Here, t is the time (in s). Plot V as a function of t.

53. $S = \dfrac{5n}{4+n}$

n	S
0	0
2	5/3
4	5/2
6	3
8	10/3



54. $V = \begin{cases} 2t, & 0 \le t < 1 \\ 4 - 2t, & 1 \le t \le 2 \end{cases}$

t	V
0	0
1/2	1
1	2
3/2	1
2	0