Chapter 5

Systems of Linear Equations

5, 6	**Systems of Linear equations**	• Linear equations and its graphs • Solving systems of two linear equations graphically and algebraically	5.1. Linear equations 5.2. Graphs of linear functions 5.3. Solving systems of two linear equations in two unknowns graphically 5.4. Solving systems of two linear equations in two unknowns algebraically

New textbook looks slightly different

5. Systems of Linear Equations; Determinants

5. Systems of Linear Equations; Determinants

5.1 Linear Equations and Graphs of Linear Functions

5.2 Systems of Equations and Graphical Solutions

5.3 Solving Systems of Two Linear Equations in Two Unknowns Algebraically

Linear Equations

- A linear equation in one unknown is written in the form:

$$\boxed{a\,x + b = 0}$$

- The solution to a linear equation is known as the root of the equation.

$$\boxed{x = -\frac{b}{a}}$$

Linear Equations

- A linear equation in two unknowns is written in the form:

$$\boxed{a\,x + b\,y = 0}$$

- The solution is any set of numbers, one for each variable, that satisfies the equation.

Linear Equations

- The solution of a linear equation in two unknowns can be graphed as a set of points.
- **Example**:
- What is the solution to $2x + y = 3$?

Solution - We find a set of points as a part of the solution to $2x + y = 3$:

This can be done by using

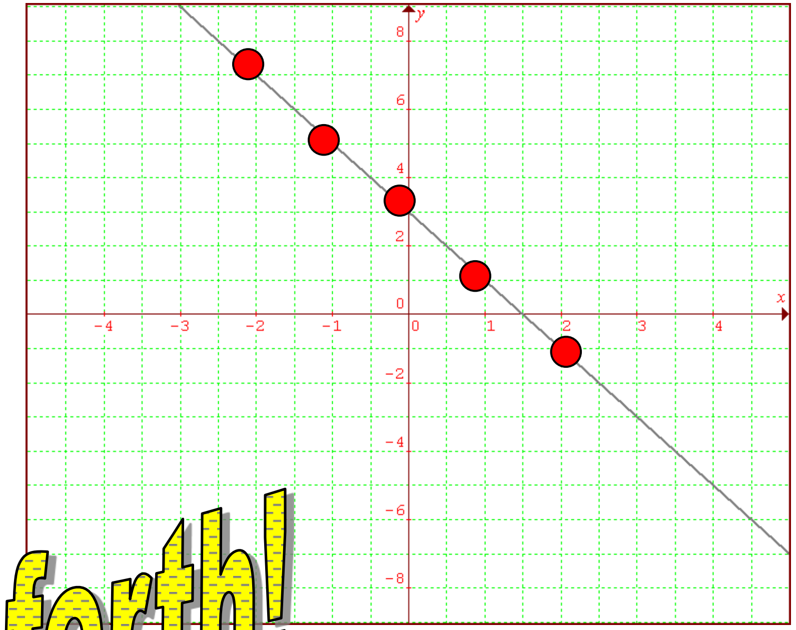
y=3-2x

pick x – calculate y - plot

(-2, 7)

(-1, 5)

- (0, 3)
- (1, 1)
- (2, -1)



and so forth!

A System of Simultaneous Linear Equations: ***2 equations in 2 unknowns***

- Two linear equations, each containing the same two unknowns:

$$\boxed{\begin{aligned} a_1 x + b_1 y &= c_1 \\ a_2 x + b_2 y &= c_2 \end{aligned}}$$

- A solution of the system is any pair of values (***x*, *y***) that satisfies **both** equations.

Example

In Exercises 9–14, for each given value of x, determine the value of y that gives a solution to the given linear equations in two unknowns.

9. $3x - 2y = 12; \quad x = 2, x = -3$

10. $-5x + 6y = 60; \quad x = -10, x = 8$

11. $x - 4y = 2; \quad x = 3, x = -0.4$

12. $3x - 2y = 9; \quad x = \frac{2}{3}, x = -3$

13. $24x - 9y = 16; \quad x = 2/3, x = -1/2$

14. $2.4y - 4.5x = -3.0; \quad x = -0.4, x = 2.0$

A

9. $3(2) - 2y = 12;\ 2y = 6 - 12 = -6;\ y = -3$

$3(-3) - 2y = 12;\ 2y = -9 - 12 = -21;\ y = -\dfrac{21}{2}$

29. The forces acting on part of a structure are shown in Fig. 5.4. An analysis of the forces leads to the equations

$$0.80F_1 + 0.50F_2 = 50$$
$$0.60F_1 - 0.87F_2 = 12$$

Are the forces 45 N and 28 N?

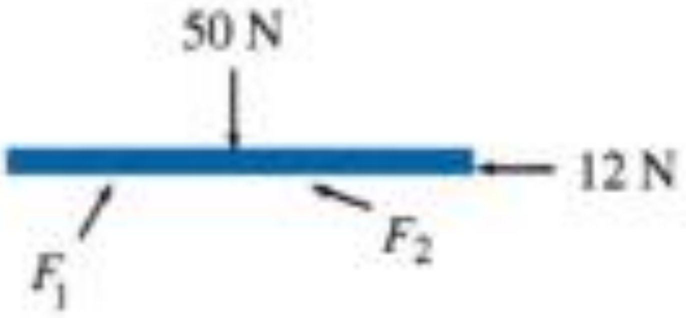


Fig. 5.4

Are BOTH conditions satisfied?

A

10. $-5(-10)+6y=60;\ 6y=60-50=10;\ y=\dfrac{5}{3}$

$-5(8)+6y=60;\ 6y=60+40=100;\ y=\dfrac{50}{3}$

11. $3-4y=2;\ -4y=-1;\ y=\dfrac{1}{4}$

$-0.4-4y=2;\ -4y=2.4;\ y=-0.6$

12. $3(2/3)-2y=9;\ 2-2y=9;\ -2y=7;\ y=-7/2$

$3(-3)-2y=9;\ -9-2y=9;\ -2y=18;\ y=-9$

13. $24(2/3)-9y=16;\ 9y=16-16=0;\ y=0$

$24(-1/2)-9y=16;\ 9y=-12-16=-28;$

$y=(-28/9)$

14. $2.4y-4.5(-0.4)=-3.0;\ 2.4y=-4.8;\ y=-2.0$

$2.4y=-4.5(2.0)=-3.0;\ 2.4y=6.0;\ y=2.5$

29. If $F_1=45$ N and $F_2=28$ N, then

$0.80(45)+0.50(28)=50;$

$0.60(45)-0.87(28)=2.64\neq 12$

Since both values do not satisfy both equations, they are not a solution.

Ch. 5.2: Graphs of Linear Equations

- A system of linear equations can be solved graphically.
- Solving graphically will produce at least one line.
- We can determine the steepness of that line by its slope.

Slope of a Line

- Given two points, $A\ (x_1, y_1)$ and $B\ (x_2, y_2)$ existing on a line, the slope, $\boldsymbol{m}$, of this line is defined as:

$$\boxed{m = \frac{y_2 - y_1}{x_2 - x_1}}$$

- Slope is often known as ***rise over run*.**

Slope of a Line

- Slope is called **positive** as x increases, y increases.
- Slope is considered **negative** when as x increases, y decreases.
- The larger the absolute value of the slope, the steeper is the line.

Slope-Intercept Form of the Equation of a Straight Line

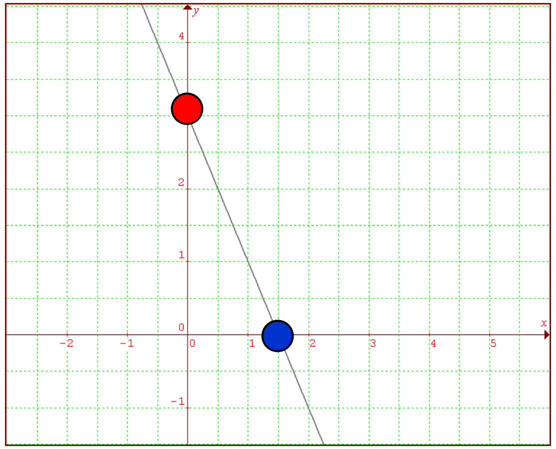
$$y = mx + b$$

Slope

y-intercept

Sketching Lines by Intercepts

- We solve the equation for $x = 0$ (the y-intercept) and $y = 0$ (the x-intercept).
- **Example**:
  - What are the x- and y-intercepts of $2x + y = 3$?
- **Answer:**
  - x-intercept: $(3/2, 0)$
  - y-intercept: $(0, 3)$



Example

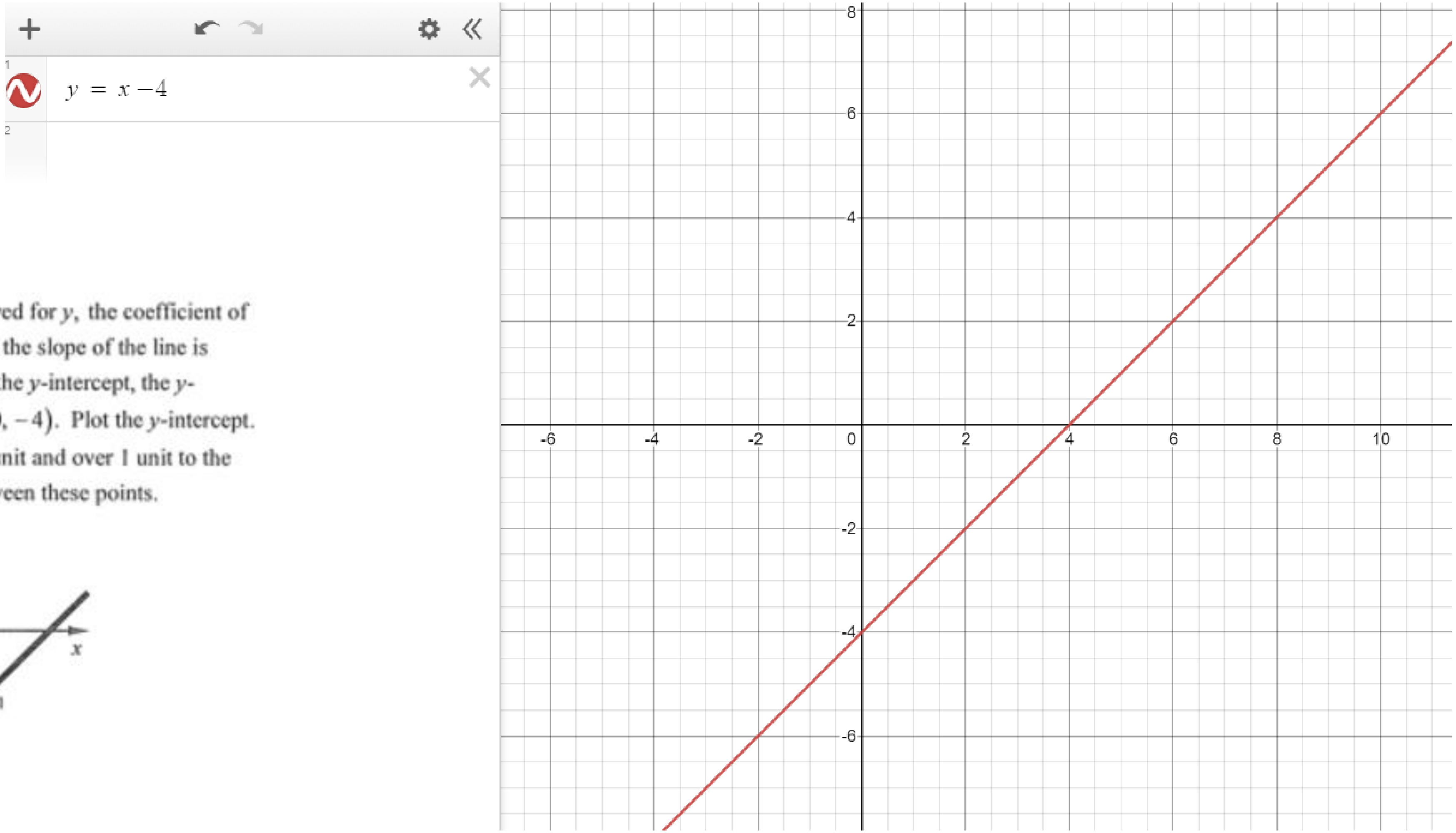
In Exercises 5–12, find the slope of the line that passes through the given points.

5. $(1, 0), (3, 8)$
6. $(3, 1), (2, -7)$
7. $(-1, 2), (-4, 17)$
8. $(-1, -2), (2, 10)$

In Exercises 21–28, find the slope and the y-intercept of the line with the given equation and sketch the graph using the slope and the y-intercept. A graphing calculator can be used to check your graph.

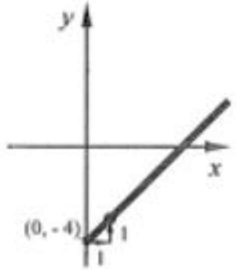
21. $y = -2x + 1$
22. $y = -4x$
23. $y = x - 4$
24. $y = \frac{4}{5}x + 2$
25. $5x - 2y = 40$
26. $-2y = 7$

https://www.desmos.com/calculator



23. $y = x - 4$

Since the equation is solved for y, the coefficient of x is the slope. Therefore the slope of the line is $1/1$. Since the b-term is the y-intercept, the y-intercept of this line is $(0, -4)$. Plot the y-intercept. From this point go up 1 unit and over 1 unit to the right. Draw the line between these points.



A

5. By taking $(3, 8)$ as (x_2, y_2) and $(1, 0)$ as (x_1, y_1)

$$m = \frac{8-0}{3-1} = \frac{8}{2} = 4$$

6. By taking $(2, -7)$ as (x_2, y_2) and $(3, 1)$ as (x_1, y_1)

$$m = \frac{-7-1}{2-3} = -8$$

=8

7. By taking $(-4, 17)$ as (x_2, y_2) and $(-1, 2)$ as (x_1, y_1)

$$m = \frac{17-2}{-4-(-1)} = \frac{15}{-4+1} = -\frac{15}{3} = -5$$

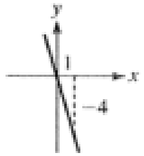
8. By taking $(2, 10)$ as (x_2, y_2) and $(-1, -2)$ as (x_1, y_1)

$$m = \frac{10-(-2)}{2-(-1)} = \frac{10+2}{2+1} = \frac{12}{3} = 4$$

22. $y = -4x$

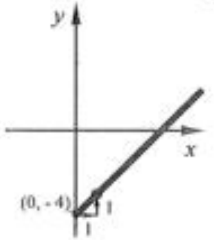
Since the equation is solved for y, the coefficient of x is the slope. Therefore the slope of the line is $-4/1$. Since the b-term is the y-intercept, the y-intercept of this line is $(0,0)$. Plot the y-intercept. From this point go down 4 units and over 1 unit to the right. Draw the line between these points.

$m = -4, b = 0$



23. $y = x - 4$

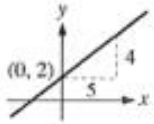
Since the equation is solved for y, the coefficient of x is the slope. Therefore the slope of the line is $1/1$. Since the b-term is the y-intercept, the y-intercept of this line is $(0, -4)$. Plot the y-intercept. From this point go up 1 unit and over 1 unit to the right. Draw the line between these points.



24. $y = \left(\frac{4}{5}\right)x + 2$

Since the equation is solved for y, the coefficient of x is the slope. Therefore the slope of the line is $4/5$. Since the b-term is the y-intercept, the y-intercept of this line is $(0, 2)$. Plot the y-intercept. From this point go up 4 units and over 5 units to the right. Draw the line between these points.

$m = \frac{4}{5}, b = 2$

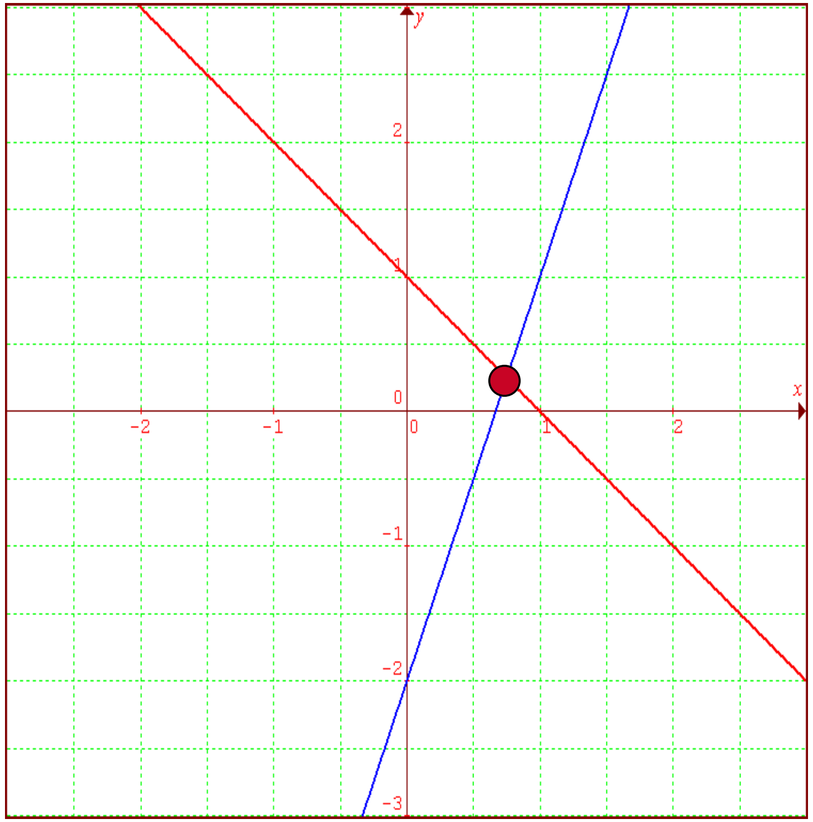


Ch. 5.3: Solving **Systems** of Linear Equations in Two Unknowns **Graphically**

- Having 2 or more equations and finding their intersection point.
- However, this will not always be possible.

The **Independent** System gives a unique solution

- Given:
  - $3x - y = 2$
  - $x + y = 1$
- One solution.
- Graphs intersect at **(¾, ¼)** .

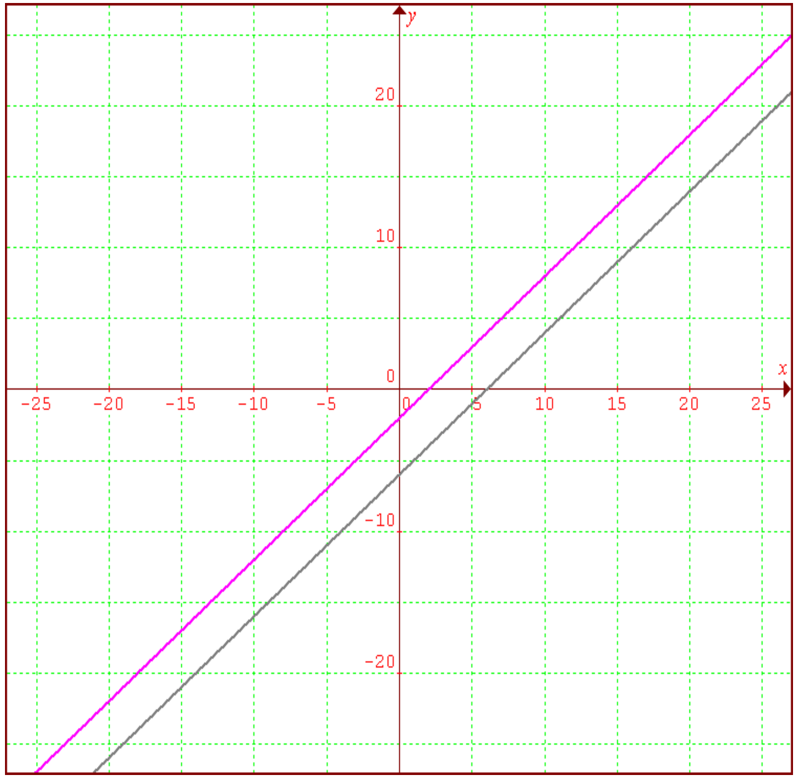


2) The **Inconsistent** System

- There is **no solution** to these types of systems of linear equations.
- Graphically, these **lines never meet** in the plane.
- If we were to work this out algebraically, we would find a false solution.
- That is: $\mathbf{0} = \boldsymbol{a}$

No solution

- Given:
  - $2x - 2y = 4$
  - $x - y = 6$
- Upon graphing this system of linear equations, we find 2 parallel lines.
- These lines do not touch. (No solution)

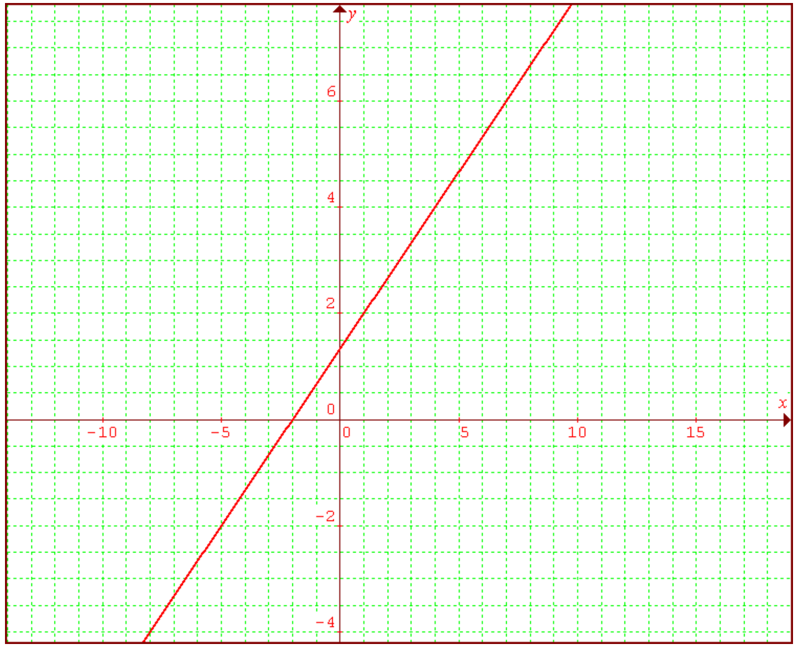


4) Similar lines (**Dependent** system)

- There are infinitely many solutions to the 2 equations.
- Graphically, the two lines are one and the same line.
- There are an infinite number of solutions (i.e., ordered pairs) to this system.
- When algebraically solving this system, we find a true statement, $\mathbf{0 = 0}$.

Infinite solutions

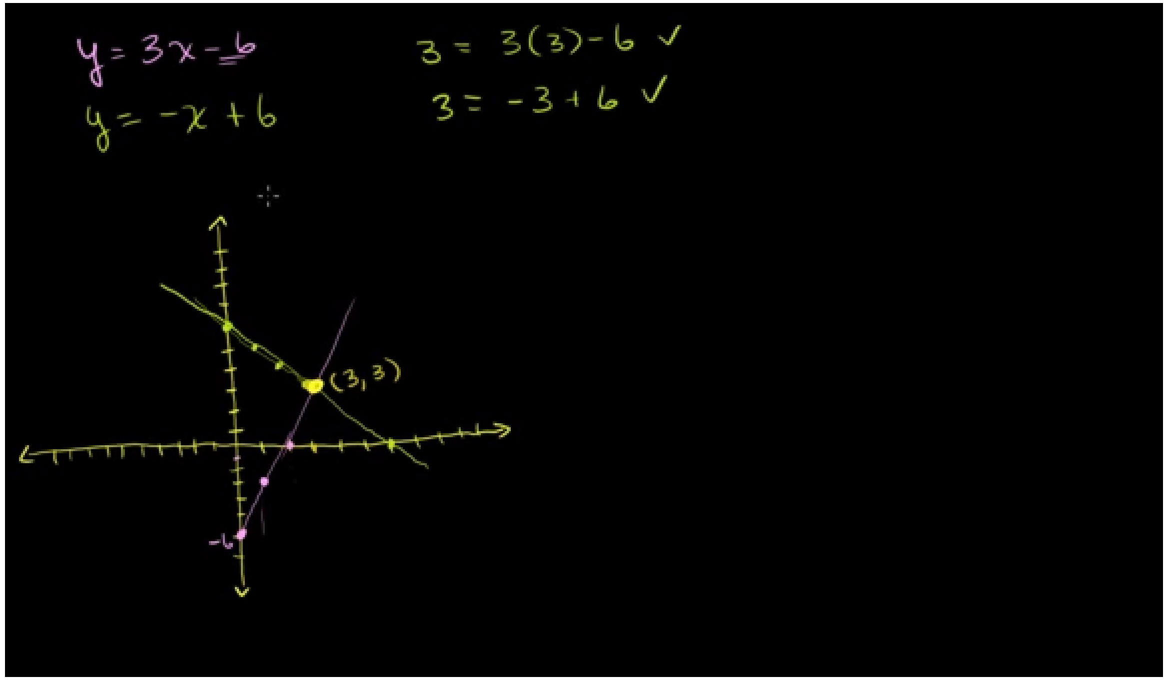
- Given:
  - $-2x + 3y = 4$
  - $4x - 6y = -8$
- By viewing the graph, we note that there is only one line.
- The first equation sits on top of the other one!



Summary

- Graphically, we find the intersection point of the given lines.
- Algebraically, we find the ordered pair that satisfies each equation if the lines are independent.
- There is no solution if the lines are parallel to each other and considered inconsistent.
- There are an infinite number of solutions if the lines lie on top of each other and considered dependent lines.

https://www.khanacademy.org/math/algebra/systems-of-eq-and-ineq/fast-systems-of-equations/v/solving-linear-systems-by-graphing



Example

In Exercises 3–20, solve each system of equations by sketching the graphs. Use the slope and the y-intercept or both intercepts. Estimate each result to the nearest 0.1 if necessary.

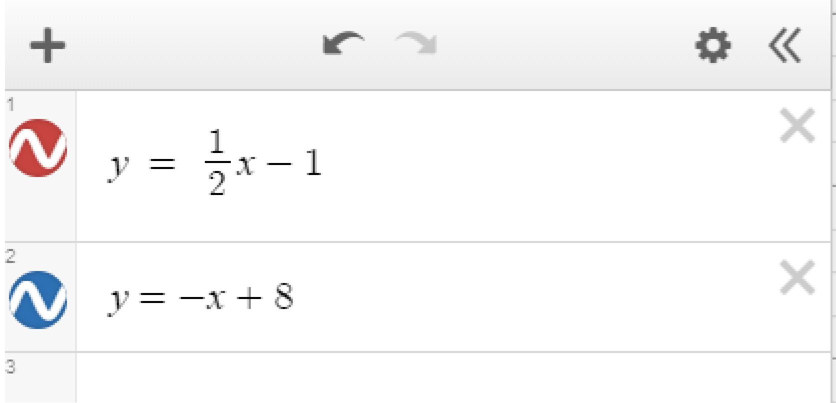
3. $y = -x + 4$
   $y = x - 2$

4. $y = \frac{1}{2}x - 1$
   $y = -x + 8$

5. $y = 2x - 6$
   $y = -\frac{1}{3}x + 1$

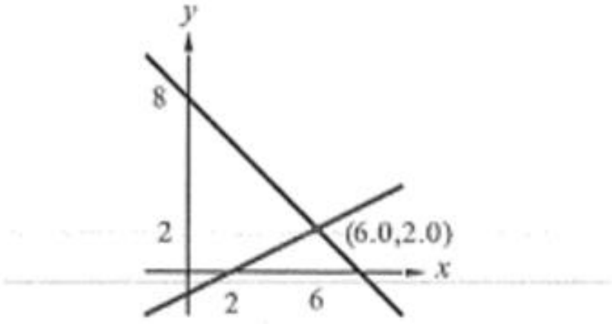
6. $y = \frac{1}{2}x - 4$
   $y = 2x + 2$

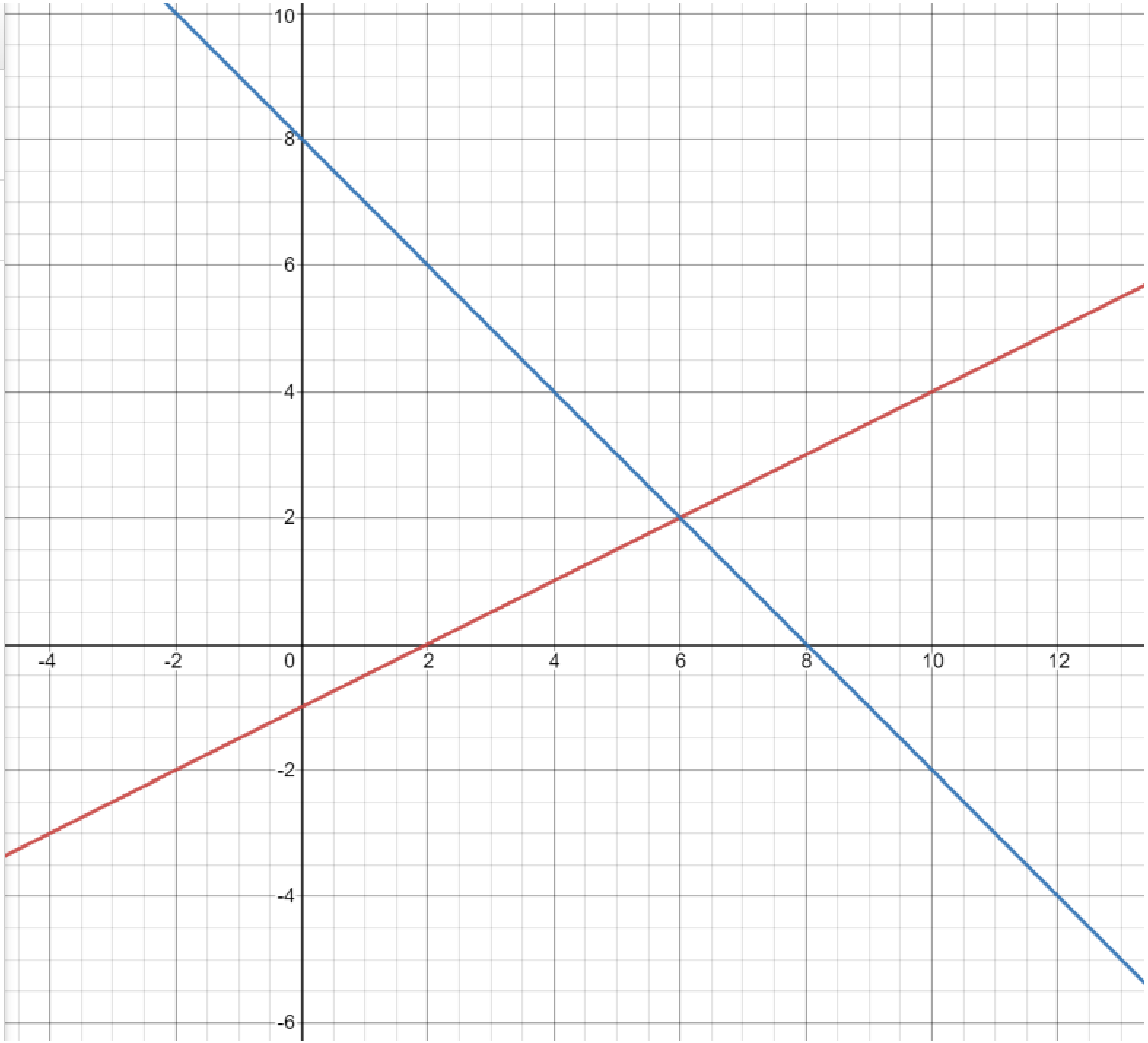
$$y = m x + b$$



4. $y = \left(\frac{1}{2}\right)x - 1;\ y = -x + 8$

The slope of the first line is $1/2$, and the y-intercept is -1. The slope of the second line is -1, and the y-intercept is 8. From the graph, the point the point of intersection is $(6.0,\ 2.0)$. Therefore, the solution of the system of equations is $x = 6.0$, $y = 2.0$.

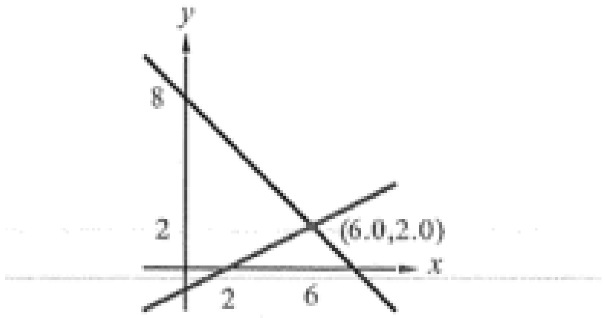




A

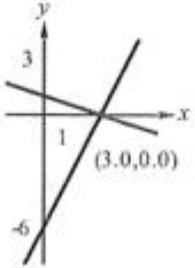
4. $y = \left(\frac{1}{2}\right)x - 1;\ y = -x + 8$

The slope of the first line is $1/2$, and the y-intercept is -1. The slope of the second line is -1, and the y-intercept is 8. From the graph, the point the point of intersection is $(6.0, 2.0)$. Therefore, the solution of the system of equations is $x = 6.0$, $y = 2.0$.



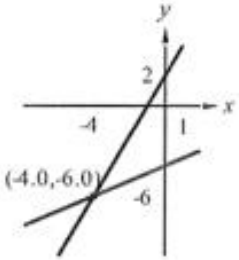
5. $y = 2x - 6;\ y = -\left(\frac{1}{3}\right)x + 1$

The slope of the first line is 2, and the y-intercept is -6. The slope of the second line is $-1/3$ and the y-intercept is 1. From the graph, the point of intersection is $(3.0, 0.0)$. Therefore, the solution of the system of equations is $x = 3.0$, $y = 0.0$.



6. $y = \left(\frac{1}{2}\right)x - 4;\ y = 2x + 2$

The slope of the first line is 1/2, and the y-intercept is -4. The slope of the second line is 2 and the y-intercept is 2. From the graph, the point of intersection is $(-4.0, -6.0)$. Therefore, the solution of the system of equations is $x = -4.0$, $y = -6.0$.



Summary Ch 5 so far…

Straight line

$$y = mx + b$$

Slope

y-intercept

Use intercepts on axis and/or gradient to draw

Solve system of linear equations by drawing a straight line and find the intercept

Ch. 5.4: Solving Systems of Linear Equations in Two Unknowns Algebraically

- Systems of linear equations can be solved algebraically to obtain exact solutions.
- These techniques include:
  - Solution by Substitution
  - Solution by Addition or Subtraction

Solution of Two Linear Equations by Substitution

1. Solve one equation for one of the unknowns.
2. Substitute this solution into the other equation to obtain a linear equation with one unknown.
3. Solve the resulting equation for the value of the unknown it contains.
4. Substitute this value into the equation of step 1 and solve for the other unknown.
5. Check the values in both original equations.

Example 1

- Solve the following system of linear equations in two unknowns by substitution.

$$4x + 3y = 18 \quad (1)$$

$$x + 5y = 13 \quad (2)$$

Solution 1

$$4x + 3y = 18 \quad (1)$$
$$x + 5y = 13 \quad (2)$$

N

1. Solve one equation for one of the unknowns.

$$x = 13 - 5y$$

2. Substitute this solution into the other equation to obtain a linear equation with one unknown.

$$4(13 - 5y) + 3y = 18$$

Solution 1 (*continued*)

1 $4x + 3y = 18$

2 $x + 5y = 13$

3. Solve the resulting equation for the value of the unknown it contains.

$$y = 2$$

4. Substitute this value into the equation of step 1 and solve for the other unknown.

$$x + 5y = 13$$

$$x + 5(2) = 13$$

$$x = 3$$

Answer: **(3, 2)**

$$\mathbf{1}\quad 4x+3y=18$$
$$\mathbf{2}\quad x+5y=13$$

Solution 1 (*continued*)

5. Check the values in both original equations.

1

$$4x+3y=18$$
$$4(3)+3(2)=18$$
$$12+6=18$$
$$18 \overset{\checkmark}{=} 18$$

2

$$x+5y=13$$
$$3+5(2)=13$$
$$13 \overset{\checkmark}{=} 13$$

- The solution checks in both equations.
- This is an independent system of linear equations.

ANOTHER METHOD
Solution of Two Linear Equations by Addition or Subtraction

1. If not already so, write the equations in the form

$$a_1 x + b_1 y = c_1$$
$$a_2 x + b_2 y = c_2$$

2. If necessary, multiply all terms of each equation by a constant chosen so that the coefficients of one unknown will be numerically the same in both equations. (They can have the same or different signs.)

Solution of Two Linear Equations by Addition or Subtraction (*continued*)

3. (a) If the numerically equal coefficients have ***different*** signs, ***add*** the terms on each side of the resulting equations.

   (b) If the numerically equal coefficients have the ***same*** sign, ***subtract*** the terms on each side of the resulting equations.

4. Solve the resulting linear equation in the other unknown.

Solution of Two Linear Equations by Addition or Subtraction (*continued*)

5. Substitute this value into one of the original equations to find the value of the other unknown.
6. Check by substituting both values into both original equations.

Example 2

- Solve the following system of linear equations in two unknowns by addition or subtraction.

1 $$2x+7y=5$$

2 $$3x+8y=20$$

1 $2x+7y=5$

2 $3x+8y=20$

Solution 2

1. We note the equations are in the correct form.

$$2x+7y=5 \quad \times 3$$
$$3x+8y=20 \quad \times 2$$

2. Multiply Equation 1 by **3** and Equation 2 by **2** to give the same first term.

$$6x+21y=15$$
$$6x+16y=40$$

Solution 2 (*continued*)

$$\mathbf{1}\quad 2x + 7y = 5$$
$$\mathbf{2}\quad 3x + 8y = 20$$

3. Since the signs of the 1st term are the same, we subtract each term.

$$\begin{array}{r} 6x + 21y = 15 \\ -\,6x - 16y = -40 \\ \hline \end{array}$$

$5y = -25$

5y = -25

4. Solve the resulting linear equation in the other unknown.

$$y = -5$$

Solution 2 (*continued*)

1. $2x + 7y = 5$
2. $3x + 8y = 20$

5. Substituting into Equation 1 to find ***x***.

$$2x + 7(-5) = 5$$

$$x = 20$$

6. Checking in both equations, the answer is: **(20, -5)**

Summary

- Be sure the systems of linear equations are in general form.
- If not, they must be rearranged into that form.
- Always check your work by substituting your answers into the original equations.

Example

5,9,13,
15,19,23

In Exercises 5–14, solve the given systems of equations by the method of elimination by substitution.

5. $x = y + 3$
   $x - 2y = 5$
6. $x = 2y + 1$
   $2x - 3y = 4$
7. $p = V - 4$
   $V + p = 10$
8. $y = 2x + 10$
   $2x + y = -2$
9. $x + y = -5$
   $2x - y = 2$
10. $3x + y = 1$
    $3x - 2y = 16$
11. $2x + 3y = 7$
    $6x - y = 1$
12. $2s + 2t = 1$
    $4s - 2t = 17$
13. $33x + 2y = 34$
    $40y = 9x + 11$
14. $3A + 3B = -1$
    $5A = -6B - 1$

In Exercises 15–24, solve the given systems of equations by the method of elimination by addition or subtraction.

15. $x + 2y = 5$
    $x - 2y = 1$
16. $x + 3y = 7$
    $2x + 3y = 5$
17. $2x - 3y = 4$
    $2x + y = -4$
18. $R - 4r = 17$
    $3R + 4r = 3$
19. $12t + 9y = 14$
    $6t = 7y - 16$
20. $3x - y = 3$
    $4x = 3y + 14$
21. $v + 2t = 7$
    $2v + 4t = 9$
22. $3x - y = 5$
    $-9x + 3y = -15$
23. $2x - 3y - 4 = 0$
    $3x + 2 = 2y$
24. $3i_1 + 5 = -4i_2$
    $3i_2 = 5i_1 - 2$

A

5. (1) $x = y + 3$

(2) $x - 2y = 5$

$(y+3) - 2y = 5$ substitute x from (1) into (2)

$-y = 2$

$y = -2$ substitute -2 for y in (1)

$x = -2 + 3 = 1$

9. (1) $x + y = -5,\ y = -x - 5$

(2) $2x - y = 2$

$2x - (-x - 5) = 2$ substitute y from (1) into (2)

$3x = -3$

$x = -1$

$-1 + y = -5$ substitute -1 for x in (1)

$y = -4$

13. (1) $33x + 2y = 34 \Rightarrow y = -\frac{33}{2}x + 17$

(2) $40y = 9x + 11$

$40\left(-\frac{33}{2}x + 17\right) = 9x + 11$ substitute y from (1) in (2)

$-660x + 680 = 9x + 11$

$-669x = -669$

15. $x + 2y = 5$

$\underline{x - 2y = 1}$

$2x = 6$

$x = 3$

$3 + 2y = 5$

$2y = 2$

$y = 1;\ (3, 1)$

19. (1) $12t + 9y = 14 \Rightarrow \quad 12t + 9y = 14$

(2) $6t = 7y - 16 \Rightarrow \underline{-12t + 14y = 32}$ add

$23y = 46$

$y = 2$

$12t + 9(2) = 14$ substitute 2 for y in (1)

$12t = -4$

$t = -\frac{1}{3}$

23.

(1)	$2x - 3y - 4 = 0$		
(2)	$3x + 2 = 2y$	put (1) in standard form	
(3)	$2x - 3y = 4$	recopy (2)	
(4)	$3x - 2y = -2$	put (2) in standard form	
(5)	$6x - 9y = 12$	(3) multiplied by 3	
(6)	$\underline{-6x + 4y = 4}$	(4) multiplied by -2	

$-5y = 16$

$y = -\frac{16}{5}$

from (1) $x = \dfrac{3y+4}{2} = \dfrac{3\left(-\frac{16}{5}\right)+4}{2} = -\dfrac{14}{5}$,

$\left(-\frac{14}{5}, -\frac{16}{5}\right)$ is the solution.

If you have time, you could also check by plotting the graph

9. (1) $x+y=-5,\ y=-x-5$

(2) $2x-y=2$

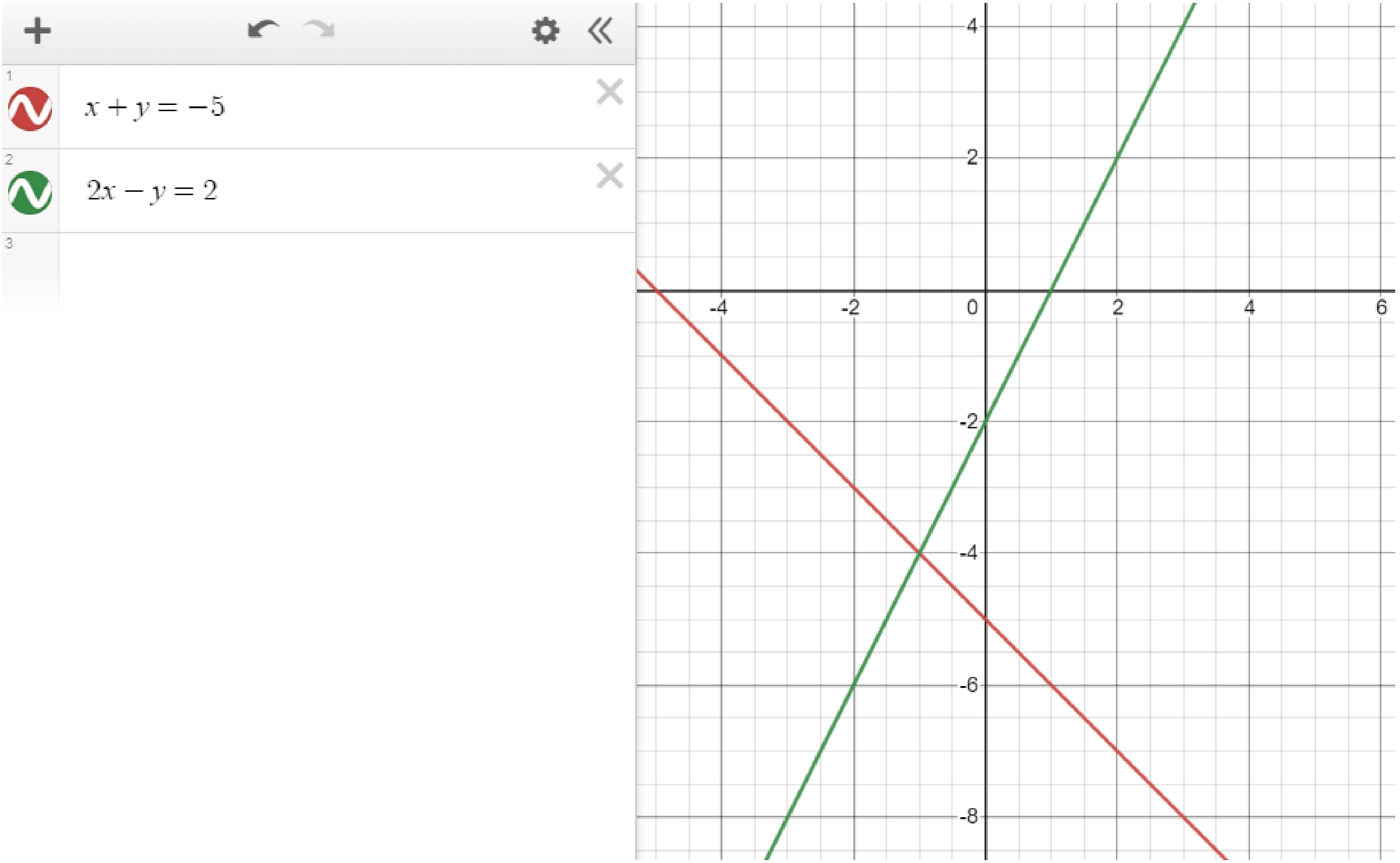
$2x-(-x-5)=2$ substitute y from (1) into (2)

$3x=-3$

$x=-1$

$-1+y=-5$ substitute -1 for x in (1)

$y=-4$



Worked examples

5.1 1,3,4

EXAMPLE 1 Meaning of linear

The equation $5x - t + 6 = 0$ is linear in x and t, but $5x^2 - t + 6 = 0$ is not linear, due to the presence of x^2.

The equation $4x + y = 8$ is linear in x and y, but $4xy + y = 8$ is not, due to the presence of xy.

The equation $x - 6y + z - 4w = 7$ is linear in x, y, z, and w, but the equation $x - \frac{6}{y} + z - 4w = 7$ is not, due to the presence of $\frac{6}{y}$, where y appears in the denominator. ■

An equation that can be written in the form

$$\boxed{ax + b = 0} \tag{1}$$

is known as a **linear equation in one unknown.** Here, a and b are constants. *The* **solution,** *or* **root,** *of the equation is* $x = -b/a$. We can also see that the solution is the same as the *zero* of the **linear function** $f(x) = ax + b$.

■ See the chapter introduction.

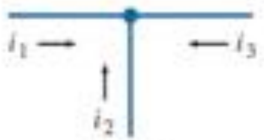


Fig. 5.1

EXAMPLE 3 Applications of linear equations

(a) A basic law of direct-current electricity, known as *Kirchhoff's current law*, may be stated as "The algebraic sum of the currents entering any junction in a circuit is zero." If three wires are joined at a junction as in Fig. 5.1, this law leads to the linear equation.

$$i_1 + i_2 + i_3 = 0$$

where i_1, i_2, and i_3 are the currents in each of the wires. (Either one or two of these currents must have a negative sign, showing that it is actually leaving the junction.)

(b) Two forces, F_1 and F_2, acting on a beam might be related by the equation

$$2F_1 + 4F_2 = 200$$

■

An equation that can be written in the form

$$\boxed{ax + by = c} \quad (5.2)$$

is a **linear equation in two unknowns.** For such equations, in Chapter 3, we found that for each value of x there is a corresponding value of y. Each of these pairs of numbers is a *solution* to the equation. *A* **solution** *is any set of numbers, one for each variable, that satisfies the equation.*

NOTE ➧

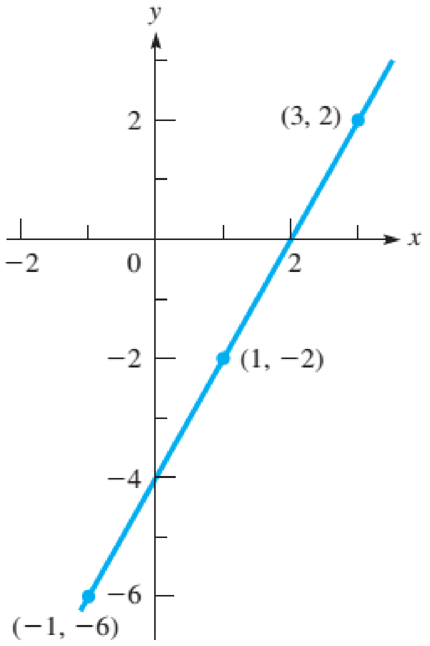


Fig. 2

EXAMPLE 4 Linear equation in two unknowns

The equation $2x - y - 4 = 0$ is a linear equation in two unknowns, x and y, because we can write it in the form of Eq. (2) as $2x - y = 4$. To graph this equation, we can write it as $y = 2x - 4$. From Fig. 2, we see that the graph is a straight line.

The coordinates of any point on the line give us a solution of this equation. For example, the point $(1, -2)$ is on the line. This means that $x = 1, y = -2$ is a solution of the equation. We can show that these values are a solution by substituting in the equation $2x - y - 4 = 0$. This gives us

$$2(1) - (-2) - 4 = 0, \qquad 2 + 2 - 4 = 0, \qquad 0 = 0$$

Because we have equality, $x = 1, y = -2$ is a solution. In the same way, we can show that $x = 3, y = 2$ is a solution.

If we are given the value of one variable, we find the value of the other variable, which gives a solution by substitution. For example, for the equation $2x - y - 4 = 0$, if $x = -1$, we have

$$\begin{aligned} 2(-1) - y - 4 &= 0 \\ -2 - y - 4 &= 0 \\ y &= -6 \end{aligned}$$

Therefore, $x = -1, y = -6$ is a solution, and the point $(-1, -6)$ is on the graph. ■

5.2 1,2,3,5,7

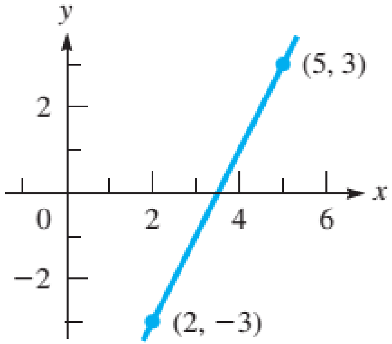


Fig. 6

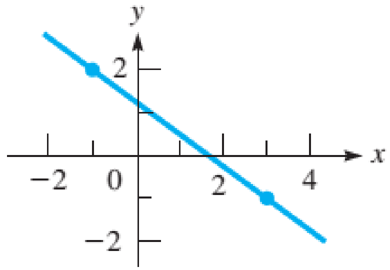


Fig. 7

EXAMPLE 1 Slope of line through two points

Find the slope of the line through the points $(2, -3)$ and $(5, 3)$.

In Fig. 6, we draw the line through the two given points. By taking $(5, 3)$ as (x_2, y_2), then (x_1, y_1) is $(2, -3)$. We may choose either point as (x_2, y_2), but *once the choice is made the order must be maintained.* Using Eq. (4), the slope is

$$m = \frac{3 - (-3)}{5 - 2} = \frac{6}{3} = 2$$

The rise is 2 units for each unit (of run) in going from left to right. ■

EXAMPLE 2 Slope of line through two points

Find the slope of the line through $(-1, 2)$ and $(3, -1)$.

In Fig. 7, we draw the line through these two points. By taking (x_2, y_2) as $(3, -1)$ and (x_1, y_1) as $(-1, 2)$, the slope is

$$m = \frac{-1 - 2}{3 - (-1)} = \frac{-3}{3 + 1} = -\frac{3}{4}$$

The line *falls* 3 units for each 4 units in going from left to right. ■

EXAMPLE 3 Slope and steepness of line

For each of the following lines shown in Fig. 8, we show the difference in the y-coordinates and in the x-coordinates between two points.

In Fig. 8(a), a line with a slope of 5 is shown. It rises sharply.

In Fig. 8(b), a line with a slope of $\frac{1}{2}$ is shown. It rises slowly.

In Fig. 8(c), a line with a slope of -5 is shown. It falls sharply.

In Fig. 8(d), a line with a slope of $-\frac{1}{2}$ is shown. It falls slowly.

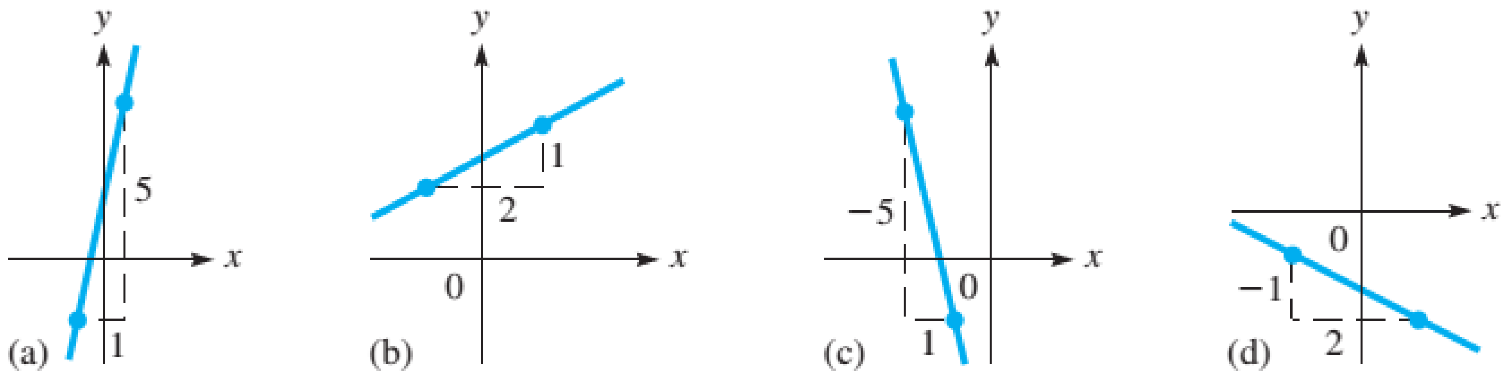


Fig. 8

EXAMPLE 5 Slope-intercept form of equation

Find the slope and the y-intercept of the line $2x + 3y = 4$.

We must first write the equation in slope-intercept form. Solving for y, we have

$$y = \underbrace{-\frac{2}{3}}_{\text{slope}}x + \underbrace{\frac{4}{3}}_{y\text{-intercept ordinate}}$$

Therefore, the slope is $-\frac{2}{3}$, and the y-intercept is the point $\left(0, \frac{4}{3}\right)$. See Fig. 11. ■

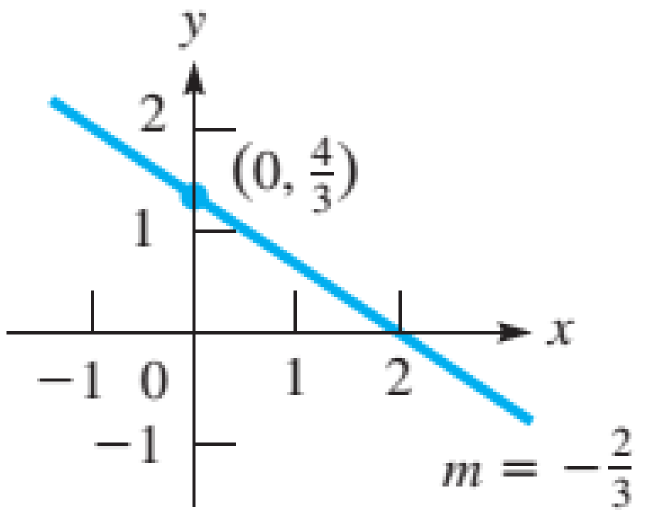


Fig. 11

EXAMPLE 7 **Sketching line by intercepts**

Sketch the graph of the line $2x - 3y = 6$ by finding its intercepts and one check point. See Fig. 13.

First, we let $x = 0$. This gives us $-3y = 6$, or $y = -2$. This gives us the y-intercept, which is the point $(0, -2)$. Next, we let $y = 0$, which gives us $2x = 6$, or $x = 3$. This means the x-intercept is the point $(3, 0)$.

The intercepts are enough to sketch the line as shown in Fig. 13. To find a check point, we can use any value for x other than 3 or any value of y other than -2. Choosing $x = 1$, we find that $y = -\frac{4}{3}$. This means that the point $\left(1, -\frac{4}{3}\right)$ should be on the line. In Fig. 13, we can see that it is on the line.

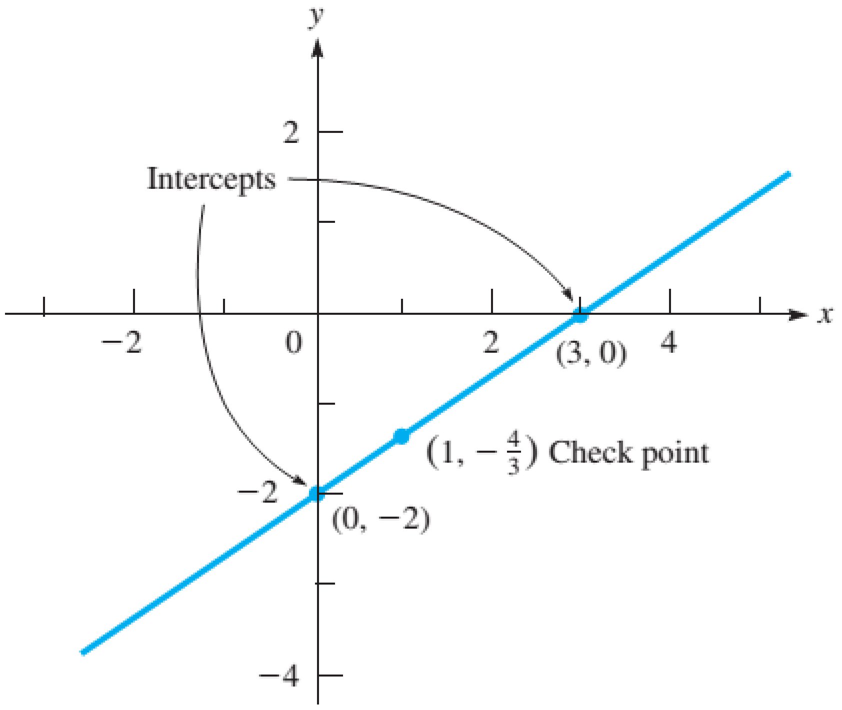


Fig. 13

5.3 1,5,6

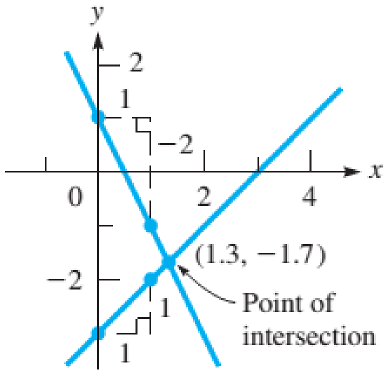


Fig. 17

EXAMPLE 1 Determine the point of intersection

Solve the system of equations

$$y = x - 3$$
$$y = -2x + 1$$

Because each of the equations is in slope-intercept form, note that $m = 1$ and $b = -3$ for the first line and that $m = -2$ and $b = 1$ for the second line. Using these values, we sketch the lines, as shown in Fig. 17.

From the figure, it can be seen that *the lines cross at about the point* $(1.3, -1.7)$. This means that the solution is approximately

$$x = 1.3 \qquad y = -1.7$$

$\left(\text{The exact solution is } x = \frac{4}{3}, y = -\frac{5}{3}.\right)$

NOTE In checking the solution, be careful to substitute the values in *both* equations. Making these substitutions gives us

$$-1.7 \stackrel{?}{=} 1.3 - 3 \quad \text{and} \quad -1.7 \stackrel{?}{=} -2(1.3) + 1$$
$$= -1.7 \qquad\qquad\qquad \approx -1.6$$

These values show that the solution checks. (The point $(1.3, -1.7)$ is *on* the first line and *almost on* the second line. The difference in values when checking the values for the second line is due to the fact that the solution is *approximate*.) ■

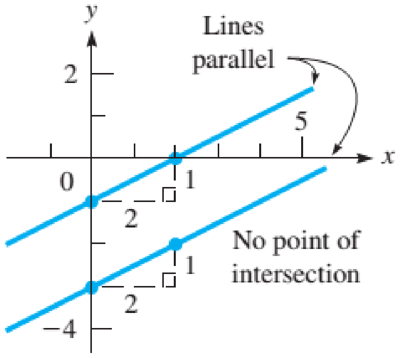


Fig. 21

Practice Exercise

2. Is the system in Example 5 inconsistent if the -6 in the second equation is changed to -12?

EXAMPLE 5 Inconsistent system

Solve the system of equations

$$x = 2y + 6$$
$$6y = 3x - 6$$

Writing each of these equations in slope-intercept form (Eq. 5), we have for the first equation

$$y = \tfrac{1}{2}x - 3$$

For the second equation, we have

$$y = \tfrac{1}{2}x - 1$$

NOTE From these, we see that each line has a slope of $\frac{1}{2}$ and that the y-intercepts are $(0, -3)$ and $(0, -1)$. Therefore, we know that the y-intercepts are different, but the *slopes are the same.* Because the slope indicates that each line rises $\frac{1}{2}$ unit for y for each unit x increases, *the lines are parallel and do not intersect,* as shown in Fig. 21. This means that ***there are no solutions*** for this system of equations. *Such a system is called* **inconsistent.** ■

EXAMPLE 6 Dependent system

Solve the system of equations

$$\begin{aligned} x - 3y &= 9 \\ -2x + 6y &= -18 \end{aligned}$$

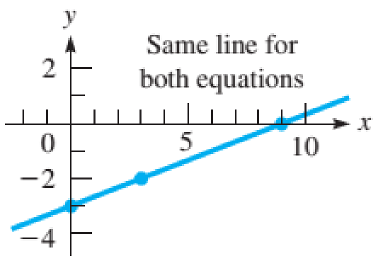


Fig. 22

We find that the intercepts and a third point for the first line are $(9, 0)$, $(0, -3)$, and $(3, -2)$. For the second line, we then find that the intercepts are the same as for the first line. We also find that the check point $(3, -2)$ also satisfies the equation of the second line. This means that the two lines are really the same line.

Another check is to write each equation in slope-intercept form. This gives us the equation $y = \frac{1}{3}x - 3$ for each line. See Fig. 22.

NOTE ▸ Because the lines are the same, *the coordinates of any point on this common line constitute a solution of the system*. Since ***no unique solution can be determined,*** *the system is called* **dependent.** ■

Later in the text, we will show algebraic ways of finding out whether a given system is consistent, inconsistent, or dependent.

5.4 1,3,6

EXAMPLE 1 Solution by substitution

Solve the following system of equations by substitution.

$$x - 3y = 6$$
$$2x + 3y = 3$$

Here, it is easiest to solve the first equation for x:

step 1

$$x = 3y + 6 \qquad \textbf{(A1)}$$

in second equation, x replaced by $3y + 6$

step 2

$$2(3y + 6) + 3y = 3 \qquad \text{substituting}$$

step 3

$$6y + 12 + 3y = 3 \qquad \text{solving for } y$$
$$9y = -9$$
$$y = -1$$

Now, put the value $y = -1$ into the first of the original equations. Because this equation is already solved for x in terms of y, Eq. (A1), we obtain

step 4

$$x = 3(-1) + 6 = 3 \qquad \text{solving for } x$$

step 5

Therefore, the solution of the system is $x = 3$, $y = -1$. As a check, substitute these values into each of the original equations. This gives us $3 - 3(-1) = 6$ and $2(3) + 3(-1) = 3$, which verifies the solution.

EXAMPLE 3 Solution by addition

Use the method of elimination by addition or subtraction to solve the system of equations.

$$x - 3y = 6 \qquad \text{already in the form of Eqs. (3)}$$
$$2x + 3y = 3$$

We look at the coefficients to determine the best way to eliminate one of the unknowns. Since the coefficients of the y-terms are *numerically the same and opposite in sign,* we may immediately *add* terms of the two equations together to eliminate y. Adding the terms of the left sides and adding terms of the right sides, we obtain

$$x + 2x - 3y + 3y = 6 + 3$$
$$3x = 9$$
$$x = 3$$

Substituting this value into the first equation, we obtain

$$3 - 3y = 6$$
$$-3y = 3$$
$$y = -1$$

The solution $x = 3, y = -1$ agrees with the results obtained for the same problem illustrated in Example 1.

EXAMPLE 6 Application—solution by subtraction

By weight, one alloy is 70% copper and 30% zinc. Another alloy is 40% copper and 60% zinc. How many grams of each are required to make 300 g of an alloy that is 60% copper and 40% zinc?

Let A = the required number of grams of the first alloy and B = the required number of grams of the second alloy. Our equations are determined from:

1. The total weight of the final alloy is 300 g: $A + B = 300$.
2. The final alloy will have 180 g of copper (60% of 300 g), and this comes from 70% of A ($0.70A$) and 40% of B ($0.40B$): $0.70A + 0.40B = 180$.

These two equations can now be solved simultaneously:

$$\begin{aligned} A + B &= 300 && \text{sum of weights is 300 g}\\ \text{copper} \longrightarrow 0.70A + 0.40B &= 180 \longleftarrow && \text{60\% of 300 g} \end{aligned}$$

70% weight of first alloy 40% weight of second alloy

$$\begin{aligned} 4A + 4B &= 1200 && \text{multiply each term of first equation by 4}\\ \underline{7A + 4B} &\underline{= 1800} && \text{multiply each term of second equation by 10}\\ 3A &= 600 && \text{subtract first equation from second equation}\\ A &= 200\text{ g} &&\\ B &= 100\text{ g} && \text{by substituting into first equation} \end{aligned}$$

Checking with the statement of the problem, using the percentages of zinc, we have $0.30(200) + 0.60(100) = 0.40(300)$, or $60\text{ g} + 60\text{ g} = 120\text{ g}$. We use zinc here since we used the percentages of copper for the equation. ■