Chapter 6

Factoring and Fractions

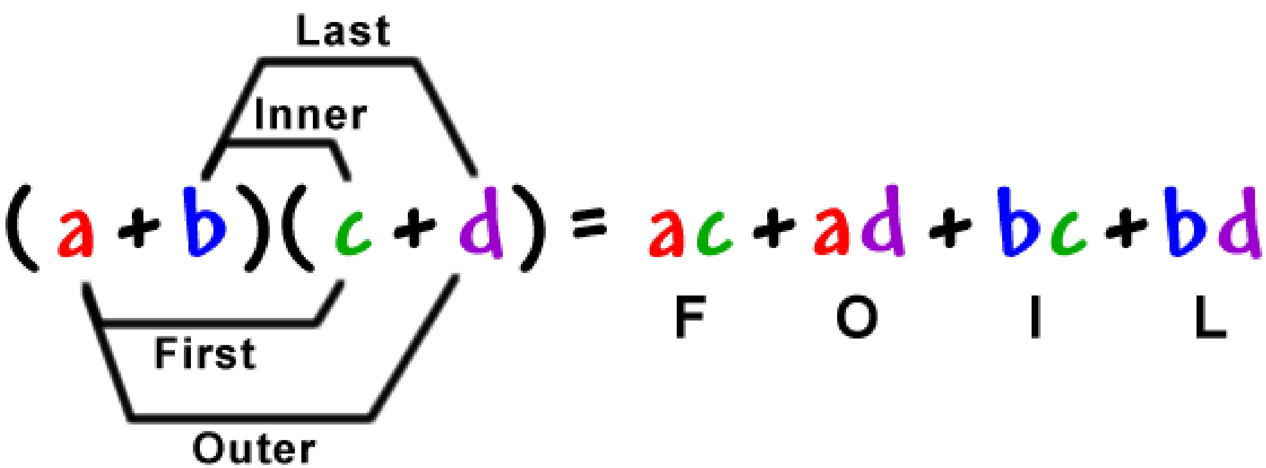
Factoring and Fractions	• Factoring: Common factors and difference between two squares • Operations on fractions	6.1 Special Products 6.2 Factoring: Common factors and difference between two squares 6.3 Factors trinomials
		6.4 Sums and Difference of cubes 6.5 Equivalent fractions 6.6. Multiplication and division of fractions 6.7 Addition and subtraction of fractions 6.8 Equations involving fractions

Ch. 6.1: Special Products

- We apply the distributive law when multiplying algebraic products.
- **We need to become familiar with the *special products* to ease our work with polynomials.**
- Work with literal numbers as you would with numbers.

General factoring method.

$$(a+b)(c+d)$$



We also have special products to help us do expansiosn

Remember these

$$a(x + y) = ax + ay \quad (1)$$

$$(x + y)(x - y) = x^2 - y^2 \quad (2)$$

$$(x + y)^2 = x^2 + 2xy + y^2 \quad (3)$$

$$(x - y)^2 = x^2 - 2xy + y^2 \quad (4)$$

$$(x + a)(x + b) = x^2 + (a + b)x + ab \quad (5)$$

$$(ax + b)(cx + d) = acx^2 + (ad + bc)x + bd \quad (6)$$

Example 1

- Multiply: $5x\,(3 - 8y)$
- We use: **1.** $a(x + y) = ax + ay$

$$5x\,(3 - 8y)$$

$$= 15x - 40xy$$

1. Multiply the outside term with the 1st term of the binomial.
2. Multiply the outside term with the 2nd term of the binomial.

- In this way, each term is multiplied together.

Example 2

- $(x + 4)^2$
- use: **3.** $(x + y)^2 = x^2 + 2xy + y^2$
- ***Solution:***

$$(x + 4)^2 = (x + 4)\,(x + 4)$$

$$= x^2 + (2\times 4)x + 4^2$$

$$= x^2 + 8x + 16$$

$$(x + y)^2 \neq x^2 + y^2$$

Example 3

- Multiply: $(x + 7)\ (x - 7)$
- We use: 5. $(x + y)(x - y) = x^2 - y^2$
- ***Solution:***

$$(x + 7)\ (x - 7) = x^2 - 7x + 7x - 7^2$$
$$= x^2 - 49$$

A Difference of Squares

Example 4

- Multiply: $(x - 8)(2x + 1)$
- We use:

  6. $(ax + b)(cx + d) = acx^2 + (ad + bc)x + bd$

- ***Solution:***

$(x - 8)(2x + 1)$

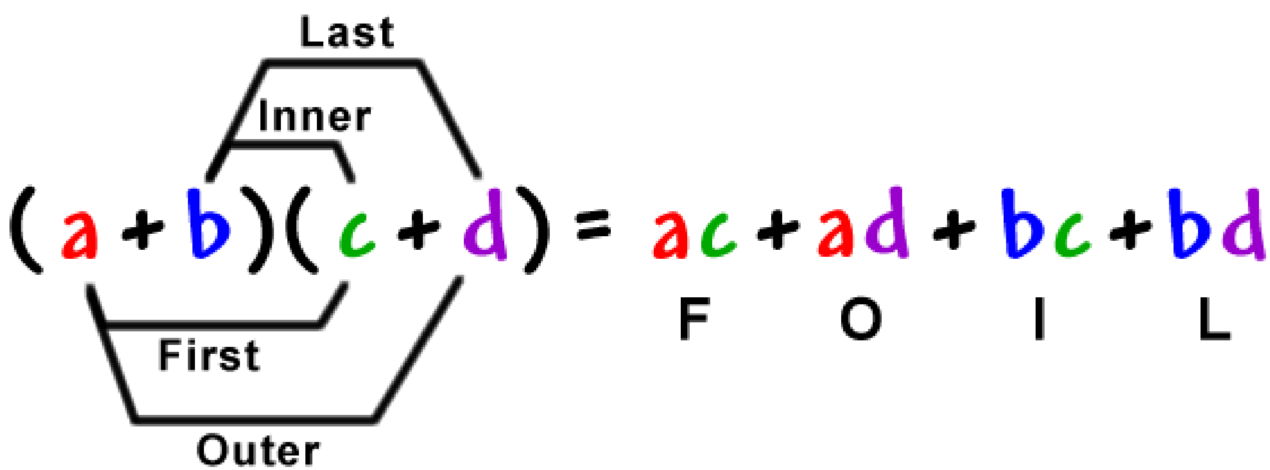
$= (1 \times 2)x^2 + (1 \times 1 - 8 \times 2)x + (-8 \times 1)$

$= 2x^2 - 15x - 8$

Each term is multiplied by each other term.

Could we just use this?

$$(a+b)(c+d)$$



1 $$(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$$

2 $$(x-y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$$

3 $$(x+y)(x^2 - xy + y^2) = x^3 + y^3$$

4 $$(x-y)(x^2 + xy + y^2) = x^3 - y^3$$

Example 5

3 $(x+y)(x^2-xy+y^2)=x^3+y^3$

- Multiply: $(x + 5)(x^2 - 5x + 25)$
- ***Solution:***
- We use **#3 recognizing** the **5** and its square **25**.

$$(x + 5)(x^2 - 5x + 25) = x^3 + 125$$

Summary

- **Recognizing** the forms that polynomials take will assist in finding their products.
- Being able to identify the products with a **difference of squares and perfect squares** and cubes will help in simplifying algebraic products.

$$a(x + y) = ax + ay \quad (6.1)$$

$$(x + y)(x - y) = x^2 - y^2 \quad (6.2)$$

$$(x + y)^2 = x^2 + 2xy + y^2 \quad (6.3)$$

$$(x - y)^2 = x^2 - 2xy + y^2 \quad (6.4)$$

$$(x + a)(x + b) = x^2 + (a + b)x + ab \quad (6.5)$$

$$(ax + b)(cx + d) = acx^2 + (ad + bc)x + bd \quad (6.6)$$

Examples

5. $40(x - y)$

6. $2x(a - 3)$

7. $2x^2(x - 4)$

8. $3a^2(2a + 7)$

9. $(T + 6)(T - 6)$

10. $(s + 2t)(s - 2t)$

11. $(3v - 2)(3v + 2)$

12. $(ab - c)(ab + c)$

17. $(5f + 4)^2$

18. $(i_1 + 3)^2$

19. $(2x + 17)^2$

20. $(9a + 8b)^2$

21. $(L^2 - 1)^2$

22. $(b^2 - 6)^2$

23. $(4a + 7xy)^2$

24. $(3A + 10z)^2$

A

$= x \quad - 12x \quad + 48x - 64$

5. $40(x - y) = 40x - 40y$

6. $2x(a - 3) = 2ax - 6x$

7. $2x^2(x - 4) = 2x^3 - 8x^2$

8. $3a^2(a + 7) = 6a^3 + 21a^2$

9. $(T + 6)(T - 6) = T^2 - 6^2 = T^2 - 36$

10. $(s + 2t)(s - 2t) = s^2 - 2st + 2st - 4t^2$
$= s^2 - 4t^2$

11. $(3v - 2)(3v + 2) = 9v^2 - 6v + 6v - 4$
$= 9v^2 - 4$

18. $(i_i + 3)^2 = (i_1)^2 + 2(i_1)(3) + 3^2$
$= i_1^2 + 6i_1 + 9$

19. $(2x + 17)^2 = (2x)^2 + 2(2x)(17) + 17^2$
$= 4x^2 + 68x + 289$

20. $(9a + 8b)^2 = (9a)^2 + 2(9a)(8b) + (8b)^2$
$= 81a^2 + 144ab + 64b^2$

21. $(L^2 - 1)^2 = (L^2)^2 - 2 \cdot L^2 \cdot 1 + 1^2$
$= L^4 - 2L^2 + 1$

22. $(b^2 - 6)^2 = b^4 + 2(-6b^2) + 36 = b^4 - 12b^2 + 36$

23. $(4a + 7xy)^2 = 16a^2 + 56axy + 49x^2y^2$

Ch. 6.2: Factoring: Common Factor and Difference of Squares

- To ***factor*** an expression, we decompose that expression into its smallest factors.
- It involves reversing the process of finding a product.
- A polynomial or a factor is called ***prime*** if it contains no factors other than **+1** or **-1** and plus or minus itself.

Factoring

$$a(x + y) = ax + ay$$
$$(x + y)(x - y) = x^2 - y^2$$
$$(x + y)^2 = x^2 + 2xy + y^2$$
$$(x - y)^2 = x^2 - 2xy + y^2$$
$$(x + a)(x + b) = x^2 + (a + b)x + ab$$
$$(ax + b)(cx + d) = acx^2 + (ad + bc)x + bd$$

- The ability to factor algebraic expressions depends heavily on the proper **recognition** of the special products.
- ***Example***:

$$14x^2 + 21x$$

$$7x \quad (2x + 3)$$

Common Monomial Factors

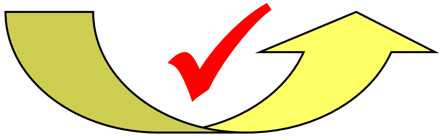
- Given: $14x^2 + 21x = 7x(2x + 3)$

$7x$ → common monomial factor

Common Monomial Factors

- We **check** the result by multiplication:

$$7x(2x + 3) = 14x^2 + 21x$$



Factoring with '**1**'

- Factor: $\mathbf{15x + 45x^2}$

  $\mathbf{= 15x(1 + 3x)}$

- When factoring out $\mathbf{15x}$, $\mathbf{1}$ is left behind to remind us that $\mathbf{15x}$ exists in the original algebraic expression.
- Multiplying back through provides the check that we need.

Factoring the **Difference of Two Squares** (this is the most useful?)

- The factors only differ by the middle sign.
- $(x + y)(x - y) = x^2 - y^2$

- So, when factoring, we know that we can decompose $x^2 - y^2$ into $(x + y)(x - y)$

Also note:

$$(ax + by)(ax - by) = (ax)^2 - (by)^2$$

$$= a^2x^2 - b^2y^2$$

Example

$$a(x + y) = ax + ay$$
$$(x + y)(x - y) = x^2 - y^2$$
$$(x + y)^2 = x^2 + 2xy + y^2$$
$$(x - y)^2 = x^2 - 2xy + y^2$$
$$(x + a)(x + b) = x^2 + (a + b)x + ab$$
$$(ax + b)(cx + d) = acx^2 + (ad + bc)x + bd$$

- Factor: $4x^2 - 9$

$= (2x)^2 - 3^2$

$= (2x + 3)(2x - 3)$ ✓

a difference of squares

Complete Factoring

$$a(x + y) = ax + ay$$
$$(x + y)(x - y) = x^2 - y^2$$
$$(x + y)^2 = x^2 + 2xy + y^2$$
$$(x - y)^2 = x^2 - 2xy + y^2$$
$$(x + a)(x + b) = x^2 + (a + b)x + ab$$
$$(ax + b)(cx + d) = acx^2 + (ad + bc)x + bd$$

- Factoring an algebraic expression **may require more than one step.**
- ***Example***:

$$6x^3 - 24x$$
$$= 6x(x^2 - 4)$$
$$= 6x\,(x + 2)(x - 2)$$

https://www.khanacademy.org/math/algebra/multiplying-factoring-expression/factoring-special-products/v/factoring-difference-of-squares

https://www.khanacademy.org/math/algebra/multiplying-factoring-expression/factoring-special-products/v/factoring-to-produce-difference-of-squares

Factoring by Grouping

- In some polynomials, terms can be grouped together to help factor the algebraic expression.
- We look for a common binomial factor in these situations.

$$a(x+y) = ax + ay$$
$$(x+y)(x-y) = x^2 - y^2$$
$$(x+y)^2 = x^2 + 2xy + y^2$$
$$(x-y)^2 = x^2 - 2xy + y^2$$
$$(x+a)(x+b) = x^2 + (a+b)x + ab$$
$$(ax+b)(cx+d) = acx^2 + (ad+bc)x + bd$$

Factor the following

Examples

Look for the common multiplication factor

12. $5a^2 - 20ax$

14. $90p^3 - 15p^2$

16. $23a - 46b + 69c$

18. $4pq - 14q^2 - 16pq^2$

20. $27a^2b - 24ab - 9a$

22. $5a + 10ax - 5ay - 20az$

12. $5a^2 - 20ax = 5a(a - 4x)$ ($5a$ is a c.m.f.)

A

12. $5a^2 - 20ax = 5a(a-4x)$ ($5a$ is a c.m.f.)

13. $288n^2 + 24n = 24n(12n+1)$ ($24n$ is a c.m.f.)

14. $90p^3 - 15p^2 = 15p^2(6p-1)$ ($15p^2$ is a c.m.f.)

15. $2x + 4y - 8z = 2(x+2y-4z)$ (2 is a c.m.f.)

16. $23a - 46b + 69c = 23(a-2b+3c)$

17. $3ab^2 - 6ab + 12ab^3 = 3ab(b-2+4b^2)$
($3ab$ is a c.m.f.)

18. $4pq - 14q^2 - 16pq^2 = 2q(2p-7q-8pq)$
($2q$ is a c.m.f.)

19. $12pq^2 - 8pq - 28pq^3 = 4pq(3q-2-7q^2)$
($4pq$ is a c.m.f.)

20. $27a^2b - 24ab - 9a = 3a(9ab-8b-3)$
($3a$ is a c.m.f.)

21. $2a^2 - 2b^2 + 4c^2 - 6d^2 = 2(a^2 - b^2 + 2c^2 - 3d^2)$
(2 is a c.m.f.)

22. $5a + 10ax - 5ay + 20az = 5a(1+2x-y+4z)$
($5a$ is a c.m.f.)

23. $x^2 - 4 = x^2 - 2^2 = (x-2)(x+2)$
(because $-2x + 2x = 0x = 0$)

24. $r^2 - 25 = r^2 - 5^2 = (r-5)(r+5)$
(because $-5r + 5r = 0r = 0$)

Ch. 6.3: Factoring Trinomials

- Recall:

$$(x+a)(x+b) = x^2 + \underbrace{(a+b)}_{\text{sum}}x + \underbrace{ab}_{\text{product}}$$

$$(x+a)(x+b) = x^2 + \underbrace{(a+b)}_{\text{sum}}x + \underbrace{ab}_{\text{product}}$$

Observations:

- We are to find integers ***a*** and ***b***, and they are found by noting that:

1. The ***coefficient of x***^{2} is **1**.
2. The ***final constant*** is the ***product*** of the constants ***a*** and ***b*** in the factors, and,
3. The ***coefficient of x*** is the ***sum*** of ***a*** and ***b***.

Example

$$(x+a)(x+b)=x^2+(a+b)x+ab$$

sum product

$$x^2 - x - 6$$

sum product

This is the hard bit

Sum is -1 Product is -6

What 2 numbers satisfy this.

$$= x^2 + (-3 + 2)\,x + (-3 \times 2)$$

$$= (x - 3)\,(x + 2)$$

Don't forget to check

Factoring General Trinomials

- Recall:

$$acx^2 + (ad + bc)x + bd = (ax + b)(cx + d)$$

- ***Observations*:**

1. The coefficient of x^2 is the product of the coefficients a and c in the factors,
2. The final constant is the product of the constants b and d in the factors, and,
3. The coefficient of x is the sum of the inner and outer products.

Summary

- Be sure to factor an expression completely.
- We first look for the common monomial factors and then check each resulting factor to see if it can be factored when we complete each step.

16. $b^2 - 12bc + 36c^2$

18. $2n^2 - 13n - 7$

20. $25x^2 + 45x - 10$

22. $7y^2 - 12y + 5$

24. $5R^4 - 3R^2 - 2$

26. $3n^2 - 20n + 20$

28. $3x^2 + xy - 14y^2$

30. $2z^2 + 13z - 5$

$$(x+a)(x+b) = x^2 + \underbrace{(a+b)}_{\text{sum}}x + \underbrace{ab}_{\text{product}}$$

$$acx^2 + (ad + bc)x + bd = (ax + b)(cx + d)$$

16. $b^2 - 12bx + 36c^2 = (b-6c)(b-6c) = (b-6c)^2$

(because $-12bc = -6bc - 6bc$)

17. $3x^2 - 5x - 2 = (3x+1)(x-2)$

(because $-5x = -6x + x$)

18. $2n^2 - 13n - 7 = (2n+1)(n-7)$

(because $-13n = -14n + n$)

19. $12y^2 - 32y - 12 = 4(3y+1)(y-3), (-8y = y - 9y)$

20. $25x^2 + 45x - 10 = 5(5x-1)(x+2)$

(because $9x = 10x - x$)

21. $2s^2 + 13s + 11 = (2s+11)(s+1)$

(because $13s = 2s + 11s$)

22. $7y^2 - 12y + 5 = (7y-5)(y-1)$

(because $-12y = -7y - 5y$)

23. $3f^4 - 16f^2 + 5 = (3f^2 - 1)(f^2 - 5)$

A

24. $5R^4 - 3R^2 - 2 = (5R^2+2)(R^2-1)$

$= (5R^2+2)(R+1)(R-1)$

25. $2t^2 + 7t - 15 = (2t-3)(t+5)$

(because $7t = 10t - 3t$)

26. $3n^2 - 20n + 20 =$ prime (cannot be further factored)

27. $3t^2 - 7tu + 4u^2 = (3t-4u)(t-u)$

(because $-7tu = -4tu - 3tu$)

28. $3x^2 + xy - 14y^2 = (3x+7y)(x-2y)$

(because $xy = -6xy + 7xy$)

29. $4x^2 - 3x - 7 = (4x-7)(x+1)$

(because $-3x = 4x - 7x$)

30. $2z^2 + 13z - 5 =$ prime (cannot be further factored)

'SC

1 $a(x+y) = ax + ay$

2 $(x+y)(x-y) = x^2 - y^2$

3 $(x+y)^2 = x^2 + 2xy + y^2$

4 $(x-y)^2 = x^2 - 2xy + y^2$

5 $(x+a)(x+b) = x^2 + (a+b)x + ab$

6 $(ax+b)(cx+d) = acx^2 + (ad+bc)x + bd$

7 $(x+a)(x+b) = x^2 + \underbrace{(a+b)}_{\text{sum}}x + \underbrace{ab}_{\text{product}}$

8 $acx^2 + (ad+bc)x + bd = (ax+b)(cx+d)$

6.4 The Sum and Difference of Cubes

$$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$$
$$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$$

Table of Cubes

$1^3 = 1$
$2^3 = 8$
$3^3 = 27$
$4^3 = 64$
$5^3 = 125$
$6^3 = 216$

$$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$$
$$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$$

EXAMPLE 1 Factoring sum and difference of cubes

(a) $x^3 + 8 = x^3 + 2^3$

$$= (x + 2)[(x)^2 - 2x + 2^2]$$
$$= (x + 2)(x^2 - 2x + 4)$$

(b) $x^3 - 1 = x^3 - 1^3$

$$= (x - 1)[(x)^2 + (1)(x) + 1^2]$$
$$= (x - 1)(x^2 + x + 1)$$

(c) $8 - 27x^3 = 2^3 - (3x)^3$ $\quad$ $8 = 2^3$ and $27x^3 = (3x)^3$

$$= (2 - 3x)[2^2 + 2(3x) + (3x)^2]$$
$$= (2 - 3x)(4 + 6x + 9x^2)$$

$$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$$
$$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$$

EXAMPLE 2 First, factor out the common factor

In factoring $ax^5 - ax^2$, we first note that each term has a common factor of ax^2. This is factored out to get $ax^2(x^3 - 1)$. However, the expression is not completely factored because $1 = 1^3$, which means that $x^3 - 1$ is the difference of cubes (see Example 1(b)). We complete the factoring by the use of Eq. (10). Therefore,

$$\begin{aligned} ax^5 - ax^2 &= ax^2(x^3 - 1) \\ &= ax^2(x - 1)(x^2 + x + 1) \end{aligned}$$

■

Examples

SUMMARY OF METHODS OF FACTORING

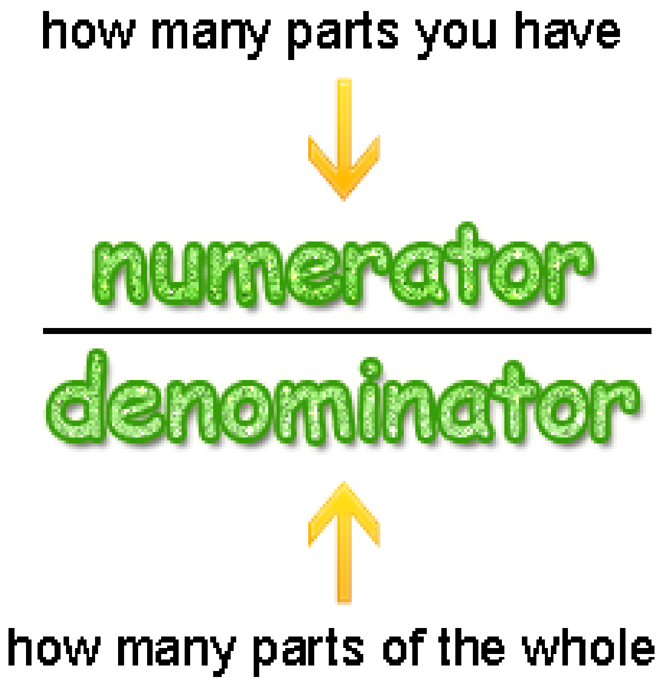
1. *Common monomial factor* **Always check for this first.**
2. *Difference of squares*
3. *Factorable trinomial*
4. *Sum or difference of cubes*
5. *Factorable by grouping*

CAUTION *Be sure the expression is factored* ***completely.***

Ch 6.5 Equivalent Fractions

3 ← numerator

5 ← denominator



Ch 6.5 Equivalent Fractions

$$\frac{6}{8} = \frac{12}{16} = \frac{3}{4}$$

When we multiply or divide the numerator and denominator of a fraction by the same number we obtain what is called an **EQUIVALENT FRACTION**

Simplest Form

Here we have an **EQUIVALENT FRACTION** since the left hand side of the equation has been multiplied by 3a to obtain the right hand side

$$\frac{ax}{2} = \frac{3a^2x}{6a}$$

Example…

In order to reduce the fraction

$$\frac{16ab^3c^2}{24ab^2c^5}$$

to its lowest terms, note that both the numerator and the denominator contain the factor $8ab^2c^2$. Therefore,

$$\frac{16ab^3c^2}{24ab^2c^5} = \frac{2b(8ab^2c^2)}{3c^3(8ab^2c^2)} = \frac{2b}{3c^3}$$

— common factor

Practice Exercise

1. Reduce to lowest terms: $\frac{9xy^5}{15x^3y^2}$

Examples to try…

In Exercises 29–64, reduce each fraction to simplest form.

29. $\dfrac{2a}{8a}$

30. $\dfrac{6x}{15x}$

31. $\dfrac{18x^2y}{24xy}$

32. $\dfrac{2a^2xy}{6axyz^2}$

33. $\dfrac{a + b}{5a^2 + 5ab}$

34. $\dfrac{t - a}{t^2 - a^2}$

35. $\dfrac{4a - 4b}{4a - 2b}$

36. $\dfrac{20s - 5r}{10r - 5s}$

37. $\dfrac{4x^2 + 1}{4x^2 - 1}$

38. $\dfrac{x^2 - y^2}{x^2 + y^2}$

39. $\dfrac{3x^2 - 6x}{x - 2}$

40. $\dfrac{10T^2 + 15T}{2T + 3}$

29. $\frac{2a}{8a}=\frac{2a}{2a\cdot 4}=\frac{1}{4}$

30. $\frac{6x}{15x}=\frac{3x(2)\div 3x}{3x(5)\div 3x}=\frac{2}{5}$

31. $\frac{18x^2y}{24xy}=\frac{6xy(3x)\div(6xy)}{6xy(4)\div(6xy)}=\frac{3x}{4}$

32. $\frac{2a^2xy}{6axyz^2}=\frac{2axy(a)\div(2axy)}{2axy(3z^2)\div(2axy)}=\frac{a}{3z^2}$

33. $\frac{a+b}{5a^2+5ab}=\frac{(a+b)}{5a(a+b)}=\frac{1}{5a}$

34. $\frac{t-a}{t^2-a^2}=\frac{(t-a)\div(t-a)}{(t-a)(t+a)\div(t-a)}=\frac{1}{t+a}$

35. $$\frac{4a-4b}{4a-2b}=\frac{4(a-b)\div 2}{2(2a-b)\div 2}=\frac{2(a-b)}{2a-b}$$

36. $$\frac{20s-5r}{10r-5s}=\frac{-5(r-4s)\div 5}{5(2r-s)\div 5}=\frac{-r+4s}{2r-s}$$

37. $$\frac{4x^2+1}{4x^2-1}=\frac{4x^2+1}{(2x-1)(2x+1)}$$

Since no cancellations can be made the fraction cannot be reduced.

38. $$\frac{x^2-y^2}{x^2+y^2}=\frac{(x-y)(x+y)}{(x^2+y^2)}$$

The x's and y's do not cancel out and the fraction cannot be reduced.

39. $$\frac{3x^2-6x}{x-2}=\frac{3x(x-2)\div(x-2)}{(x-2)\div(x-2)}=3x$$

40. $$\frac{10T^2+15T}{2T+3}=\frac{(2T+3)5T\div(2T+3)}{(2T+3)\div(2T+3)}=5T$$

More Examples

41. $\dfrac{3 + 2y}{4y^3 + 6y^2}$

42. $\dfrac{6 - 3t}{4t^3 - 8t^2}$

43. $\dfrac{x^2 - 8x + 16}{x^2 - 16}$

44. $\dfrac{4a^2 + 12ab + 9b^2}{4a^2 + 6ab}$

45. $\dfrac{2w^4 + 5w^2 - 3}{w^4 + 11w^2 + 24}$

46. $\dfrac{3y^3 + 7y^2 + 4y}{y^2 + 5y + 4}$

47. $\dfrac{5x^2 - 6x - 8}{x^3 + x^2 - 6x}$

48. $\dfrac{5s^2 + 8rs - 4s^2}{6r^2 - 17rs + 5s^2}$

49. $\dfrac{N^4 - 16}{8N - 16}$

50. $\dfrac{3 + x(4 + x)}{3 + x}$

51. $\dfrac{t + 4}{(2t + 9)t + 4}$

52. $\dfrac{2A^3 + 8A^4 + 8A^5}{4A + 2}$

41. $$\frac{3+2y}{4y^3+6y^2}=\frac{(2y+3)}{2y^2(2y+3)}=\frac{1}{2y^2}$$

42. $$\frac{6-3t}{4t^3-8t^2}=\frac{-3(t-2)\div(t-2)}{4t^2(t-2)\div(t-2)}=\frac{-3}{4t^2}$$

43. $$\frac{x^2-8x-16}{x^2-16}=\frac{(x-4)(x-4)\div(x-4)}{(x+4)(x-4)\div(x-4)}=\frac{x-4}{x+4}$$

44. $$\frac{4a^2+12ab+9b^2}{4a^2+6ab}=\frac{(2a+3b)(2a+3b)\div(2a+3b)}{2a(2a+3b)\div(2a+3b)}$$

$$=\frac{2a+3b}{2a}$$

45. $$\frac{2w^4 5w^2-3}{w^4+11w^2+24}=\frac{(2w^2-1)(w^2+3)}{(w^2+8)(w^2+3)}=\frac{2w^2-1}{w^2+8}$$

Ch 6.6 Multiplication and Division of Fractions

$$\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$$

Multiplying fractions…

Dividing fractions…

$$\frac{a}{b} \div \frac{c}{d} = \frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$$

Examples… Multiplying fractions

(a) $\frac{3}{5} \times \frac{2}{7} = \frac{(3)(2)}{(5)(7)} = \frac{6}{35}$ ← multiply numerators; ← multiply denominators

(b) $\frac{3a}{5b} \times \frac{15b^2}{a} = \frac{(3a)(15b^2)}{(5b)(a)}$

$$= \frac{45ab^2}{5ab} = \frac{9b}{1}$$

$$= 9b$$

(c) $(6x)\left(\frac{2y}{3x^2}\right) = \left(\frac{6x}{1}\right)\left(\frac{2y}{3x^2}\right)$

$$= \frac{12xy}{3x^2} = \frac{4y}{x}$$

Examples… Dividing fractions

(a) $\frac{6x}{7} \div \frac{5}{3} = \frac{6x}{7} \times \frac{3}{5} = \frac{18x}{35}$

multiply

invert

(b) $\frac{\frac{3a^2}{5c}}{\frac{2c^2}{a}} = \frac{3a^2}{5c} \times \frac{a}{2c^2} = \frac{3a^3}{10c^3}$

multiply

invert

$$(x + y) \div \frac{2x + 2y}{6x + 15y} = \frac{x + y}{1} \times \frac{6x + 15y}{2x + 2y} = \frac{\cancel{(x + y)}(3)(2x + 5y)}{2\cancel{(x + y)}}$$

invert

$$= \frac{3(2x + 5y)}{2}$$

simplify

$$\frac{3(x-y)}{(x-y)^2} \times \frac{(x^2-y^2)}{6x+9y} = \frac{3(x-y)(x^2-y^2)}{(x-y)^2(6x+9y)}$$

$$= \frac{3(x-y)(x+y)(x-y)}{(x-y)^2(3)(2x+3y)}$$

$$= \frac{\cancel{3(x-y)^2}(x+y)}{\cancel{3(x-y)^2}(2x+3y)}$$

$$= \frac{x+y}{2x+3y}$$

The common factor $3(x-y)^2$ is readily recognized using this procedure.

Examples

Ch 6.7 Addition and Subtraction of Fractions

(a) $\frac{5}{9} + \frac{2}{9} - \frac{4}{9} = \frac{5 + 2 - 4}{9}$ ← sum of numerators ← same denominators

$= \frac{3}{9} = \frac{1}{3}$ ← final result in lowest terms

CAUTION

use parentheses to show subtraction of both terms

(b) $\frac{b}{ax} + \frac{1}{ax} - \frac{2b - 1}{ax} = \frac{b + 1 - (2b - 1)}{ax} = \frac{b + 1 - 2b + 1}{ax}$

$= \frac{2 - b}{ax}$

Lowest Common Denominator (LCD)

https://www.khanacademy.org/math/cc-fourth-grade-math/comparing-fractions-and-equivalent-fractions/imp-common-denominators/v/finding-common-denominators

Highest of each factor – eg 3, r^2, s^3

$$\frac{2}{3r^2} + \frac{4}{rs^3} - \frac{5}{3s}$$

Lowest Common Denominator (LCD)

$$\frac{2}{3r^2} + \frac{4}{rs^3} - \frac{5}{3s} = \frac{2(s^3)}{(3r^2)(s^3)} + \frac{4(3r)}{(rs^3)(3r)} - \frac{5(r^2s^2)}{(3s)(r^2s^2)}$$

change to equivalent fractions with LCD

factors needed in each

$$= \frac{2s^3}{3r^2s^3} + \frac{12r}{3r^2s^3} - \frac{5r^2s^2}{3r^2s^3}$$

$$= \frac{2s^3 + 12r - 5r^2s^2}{3r^2s^3}$$

combine numerators over LCD

LCD… adding fractions

$$\frac{a}{x-1} + \frac{a}{x+1} = \frac{a(x+1)}{(x-1)(x+1)} + \frac{a(x-1)}{(x+1)(x-1)}$$

change to equivalent fractions with LCD

factors needed

$$= \frac{ax + a + ax - a}{(x+1)(x-1)}$$

combine numerators over LCD

$$= \frac{2ax}{(x+1)(x-1)}$$

simplify

examples

In Exercises 5–44, perform the indicated operations and simplify. For Exercises 33, 34, 39, and 40, check the solution with a graphing calculator.

5. $\frac{3}{5} + \frac{6}{5}$

6. $\frac{2}{13} + \frac{6}{13}$

7. $\frac{1}{x} + \frac{7}{x}$

8. $\frac{2}{a} + \frac{3}{a}$

9. $\frac{1}{2} + \frac{3}{4}$

10. $\frac{5}{9} - \frac{1}{3}$

11. $\frac{3}{4x} + \frac{7a}{4} + 2$

12. $\frac{t-3}{a} - \frac{t}{2a}$

13. $\frac{a}{x} - \frac{b}{x^2}$

14. $\frac{3}{2s^2} + \frac{5}{4s}$

15. $\frac{6}{5x^3} + \frac{a}{25x}$

16. $\frac{a}{6y} - \frac{2b}{3y^4}$

17. $\frac{2}{5a} + \frac{1}{a} - \frac{a}{10}$

18. $\frac{1}{2A} - \frac{6}{B} - \frac{9}{4C}$

EXAMPLE 8 Complex fraction

$$\frac{\frac{2}{x}}{1-\frac{4}{x}} = \frac{\frac{2}{x}}{\frac{x-4}{x}}$$ first, perform subtraction in denominator

$$= \frac{2}{x} \times \frac{x}{x-4} = \frac{2\cancel{x}}{\cancel{x}(x-4)}$$ invert divisor and multiply

$$= \frac{2}{x-4}$$ simplify ■

■ The original complex fraction can be written as a division as follows:

$$\frac{2}{x} \div \left(1 - \frac{4}{x}\right)$$

Examples

Ch 6.8 Equations involving Fractions

To give a whole number

Solve for x: $\frac{x}{12} - \frac{1}{8} = \frac{x+2}{6}$.

First, note that the LCD of the terms of the equation is 24. Therefore, multiply each term by 24. This gives

$$\frac{24(x)}{12} - \frac{24(1)}{8} = \frac{24(x+2)}{6} \qquad \text{each term multiplied by LCD}$$

Reduce each term to its lowest terms and solve the resulting equation:

$$\begin{aligned} 2x - 3 &= 4(x+2) \qquad \text{each term reduced} \\ 2x - 3 &= 4x + 8 \\ -2x &= 11 \\ x &= -\frac{11}{2} \end{aligned}$$

Continued…

Solve for x: $\frac{x}{2} - \frac{1}{b^2} = \frac{x}{2b}$.

First, determine that the LCD of the terms of the equation is $2b^2$. Then multiply each term by $2b^2$ and continue with the solution:

$$\frac{2b^2(x)}{2} - \frac{2b^2(1)}{b^2} = \frac{2b^2(x)}{2b} \quad \text{each term multiplied by LCD}$$

$$b^2x - 2 = bx \quad \text{each term reduced}$$

$$b^2x - bx = 2$$

$$x(b^2 - b) = 2 \quad \text{factor}$$

$$x = \frac{2}{b^2 - b}$$

Examples

EQUATIONS

$$a(x + y) = ax + ay$$

$$(x + y)(x - y) = x^2 - y^2$$

$$(x + y)^2 = x^2 + 2xy + y^2$$

$$(x - y)^2 = x^2 - 2xy + y^2$$

$$(x + a)(x + b) = x^2 + (a + b)x + ab$$

$$(ax + b)(cx + d) = acx^2 + (ad + bc)x + bd$$

$$(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$$

$$(x - y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$$

$$(x + y)(x^2 - xy + y^2) = x^3 + y^3$$

$$(x - y)(x^2 + xy + y^2) = x^3 - y^3$$

$$x - y = -(y - x)$$

Worked examples

6.1 1,2,3,5,8

$$a(x+y) = ax + ay \quad (1)$$
$$(x+y)(x-y) = x^2 - y^2 \quad (2)$$
$$(x+y)^2 = x^2 + 2xy + y^2 \quad (3)$$
$$(x-y)^2 = x^2 - 2xy + y^2 \quad (4)$$
$$(x+a)(x+b) = x^2 + (a+b)x + ab \quad (5)$$
$$(ax+b)(cx+d) = acx^2 + (ad+bc)x + bd \quad (6)$$

Some worked examples to help you

EXAMPLE 1 Using special products 1 and 2

(a) Using Eq. (1) in the following product, we have

$$6(3r+2s) = 6(3r) + 6(2s) = 18r + 12s$$

(b) Using Eq. (2), we have

$$(3r+2s)(3r-2s) = (3r)^2 - (2s)^2 = 9r^2 - 4s^2$$

sum of $3r$ and $2s$

difference of $3r$ and $2s$

difference of squares

Using Eq. (1) in (a), we have $a = 6$. In (a) and (b), $3r = x$ and $2s = y$.

$$(x + y)^2 = x^2 + 2xy + y^2 \quad (3)$$

$$(x - y)^2 = x^2 - 2xy + y^2 \quad (4)$$

EXAMPLE 2 Using special products 3 and 4

Using Eqs. (3) and (4) in the following products, we have

(a) $(5a + 2)^2 = (5a)^2 + 2(5a)(2) + 2^2 = 25a^2 + 20a + 4$ Eq. (3)

square, twice product, square

(b) $(5a - 2)^2 = (5a)^2 - 2(5a)(2) + 2^2 = 25a^2 - 20a + 4$ Eq. (4)

In these illustrations, we let $x = 5a$ and $y = 2$. *It should be emphasized that*

$(5a + 2)^2$ is **not** $(5a)^2 + 2^2$, or $25a^2 + 4$

We must very carefully follow the forms of Eqs. (3) and (4) and be certain to ***include the middle term,*** 20a. (See Example 5)

$$(x + a)(x + b) = x^2 + (a + b)x + ab \quad (5)$$

$$(ax + b)(cx + d) = acx^2 + (ad + bc)x + bd \quad (6)$$

EXAMPLE 3 Using special products 5 and 6

Using Eqs. (5) and (6) in the following products, we have

(a) $(x + 5)(x - 3) = x^2 + [5 + (-3)]x + (5)(-3) = x^2 + 2x - 15$ Eq. (5)

(b) $(4x + 5)(2x - 3) = (4x)(2x) + [(4)(-3) + (5)(2)]x + (5)(-3)$ Eq. (6)

$$= 8x^2 - 2x - 15$$

■

Generally, when we use these special products, we find the middle term mentally and write down the result directly, as shown in the next example.

EXAMPLE 5 Applications of special products

(a) When analyzing the forces on a certain type of beam, the expression $Fa(L - a)(L + a)$ occurs. In expanding this expression, we first multiply $L - a$ by $L + a$ by use of Eq. (2). The expansion is completed by using Eq. (1), the distributive law.

$$Fa(L - a)(L + a) = Fa(L^2 - a^2) \qquad \text{Eq. (2)}$$
$$= FaL^2 - Fa^3 \qquad \text{Eq. (1)}$$

(b) The electrical power delivered to the resistor R in Fig. (1) is $R(i_1 + i_2)^2$. Here, i_1 and i_2 are electric currents. To expand this expression, we first perform the square by use of Eq. (3) and then complete the expansion by use of Eq. (1).

$$R(i_1 + i_2)^2 = R(i_1^2 + 2i_1i_2 + i_2^2) \qquad \text{Eq. (3)}$$
$$= Ri_1^2 + 2Ri_1i_2 + Ri_2^2 \qquad \text{Eq. (1)}$$

■

$$(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3 \tag{7}$$
$$(x - y)^3 = x^3 - 3x^2y + 3xy^2 - y^3 \tag{8}$$
$$(x + y)(x^2 - xy + y^2) = x^3 + y^3 \tag{9}$$
$$(x - y)(x^2 + xy + y^2) = x^3 - y^3 \tag{10}$$

EXAMPLE 8 Using special products 8, 9, and 10

(a) $(2x - 5)^3 = (2x)^3 - 3(2x)^2(5) + 3(2x)(5^2) - 5^3$ Eq. (8)

$= 8x^3 - 60x^2 + 150x - 125$

(b) $(x + 3)(x^2 - 3x + 9) = x^3 + 3^3$ Eq. (9)

$= x^3 + 27$

(c) $(x - 2)(x^2 + 2x + 4) = x^3 - 2^3$ Eq. (10)

$= x^3 - 8$

6.2 2,3,4,6,8,9

EXAMPLE 2 Common monomial factor

In factoring $6x - 2y$, we note each term contains a factor of 2:

$$6x - 2y = 2(3x) - 2y = 2(3x - y)$$

Here, 2 is the common monomial factor, and $2(3x - y)$ is the required factored form of $6x - 2y$. Once the common factor has been identified, it is not actually necessary to write a term like $6x$ as $2(3x)$. The result can be written directly.

We check the result by multiplication. In this case,

$$2(3x - y) = 6x - 2y$$

Since the result of the multiplication gives the original expression, the factored form is correct. ■

Some worked examples to help you

EXAMPLE 3 Common factor same as term

Factor: $4ax^2 + 2ax$.

The numerical factor 2 and the literal factors a and x are common to each term. Therefore, the common monomial factor of $4ax^2 + 2ax$ is $2ax$. This means that

$$4ax^2 + 2ax = 2ax(2x) + 2ax(1) = 2ax(2x + 1)$$

Note the presence of the 1 in the factored form. When we divide $4ax^2 + 2ax$ by $2ax$, we get

$$\frac{4ax^2 + 2ax}{2ax} = \frac{4ax^2}{2ax} + \frac{2ax}{2ax}$$
$$= 2x + 1$$

EXAMPLE 4 Common factor by inspection

Factor: $6a^5x^2 - 9a^3x^3 + 3a^3x^2$.

After inspecting each term, we determine that each contains a factor of 3, a^3, and x^2. Thus, the common monomial factor is $3a^3x^2$. This means that

$$6a^5x^2 - 9a^3x^3 + 3a^3x^2 = 3a^3x^2(2a^2 - 3x + 1)$$

EXAMPLE 6 Factoring difference of two squares

In factoring $x^2 - 16$, note that x^2 is the square of x and 16 is the square of 4. Therefore,

squares

$$x^2 - 16 = x^2 - 4^2 = (x + 4)(x - 4)$$

difference sum difference

EXAMPLE 8 Complete factoring—application

(a) In factoring $20x^2 - 45$, note a common factor of 5 in each term. Therefore, $20x^2 - 45 = 5(4x^2 - 9)$. However, the factor $4x^2 - 9$ itself is the difference of squares. Therefore, $20x^2 - 45$ is completely factored as

common factor

$$20x^2 - 45 = 5(4x^2 - 9) = 5(2x + 3)(2x - 3)$$

difference of squares

EXAMPLE 9 Factoring by grouping

Factor: $2x - 2y + ax - ay$.

We see that there is no common factor to all four terms, but that each of the first two terms contains a factor of 2, and each of the third and fourth terms contains a factor of a. Grouping terms this way and then factoring each group, we have

$$\begin{aligned} 2x - 2y + ax - ay &= (2x - 2y) + (ax - ay) \\ &= 2(x - y) + a(x - y) \quad \text{now note the common factor of } (x - y) \\ &= (x - y)(2 + a) \end{aligned}$$

■

6.5 3,4,6,7,10

EXAMPLE 3 **Reducing fraction to lowest terms**

In order to reduce the fraction

$$\frac{16ab^3c^2}{24ab^2c^5}$$

to its lowest terms, note that both the numerator and the denominator contain the factor $8ab^2c^2$. Therefore,

$$\frac{16ab^3c^2}{24ab^2c^5} = \frac{2b(8ab^2c^2)}{3c^3(8ab^2c^2)} = \frac{2b}{3c^3}$$

common factor

Notice: difference of squares

EXAMPLE 4 Cancel factors only

When simplifying the expression

$$\frac{x^2(x-2)}{x^2-4}$$

a term, but not a factor, of the denominator

many students would "cancel" the x^2 from the numerator and the denominator. This is incorrect, because $\boldsymbol{x^2}$ ***is a term only*** of the denominator.

In order to simplify the above fraction properly, we should factor the denominator. We get

$$\frac{x^2\cancel{(x-2)}^{1}}{\cancel{(x-2)}_{1}(x+2)} = \frac{x^2}{x+2}$$

Here, the common ***factor*** $x - 2$ has been divided out. ■

EXAMPLE 6 Remaining factor of 1 in denominator

$$\frac{2x^2 + 8x}{x + 4} = \frac{2x\cancel{(x + 4)}^{1}}{\cancel{(x + 4)}_{1}} = \frac{2x}{1}$$
$$= 2x$$

The numerator and the denominator were each divided by $x + 4$ after factoring the numerator. The only remaining factor in the denominator is 1, and it is generally not written in the final result. Another way of writing the denominator is $1(x + 4)$, which shows the *factor* of 1 more clearly. ■

EXAMPLE 7 Cancel factors only

$$\frac{x^2 - 4x + 4}{x^2 - 4} = \frac{(x - 2)\cancel{(x - 2)}^{1}}{(x + 2)\cancel{(x - 2)}_{1}}$$
$$= \frac{x - 2}{x + 2}$$

x is a term but not a factor

$$(x - y)^2 = x^2 - 2xy + y^2$$

$$(x + y)(x - y) = x^2 - y^2$$

Here, the numerator and the denominator have each been *factored first and then the common factor $x - 2$ has been divided out.* In the final form, neither the x's nor the 2's may be canceled, because they are not common *factors*. ■

$$(x - y)(x^2 + xy + y^2) = x^3 - y^3$$

EXAMPLE 10 Factors differ only in sign

$$\frac{2x^4 - 128x}{20 + 7x - 3x^2} = \frac{2x(x^3 - 64)}{(4 - x)(5 + 3x)} = \frac{2x\cancel{(x - 4)}(x^2 + 4x + 16)}{-\cancel{(x - 4)}(3x + 5)}$$

$$= -\frac{2x(x^2 + 4x + 16)}{3x + 5}$$

Again, the factor $4 - x$ has been replaced by the equal expression $-(x - 4)$. This allows us to recognize the common factor of $x - 4$.

Also, note that the order of the terms of the factor $5 + 3x$ was changed in writing the third fraction. This was done only to write the terms in the more standard form with the x-term first. However, because both terms are *positive*, it is simply an application of the commutative law of addition, and the factor itself is not actually changed. ■

$$(ax + b)(cx + d) = acx^2 + (ad + bc)x + bd$$

EXAMPLE 3 Multiplying algebraic fractions

$$\left(\frac{2x-4}{4x+12}\right)\left(\frac{2x^2+x-15}{3x-1}\right) = \frac{2(x-2)(2x-5)(x+3)}{4(x+3)(3x-1)}$$

multiplications indicated

$$= \frac{(x-2)(2x-5)}{2(3x-1)}$$

Here, the common factor is $2(x+3)$. It is permissible to multiply out the final form of the numerator and the denominator, but it is often preferable to leave the numerator and the denominator in factored form, as indicated. ■

EXAMPLE 5 Division by a fraction—application

When finding the center of gravity (c.g.) of a uniform flat semicircular metal plate, the equation $X = \frac{4\pi r^3}{3} \div \left(\frac{\pi r^2}{2} \times 2\pi\right)$ is derived. Simplify the right side of this equation to find X as a function of r in simplest form. See Fig. 6.

The parentheses indicate that we should perform the multiplication first:

$$X = \frac{4\pi r^3}{3} \div \left(\frac{\pi r^2}{2} \times 2\pi\right) = \frac{4\pi r^3}{3} \div \left(\frac{2\pi^2 r^2}{2}\right) \qquad 2\pi = \frac{2\pi}{1}$$

$$= \frac{4\pi r^3}{3} \div (\pi^2 r^2) = \frac{4\pi r^3}{3} \times \frac{1}{\pi^2 r^2}$$

$$= \frac{4\pi r^3}{3\pi^2 r^2} = \frac{4r}{3\pi} \qquad \text{divide out the common factor of } \pi r^2$$

This is the exact solution. Approximately, $X = 0.424r$. ■

EXAMPLE 6 Dividing algebraic fractions

$$(x+y) \div \frac{2x+2y}{6x+15y} = \frac{x+y}{1} \times \frac{6x+15y}{2x+2y} = \frac{\cancel{(x+y)}(3)(2x+5y)}{2\cancel{(x+y)}}$$ indicate multiplication

invert

$$= \frac{3(2x+5y)}{2}$$ simplify ■

Practice Exercise

2. Divide: $\frac{3x}{a+1} \div \frac{x^2+2x}{a^2+a}$

EXAMPLE 7 Dividing algebraic fractions

invert

$$\frac{\frac{4-x^2}{x^2-3x+2}}{\frac{x+2}{x^2-9}} = \frac{4-x^2}{x^2-3x+2} \times \frac{x^2-9}{x+2} = \frac{(2-x)(2+x)(x-3)(x+3)}{(x-2)(x-1)(x+2)}$$ factor and indicate multiplications

$$= \frac{-\cancel{(x-2)}\cancel{(x+2)}(x-3)(x+3)}{\cancel{(x-2)}(x-1)\cancel{(x+2)}}$$ replace $(2-x)$ with $-(x-2)$ and $(2+x)$ with $(x+2)$

$$= -\frac{(x-3)(x+3)}{x-1} \quad \text{or} \quad \frac{(x-3)(x+3)}{1-x}$$ simplify

Note the use of Eq. (11) when the factor $(2-x)$ was replaced by $-(x-2)$ to get the first form of the answer. As shown, it can also be used to get the alternate form of the answer, although it is not necessary to give this form. ■

6.7 3,4,6,8

EXAMPLE 3 Lowest common denominator

Find the LCD of the following fractions:

$$\frac{x-4}{x^2-2x+1} \qquad \frac{1}{x^2-1} \qquad \frac{x+3}{x^2-x}$$

Factoring each of the denominators, we find that the fractions are

$$\frac{x-4}{(x-1)^2} \qquad \frac{1}{(x-1)(x+1)} \qquad \frac{x+3}{x(x-1)}$$

The factor $(x-1)$ appears in all the denominators. It is squared in the first fraction and appears only to the first power in the other two fractions. Thus, we must have $(x-1)^2$ as a factor in the LCD. We do not need a higher power of $(x-1)$ because, as far as this factor is concerned, each denominator will divide into it evenly. Next, the second denominator has a factor of $(x+1)$. Therefore, the LCD must also have a factor of $(x+1)$; otherwise, the second denominator would not divide into it exactly. Finally, the third denominator shows that a factor of x is also needed. The LCD is therefore $x(x+1)(x-1)^2$. All three denominators will divide exactly into this expression, and there is no simpler expression for which this is true. ■

EXAMPLE 4 Finding LCD—combining fractions

Combine: $\frac{2}{3r^2} + \frac{4}{rs^3} - \frac{5}{3s}$. ← **Highest of each factor**

By looking at the denominators, notice that the factors necessary in the LCD are 3, r, and s. The 3 appears only to the first power, the largest exponent of r is 2, and the largest exponent of s is 3. Therefore, the LCD is $3r^2s^3$. Now, write each fraction with this quantity as the denominator. Since the denominator of the first fraction already contains factors of 3 and r^2, ***it is necessary to introduce the factor of s^3.*** In other words, we must multiply the numerator and the denominator of this fraction by s^3. For similar reasons, we must multiply the numerators and the denominators of the second and third fractions by $3r$ and r^2s^2, respectively. This leads to

$$\frac{2}{3r^2} + \frac{4}{rs^3} - \frac{5}{3s} = \frac{2(s^3)}{(3r^2)(s^3)} + \frac{4(3r)}{(rs^3)(3r)} - \frac{5(r^2s^2)}{(3s)(r^2s^2)}$$

change to equivalent fractions with LCD

factors needed in each

$$= \frac{2s^3}{3r^2s^3} + \frac{12r}{3r^2s^3} - \frac{5r^2s^2}{3r^2s^3}$$

$$= \frac{2s^3 + 12r - 5r^2s^2}{3r^2s^3}$$

combine numerators over LCD

Note that the term $-\frac{5r^2s^2}{3r^2s^3}$ was treated as $+\frac{-5r^2s^2}{3r^2s^3}$, therefore leading to the last term $-5r^2s^2$ in the resulting numerator. ■

Do this example

Then try exercise

EXAMPLE 6 Combining fractions

$$\frac{3}{2x^2 - 2x - 24} - \frac{x-1}{x^2 - 8x + 16} = \frac{3x}{2(x-4)(x+3)} - \frac{x-1}{(x-4)^2}$$ factor denominators

$$= \frac{3x(x-4) - (x-1)(2)(x+3)}{2(x-4)^2(x+3)}$$ change to equivalent fraction with LCD

$$= \frac{(3x^2 - 12x) - (2x^2 + 4x - 6)}{2(x-4)^2(x+3)}$$ expand in numerator

$$= \frac{3x^2 - 12x - 2x^2 - 4x + 6}{2(x-4)^2(x+3)} = \frac{x^2 - 16x + 6}{2(x-4)^2(x+3)}$$ simplify

In doing this kind of problem, many errors may arise in the use of the minus sign.

CAUTION ➧ Remember, *if a minus sign precedes an expression, the* ***signs of all terms must be changed*** before they can be combined with other terms. ■

Practice Exercise

4. Combine:

$$\frac{3}{2x+2} - \frac{2}{x^2 + 2x + 1}$$