Chapter 7

Quadratic Equations

Solving Quadratic equations	• Solving quadratic equations by: factoring, and quadratic formula	7.1 Quadratic Equations: Solution by factoring. 7.3 The quadratic formula 7.4 The graph of the quadratic function

Ch. 7.1: Quadratic Equations; Solution by Factoring

- The general form of the quadratic equation in x is:
  - Given that a, b and c are constants ($a \neq 0$), the equation:

$$ax^2 + bx + c = 0$$

Example 1

- Identify ***a***, ***b*** and ***c*** in the quadratic equation:

$$x^2 - 5x - 11 = 0$$

$a = 1$

$b = -5$

$c = -11$

Identifying Quadratic Equations

- Examples of equations which are not quadratic:

$$2x - 9 = 0$$

- No x^2 terms.

$$x^4 - 4x^3 - x^2 + 3 = 0$$

- No power of x should be higher than 2.

$$x^2 + 7x = x^2$$

- Simplifying produces a linear equation.

Solutions of a Quadratic Equation

- The solution of an equation consists of all numbers (**roots**) which, when substituted in the equation, give equality.
- There are 2 roots in a solution of a quadratic equation.

Examples of Solutions of a Quadratic Equation

① $x^2 - 5x + 6 = 0$

$x = 2, x = 3$

real roots

Examples of Solutions of a Quadratic Equation (*continued*)

② $x^2 - 4x + 4 = 0$

$x = 2, x = 2$

double roots

Examples of Solutions of a Quadratic Equation *(continued)*

③ $x^2 + 1 = 0$

$$x = \sqrt{-1},\ x = -\sqrt{-1}$$

imaginary roots

Procedure for Solving a Quadratic Equation by Factoring

1. Collect all terms on the left & simplify into the general quadratic equation form.
2. Factor the quadratic expression.
3. Set each factor equal to zero.
4. Solve the resulting linear equations. These numbers are the roots of the quadratic equation.
5. Check the solutions in the original equations.

Example 2

Can use special products

- Solve the quadratic equation by factoring.

$$x^2 + 2x - 15 = 0$$

factor

$$(x+5)(x-3) = 0$$

set each factor to 0

$\boldsymbol{x} + 5 = 0$ $\boldsymbol{x} - 3 = 0$

solve

$\boldsymbol{x} = -5$ $\boldsymbol{x} = 3$

Solving a Quadratic Equation by Factoring

- Remember to check the solutions by substituting into the original equation.
- The answer should be **'0'**.
- It is essential for the quadratic expression on the left to be equal to zero on the right.
- If a product equals a nonzero number, it is probable that neither factor will give a correct root.

? Ex 7.1

In Exercises 3–8, determine whether or not the given equations are quadratic. If the resulting form is quadratic, identify a, b, and c, with $a > 0$. Otherwise, explain why the resulting form is not quadratic.

3. $x(x - 2) = 4$

4. $(3x - 2)^2 = 2$

5. $x^2 = (x + 2)^2$

6. $x(2x^2 + 5) = 7 + 2x^2$

7. $n(n^2 + n - 1) = n^3$

8. $(T - 7)^2 = (2T + 3)^2$

In Exercises 9–44, solve the given quadratic equations by factoring.

9. $x^2 - 4 = 0$

10. $B^2 - 400 = 0$

11. $4x^2 = 9$

12. $x^2 = 0.16$

13. $x^2 - 8x - 9 = 0$

14. $x^2 + x - 6 = 0$

3. $x(x-2)=4;\ x^2-2x-4=0;\ a=1,\ b=-2,$
$c=-4$

A

4. $(3x-2)^2=2;\ 9x^2-12x+4=2;\ 9x^2-12x+2=0$
$a=9,\ b=-12,\ c=2$

5. $x^2=(x+2)^2$
$x^2=x^2+4x+4$
$4x+4=0,$ no x^2 term, not quadratic

6. $x(2x^2+5)=7+2x^2;\ 2x^3+5x=7+2x^2$
Not quadratic; there is an x^3 term.

7. $n(n^2+n-1)=n^3;\ n^3+n^2-n=n^3;\ n^2-n=0$
$a=1,\ b=-1,\ c=0$

8. $(T-7)^2=(2T+3)^2$
$T^2-14t+49=4T^2+12T+9$
$-3T^2-26T+40=0;\ 3T^2+26T-40=0$
$a=3,\ b=26,\ c=-40$

9.
$$x^2-4=0$$
$$(x+2)(x-2)=0$$
$$x+2=0 \quad \text{or} \quad x-2=0$$
$$x=-2 \qquad\qquad x=2$$

10. $B^2-400=0;\ (B-20)(B+20)=0$
$B-20=0,\ B=20$ or $B+20=0,\ B=-20$

11. $4x^2=9$
$4x^2-9=0;\ (2x+3)(2x-3)=0$
$2x+3=0;\ 2x=-3,\ x=-\dfrac{3}{2}$ or
$2x-3=0;\ 2x=3,\ x=\dfrac{3}{2}$

12.
$$x^2=0.16$$
$$x^2-0.16=0$$
$$(x-0.4)(x+0.4)=0$$
$$x-0.4=0;\ x=0.4$$
$$\ldots=0;\ x=-0.4$$

13.
$$x^2-8x-9=0$$
$$(x-9)(x+1)=0$$
$$x-9=0 \quad \text{or} \quad x+1=0$$
$$x=9 \qquad\qquad x=-1$$

14. $x^2+x-6=0;\ (x+3)(x-2)=0$
$x+3=0,\ x=-3$ or $x-2=0,\ x=2$

https://www.khanacademy.org/math/algebra/quadratics/factoring_quadratics/v/Example%201:%20Solving%20a%20quadratic%20equation%20by%20factoring

https://www.khanacademy.org/math/algebra/quadratics/quadratics-square-root/v/solving-quadratic-equations-by-square-roots

Ch. 7.3: The Quadratic Formula

- The quadratic formula can be used to find solutions (roots) to a quadratic equation.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Info only > Following completing the square derives this (p220)

Show

https://www.khanacademy.org/math/algebra/quadratics/quadratic-formula/v/quadratic-formula-1

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example

- Solve the quadratic equation by the quadratic formula.

$$2x^2 + 5x - 3 = 0$$

Solution

- Identify ***a***, ***b*** and ***c*** in the quadratic equation:

$$2x^2 + 5x - 3 = 0$$

$a = 2$ $\qquad$ $b = 5$ $\qquad$ $c = -3$

$a = 2$ $b = 5$ $c = -3$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Solution (*continued*)

- Using the quadratic formula:

$$x = \frac{-5 \pm \sqrt{5^2 - 4(2)(-3)}}{2(2)}$$

- Answer: $x = -3, x = 0.5$

Characteristics of the Roots of a Quadratic Equation

1. If $b^2 - 4ac$ is positive & a perfect square, the roots are real, rational, & unequal.
2. If $b^2 - 4ac$ is positive but not a perfect square, the roots are real, irrational, & unequal.
3. If $b^2 - 4ac = 0$, the roots are real, rational & equal.
4. If $b^2 - 4ac < 0$, the roots contain imaginary numbers & are unequal.

?Ex 7.3. 5-9

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

EXERCISES 7.3

In Exercises 1–4, make the given changes in the indicated examples of this section and then solve the resulting equations by the quadratic formula.

1. In Example 1, change the − sign before $5x$ to +.
2. In Example 2, change the coefficient of x^2 from 2 to 3.
3. In Example 3, change the + sign before $24x$ to −.
4. In Example 4, change 4 to 3.

In Exercises 5–36, solve the given quadratic equations, using the quadratic formula. Exercises 5–8 are the same as Exercises 11–14 of Section 7.2.

5. $x^2 + 2x - 8 = 0$
6. $x^2 - 8x - 20 = 0$
7. $D^2 + 3D + 2 = 0$
8. $t^2 + 5t - 6 = 0$

9. $x^2 - 4x + 2 = 0$

11. $v^2 = 15 - 2v$

13. $2s^2 + 5s = 3$

15. $3y^2 = 3y + 2$

17. $y + 2 = 2y^2$

19. $30y^2 + 23y - 40 = 0$

21. $8t^2 + 61t = -120$

23. $s^2 = 9 + s(1 - 2s)$

25. $25y^2 = 121$

27. $15 + 4z = 32z^2$

29. $x^2 - 0.20x - 0.40 = 0$

31. $0.29Z^2 - 0.18 = 0.63Z$

$ax^2 + bx + c = 0$

A

5. $x^2+2x-8=0;\ a=1,\ b=2,\ c=-8$

$$x=\frac{-2\pm\sqrt{2^2-4(1)(-8)}}{2(1)}=\frac{-2\pm\sqrt{36}}{2}$$

$$=\frac{-2\pm 6}{2}$$

$x=2$ or $x=-4$

6. $x^2-8x-20=0;\ a=1,\ b=-8,\ c=-20$

$$x=\frac{8\pm\sqrt{64-4(1)(-20)}}{2(1)}=\frac{8\pm\sqrt{144}}{2}$$

$$=\frac{8\pm\sqrt{144}}{2}$$

$$=\frac{8\pm 12}{2}=-2,\ 10$$

7. $D^2+3D+2=0;\ a=1,\ b=3,\ c=2$

$$D=\frac{-3\pm\sqrt{9-4(1)(2)}}{2(1)}=\frac{-3\pm\sqrt{1}}{2}$$

$$=\frac{-3\pm 1}{2}=-2,\ -1$$

8. $t^2+5t-6=0;\ a=1,\ b=5,\ c=-6$

$$t=\frac{-5\pm\sqrt{25-4(1)(-4)}}{2(1)}=\frac{-5\pm\sqrt{49}}{2}$$

$$=\frac{-5\pm 7}{2}=6,\ 1$$

9. $x^2-4x+2=0;\ a=1,\ b=-4,\ c=2$

$$x=\frac{-(-4)\pm\sqrt{(-4)^2-4(1)(2)}}{(2)}$$

$$=\frac{4\pm\sqrt{8}}{2}$$

$$=\frac{4\pm 2\sqrt{2}}{2}$$

$$=2\pm\sqrt{2}$$

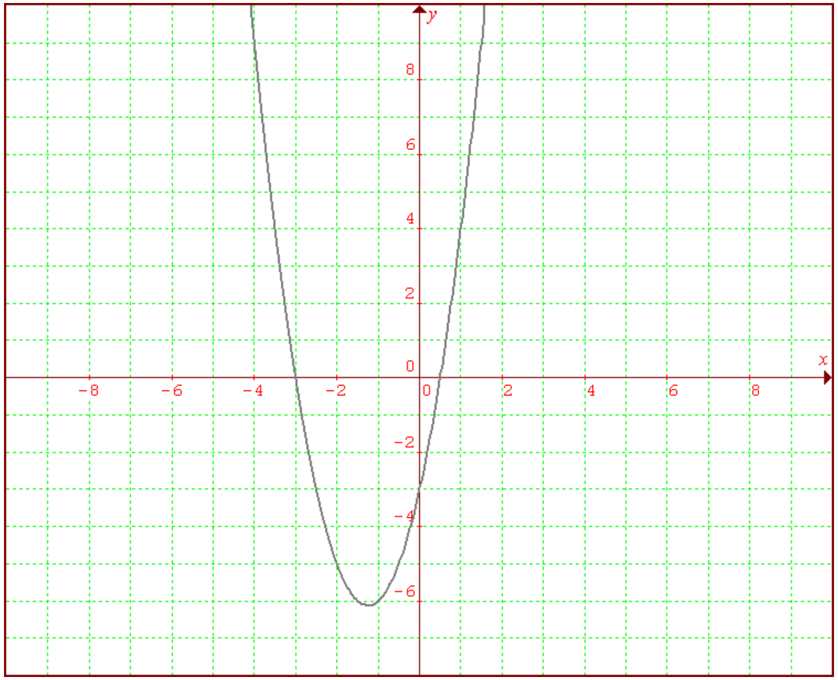
Ch. 7.4: The Graph of the Quadratic Equation

- By graphing the function $ax^2 + bx + c$, we can find its solution.
- First we let: $y = ax^2 + bx + c$

The Graph of the Quadratic Equation

- Using a graphing utility, graph:

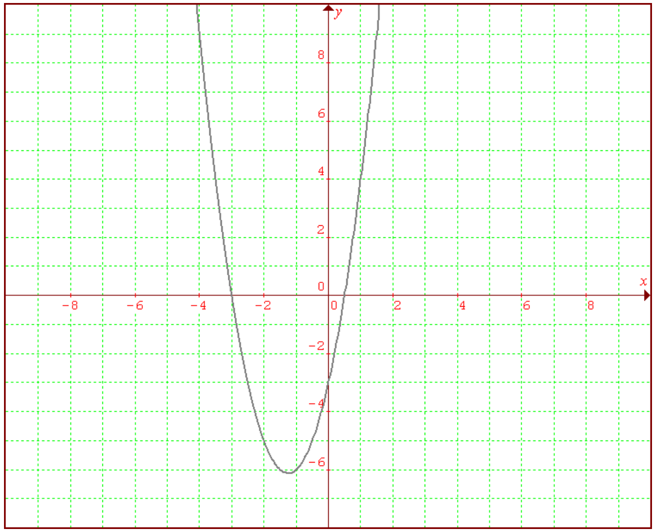
$$2x^2 + 5x - 3 = 0$$



https://www.khanacademy.org/math/algebra/quadratics/solving_graphing_quadratics/v/graphing-a-quadratic-function

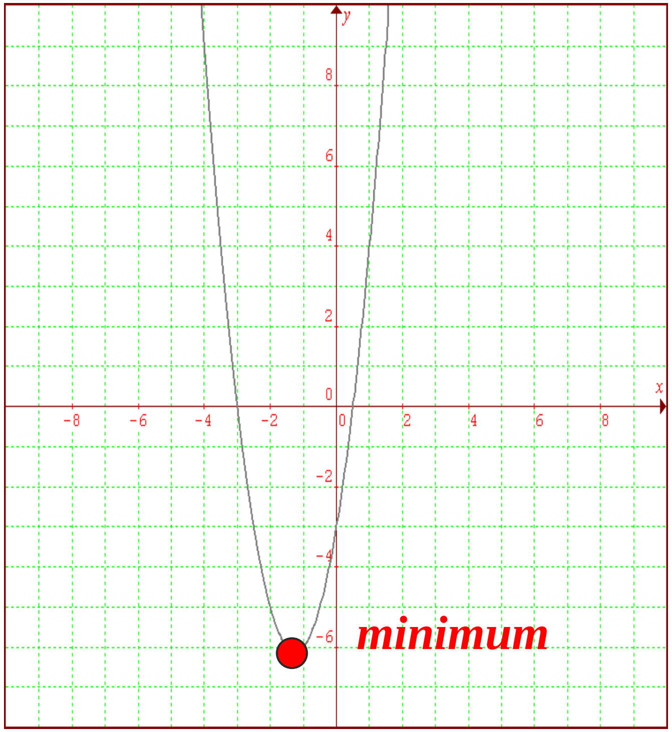
The Graph of the Quadratic Equation

- The graph of any quadratic function $y = ax^2 + bx + c$ will have the same basic shape, the shape of a parabola.



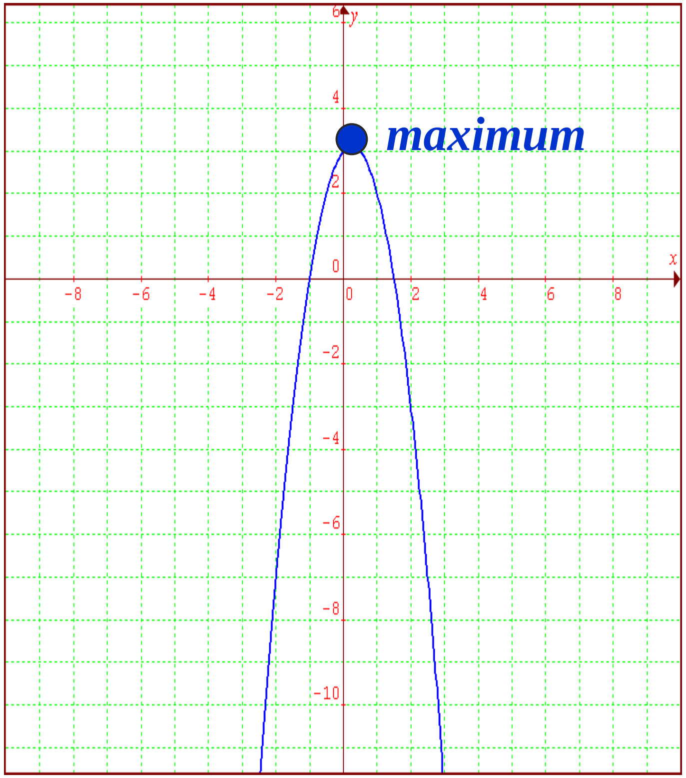
The Graph of the Quadratic Equation

- All parabolas have an ***extreme point***.
- This point is known as a ***vertex***.
- If $\boldsymbol{a > 0} \rightarrow$ ***minimum***.
- The graph opens upward.



The Graph of the Quadratic Equation

- All parabolas have an ***extreme point***.
- This point is known as a ***vertex***.
- If $\boldsymbol{a < 0} \rightarrow$ ***maximum***.
- The graph opens downward.



Finding the Vertex

- We can find the x-coordinate of the vertex with:

$$x = \frac{-b}{2a}$$

- Substituting this value into the given equation, we can find the y-coordinate of the vertex.

Solving Quadratic Equations Graphically

- When a quadratic equation is graphed, the roots of the equation are the x-coordinates of the points for which $y = 0$ (the x-intercepts).
- Knowing when a quadratic curve is at a maximum or a minimum is a useful concept in maximization problems.

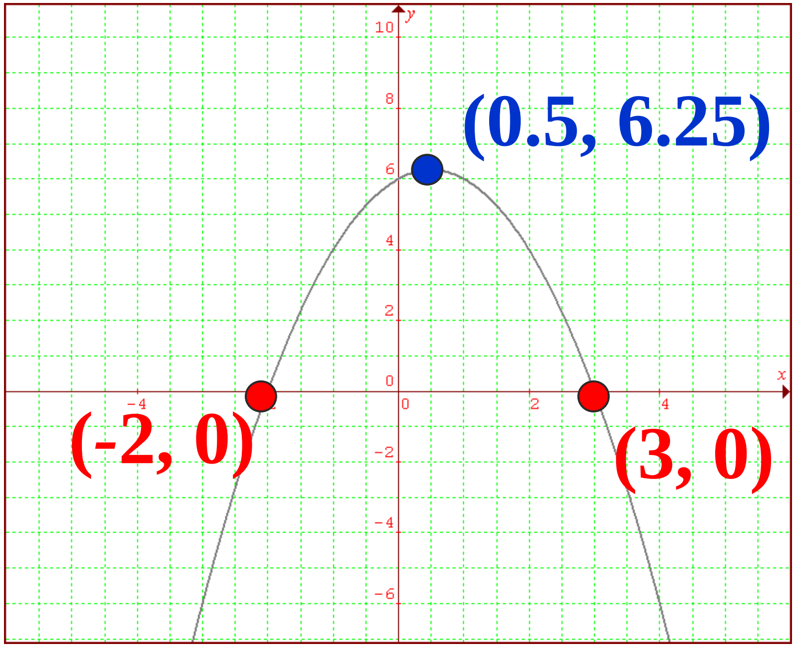
https://www.khanacademy.org/math/algebra/quadratics/solving_graphing_quadratics/v/quadratic-functions-2

Explanation of vertex etc > for information only

Example

- From the graph of $y = -x^2 + x + 6$, identify the roots of the equation.
- $x = -2$
- $x = 3$
- Vertex: **(0.5, 6.25)**
- Since $a < 0$, the vertex is at a maximum.

$$x = \frac{-b}{2a}$$



Summary

- We can sketch a quadratic equation when we know the vertex, the x-intercepts and the y-intercepts.
- We also note, the curve is symmetric to a vertical line through the vertex.
- Knowing when the vertex of the curve is a maximum or a minimum point is a useful concept in maximization problems.

https://www.desmos.com/calculator

? Ex 7.4 q 3&4

In Exercises 3–8, sketch the graph of each parabola by using only the vertex and the y-intercept. Check the graph using a graphing calculator.

3. $y = x^2 - 6x + 5$

4. $y = -x^2 - 4x - 3$

5. $y = -3x^2 + 10x - 4$

6. $s = 2t^2 + 8t - 5$

7. $R = v^2 - 4v$

8. $y = -2x^2 - 5x$

In Exercises 9–12, sketch the graph of each parabola by using the vertex, the y-intercept, and the x-intercepts. Check the graph using a graphing calculator.

9. $y = x^2 - 4$

10. $y = x^2 + 3x$

11. $y = -2x^2 - 6x + 8$

12. $u = -3v^2 + 12v - 5$

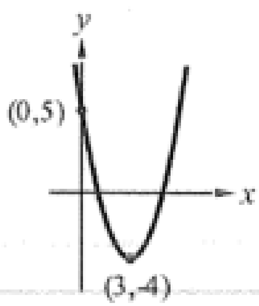
3. $y = x^2 - 6x + 5;\ a = 1,\ b = -6,\ c = 5$

The x-coordinate of the extreme point is

$\frac{-b}{2a} = \frac{-(-6)}{2(1)} = 3$, and the y-coordinate is

$y = 3^2 - 6(3) + 5 = -4$

The extreme point is $(3, -4)$. Since $a > 0$ it is a minimum point.



Since $c = 5$, the y-intercept is $(0, 5)$. Use the minimum point $(3, -4)$ and the y-intercept $(0, 5)$, and the fact that the graph is a parabola, to sketch the graph.

A

4. $y = -x^2 - 4x - 3;\ a = -1,\ b = -4.$

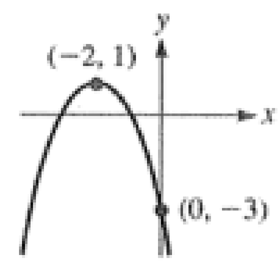
This means that the x-coordinate of the extreme is

$$\frac{-b}{2a} = \frac{-(-4)}{2(-1)} = -2,$$

and the y-coordinate is

$$y = -(-2)^2 - 4(-2) - 3 = 1.$$

Thus the extreme point is $(-2, 1)$. Since $a < 0$, it is a maximum point.



Since $c = -3$, the y-intercept is $(0, -3)$. Use the maximum point $(-2, 1)$ and the y-intercept $(0, -3)$, and the fact that the graph is a parabola, to sketch the graph.

Ch 7 Quadratic Summary

$$ax^2 + bx + c = 0$$

- Factor
- Completing the squares
- Quadratic Equation
- Graph

$$x^2 + 2x - 15 = 0$$ factor

$$(x+5)(x-3) = 0$$ set each factor to 0

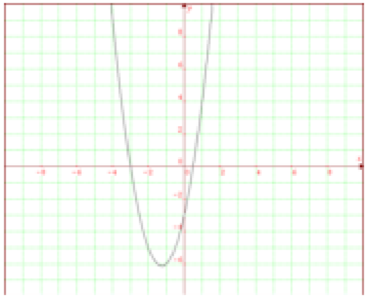
$x + 5 = 0$ $\quad$ $x - 3 = 0$ $\quad$ solve

$x = -5$ $\quad$ $x = 3$

1. Divide each side by a (coefficient of x^2).
2. Rewrite the equation with the constant on the right side.
3. Complete the square: add the square of ½ of the coefficient of x to both sides.
4. Write the left side as a square & simplify the right side.
5. Equate the square root of the left side to the principal square root of the right side & to its negative.
6. Solve the 2 resulting linear equations.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The graph of any quadratic function $y = ax^2 + bx + c$ will have the same basic shape, the shape of a parabola.



$$x = \frac{-b}{2a}$$

Worked examples

EXAMPLE 1 Examples of quadratic equations

The following are quadratic equations.

$$\underset{a=1}{x^2} - \underset{b=-4}{4x} - \underset{c=-5}{5} = 0$$

To show this equation in the form of Eq. (1), it can be written as $1x^2 + (-4)x + (-5) = 0$.

$$\underset{a=3}{3x^2} - \underset{c=-6}{6} = 0$$

Because there is no x-term, $b = 0$.

$$\underset{a=2}{2x^2} + \underset{b=7}{7x} = 0$$

Because no constant appears, $c = 0$.

$$(m - 3)x^2 - mx + 7 = 0$$

The constants in Eq. (1) may include literal expressions. In this case, $m - 3$ takes the place of a, $-m$ takes the place of b, and $c = 7$.

$$4x^2 - 2x = x^2$$

After all nonzero terms have been collected on the left side, the equation becomes $3x^2 - 2x = 0$.

$$(x + 1)^2 = 4$$

Expanding the left side and collecting all nonzero terms on the left, we have $x^2 + 2x - 3 = 0$. ■

EXAMPLE 2 Examples of equations not quadratic

The following are not quadratic equations.

$bx - 6 = 0$	There is no x^2-term.
$x^3 - x^2 - 5 = 0$	There should be no term of degree higher than 2. Thus, there can be no x^3-term in a quadratic equation.
$x^2 + x - 7 = x^2$	When terms are collected, there will be no x^2-term. ■

EXAMPLE 3 Solutions (roots) of a quadratic equation

(a) The quadratic equation $3x^2 - 7x + 2 = 0$ has roots $x = 1/3$ and $x = 2$. This is seen by substituting these numbers in the equation.

$$3\left(\tfrac{1}{3}\right)^2 - 7\left(\tfrac{1}{3}\right) + 2 = 3\left(\tfrac{1}{9}\right) - \tfrac{7}{3} + 2 = \tfrac{1}{3} - \tfrac{7}{3} + 2 = \tfrac{0}{3} = 0$$

$$3(2)^2 - 7(2) + 2 = 3(4) - 14 + 2 = 14 - 14 = 0$$

(b) The quadratic equation $4x^2 - 4x + 1 = 0$ has a **double root** (*both roots are the same*) of $x = 1/2$. Showing that this number is a solution, we have

$$4\left(\tfrac{1}{2}\right)^2 - 4\left(\tfrac{1}{2}\right) + 1 = 4\left(\tfrac{1}{4}\right) - 2 + 1 = 1 - 2 + 1 = 0$$

(c) The quadratic equation $x^2 + 9 = 0$ has the imaginary roots $x = 3j$ and $x = -3j$, which means $x = 3\sqrt{-1}$ and $x = -3\sqrt{-1}$. ■

EXAMPLE 6 Quadratic equations with $b = 0$ or $c = 0$

(a) In solving the equation $3x^2 - 12 = 0$, we note that $b = 0$ (there is no x-term). However, we can solve it by factoring. First we note the common factor of 3. Because it is a constant, we can first divide all terms by 3, and proceed with the solution.

$$3x^2 - 12 = 0$$

$$x^2 - 4 = 0 \qquad \text{divide each term by 3}$$

$$(x - 2)(x + 2) = 0 \qquad \text{factor}$$

$$x - 2 = 0, \qquad x = 2$$

$$x + 2 = 0, \qquad x = -2$$

The roots 2 and -2 check. We could also have first factored the 3 from each term, but the results would be the same since 3 is a constant, and the only two factors that can be set equal to zero are $x - 2$ and $x + 2$.

(b) In solving the equation $3x^2 - 12x = 0$, we note that $c = 0$ (there is no constant term). However, we can solve it by factoring. We note the factor of 3, and because 3 is a constant, we can divide each term by 3. We also note the common factor of x, but because we are solving for x, ***we cannot divide each term by*** x. If we divide by x, we lose one of the two roots. Therefore,

$$3x^2 - 12x = 0$$

$$x^2 - 4x = 0 \qquad \text{divide each term by 3, but not by } x$$

$$x(x - 4) = 0 \qquad \text{factor}$$

$$x = 0, 4$$

These roots check. Again, if we had divided out the x, we would not have found the root $x = 0$, and therefore had an incomplete solution. ■

EXAMPLE 7 Quadratic equation—application

For a certain fire hose, the pressure loss P (in lb/in.^{2} per 100 ft of hose) is $P = 2Q^2 + Q$, where Q is the flow rate (in 100 gal/min). Find Q for $P = 15$ lb/in.^{2} per 100 ft.

Substituting, we have the following equation and solution.

$$2Q^2 + Q = 15$$

$$2Q^2 + Q - 15 = 0$$

$$(2Q - 5)(Q + 3) = 0$$

$$2Q - 5 = 0, \qquad Q = 2.5 \;\; 100\text{ gal/min}$$

$$Q + 5 = 0, \qquad Q = -5 \;\; 100\text{ gal/min}$$

not realistically possible

These roots check, but the negative root is not realistically possible, which means the only real solution is $Q = 2.5$ 100 gal/min, or 250 gal/min.

EXAMPLE 1 Quadratic formula—rational roots

$$\text{Solve: } \underset{a=1}{\underset{\uparrow}{x^2}} - \underset{b=-5}{\underset{\uparrow}{5x}} + \underset{c=6}{\underset{\uparrow}{6}} = 0.$$

Here, using the indicated values of a, b, and c in the quadratic formula, we have

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(6)}}{2(1)} = \frac{5 \pm \sqrt{25 - 24}}{2} = \frac{5 \pm 1}{2}$$

$$x = \frac{5+1}{2} = 3 \quad \text{or} \quad x = \frac{5-1}{2} = 2$$

The roots $x = 3$ and $x = 2$ check when substituted in the original equation. ■

EXAMPLE 3 Quadratic formula—double root

Solve: $9x^2 + 24x + 16 = 0$.

In this example, $a = 9$, $b = 24$, and $c = 16$. Thus,

$$x = \frac{-24 \pm \sqrt{24^2 - 4(9)(16)}}{2(9)} = \frac{-24 \pm \sqrt{576 - 576}}{18} = \frac{-24 \pm 0}{18} = -\frac{4}{3}$$

Here, both roots are $-\frac{4}{3}$, and we write the result as $x = -\frac{4}{3}$ and $x = -\frac{4}{3}$. We will get a double root when $b^2 = 4ac$, as in this case.

EXAMPLE 4 **Quadratic formula—imaginary roots**

Solve: $3x^2 - 5x + 4 = 0$.

In this example, $a = 3$, $b = -5$, and $c = 4$. Therefore,

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(3)(4)}}{2(3)} = \frac{5 \pm \sqrt{25 - 48}}{6} = \frac{5 \pm \sqrt{-23}}{6}$$

These roots contain imaginary numbers. This happens if $b^2 < 4ac$.

EXAMPLE 5 Quadratic formula—literal numbers

The equation $s = s_0 + v_0 t - \frac{1}{2}gt^2$ is used in the analysis of projectile motion (see Fig. 5). Solve for t.

$$gt^2 - 2v_0 t - 2(s_0 - s) = 0 \qquad \text{multiply by } -2\text{, put in form of Eq. (1)}$$

In this form, we see that $a = g$, $b = -2v_0$, and $c = -2(s_0 - s)$:

$$\begin{aligned} t &= \frac{-(-2v_0) \pm \sqrt{(-2v_0)^2 - 4g(-2)(s_0 - s)}}{2g} \\ &= \frac{2v_0 \pm \sqrt{4(v_0^2 + 2gs_0 - 2gs)}}{2g} \\ &= \frac{2v_0 \pm 2\sqrt{v_0^2 + 2gs_0 - 2gs}}{2g} \\ &= \frac{v_0 \pm \sqrt{v_0^2 + 2gs_0 - 2gs}}{g} \end{aligned}$$

EXAMPLE 1 Graphing a quadratic function

Graph the function $f(x) = x^2 + 2x - 3$.

First, let $y = x^2 + 2x - 3$. Then set up a table of values and graph the function as shown in Fig. 10. We can also display it on a calculator as shown in Fig. 11.

x	y
-4	5
-3	0
-2	-3
-1	-4
0	-3
1	0
2	5

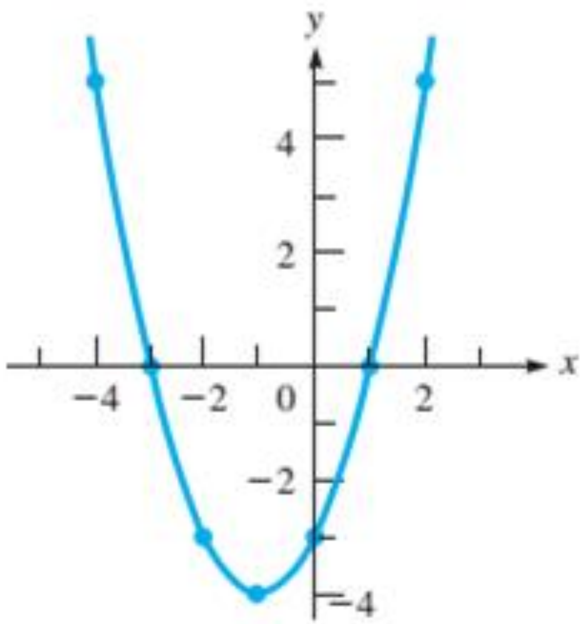


Fig. 10

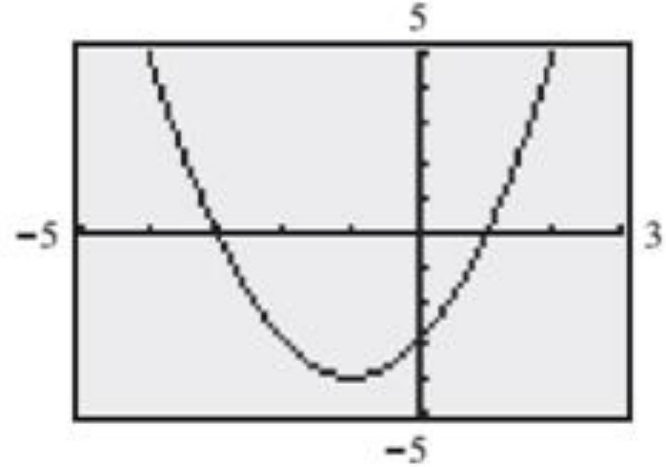


Fig. 11

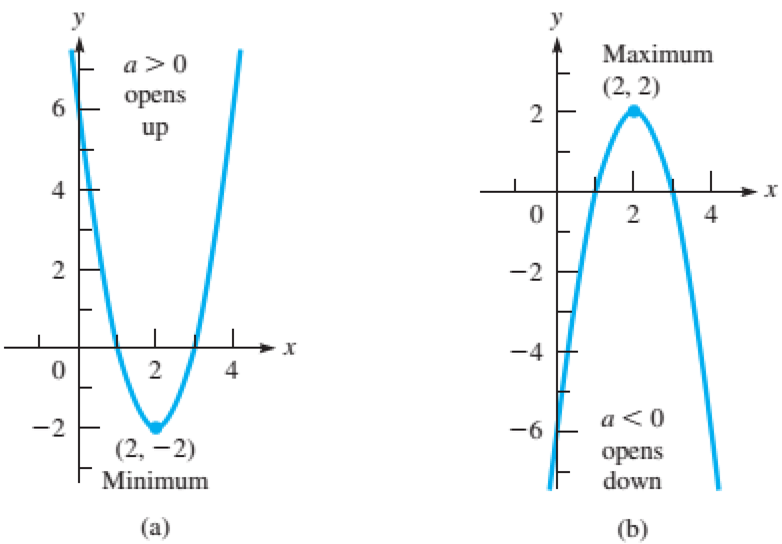


Fig. 12

EXAMPLE 2 Parabola—extreme points

The graph of $y = 2x^2 - 8x + 6$ is shown in Fig. 12(a). For this parabola, $a = 2$ ($a > 0$) and it opens upward. The vertex (a minimum point) is $(2, -2)$.

The graphs of $y = -2x^2 + 8x - 6$ is shown in Fig. 12(b). For this parabola, $a = -2$ ($a < 0$) and it opens downward. The vertex (a maximum point) is $(2, 2)$. ■

We can sketch the graphs of parabolas like these by using its basic shape and knowing two or three points, including the vertex. Even when using a graphing calculator, we can get a check on the graph by knowing the vertex and how the parabola opens.

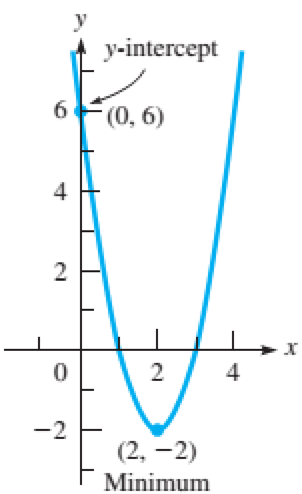


Fig. 13

Practice Exercise

EXAMPLE 3 Graphing a parabola—vertex—y-intercept

For the graph of the function $y = 2x^2 - 8x + 6$, find the vertex and y-intercept and sketch the graph. (This function is also used in Example 2.)

First, $a = 2$ and $b = -8$. This means that the x-coordinate of the vertex is

$$\frac{-b}{2a} = \frac{-(-8)}{2(2)} = \frac{8}{4} = 2$$

and the y-coordinate is

$$y = 2(2^2) - 8(2) + 6 = -2$$

Thus, the vertex is $(2, -2)$. Because $a > 0$, it is a minimum point.

Because $c = 6$, the y-intercept is $(0, 6)$.

We can use the minimum point $(2, -2)$ and the y-intercept $(0, 6)$, along with the fact that the graph is a parabola, to get an approximate sketch of the graph. Noting that a parabola increases (or decreases) away from the vertex in the same way on each side of it (it is *symmetric* to a vertical line through the vertex), we sketch the graph in Fig. 13. It is the same graph as that shown in Fig. 12(a). ■

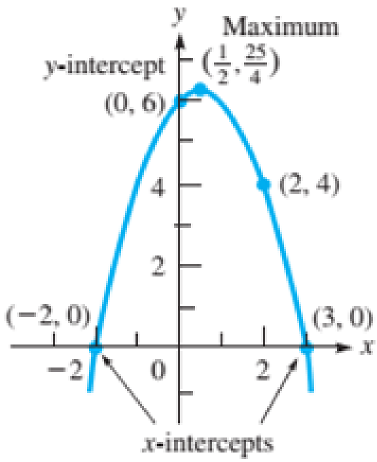


Fig. 14

EXAMPLE 4 Graphing a parabola—vertex—y-intercept

Sketch the graph of $y = -x^2 + x + 6$.

We first note that $a = -1$ and $b = 1$. Therefore, the x-coordinate of the maximum point $(a < 0)$ is $-\frac{1}{2(-1)} = \frac{1}{2}$. The y-coordinate is $-\left(\frac{1}{2}\right)^2 + \frac{1}{2} + 6 = \frac{25}{4}$. This means that the maximum point is $\left(\frac{1}{2}, \frac{25}{4}\right)$.

The y-intercept is $(0, 6)$.

Using these points in Fig. 14, because they are close together, they do not give a good idea of how wide the parabola opens. Therefore, setting $y = 0$, we solve the equation

$$-x^2 + x + 6 = 0 \qquad \text{multiply each term by } -1$$

or

$$x^2 - x - 6 = 0$$

This equation is factorable. Thus,

$$(x - 3)(x + 2) = 0$$
$$x = 3, -2$$

This means that the x-intercepts are $(3, 0)$ and $(-2, 0)$, as shown in Fig. 14.

Also, rather than finding the x-intercepts, we can let $x = 2$ (or some value to the right of the vertex) and then use the point$(2, 4)$. ■