Mathematics I (Maths 100)

Course Topics

1. **Basic algebraic operations**. Transposition. Rounding errors. Significant figures. Measurements. Scientific and engineering notation. Calculations. Conversion of numbers.

1. **Geometry**. Geometrical shapes and solids.

1. **Dimensional analysis**. Unit conversions, unit modifiers, and metric measurements such as conversion between metric and the U.S. customary system.

1. **Logarithms**. Laws of indices. Solving for the exponent. Solving for any variable, base e, and base 10. Exponential growth and decay.

1. **Quadratic equations**. Methods of solutions.

1. **Graphical solution of linear equations**.

Assessments

Weekly quizzes, weekly assignments, midterm examinations, and final examination.

Grading policy

Grading policy:

- Weekly Quizzes	(10%)
- Assignments	(10%)
- Midterm Exam 1	(20%)
- Midterm Exam 1	(20%)
- Final Exam	(40%)
Total	(100%)

At the end of Week 1

- I know what the Real Number System is.
- I know commutative, associated and distributed laws are.
- I can give examples of exact and approximate numbers.
- I can quote values to a number of significant figures.
- I can solve problems with exponents.
- I can solve problems with roots and radicals
- I can define the terminology in algebraic expressions.

Ch. 1.1: Numbers

- The Real Number System includes:
  - Natural numbers,
  - Whole numbers,
  - Positive & negative integers,
  - Rational & irrational numbers.
- We also work with complex numbers.

The Real Number System

Irrational Numbers:

$\sqrt{2}$ π e

Rationals: -1/2 1/8 0.56 0.999…

Whole Numbers.: 0, 1, 2, 3,

Naturals: 1, 2, 3, …

Pos./Neg. Integers:

…, -3, -2, -1, 0, 1, 2, 3, …

The Number Line

- Real numbers can be represented as points on a line.

Absolute Value

- The absolute value of a ***positive*** number is the number itself.
- The absolute value of a ***negative*** number is the corresponding positive number.
- It represents the ***distance*** from the number and zero on a number line.
- ***Example***: **|-3| = 3**

Signs of Inequality

- ***Greater than***:
- ***Example***: **5 > -1**

Signs of Inequality *(continued)*

- ***Less than***:
- ***Example***: **-3 < -1**

Example

Basic Technical maths(9)
Ex 1.1 q5-16

5. 3: integer, rational $\left(\frac{3}{1}\right)$, real

$\sqrt{-4}$: imaginary

$-\frac{\pi}{6}$: irrational, real

9. $6 < 8$

Answers

5. 3: integer, rational $\left(\frac{3}{1}\right)$, real

$\sqrt{-4}$: imaginary

$-\frac{\pi}{6}$: irrational, real

6. $-\sqrt{-6}$: imaginary

$-2.33 = -\frac{233}{100}$: rational, real

$\frac{\sqrt{7}}{3}$: irrational

7. $|3| = 3$, $|-4| = 4$, $\left|-\frac{\pi}{2}\right| = \frac{\pi}{2}$

8. $|-0.857| = 0.857$, $|\sqrt{2}| = \sqrt{2}$, $\left|-\frac{19}{4}\right| = \frac{19}{4}$

9. $6 < 8$

10. $7 > 5$

11. $\pi > -3.2$, $\pi = 3.14$ is greater than -3.2.

12. $-4 < 0$

13. $-|-3| = -3 \Rightarrow -4 < -3 = -|-3|$

14. $-\sqrt{2} > -1.42$

15. $-\frac{1}{3} > -\frac{1}{2}$

16. $-0.6 < 0.2$

Reciprocals

- Every number, except zero, has a reciprocal.
- The reciprocal of a number is **1** divided by the number.
- ***Example***:
- The reciprocal of **12** is $\frac{1}{12}$

a unit fraction

Denominate Numbers

- Numbers which represent a measurement and are written with units.
- ***Example***:
- A temperature of **25** degrees Celsius

25°C

Literal Numbers

- We use literal numbers to represent the wording of numbers.
- ***Example***:
- Five times a ***number***, where ***x*** is that number

5*x*

Example

Ex 1.1 q 17 and 18

17. The reciprocal of $3 = \frac{1}{3}$. The reciprocal of

$-\frac{4}{\sqrt{3}}$ is $\frac{1}{-\frac{4}{\sqrt{3}}} = -\frac{\sqrt{3}}{4}$.

The reciprocal of $\frac{y}{b}$ is $\frac{1}{\frac{y}{b}} = \frac{b}{y}$.

18. The reciprocal of $-\frac{1}{3}$ is $\frac{1}{-\frac{1}{3}} = -3$.

The reciprocal of 0.25 is $\frac{1}{0.25} = 4$.

The reciprocal of x is $\frac{1}{x}$.

Ch. 1.2: Fundamental Operations of Algebra

- The commutative law of addition:
  - ***Definition***:
  - $a + b = b + a$

- The associative law of addition:
  - ***Definition***:
  - $(a + b) + c = a + (b + c)$

Fundamental Operations of Algebra *(continued)*

- The commutative law of multiplication:
  - ***Definition***:
  - $a \times b = b \times a$
- The associative law of multiplication:
  - ***Definition***:
  - $(a \times b) \times c = a \times (b \times c)$

Fundamental Operations of Algebra *(continued)*

- The distributive law of multiplication over addition:
  - ***Definition***:
  - $a\,(b + c) = ab + ac$

Operations on Positive & Negative Numbers

- Addition of 2 numbers of the same sign:
  - Add their absolute values & assign the sum their common sign.
- Addition of 2 numbers of different signs:
  - Subtract the number of smaller absolute value from the number of larger absolute value & assign to the result the sign of the number of larger absolute value.

Operations on Positive & Negative Numbers *(continued)*

- Subtraction of 1 number from another:
  - Change the sign of the number being subtracted & change the subtraction to addition.
  - Perform the addition.

Operations on Positive & Negative Numbers *(continued)*

- Multiplication & division of 2 numbers:
  - The product (or quotient) of 2 numbers of the same sign is ***positive***.
  - The product (or quotient) of 2 numbers of different signs is ***negative***.

Order of Operations

1. Operations within specific groupings are done first.
2. Perform multiplications and divisions (from left to right).
3. Then perform additions and subtractions (from left to right).

Operations with Zero

- Working with addition & subtraction:
  - $a + 0 = a$
  - $a - 0 = a$; $0 - a = -a$
- Working with multiplication & division:
  - $a \times 0 = 0$
  - $0 \div a = 0$; (if $a \neq 0$)

Division by zero is undefined.

Example

Ex 1.2 q 5-10 and 33 to 36

5. $8+(-4)=8-4=4$

33. $-7-\dfrac{|-14|}{2(2-3)}-3|6-8|=-7-\dfrac{14}{2(-1)}-3|-2|$

$$=-7-\frac{14}{-2}-3(2)$$

$$=-7-(-7)-6$$

$$=0-6$$

$$=-6$$

Answers

5. $8+(-4)=8-4=4$

6. $-4+(-7)=-11$

7. $-3+9=-(3-9)=-(-6)=6$

8. $18-21=-3$

9. $-19-(-16)=-19+16=-3$

10. $8-(-4)=8+4=12$

33. $$-7-\frac{|-14|}{2(2-3)}-3|6-8|=-7-\frac{14}{2(-1)}-3|-2|$$
$$=-7-\frac{14}{-2}-3(2)$$
$$=-7-(-7)-6$$
$$=0-6$$
$$=-6$$

34. $$-7(-3)+\frac{6}{-3}-(-9)=21+(-2)+9$$
$$=19+9=28$$

35. $$\frac{3(-9)-2(-3)}{3-10}=\frac{-(3\times 9)+(2\times 3)}{-7}$$
$$=\frac{-27+6}{-7}$$
$$=+\left(\frac{27-6}{7}\right)$$
$$=\frac{21}{7}=3$$

36. $$\frac{20(-12)-40(-15)}{98-|-98|}=\frac{-240-(-600)}{98-98}$$
$$=\frac{360}{0}\text{ is undefined}$$

Ch. 1.3: Calculators & Approximate Numbers

- You will want to use one of two general types of calculators:
  - A scientific calculator
  - A graphing calculator
- Be sure to keep the manual for reference.
- Otherwise, check for one on the internet.

Approximate & Exact Numbers

- The final result of a calculation should not be written with any more accuracy than is proper.
- We use:
  - ***Approximate numbers***
  - ***Exact numbers***

Approximate Numbers

- ***Approximate numbers*** are determined by some measurement.
  - ***Example***: a shaft is approximately **1.75 *m*** in diameter.
- Many fractions are approximate.
  - ***Example***: **2/3 = 0.6667**
- Irrational numbers are approximate.
  - ***Example***: π = **3.1415927…**

Exact Numbers

- ***Exact numbers*** are determined by definition or by counting.
  - ***Example***: There are **24** hours in a day, no more no less.
  - ***Example***: A car has exactly **4** wheels.
- On the other hand, a certain town has population of approximately **3500** people***.***

Significant Digits

- Zeros are used in approximate numbers to properly locate the decimal point.
- Except for these zeros, all other digits are called ***significant digits***.

Significant Digits

- For example, the following numbers have **4 significant digits**.

497.3

39.05

8003

2.008

Notice how the zeros are surrounded by natural numbers.

Approximate & Exact Numbers

- When adding or subtracting approximate numbers, keep as many decimal places in your answer as contained in the number having the fewest decimal places.
- When multiplying 2 or more approximate numbers, round the result to as many digits as are in the factor having the fewest significant digits.

Accuracy & Precision

- Accuracy:
  - the number of significant digits a number has.
- Precision:
  - the decimal position of the last significant digit.

Example

- A measurement of **1.125** is more precise than a measurement of **1.13**.
- It (**1.125**) is also considered more precise since it is more accurate to **3** significant digits and **1.13** has **2** significant digits.

Accuracy:

the number of significant digits a number has.

Precision:

the decimal position of the last significant digit.

THIS IS FOR **APPROXIMATE** NUMBERS.

- When **adding or subtracting** approximate numbers, keep as many decimal places in your answer as contained in the number having the **fewest decimal places.**
- When **multiplying** 2 or more approximate numbers, round the result to as many digits as are in the factor having the **fewest significant figures.**

Example

2041.2 has 5 significant figures and 1 decimal place. 0.005 has 1 significant figure an 3 decimal places

Adding them

2041.2 + 0.005 = 2041.205

BUT the fewest decimal places is 1 (2041.2) so our answer is quoted to 1 decimal place = **2041.2**

Multiply them

2041.2 * 0.005 = 10.206

BUT 0.005 has only **1** significant figure so the answer is = **10**

Highest = 2041.244 * 0.00544 = 11.104

Lowest = 2041.150 * 0.00450 = 9.185

Rounding

- To round off a number to a specified number of significant digits, discard all digits to the right of the last significant digit (replace them with zeros if needed to properly place the decimal point).
    - If the first digit discarded is 5 or more, increase the last significant digit by 1 (***round up***).
    - If the first digit discarded is less than 5, do not change the last significant digit (***round down***).

Operations with Approximate Numbers

1. When approximate number are added or subtracted, the result is expressed with the precision of the least precise number.
2. When approximate numbers are multiplied or divided, the result is express with the accuracy of the least accurate number.

Operations with Approximate Numbers

3. When the root of an approximate number is found, the result is expressed with the accuracy of the number. (number of significant figures)
4. When approximate numbers and exact numbers are involved, the accuracy of the result is limited only by the approximate numbers.

Example

Ex 1.3 q 9-20

9. 107 has 3 significant digits. 3004 has 4 significant digits.

9. 107 has 3 significant digits. 3004 has 4 significant digits.

10. 3600 has 2 significant digits
730 has 2 significant digits

11. 6.80 has 3 significant digits; the zero indicates precision.
6.08 has 3 significant digits; the zero is not used for decimal location, and is not a place-holder only.

12. 0.8735 has 4 significant digits
0.0075 has 2 significant digits

13. 3000 has 1 significant digit. 3000.1 has 5 significant digits.

14. 1.00 has 3 significant digits
0.01 has 1 significant digit

15. (a) 0.01 is more precise (more decimal places).
(b) 30.8 is more accurate (more significant digits).

16. (a) 0.041 and 7.673 have the same precision
(b) 7.673 is more accurate than 0.041

17. (a) Both numbers have the same precision with digits in the tenths place.
(b) 78.0 with 3 significant digits is more accuate than 0.1 with 1 significant digit.

18. (a) 0.004 is more precise than 7040
(b) 7040 is more accurate than 0.004

19. (a) 0.004 is more precise (more decimal places).
(b) Both have the same accuracy

20. (a) 50.060 and 8.914 have the same precision
(b) 50.060 is more accurate than 8.914

Ch. 1.4: Exponents

- We use exponents to demonstrate when a number is multiplied by itself n times.

base → a^n ← exponent

- Only exponents of the same base may be combined.

Laws of Exponents

- Product Law: $a^m \times a^n = a^{m+n}$

- Quotient Law: $\dfrac{a^m}{a^n} = a^{m-n}$ $m > n,\ a \neq 0$

$$\frac{a^m}{a^n} = \frac{1}{a^{n-m}} \qquad m < n,\ a \neq 0$$

Laws of Exponents (*continued*)

- Power Law:

$$(a^m)^n = a^{mn}$$

$$(ab)^n = a^n b^n$$

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n} \qquad b \neq 0$$

Zero & Negative Exponents

- Any number or variable raised to a zero exponent **0** is equal to **1**.
  - That is: $a^0 = 1$, $a \neq 0$.
- A negative exponent is defined by:

$$a^{-n} = \frac{1}{a^n}$$

Order of Operations

1. Operations within specific groupings
2. Powers
3. Multiplications and divisions (from left to right)
4. Additions and divisions (from left to right)

Evaluating Algebraic Expressions

- An algebraic expression is evaluated by substituting given values of the literal numbers in the expression and calculating the result.

Example

- Evaluate the following algebraic expression when $x = -1$.

$$5x^3 + 7x^2 - 2x + 1$$

$$= 5(-1)^3 + 7(-1)^2 - 2(-1) + 1$$

$$= -5 + 7 + 2 + 1$$

$$= 5$$

Example

Ex 1.4 q 5 -12

5. $x^3 \cdot x^4 = x^{3+4} = x^7$

5. $x^3 \cdot x^4 = x^{3+4} = x^7$

6. $y^2 y^7 = y^{2+7} = y^9$

7. $2b^4 b^2 = 2b^{4+2} = 2b^6$

8. $3k^5(k) = 3k^{5+1} = 3k^6$

9. $\frac{m^5}{m^3} = m^{5-3} = m^2$

10. $\frac{x^6}{x} = 2x^{6-1} = 2x^5$

11. $\frac{n^5}{7n^9} = \frac{n^{5-9}}{7} = \frac{n^{-4}}{7} = \frac{1}{7n^4}$

12. $\frac{3s}{s^4} = 3s^{1-4} = 3s^{-3} = \frac{3}{s^3}$

Ch. 1.5: Scientific Notation

- How is a very large or a very small number expressed?
- We express the number in ***scientific notation***.
- We use $\boldsymbol{P} \times \mathbf{10}^{\boldsymbol{k}}$
- The exponent of **10** tells us how many decimal places are in the number.

Examples

1. $6.873 \times 10^{11} = 687\ 300\ 000\ 000$
2. $5.67 \times 10^{-6} = .000\ 005\ 67$

- Check to see how you can use your calculator to express numbers in scientific notation and perform multiplication & division with numbers that are expressed in scientific notation.

Example

Ex 1.5 q 29 to 36

29. $1280(865{,}000)(43.8) = 4.85 \times 10^{10}$

29. $1280(865{,}000)(43.8) = 4.85\times10^{10}$

30. $0.0000569(3{,}190{,}000) = 1.82\times10^{2}$

31. $\dfrac{0.0732(6710)}{0.00134(0.0231)} = \dfrac{7.32\times10^{-2}\times6.71\times10^{3}}{1.34\times10^{-3}\times2.31\times10^{-2}}$

$= 1.59\times10^{7}$

32. $\dfrac{0.00452}{2430(97{,}100)} = 1.92\times10^{-11}$

33. $(3.642\times10^{-8})(2.736\times10^{5}) = 9.965\times10^{-3}$

34. $\dfrac{(7.309\times10^{-1})^{2}}{5.9843(2.5036\times10^{-20})} = 3.566\times10^{18}$

35. $\dfrac{(3.69\times10^{-7})(4.61\times10^{21})}{0.0504} = 3.40\times10^{16}$

36. $\dfrac{(9.907\times10^{7})(1.08\times10^{12})^{2}}{(3.603\times10^{-5})(2054)} = 1.56\times10^{33}$

Ch. 1.6: Roots & Radicals

- The **square** root of a number x is one of **two** equal factors whose product is x.

$$\sqrt{144} = \sqrt{2 \times 2 \times 2 \times 2 \times 3 \times 3}$$
$$= \sqrt{4} \times \sqrt{4} \times \sqrt{9}$$
$$= 2 \times 2 \times 3 = \mathbf{12}$$

Because 12 x 12 = 144

Roots & Radicals

- The **cube** root of a number *x* is one of **three** equal factors whose product is *x*.

$$\sqrt[3]{216} = \sqrt{2 \times 2 \times 2 \times 3 \times 3 \times 3}$$
$$= \sqrt[3]{8} \times \sqrt[3]{27}$$
$$= 2 \times 3 = \mathbf{6}$$

Because 6 x 6 x 6 = 216

We usually use prime numbers to give us our simplest answer.

A **prime number** (or a prime) is a natural number greater than 1 that has no positive divisors other than 1 and itself.

n Roots

- The ***n***th root of a number ***a*** is one of ***n*** equal factors whose product is ***a***. This is denoted by:

$$\sqrt[n]{a}$$

index

radicand

radical sign

Properties of Roots & Radicals

1. We define the principal n^{th} root of a to be positive if a is positive & to be negative if a is negative and n is odd.
2. The square root of a product of positive numbers is the product of their square roots.

$$\sqrt{ab} = \sqrt{a}\sqrt{b}$$

Example

Ex 1.6 q 17 to 24

17. $\left(\sqrt{5}\right)^2 = 5$

16. $\sqrt[5]{-32} = -2$

17. $\left(\sqrt{5}\right)^2 = 5$

18. $\left(\sqrt[3]{31}\right)^3 = \sqrt[3]{31}\sqrt[3]{31}\sqrt[3]{31} = 31$

19. $\left(-\sqrt[3]{-47}\right)^3 = (-1)^3\left(\sqrt[3]{-47}\right)^3 = -1(-47) = 47$

20. $\left(\sqrt[5]{-23}\right)^5 = -23$

21. $\left(-\sqrt[4]{53}\right)^4 = 53$

22. $-\sqrt{32} = -\sqrt{16\cdot 2} = -4\sqrt{2}$

23. $\sqrt{1200} = \sqrt{400(3)} = 20\sqrt{3}$

24. $\sqrt{50} = \sqrt{25\cdot 2} = 5\sqrt{2}$

Ch. 1.7: Addition & Subtraction of Algebraic Expressions

- We can work with algebraic expressions as we would with any real numbers.
- Be sure to follow the order of operations.
- Collect like terms watching for any negative terms.

Terminology used with Algebraic Expressions

- Monomial:
  - an algebraic expression containing only one term.
  - ***Examples***: $3\boldsymbol{x}^5$, 7, $-15\boldsymbol{x}^2$
- Binomial:
  - an algebraic expression containing two terms.
  - ***Examples*** : $3\boldsymbol{x}^5 - 4\boldsymbol{x}$, $2\boldsymbol{x} + 7$, $-15\boldsymbol{x}^2 - 20$

Terminology used with Algebraic Expressions (*continued*)

- Trinomial:
  - an algebraic expression containing three terms.
  - ***Examples***: $3\boldsymbol{x}^5 - 4\boldsymbol{x} + 1$, $\boldsymbol{x}^3 - 2\boldsymbol{x} + 7$
- Multinomial:
  - Any expression containing two or more terms.
  - ***Examples*** : $3\boldsymbol{x}^5 - 8\boldsymbol{x}^2 + 4\boldsymbol{x} + 1$

In mathematics, a **coefficient** is a multiplicative factor in some term of a polynomial, a series or any expression; it is usually a number, but in any case does not involve any variables of the expression. For instance in

$$7x^2 - 3xy + 1.5 + y$$

the first two terms respectively have the coefficients 7 and −3. The third term 1.5 is a constant. The final term does not have any explicitly written coefficient, but is considered to have coefficient 1,

Terminology used with Algebraic Expressions (*continued*)

- Coefficient:
  - The numbers & literal symbols multiplying any given factor in an algebraic expression.
- Numerical coefficient:
  - The product of all the numbers in explicit form.
- Similar or like terms:
  - All terms that differ at most in their numerical coefficients.

Addition & Subtraction of Algebraic Expressions

- In adding and subtracting algebraic expressions, we combine similar (or like) terms into a single term.
- The final simplified expression will contain only terms that are not similar.

Example

- Simplify by collecting like terms:

Notice how the sign of each term in the second algebraic expression is reversed.

$$(-6x^4 + 3x^2 + 6) - (2x^4 + 5x^3 - 5x^2 + 7)$$

$$= -6x^4 + 3x^2 + 6 - 2x^4 - 5x^3 + 5x^2 - 7$$

$$= (-6x^4 - 2x^4) - 5x^3 + (3x^2 + 5x^2) + (6 - 7)$$

$$= -8x^4 - 5x^3 + 8x^2 - 1$$

Example

Ex 1.7 q 5 to 12

5. $5x + 7x - 4x = 12x - 4x = 8x$

5. $5x+7x-4x=12x-4x=8x$

6. $6t-3t-4t=-t$

7. $2y-y+4x=y(2-1)+4x=y+4x$

8. $4C+L-6C=-2C+L$

9. $2F-2T-2+3F-T=5F-3T-2$

10. $x-2y+3x-y+z=4x-3y+z$

11. $a^2b-a^2b^2-2a^2b=a^2b-2a^2b-a^2b^2=-a^2b-a^2b^2$

12. $xy^2-3x^2y^2+2xy^2=3xy^2-3x^2y^2$

Ch. 1.8: Multiplication of Algebraic Expressions

- To find the products of two or more monomials, we use the laws of exponents & the laws for multiplying signed numbers.
  1. Multiply the numerical coefficients.
  2. Multiply the literal numbers.
  3. Combine any exponents when bases are the same.

Example

- Multiply:

1. Multiply the numerical coefficients.

$$5x^3(9x^2 - 7x)$$

2. Multiply the literal numbers.

$$= 45x^5 - 35x^4$$

3. Combine any exponents when bases are the same.

Multiplication of Algebraic Expressions

- When working with multinomials, be sure to use the distributive property over each term.
- ***Example***:

$$(x-3)^2 = (x-3)(x-3)$$

$$= x^2 - 3x - 3x + 9$$

$$= x^2 - 6x + 9$$

Ex 1.8 q 19-22, 55 and 56

19. $$\begin{aligned} ax\left(cx^2\right)\left(x+y^3\right) &= ax\left(cx^2\left(x\right)+cx^2\left(y^3\right)\right) \\ &= ax\left(cx^3\right)+ax\left(cx^2y^3\right) \\ &= acx^4+acx^3y^3 \end{aligned}$$

20. $-2\left(-3st^3\right)(3s-4t)=6st^3(3s-4t)=18s^2t^3-24st^4$

21. $(x-3)(x+5)=x^2+5x-3x-15=x^2+2x-15$

22. $(a+7)(a+1)=a^2+8a+7$

55. $$\begin{aligned}(x+y)^3&=(x+y)(x+y)(x+y)\\&=\left(x^2+2xy+y^2\right)(x+y)\\&=x^3+x^2y+2x^2y+2xy^2+xy^2+y^3\\&=x^3+3x^2y+3xy^2+y^3\neq x^3+y^3\end{aligned}$$

56. $$\begin{aligned}&(x+y)\left(x^2-xy+y^2\right)\\&=x^3-x^2y+xy^2+x^2y-xy^2+y^3\\&=x^3+y^3\end{aligned}$$

Ch. 1.9: Division of Algebraic Expressions

- When dividing algebraic expressions once again use the laws of exponents and the laws for dividing signed numbers.
- Combine the exponents if the bases are the same.

Division of Algebraic Expressions

- The quotient of a multinomial divided by a monomial is found by dividing each term of the multinomial by the monomial and adding the results.
- That is:

$$\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$$

Polynomials in x

- If each term in an algebraic sum is a number or is of the form ax^n, where n is a nonnegative integer.
- ***Degree of the polynomial***:
  - The greatest value of the exponent *n* that appears.

Division of One Polynomial by Another

This is long division using polynomials

First watch a video on long division

http://www.youtube.com/watch?v=FXgV9ySNusc

Then we will look at a procedure.

Division of One Polynomial by Another

- Arrange the dividend & the divisor in descending powers of the variable.
- We divide similarly as we would with long division.

Example

- We are asked to solve

$$\left(6x^2 - 13x + 7\right) \div \left(x + 1\right)$$

- Note that each polynomial is arranged in descending order of powers.

quotient.

divisor.

$$\begin{array}{r} 6x - 19 \\ x + 1 \overline{\smash{)}\, 6x^2 - 13x + 7} \\ \underline{6x^2 + 6x \phantom{{}+7}} \\ -19x + 7 \\ \underline{-19x - 19} \\ 26 \end{array}$$

dividend

remainder → 26

1. Arrange the dividend & the divisor in descending powers of the variable. We divide similarly as we would with long division.
2. Divide the first term of the dividend by the first term of the divisor. This gives us our first term in the quotient.
3. Multiply the entire divisor by the first term of the quotient. Subtract this product from the dividend.
4. Divide the first term of the difference by the first term of the divisor. This gives us the second term.
5. Multiply the entire divisor by this term of the quotient. Subtract this product from the difference.
6. This gives the remainder (repeat if needed)

Solution

$$
\begin{array}{r|l}
 & 6x - 19 \\
\hline
x + 1 & 6x^2 - 13x + 7 \\
 & 6x^2 + 6x \\
\cline{2-2}
 & \quad -19x + 7 \\
 & \quad -19x - 19 \\
\cline{2-2}
\text{remainder} \rightarrow & \qquad 26
\end{array}
$$

Final Answer

- Therefore, the quotient for:

$$\left(6x^2 - 13x + 7\right) \div \left(x + 1\right)$$

$$= 6x - 19 + \frac{26}{x+1}$$

remainder

① make sure in order $x^2 \to x^1 \to x^0$

RECAP

Solution

Quotient

$$\begin{array}{r} 6x - 19 \\ x+1 \overline{\big) 6x^2 - 13x + 7} \\ 6x^2 + 6x \\ \hline -19x + 7 \\ -19x - 19 \\ \hline 26 \end{array}$$

← Divident

Devisor

remainder → 26

② $\frac{6x^2}{x} = 6x$ (first term of dividend ÷ first term of divisor)

③ multiply divisor by 1st term quotient $6x(x+1)$

subtract this from above

④ Next term is $\frac{-19x}{x} = -19$

⑤ multiply devisor by this term $-19(x+1)$ then subtract

⑥ this gives remainder [If no more x terms stop]

ans $= 6x - 19 + \frac{26}{x+1}$

Ex 1.9 questions 7,8,9 and 32

7. $$\frac{-16r^3t^5}{-4r^5t} = \frac{4t^{5-1}}{r^{5-3}} = \frac{4t^4}{r^2}$$

7. $\dfrac{-16r^3t^5}{-4r^5t} = \dfrac{4t^{5-1}}{r^{5-3}} = \dfrac{4t^4}{r^2}$

8. $\dfrac{51mn^5}{17m^2n^2} = \dfrac{3n^3}{m}$

9. $\dfrac{(15x^2)(4bx)(2y)}{30bxy} = 4x^2$

32.
$$\begin{array}{r} 2x+1 \\ 3x-4\,\big)\,6x^2-5x-9 \\ \underline{6x^2-8x} \\ 3x-9 \\ \underline{3x-4} \\ -5 \end{array}$$

$$(6x^2-5x-9)\div(3x-4) = 2x+1+\frac{-5}{3x-4}$$

Ch. 1.10: Solving Equations

- An equation is an algebraic statement that two algebraic expressions are equal.
- Any value of the literal numbers representing the ***unknown*** that produces equality when ***substituted*** in the equation is said to ***satisfy*** the equation.
- An equation valid only for certain values of the unknown is a ***conditional equation***.

Solving Equations

- To solve an equation we find the values of the unknown that satisfy it.
- ***Key Rule*** when solving equations.

Perform the same operation

on both sides of the equation.

Procedure for Solving Equations

1. Remove grouping symbols (distributive law).
2. Combine any like terms of each side (also after *step 3*).
3. Perform the same operations on both sides, until ***x = result*** is obtained.
4. Check the solution in the original equation.

Example

- Solve this linear equation:

$$3(x-4)-6(1-3x)=20$$

$$3x-12-6+18x=20$$

$$21x=38$$

$$x=\frac{38}{21}\cong 1.81$$

1. Remove grouping symbols (distributive law).
2. Combine any like terms of each side (also after *step 3*).
3. Perform the same operations on both sides, until ***x = result*** is obtained.

Check

- Solve this linear equation:

$$3\left(\frac{38}{21}-4\right)-6\left(1-3\times\frac{38}{21}\right)\overset{?}{=}20$$

$$20 \overset{\checkmark}{=} 20$$

Ex 1.10 questions 29-32, 50

29. $$\frac{4x-2(x-4)}{3}=8$$

$$4x-2(x-4)=24$$

$$4x-2x+8=24$$

$$2x=16$$

$$x=8$$

29. $\dfrac{4x-2(x-4)}{3}=8$

$$4x-2(x-4)=24$$
$$4x-2x+8=24$$
$$2x=16$$
$$x=8$$

30. $2x=\dfrac{3-5(7-3x)}{4}$

$$8x=3-5(7-3x)$$
$$8x=3-35+15x$$
$$-7x=-32$$
$$x=\frac{32}{7}$$

31. $|x|-1=8$

$$|x|=9$$
$$x=-9 \text{ or } x=9$$

32. $2-|x|=4$

$$-|x|=2$$
$$|x|=-2, \text{ no solution}$$

50. $210(3x)=55.3x+38.5(8.25-3x)$

$$630x=55.3x+317.625-115.5x$$
$$690.2x=317.625$$
$$x=0.46 \text{ m}$$

Ch. 1.11: Formulas & Literal Equations

- A ***formula*** is an equation that expresses the relationship between two or more related quantities.
- We can isolate the desired symbol by using algebraic operations on the literal numbers.
- When expected to substitute a given value into the formula, we should first isolate the given variable.

Example

- In the given formula, isolate for ***e***.

$$C = \frac{2eAk_1k_2}{d\left(k_1 + k_2\right)}$$

- ***Solution***:

1. We multiply both sides by $\boldsymbol{d(k_1 + k_2)}$.
2. Divide both sides by $\boldsymbol{2Ak_1k_2}$.

Solution

1. We multiply both sides by $d(k_1 + k_2)$
2. Divide both sides by $2Ak_1k_2$.

$$C = \frac{2eAk_1k_2}{d\left(k_1 + k_2\right)} \Rightarrow Cd\left(k_1 + k_2\right) = 2eAk_1k_2$$

$$\frac{Cd\left(k_1 + k_2\right)}{2Ak_1k_2} = e$$

Ex 1.11 questions 5-8, 35

5. $E = IR$

$$\frac{E}{I} = \frac{IR}{I}$$

$$R = \frac{E}{I}$$

5. $E = IR$

$$\frac{E}{I} = \frac{IR}{I}$$

$$R = \frac{E}{I}$$

6. $PV = nRT$

$$T = \frac{PV}{nR}$$

35. $$P = \frac{V_1(V_2 - V_1)}{gJ}$$

$$gJP = V_1V_2 - V_1^2$$

$$gJP + V_1^2 = V_1V_2$$

$$V_2 = \frac{gJP + V_1^2}{V_1}$$

7. $rL = g_2 - g_1$

$$g_1 = g_2 - rL$$

8. $W = S_dT - Q$

$$Q = S_dT - W$$

Ch. 1.12: Applied Word Problems

- Mathematical questions in science, biology, etc. do not present themselves in neat, tidy equations.
- They are often presented as word problems which must be solved.
- The following is a step-by-step approach that you can use to solve word problems.

Procedure for Solving Word Problems

1. Read the statement of the problem. First, read it for a general overview. Then read it slowly & carefully, listing the information provided.
2. Clearly identify the unknown quantities & then assign an appropriate letter to represent one of them, stating this choice clearly.
3. Specify the other unknown quantities in terms of the one in step 2.

Procedure for Solving Word Problems (*continued*)

4. If possible, make a sketch using the known & unknown quantities.
5. Analyze the statement of the problem & write the necessary equation.
6. Solve the equation, clearly stating the solution.
7. Check the solution with the original statement of the problem.

Example

- The sum of 3 electric currents that come together at a point in an integrated circuit is zero. If the second current is double the first & the third current is 9.2 μA more than the first, what are the currents?
- (The sign of a current indicates the direction of flow.)

Solution

1. Read the statement of the problem. First, read it for a general overview. Then read it again slowly & carefully, listing the information provided.

- ***Information provided***:
  - ✓ 3 separate electric currents.
  - ✓ Their sum total is zero.
  - ✓ The currents can be positive or negative.

Solution *(continued)*

2. Clearly identify the unknown quantities & then assign an appropriate letter to represent one of them, stating this choice clearly.

- ***Unknown quantities***:
  - ✓ First current: x
  - ✓ Each current listed is in terms of the first electric current.

Solution *(continued)*

3. Specify the other unknown quantities in terms of the one in step 2.

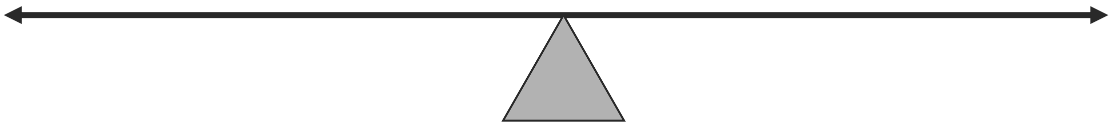
- ***Unknown quantities***:
  - ✓ First current: x
  - ✓ Second current: $2x$
  - ✓ Third current: $x + 9.2$

Solution *(continued)*

4. If possible, make a sketch using the known & unknown quantities.

- ***Sketch***:

First current + Second current + Third current = 0



Solution *(continued)*

5. Analyze the statement of the problem & write the necessary equation.

- ***The equation***:
  - ✓ $\boldsymbol{x} + 2\boldsymbol{x} + (\boldsymbol{x} + 9.2) = \mathbf{0}$

Solution *(continued)*

6. Solve the equation, clearly stating the solution.

- ***The equation***:

$x + 2x + (x + 9.2) = 0$

$x = -2.3$

  - ***Therefore***,
    - ✓ First current: $x = -2.3$
    - ✓ Second current: $2(-2.3) = -4.6$
    - ✓ Third current: $(-2.3 + 9.2) = 6.9$

Solution *(continued)*

7. Check the solution with the original statement of the problem.

- ***Check***:

$$(-2.3) + (-4.6) + (-6.9) \overset{?}{=} 0$$

$$0 \overset{\checkmark}{=} 0$$

- ***Conclusion***: The solution is correct.

Ex 1.12 questions 5-6, 18

5. $x = \text{cost 6 years ago}$

$$x + 5000 = \text{cost today}$$

$$x + (x + 5000) = 49{,}000$$

$$2x + 5000 = 49{,}000$$

$$2x = 44{,}000$$

$$x = 22{,}000$$

$$x + 5000 = 27{,}000$$

\$22,000 six years ago   \$27,000 is the cost today

6. Let x = flow rate of first stream in ft^3/s;

$x+1700$ = flow rate of second stream in ft^3/s

$$(x+x+1700)\cdot 3600 = 1.98\times 10^7$$

$$x = 1900\ \text{ft}^3/\text{s}$$

$$x+1700 = 3600\ \text{ft}^3/\text{s}$$

18.

$$G_1+G_2 = 750 \Rightarrow G_2 = 750-G_1$$

$$0.65G_1+0.75G_2 = 530$$

$$0.65G_1+0.75(750-G_1) = 530$$

$$0.65G_1+562.2-0.75G_1 = 530$$

$$-0.10G_1 = -32.5$$

$$G_1 = 325\ MW$$

$$G_2 = 750-G_1 = 425\ MW$$

SUMMARY BELOW

Irrational Numbers:

$\sqrt{2}$ π e

Can't be written as one integer divided by another

Rationals: -1/2 1/8 0.56 0.999…

Whole Numbers.: 0, 1, 2, 3,

Naturals: 1, 2, 3, …

Pos./Neg. Integers:

…, -3, -2, -1, 0, 1, 2, 3, …

ORDER of operations

1. Operations within specific groupings
2. Powers
3. Multiplications and divisions (from left to right)
4. Additions and divisions (from left to right)

.

- commutative $a + b = b + a$
- associative $(a + b) + c = a + (b + c)$
- distributive $a (b + c) = ab + ac$

$\sqrt[n]{a}$ — index, radicand, radical sign

Division by zero is undefined

Mon = 1
Bi = 2
Tri = 3

$$\sqrt{ab} = \sqrt{a}\sqrt{b}$$

Product Law: $a^m \times a^n = a^{m+n}$

Accuracy: number of significant digits
Precision: decimal position of the last significant digit.

Quotient Law: $\frac{a^m}{a^n} = a^{m-n}$ $m > n,$

base → a^n ← exponent

Approx = measure
Exact = count

Power Law: $(a^m)^n = a^{mn}$

$(ab)^n = a^n b^n$

Scientific notation $P \times 10^k$

Accuracy:

the number of significant digits a number has.

Precision:

the decimal position of the last significant digit.

When **adding or subtracting approximate** numbers, keep as many decimal places in your answer as contained in the number having the **fewest decimal places.**

When **multiplying** 2 or more approximate numbers, round the result to as many digits as are in the factor having the **fewest significant digits.**

THIS IS FOR **APPROXIMATE** NUMBERS.

Eg

2041.2 has 5 significant figures and 1 decimal place

0.005 has 1 significant figure an 3 decimal places

So to add them

2041.2 + 0.005 = 2041.205 BUT the fewest decimal places is 1 (2041.2) so our answer is quoted to 1 decimal place = **2041.2**

Multiply them

2041.2 * 0.005 = 10.206 BUT 0.005 has only **1** significant digit so the answer is = **10**

(Highest = 2041.244 * 0.00544 = 11.104

Lowest = 2041.150 * 0.00450 = 9.185**)**

- Multiply:

$$5x^3(9x^2 - 7x)$$

1. Multiply the numerical coefficients.

2. Multiply the literal numbers.

3. Combine any exponents when bases are the same.

$$= 45x^5 - 35x^4$$

$$\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$$

$$\begin{array}{r|l} & 6x \quad -19 \\ \hline x+1 & 6x^2 - 13x + 7 \\ & 6x^2 + 6x \\ \hline & \quad -19x + 7 \\ & \quad -19x - 19 \\ \hline & \quad\quad 26 \end{array}$$

remainder → 26

Perform the same operation on both sides of the equation.

1. Remove grouping symbols (distributive law).
2. Combine any like terms of each side (also after *step 3*).
3. Perform the same operations on both sides, until $x = result$ is obtained.
4. Check the solution in the original equation.

M1 W2

Ch 1