General Physics II

Electromagnetism

General Physics II: Electricity & Magnetism

I. Course Objective

This course gives an introduction to the electromagnetism. The objective of this course is to develop a deeper understanding of electromagnetism fields and techniques used to solve engineering problems in electromagnetism.

II. Course Description

This course will cover: Coulomb's law, the electrostatic field, Gauss’s Law, the electrostatic potential, capacitance and dielectrics, electric current, resistance and electromotive force, direct current circuits, magnetic field and magnetic forces, sources of magnetic fields, Ampere's Law, Faraday's Law, induction and Maxwell's equations.

III. Course Objectives

By the end of this course, students should be able to:

1. Describe the properties of electric charge, conductors and insulators, charge conservation and quantization.
2. Perform calculations on Coulomb's law, electric fields, electric forces and Gauss's law.
3. Demonstrate knowledge of the electric potential, equipotential surfaces, capacitors and types of capacitors.
4. Perform calculations on electric potential, capacitors and series/parallel capacitors in circuits.
5. Calculate current, potentials, resistances, and electromotive forces for simple circuits.
6. Describe the magnetic fields, forces, and potentials involved in the interaction of point charges and of currents.
7. Apply Faraday's Law, Ohm's Law, Kirchhoff's rules and Lenz's Law to solve problems in electromagnetism.

Textbook: Please take notes in class and use the textbook…

College Physics, Hugh D. Young

Course Topics

- **Electric Charge and Electric Field:** Properties of electric charge, Forces between electric charges and Coulomb's law, Definition of electric field, Calculation of electric fields, Motion of a charged particle in an electric field, Electric flux, Applications of Gauss's Law.
- **Electric Potential and Capacitance:** Electrical potential energy, Calculation of electric potential, Calculation of capacitance, Analysis of effects of dielectrics in capacitors.
- **Current, Resistance, and Dielectric Current Circuits:** Examination of current, current density, resistance and resistivity, Interpretation of Ohm's law, Analysis of electric networks via Kirchhof's rules.

- **Magnetic Field and Magnetic Forces:** Motion point charges under crossed electric and magnetic fields, Biot-Savart Law and its use for calculation of magnetic fields, Ampere’s Law and applications.
- **Electromagnetic Induction:** Use of Faraday’s Law and Lenz’s Law in calculating induced emf’s and currents, Motional emf, Mutual inductance, self-inductance and inductors.

Grading Policy

Your grade will be judged on your performance in Home work, Quizzes, one midterm exam and the final exam. Points will be allocated to each of these in the following manner:

GRADING SCALE:

Grade Component	Weight (%)
Assignments	10
Quizzes	20
Midterm Exam	30
Final Exam	40
Total	100

17 Electric Charge and Electric Field

Introduction

- Knowledge of electricity dates back to Greek antiquity (600 BC).
- Began with the realization that amber when rubbed with wool, attracts small objects.
- This phenomenon is not restricted to amber/wool but may occur whenever two non-conducting substances are rubbed together.

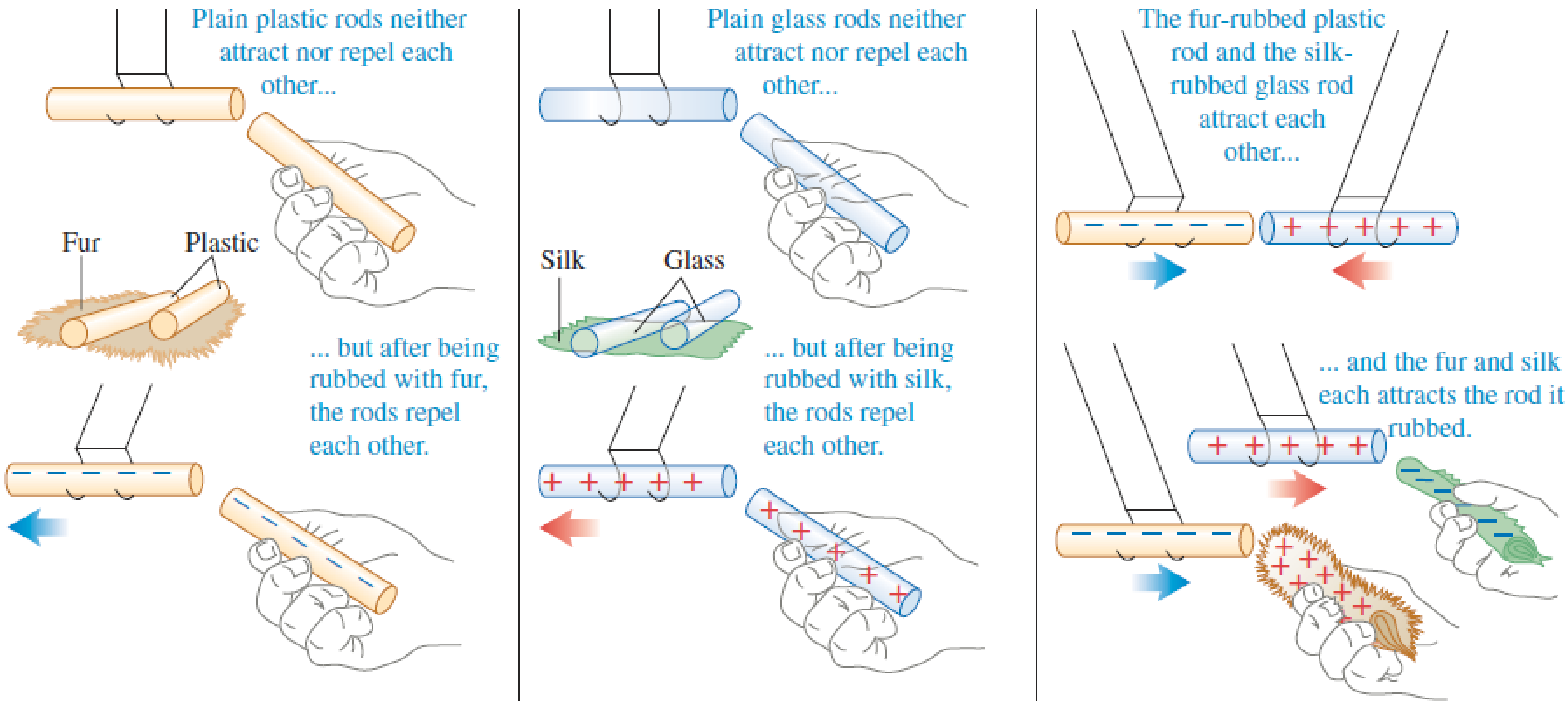
Charged Objects

When two objects are rubbed together, some electrons from one object move to another object. For example, when a plastic bar is rubbed with fur, electrons will move from the fur to the plastic stick. Therefore, the plastic bar will be negatively charged and the fur will be positively charged.

https://phet.colorado.edu/sims/html/balloons-and-static-electricity/latest/balloons-and-static-electricity_en.html

https://phet.colorado.edu/sims/html/john-travoltage/latest/john-travoltage_en.html

Transferring Charge



(a) Interaction between plastic rods rubbed on fur

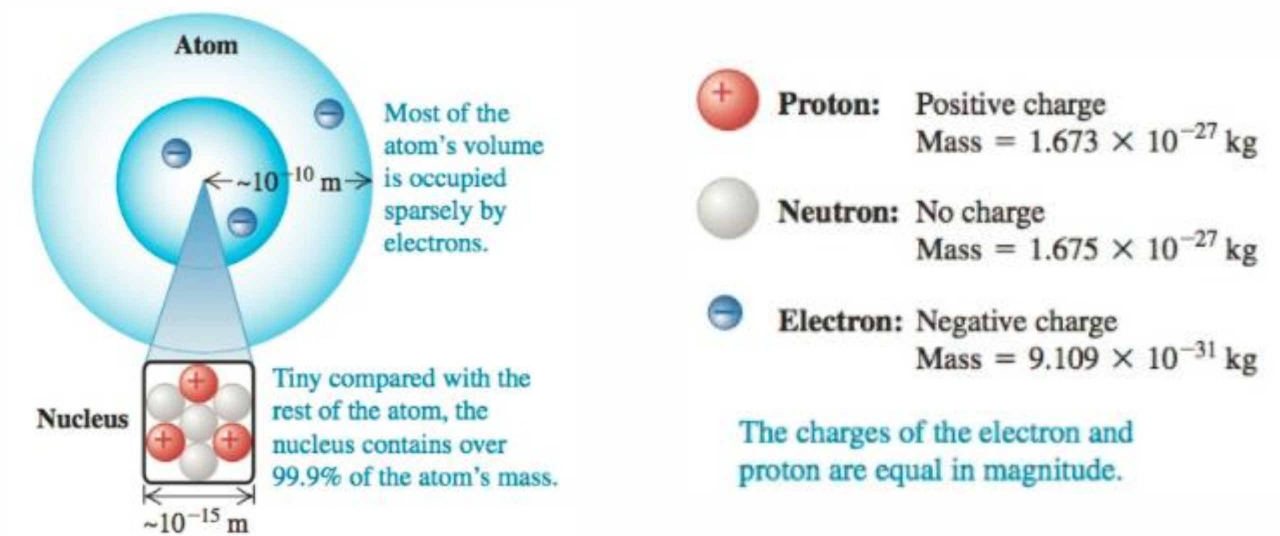
(b) Interaction between glass rods rubbed on silk

(c) Interaction between objects with opposite charges

17.1 Electrical Charge

Matter is made of atoms: An Atom is composed of three different components : electrons, protons, and neutrons. An electron can be removed easily from an atom

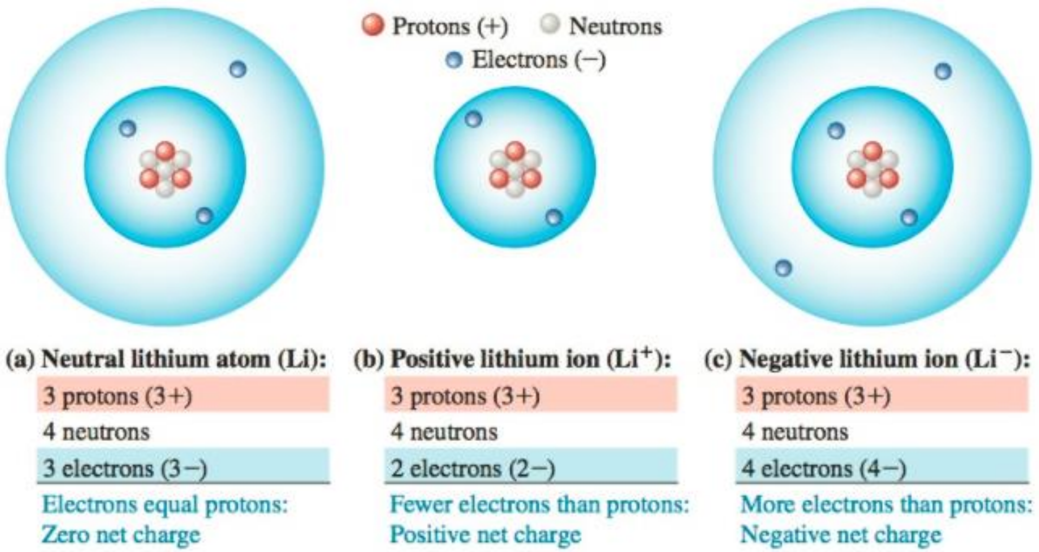
Normally, an atom is electrically neutral, which means that there are equal numbers of protons and electrons. Positive charge of protons is balanced by negative charge of electrons. It has no net electrical charge.



Charges and Atomic Structure

There are two kinds of electrical charges, positive and negative. Same charges (+ and +, or - and -) repel each other and opposite charges (+ and -) attract each other.

- A positive ion is an atom that has lost electrons.
- A negative ion is an atom that has gained electrons.



17.2 Conductors and Insulators

(Material classification)

- Materials/substances may be classified according to their capacity to carry or ***conduct*** electric charge:

- Conductors are material in which electric charges move freely.
  - Metals are good conductors: Copper, aluminum, and silver.

- Insulator are materials in which electrical charge do not move freely.
  - Most nonmetals are insulator: Glass, Rubber are good insulators.

- Semiconductors are a third class of materials with electrical properties somewhere between those of insulators and conductors.
  - Silicon and germanium are semiconductors used widely in the fabrication of electronic devices.

17.3 Conservation and Quantization of Charge

The Law of Conservation of Charge

The Law of conservation of charge states that the net charge of an isolated system remains constant.

Charge may only be transferred from one object to another.

17.3 Quantization

- Robert Millikan found, in 1909, that charged objects may only have an integer multiple of a fundamental unit of charge.
  - Charge is ***quantized***.
  - An object may have a charge ± 1e, or $\pm$ 2e, or $\pm$ 3e,
  - Not $\pm$ 1.5e.
  - Proton has a charge ***+1e***.
  - Electron has a charge ***–1e***.
  - Some particles such as ***neutrons*** have no (zero) charge.

"charge is quantized" in terms of an equation, we say:
q = n e

Unit of Electrical Charge: The Coulomb " C "

The symbol for electric charge is written q, - q or Q.

The unit of electric charge is coulomb "C". The charge of one electron is equal and opposite to the charge of one proton, which is **$1.6 * 10^{-19}$ C**.

This number is given a symbol "e".

$$\boldsymbol{e = 1.6 \times 10^{-19} C}$$

Extra examples:

1. **How many protons (units of charge) are there in 1 *C* of charge?**

2. **A metal sphere is insulated electrically and is given a charge. If 50 electrons are added to the sphere, how many Coulombs are added to the sphere?**

17.4 Coulomb's law

Coulomb's Law

Coulomb discovered in 1785 the fundamental law
of electrical force between two stationary charged particles.

An electric force has the following properties:

- Inversely proportional to the square of the separation, r, between the particles, and is along a line joining them.
- Proportional to the product of the magnitudes of the charges $|q_1|$ and $|q_2|$ on the two particles.
- Attractive if the charges are of opposite sign and repulsive if the charges have the same sign.

Coulomb's Law (Mathematical Formulation)

$$F \propto \frac{1}{r^2}$$

$$F \propto |q_1||q_2|$$

$$\Rightarrow$$

$$\boxed{F = k_e \frac{|q_1||q_2|}{r^2}}$$

- k_e known as the Coulomb constant.
- Value of k_e depends on the choice of units.
- SI units
  - **Force: the Newton (N)**
  - **Distance: the meter (m).**
  - **Charge: the coulomb (C).**

Coulomb's Constant $'K_e'$

$$K_e = \frac{1}{4\pi\varepsilon_\circ} = \frac{1}{4\pi \times 8.85\times 10^{-12}} = 9\times 10^{9}\, Nm^{2}/C^{2}$$

where ε_o is known as the *Permittivity constant of free space.*
$\varepsilon_o = 8.85 \times 10^{-12}\ C^2/N.m^2$ (or F/m)

Experimentally measurement: *$k_e = 8.9875\times10^9\ Nm^2/C^2$.*
Reasonable approximate value: $k_e = 8.99\times10^9\ Nm^2/C^2$.

Example: the Coulomb constant unit

$$F = k_e \frac{|q_1||q_2|}{r^2} \Rightarrow [F] = [k_e]\frac{[q_1][q_2]}{[r]^2}$$

$$[Newton] = [k_e]\frac{[Coulomb][Coulomb]}{[meter]^2}$$

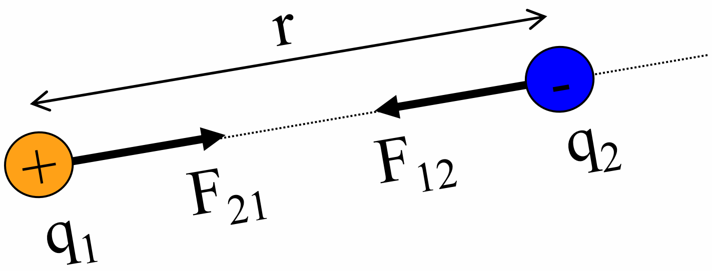
Then the Coulomb constant unit is

$$[k_e] = N \cdot m^2 / C^2$$

The electrostatic force

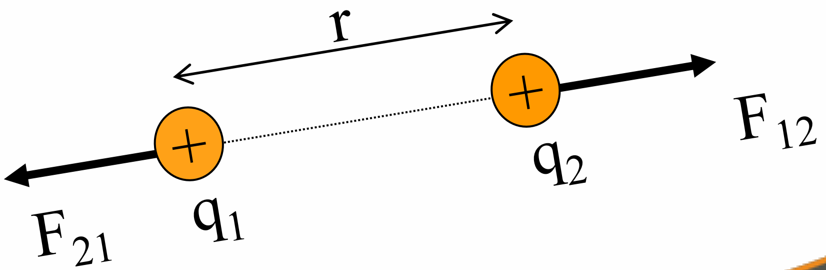
- The electrostatic force is often called Coulomb force.
- It is a force (thus, a vector):
  - a magnitude
  - a direction.

  - F_{21}: Force **from** 2 **on** 1



$$|F_{12}| = |F_{21}|$$

$$\vec{F}_{12} = -\vec{F}_{21}$$



Example

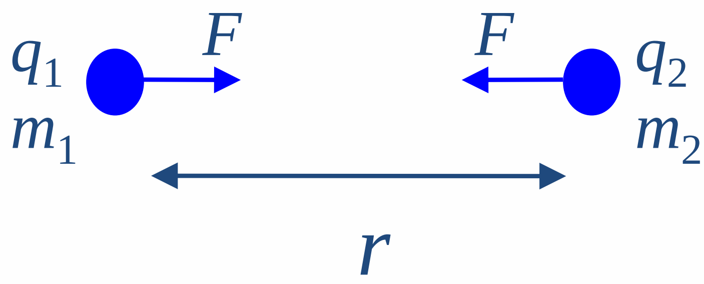
The electron and proton of a hydrogen atom are separated (on the average) by a distance of approximately $5.3\text{x}10^{-11}$ m. Find the magnitude of the electric force that each particle exerts on the other.

$q_1 = -1.60\text{x}10^{-19}$ C
$q_2 = 1.60\text{x}10^{-19}$ C
$r = 5.3\text{x}10^{-11}$ m

$$F_e = k_e \frac{|e|^2}{r^2} = 9\times10^9 \tfrac{Nm^2}{C^2} \frac{\left(1.6\times10^{-19}C\right)^2}{\left(5.3\times10^{-11}m\right)^2} = 8.2\times10^{-8} N$$

Attractive force with a magnitude of 8.2×10^{-8} N

Gravitational Force vs. Electrical Force between Electrons/Protons



$$F_{elec} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$$

$$F_{grav} = G\frac{m_1 m_2}{r^2}$$

$$\rightarrow \quad \frac{F_{elec}}{F_{grav}} = \frac{q_1 q_2}{m_1 m_2}\,\frac{\frac{1}{4\pi\varepsilon_0}}{G}$$

For an electron/proton:

$|q| = 1.6 \times 10^{-19}$ C

$m_e = 9.11 \times 10^{-31}$ kg

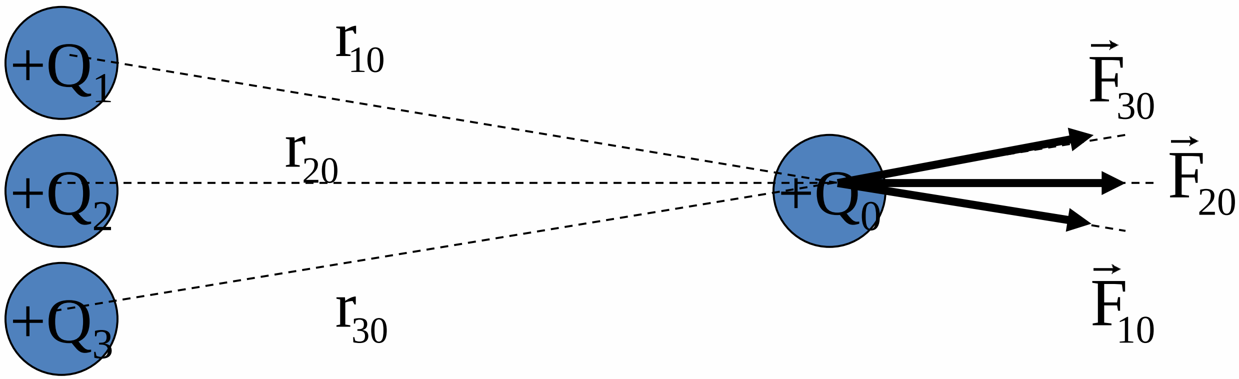
$m_p = 1.67 \times 10^{-27}$ kg

$G = 6.674 \times 10^{-11}$ Nm^2/kg^2

$$\rightarrow \quad \frac{F_{elec}}{F_{grav}} = 2.27 \times 10^{39}$$

Superposition of Forces

$$\vec{F}_T = \vec{F}_{10} + \vec{F}_{20} + \vec{F}_{30} + \dots$$

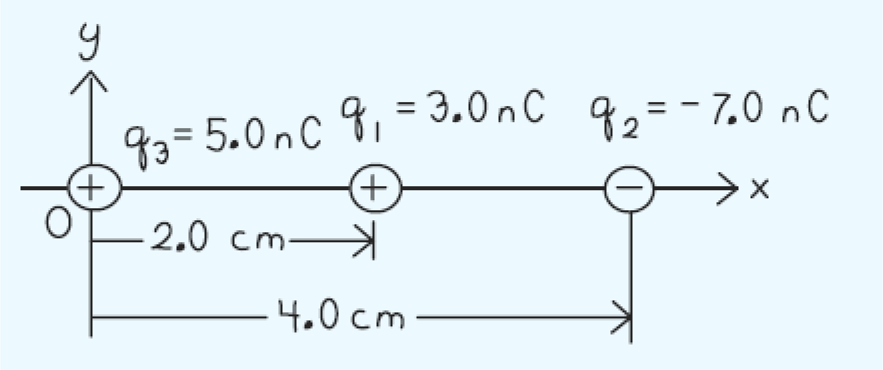


$$\vec{F}_T = \frac{kq_0q_1}{r_{10}^2}\hat{r}_{10} + \frac{kq_0q_2}{r_{20}^2}\hat{r}_{20} + \frac{kq_0q_3}{r_{30}^2}\hat{r}_{30} + \dots$$

$$\vec{F}_T = kq_0\left(\frac{q_1}{r_{10}^2}\hat{r}_{10} + \frac{q_2}{r_{20}^2}\hat{r}_{20} + \frac{q_3}{r_{30}^2}\hat{r}_{30} + \dots\right) = kq_0\sum_{i=1}^{N}\frac{q_i}{r_{i0}^2}\hat{r}_{i0}$$

Example

Two point charges are located on the positive x axis of a coordinate system. Charge $q_1 = 3$ nC is 2.0 cm from the origin, and charge $q_2 = -7$ nC is 4.0 cm from the origin. What is the total force (magnitude and direction) exerted by these two charges on a third point charge $q_3 = 5.0$ nC located at the origin?



We use Coulomb's law to find the magnitudes of the forces F_{13} and F_{23}

$$F_{13} = k\frac{|q_1 q_3|}{r_{12}^{\ 2}}$$

$$= (8.99 \times 10^9\ \mathrm{N \cdot m^2/C^2})\frac{(3.0 \times 10^{-9}\ \mathrm{C})(5.0 \times 10^{-9}\ \mathrm{C})}{(0.020\ \mathrm{m})^2}$$

$$= 3.37 \times 10^{-4}\ \mathrm{N}, \quad \text{(In the -x direction)}$$

$$F_{23} = k\frac{|q_2 q_3|}{r_{23}^{\ 2}}$$

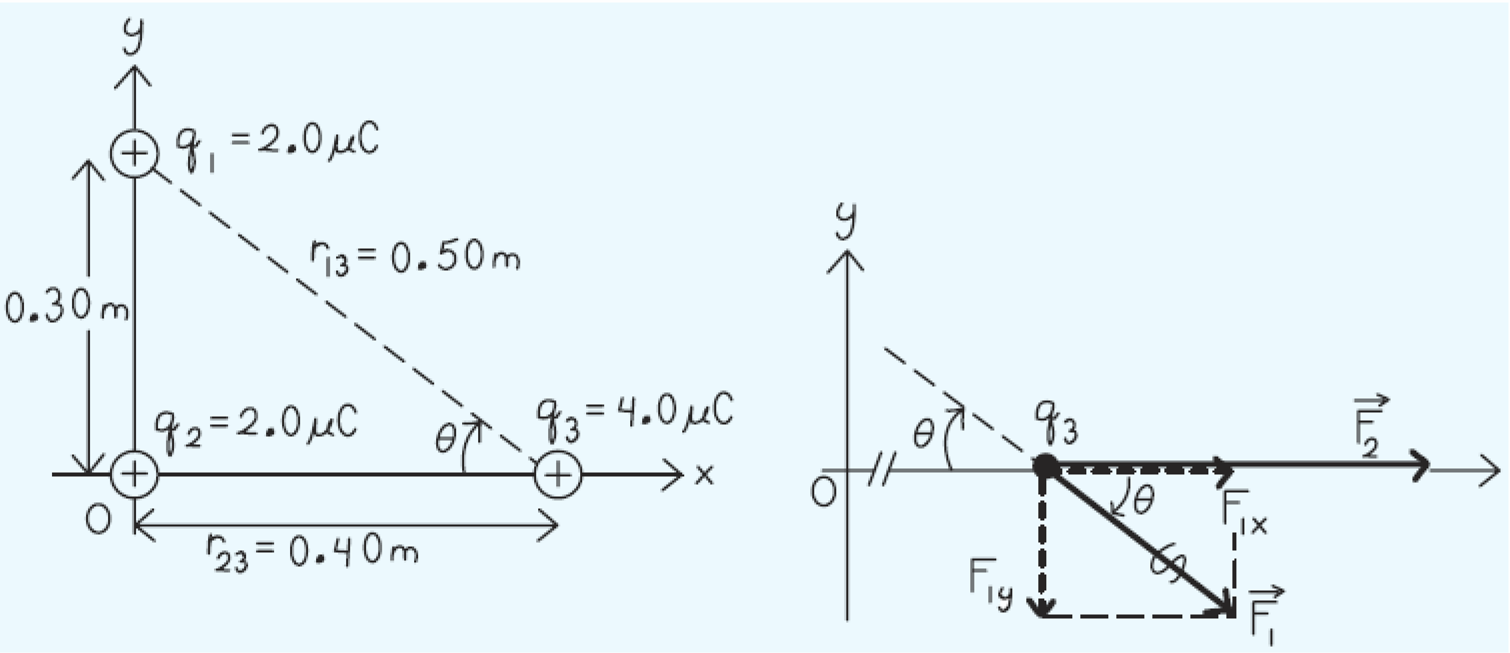
$$= (8.99 \times 10^9\ \mathrm{N \cdot m^2/C^2})\frac{(7.0 \times 10^{-9}\ \mathrm{C})(5.0 \times 10^{-9}\ \mathrm{C})}{(0.040\ \mathrm{m})^2}$$

$$= 1.97 \times 10^{-4}\ \mathrm{N}. \quad \text{(In the +x direction)}$$

$$F_{\text{total},x} = -3.37 \times 10^{-4}\ \mathrm{N} + 1.97 \times 10^{-4}\ \mathrm{N} = -1.40 \times 10^{-4}\ \mathrm{N}.$$

Example

A point charge $q_1 = 2.0\ \mu C$ is located on the positive *y* axis at $y = 0.30\ m$, and an identical charge q_2 at the origin. Find the magnitude and direction of the total force that these two charges exert on a third charge $q_3 = 4.0\ \mu C$ that is on the positive *x* axis at $x = 0.40\ m$.



$$F_1 = k\frac{|q_1 q_3|}{r_{13}{}^2}$$

$$= (8.99 \times 10^9\ \mathrm{N \cdot m^2/C^2})\frac{(2.0 \times 10^{-6}\ \mathrm{C})(4.0 \times 10^{-6}\ \mathrm{C})}{(0.50\ \mathrm{m})^2}$$

$$= 0.288\ \mathrm{N},$$

$$F_2 = k\frac{|q_2 q_3|}{r_{23}{}^2}$$

$$= (8.99 \times 10^9\ \mathrm{N \cdot m^2/C^2})\frac{(2.0 \times 10^{-6}\ \mathrm{C})(4.0 \times 10^{-6}\ \mathrm{C})}{(0.40\ \mathrm{m})^2}$$

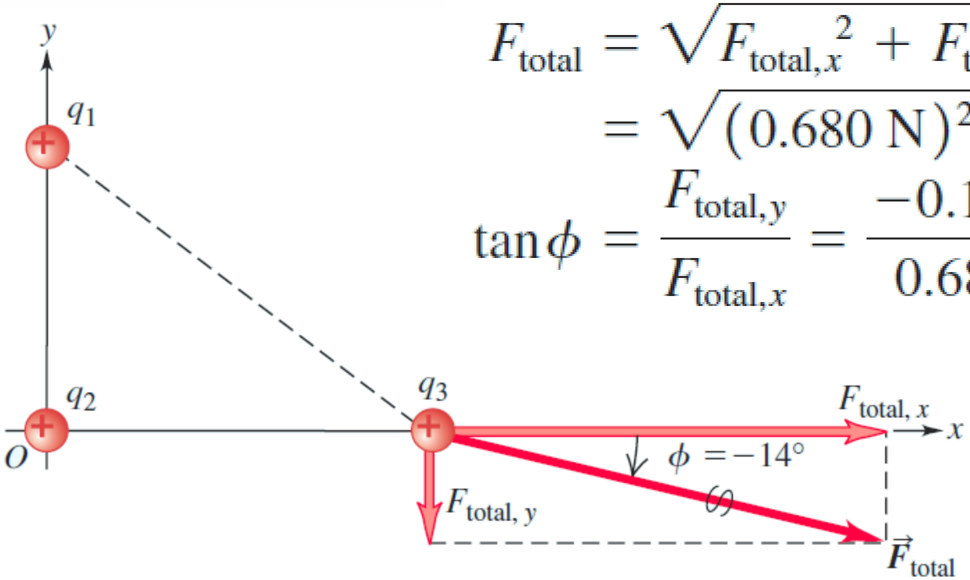
$$= 0.450\ \mathrm{N}.$$

$$\begin{aligned}\sum F_{\text{total},x} &= F_{1x} + F_{2x}\\ &= (0.288\ \text{N})\cos\theta + 0.450\ \text{N}\\ &= (0.288\ \text{N})(0.80) + 0.450\ \text{N} = 0.680\ \text{N},\\ \sum F_{\text{total},y} &= F_{1y} + F_{2y} = -(0.288\ \text{N})\sin\theta + 0\\ &= -(0.288\ \text{N})(0.60) = -0.173\ \text{N}.\end{aligned}$$

- These components combine to form the resultant force:

$$\begin{aligned}F_{\text{total}} &= \sqrt{F_{\text{total},x}{}^2 + F_{\text{total},y}{}^2}\\ &= \sqrt{(0.680\ \text{N})^2 + (-0.173\ \text{N})^2} = 0.70\ \text{N},\end{aligned}$$

$$\tan\phi = \frac{F_{\text{total},y}}{F_{\text{total},x}} = \frac{-0.173\ \text{N}}{0.680\ N} \quad \text{and} \quad \phi = -14°.$$



Extra examples:

1. **A proton is located 0.12 mm from a second charge. They exert a force of attraction on each other equal to 5.0 µN. What is the magnitude of the second charge?**

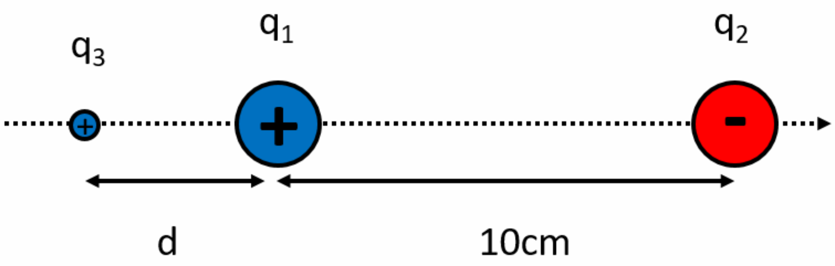
2. **Two electrons produce a force of 12 pN on each other. Calculate the distance of separation between the charges.**

Example

Two fixed charge, 1μC and -3μC are separated by 10 cm. Where may a third charge be located so that no net force acts on it ?

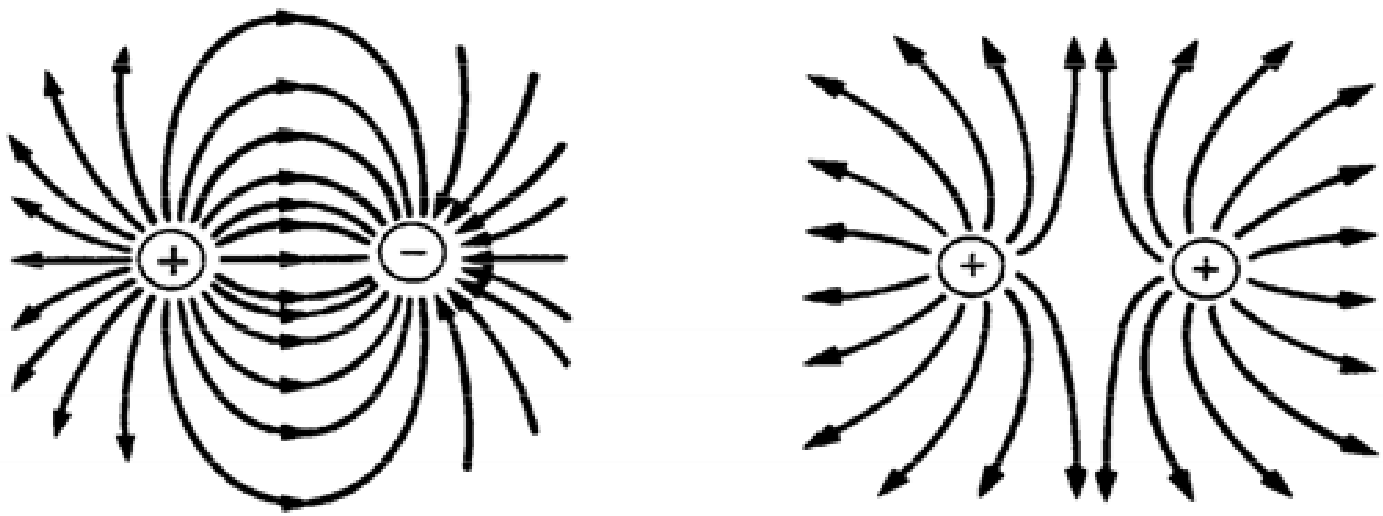
$$F_{13} = F_{23}$$

$$k_e \frac{|q_1||q_3|}{r^2{}_{13}} = k_e \frac{|q_2||q_3|}{r^2{}_{23}}$$



EXERCISE: Solve the equation to find d

The Electric Field



Electric Field

- Suggests the notion of ***electrical field*** (first introduced by Michael Faraday (1791-1867).
- An electric field is said to exist in a region of space surrounding a charged object.
- If another charged object enters a region where an electrical field is present, it will be subject to an electrical force.

17.5 Electric Field & Electric Force

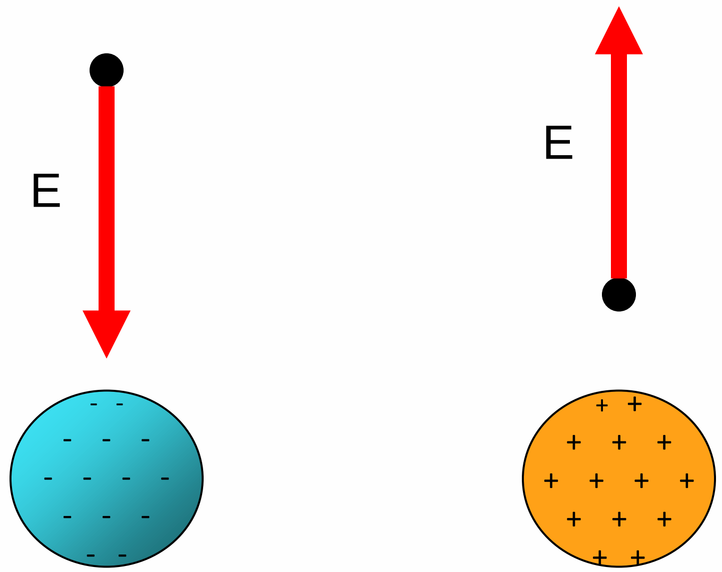
Consider a small test charge q_0 near a larger charge Q. We define the electric field E at the location of the small test charge as a ratio of the electric force F acting on it and the test charge q_0

$$\vec{E} = \frac{\vec{F}}{q_0}$$

This is the electric field produced by charge Q (the force from Q on q_0) at the location of q_o

Electric Field Direction

- The direction of the Electric Field, E at a point is the direction of the electric force that would be exerted on a small positive test charge placed at that point.



https://phet.colorado.edu/en/simulation/charges-and-fields

17.6 Calculating Electric Fields

Electric Field from a Point Charge

Suppose we have two charges, q and q_0, separated by a distance r. The electric force between the two charges is

$$F = k_e \frac{|q||q_o|}{r^2}$$

We can consider q_0 to be a test charge, and determine the electric field from charge q as

$$E = k_e \frac{|q|}{r^2}$$

Questions

Extra examples:

1. **Sketch the electric field lines between the charges shown below.**

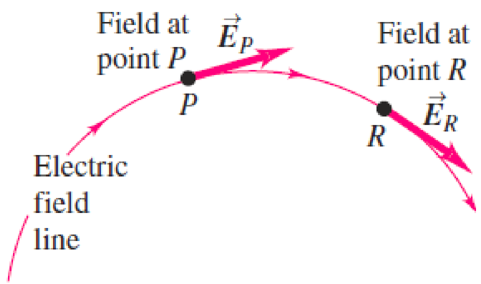
2. **Calculate the magnitude of the electric field produced by an electron at a distance of 3.0 nm.**

3. **A charge 2Q produces an electric field E_1 at a point P. Point P is located a distance r away from the charge. Draw diagrams and formulate an expression for the electric field E_2 in terms of E_1 at a new distance, 4r away and with a new charge, 4Q.**

4. **At a certain distance 'r', a charge Q produces an electric field equal to 4 N/C. What is the electric field at a new distance of '2r'?**

17.7 Electric Field Lines

To visualize electric field patterns, one can draw lines pointing in the direction of the electric field vector at any point.
These lines are called electric field lines.



1. The electric field vector (direction) is tangent to the electric field lines at each point.
2. The number of lines per unit area through a surface perpendicular to the lines is proportional to the strength of the electric field in a given region.
3. No two field lines can cross each other . Why?

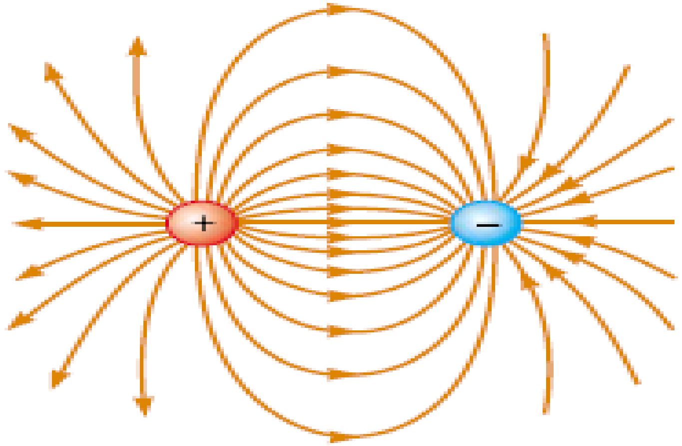
The Electric Field has a unique direction!

Electric Field from an Electric Dipole

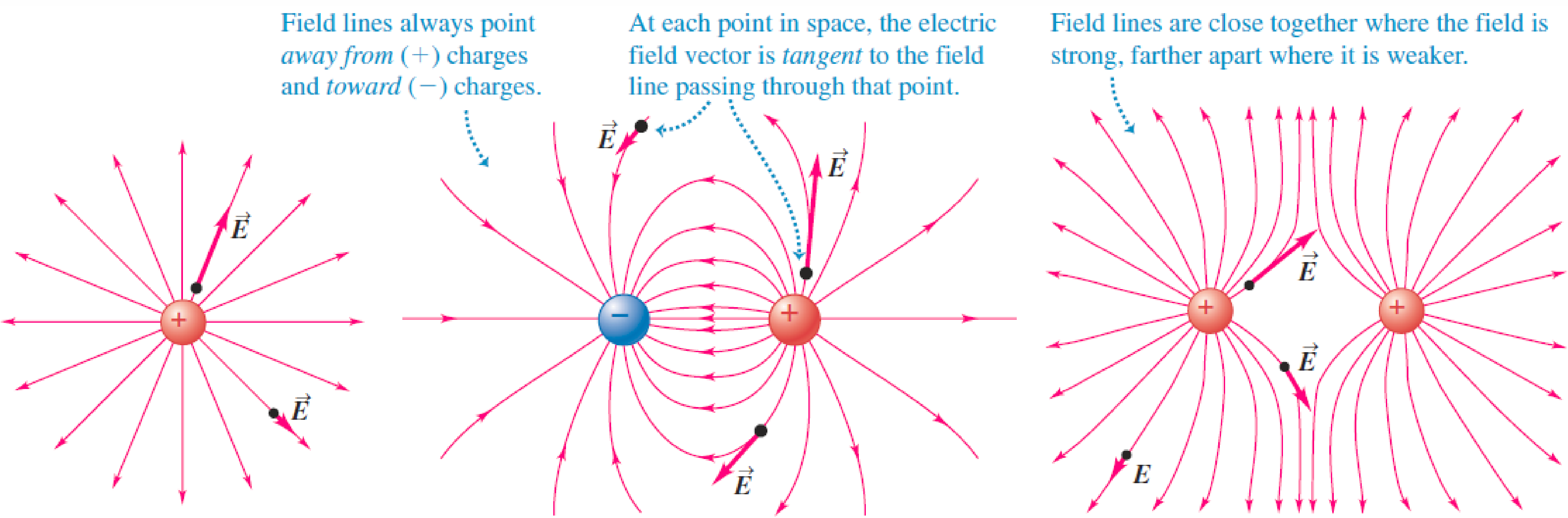
A system of two oppositely charged point particles is called an electric dipole.

The vector sum of the electric field from the two charges gives the electric field of the dipole (superposition principle).

We have shown the electric field lines from a dipole



Electric field lines for several charge distributions

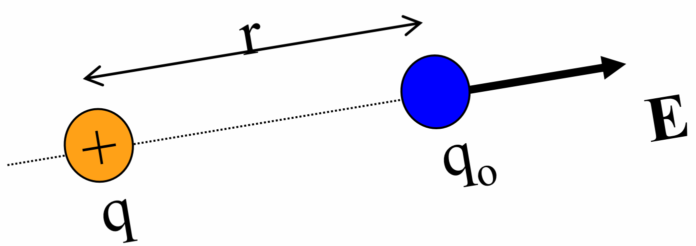


(a) A single positive charge

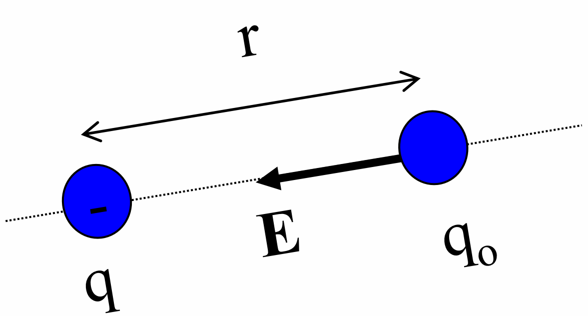
(b) Two equal and opposite charges (a dipole)

(c) Two equal positive charges

- If q is +ve, the electric field at a given point is ***radially outward*** away from q.



- If q is -ve, the electric field at a given point is ***radially inward*** towards q.

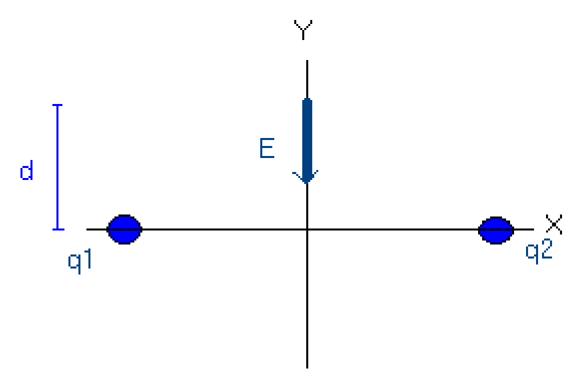


Question

Two charges q_1 and $q_{2,}$ fixed along the x-axis as shown, produce an electric field E at the point (x,y) = (0,d), which is the directed along the negative y-axis.

Which of the following is true?

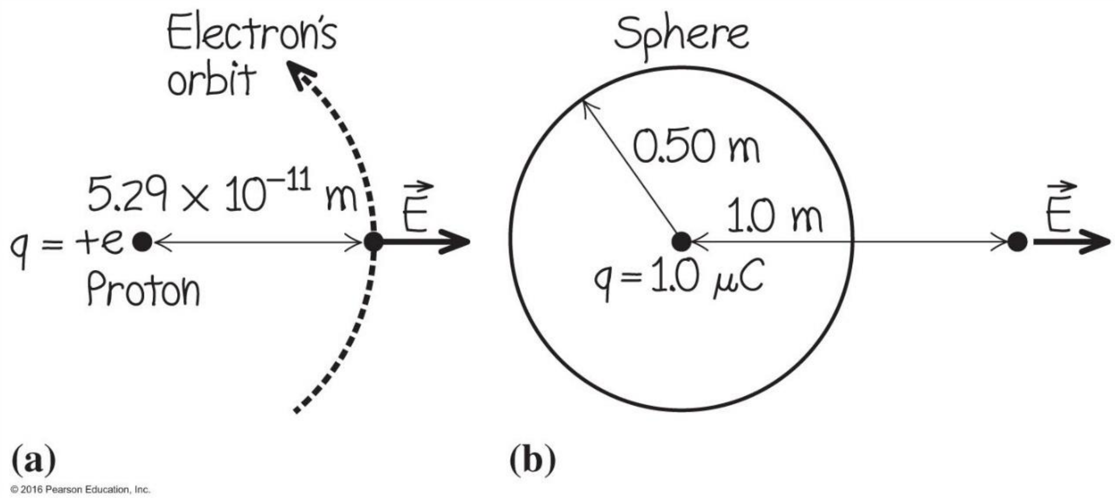
1. Both charges are positive
2. Both charges are negative
3. The charges have opposite signs



Examples

(a) In the hydrogen atom show below the distance between the proton and electron is approximately 5.29 x 10^{-11} m. Find the electric field from the proton at the location of the electon.

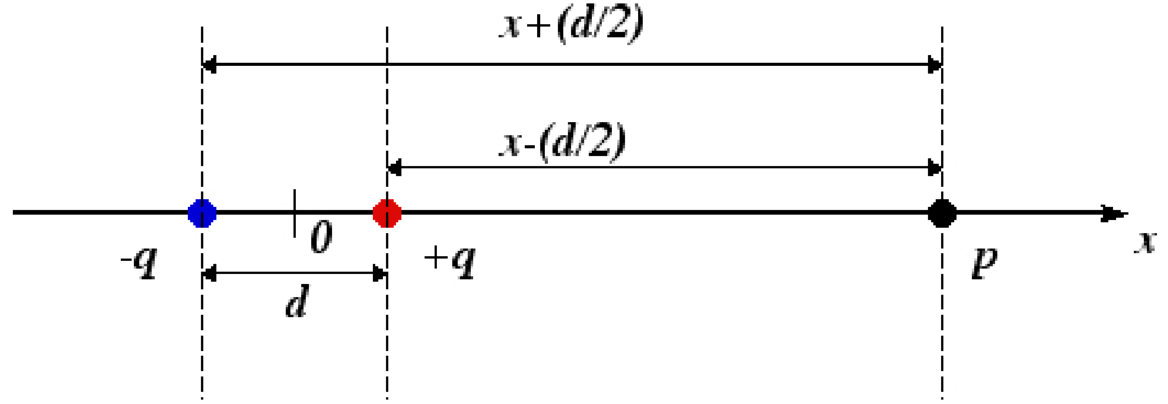
(b) A Van de Graaf generator can build up a large static charge on a metal sphere. Suppose it has a radius of 0.50 m and a net charge of 1.0 µC. What is the magnitude of the electric field 1.0m from the centre of the sphere?



Example

- Two charges on the x-axis a distance
  d apart
  - Put $-q$ at $x = -d/2$
  - Put $+q$ at $x = +d/2$

Calculate the electric field at a point P a distance x from the origin



Principle of superposition: The electric field at any point *x* is the sum of the electric fields from +*q* and -*q*

$$\vec{E} = \vec{E}_{+} + \vec{E}_{-} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r_{+}^{2}} - \frac{1}{4\pi\varepsilon_0}\frac{q}{r_{-}^{2}}$$

Replacing r_+ and r_- we get

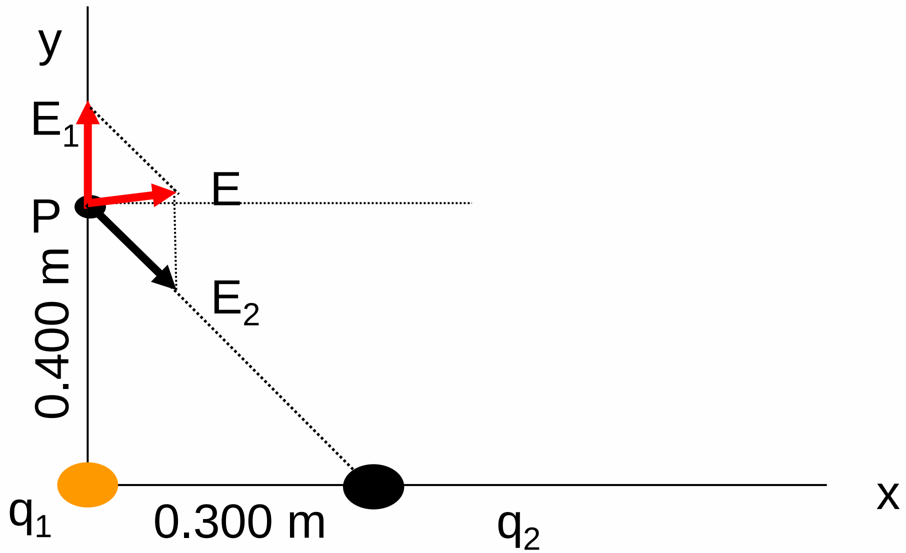
$$E = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{\left(x - \frac{1}{2}d\right)^2} - \frac{1}{\left(x + \frac{1}{2}d\right)^2}\right]$$

This equation gives the electric field everywhere on the *x*-axis (except for $x = \pm d/2$)

Example

Electric Field Due to Two Point Charges

Charge q_1=7.00 μC is at the origin, and charge q_2=-10.00 μC is on the x axis, 0.300 m from the origin. Find the electric field at point P, which has coordinates (0, 0.400) m.



$$E_1 = k_e \frac{|q_1|}{r_1^2} = 8.99\times10^9 \tfrac{Nm^2}{C^2} \frac{\left(7.00\times10^{-6}C\right)}{\left(0.400m\right)^2} = 3.93\times10^5 N/C$$

$$E_2 = k_e \frac{|q_2|}{r_2^2} = 8.99\times10^9 \tfrac{Nm^2}{C^2} \frac{\left(10.00\times10^{-6}C\right)}{\left(0.500m\right)^2} = 3.60\times10^5 N/C$$

$$E_x = \tfrac{3}{5} E_2 = 2.16\times10^5 N/C$$

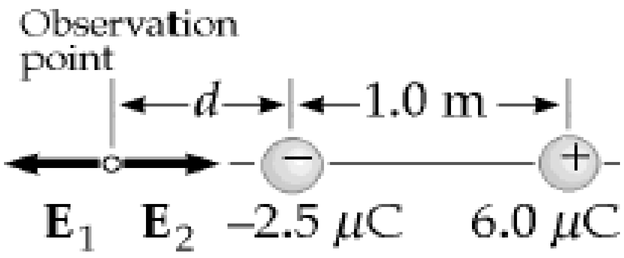
$$E_y = E_1 - E_2 \sin\theta = E_1 - \tfrac{4}{5} E_2 = 1.05\times10^5 N/C$$

$$E = \sqrt{E_x^2 + E_y^2} = 2.4\times10^5 N/C$$

$$\phi = \tan^{-1}(E_y / E_x) = 25.9^o$$

Example

In the figure, determine the point (other than infinity) at which the total electric field is zero.



Solution: The sum of two vectors can be zero only if the two vectors have the same magnitude and opposite directions.

require that $\left|\mathbf{E}_2\right| = \left|\mathbf{E}_1\right|$ or $\dfrac{k_e\left|-2.5\ \mu C\right|}{d^2} = \dfrac{k_e\left|6.0\ \mu C\right|}{(d+1.0\ \text{m})^2}$

This reduces to $2.5(d+1.0\ \text{m})^2 = 6.0d^2$ or $d + 1.0\ \text{m} = \pm\sqrt{\dfrac{6.0}{2.5}}\,d = \pm 1.55d$

Solving, $d = \dfrac{1.0\ \text{m}}{+1.55-1} = 1.8\ \text{m}$ and $d = \dfrac{1.0\ \text{m}}{-1.55-1} = -0.39\ \text{m}$

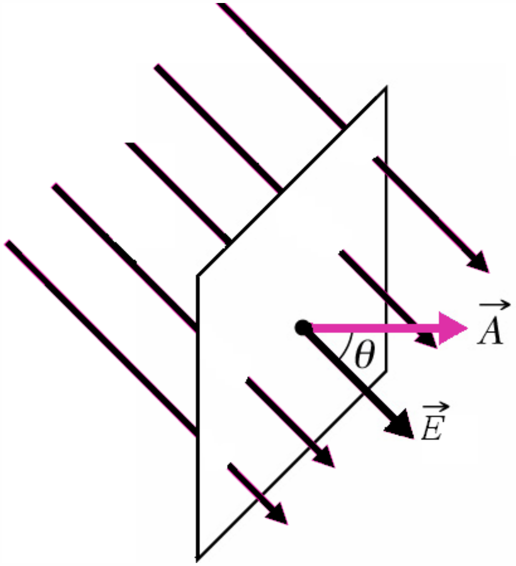
17.8 Gauss's Law and Field Calculations

Gauss's law is an alternative formulation of the principles of electrostatics. It is mathematically equivalent to Coulomb's law, but for some problems it provides a useful alternative approach to calculating electric fields. Coulomb's law enables us to find the field at a point P caused by a single point charge q.

Electric Flux

Electric flux quantifies the notion "number of field lines crossing a surface."

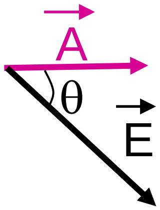
The electric flux Φ through a flat surface in a uniform electric field depends on the **field strength *E***, the **surface area *A***, and the **angle θ** between the field and the normal to the surface.

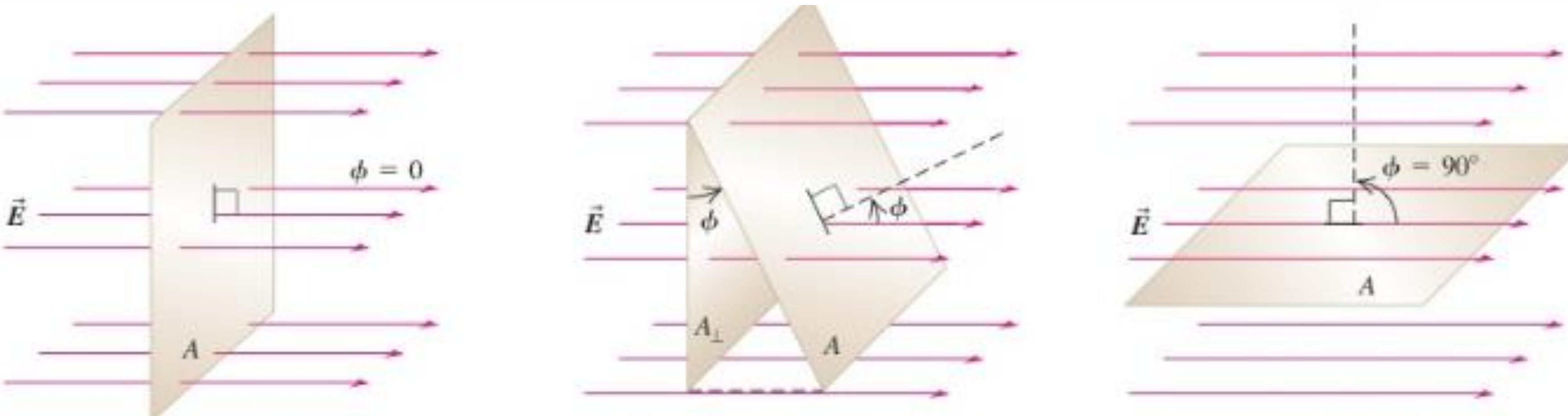


Mathematically, the flux is given by

$$\Phi = EA\cos\theta = \vec{E}\cdot\vec{A}.$$

Here **A** is a vector whose magnitude is the surface area *A* and whose orientation is normal to the surface.





Electric field $\vec{E}$ is perpendicular to area A; the angle between $\vec{E}$ and a line perpendicular to the surface is zero. The flux is $\Phi_E = EA$.

Area A is tilted at an angle ϕ from the perpendicular to $\vec{E}$. The flux is $\Phi_E = EA\cos\phi$.

Area A is parallel to $\vec{E}$ (tilted at 90° from the perpendicular to $\vec{E}$). The flux is $\Phi_E = EA\cos 90° = 0$.

Roughly speaking, we can picture Φ_E in terms of the number of field lines that pass through A. More area means more lines through the area, and a stronger field means more closely spaced lines and therefore more lines per unit area.

- When $\theta < 90^\circ$, the flux is positive (out of the surface), and when $\theta > 90^\circ$, the flux is negative.

- Units: Nm^2/C in SI units, **the electric flux is a SCALAR quantity**

- Find the electric flux through the area $A = 2\ m^2$, which is perpendicular to an electric field E=22 N/C

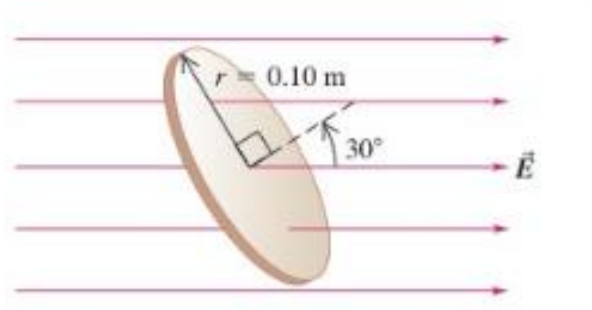
Answer: $\Phi = 44\ Nm^2/C$.

Example

- Calculate the electric flux through the disc. The disc is orientated 30° away from the electric field where E = 2000 N/C

- Calculate the electric flux if the axis of the disc is orientated perpendicular to the electric field

- Calculate the electric flux if the axis of the disc is orientated parallel to the electric field



بوليتكنك POLYTECHNIC

Solution:

SET UP AND SOLVE Part (a): The area is $A = \pi(0.10\ \mathrm{m})^2 = 0.0314\ \mathrm{m}^2$. From Equation 17.4,

$$\Phi_E = EA\cos\phi = (2.0 \times 10^3\ \mathrm{N/C})(0.0314\ \mathrm{m}^2)(\cos 30^\circ)$$
$$= 54\ \mathrm{N \cdot m^2/C}.$$

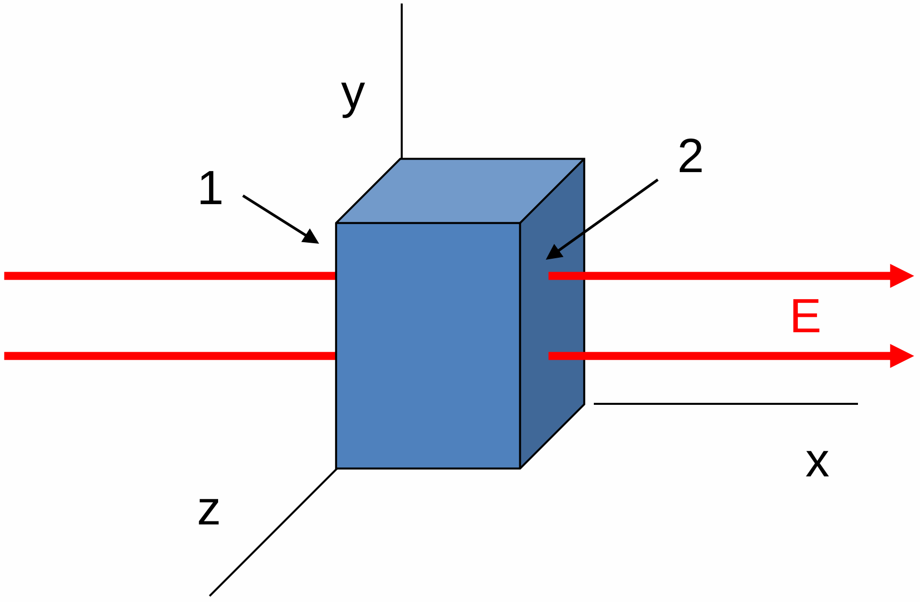
Part (b): The axis of the disk is now perpendicular to $\vec{E}$, so $\phi = 90^\circ$, $\cos\phi = 0$, and $\Phi_E = 0$.

Part (c): The axis of the disk is parallel to $\vec{E}$, so $\phi = 0$, $\cos\phi = 1$, and, from Equation 17.4,

$$\Phi_E = EA\cos\phi = (2.0 \times 10^3\ \mathrm{N/C})(0.0314\ \mathrm{m}^2)(1)$$
$$= 63\ \mathrm{N \cdot m^2/C}.$$

Example

Calculate the flux of a constant E field (along x) through a cube of side "L".



Solution:

$$\Phi_1 = EA_1 cos\theta_1 = -EL^2$$

$$\Phi_2 = EA_2 cos\theta_2 = EL^2$$

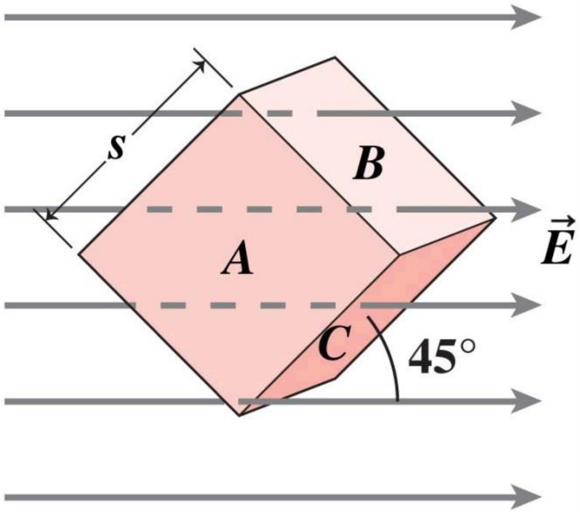
$$\Phi_{net} = -EL^2 + EL^2 = 0$$

NB: Area vectors always point **OUT** of a surface, hence, $\overrightarrow{A_1} = -1$ and $\overrightarrow{A_2} = 1$

Question

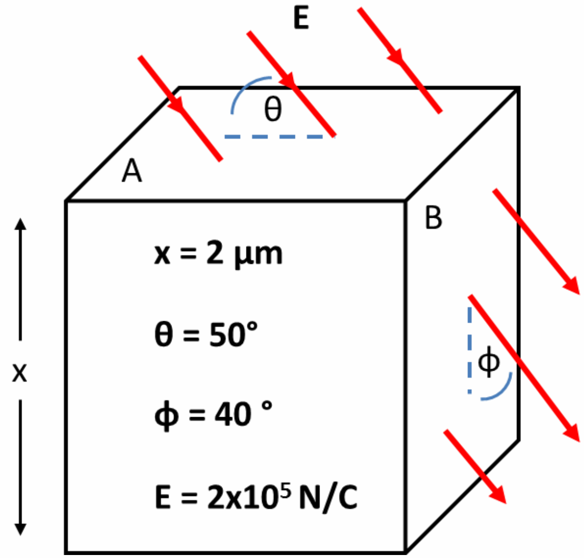
The flux through side *B* of the cube in the figure is the same as the flux through side *C*. What is a correct expression for the sum of the flux through side B and side C?

$$\Phi = 2s^2\vec{E}cos45°$$



Question

Calculate the flux through side A and B of the cube shown



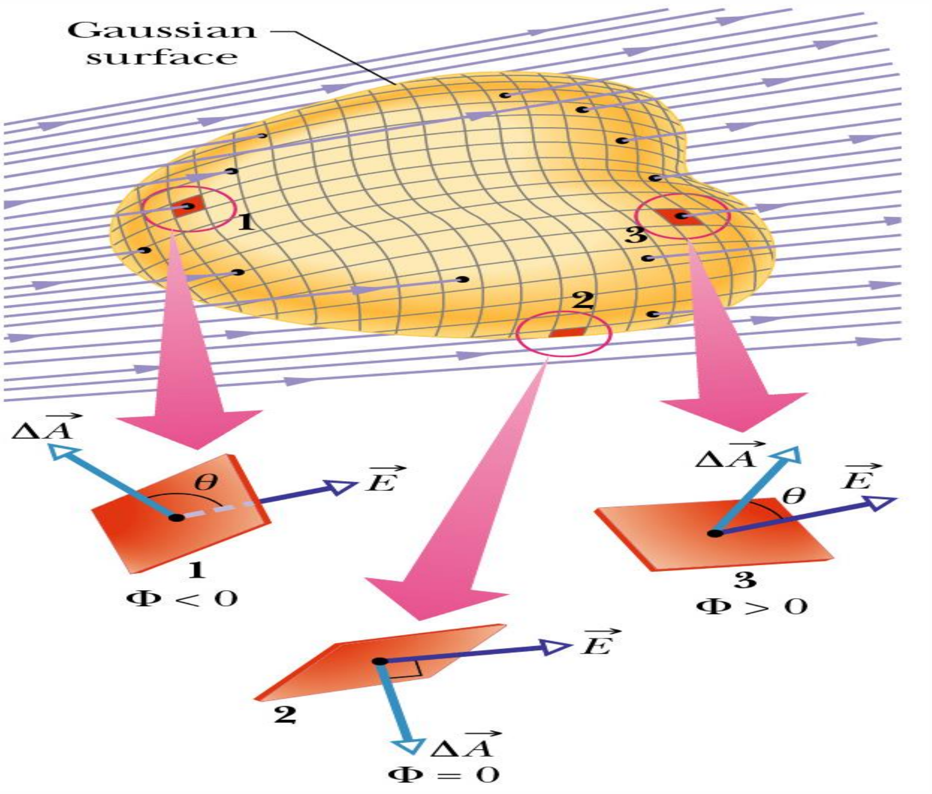
$$\boldsymbol{\Phi_T = \Phi_A + \Phi_B}$$

$$= \boldsymbol{EAcos(180 - (90° - \theta)) + EAcos(90° - \phi)}$$

$$= \boldsymbol{-6.13 \times 10^{-7} + 5.14 \times 10^{-7}}$$

$$= \boldsymbol{-9.9 \times 10^{-8}\ Nm^2/C}$$

When we have a complicated surface, we can divide it up into tiny elemental areas:



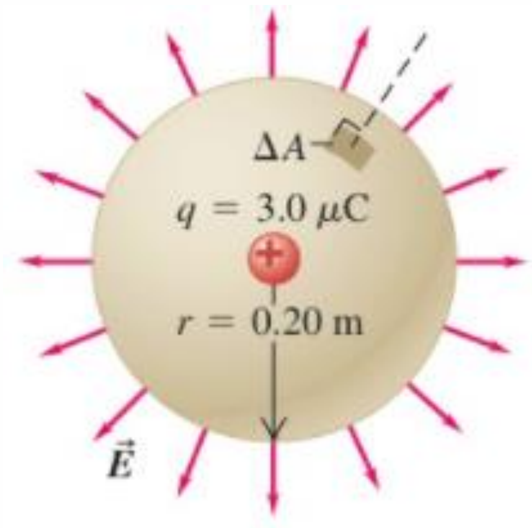
$$d\Phi = \vec{E}\cdot d\vec{A} = E\,dA\cos\theta \quad \Rightarrow \quad \Phi = \oint \vec{E}.d\vec{A}$$

Charge INSIDE a Surface

We can calculate the flux through a surface that **encloses** a charge. Suppose a positive point charge with magnitude 3.0 μC is placed at the center of a sphere with radius 0.20 m. Find the electric flux through the sphere due to this charge.

$$\boldsymbol{E = k\frac{q}{r^2} = (9 \times 10^9)\frac{(3 \times 10^{-6})}{(0.2)^2}}$$

$$\boldsymbol{= 6.75 \times 10^5 \; N/C}$$



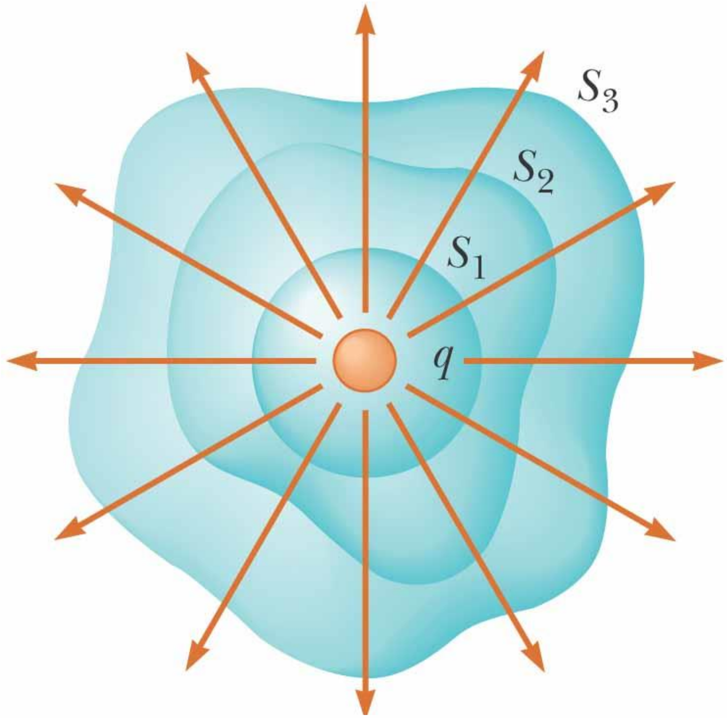
$$\boldsymbol{\Phi_E = EA = (6.75 \times 10^5)(4\pi(0.2)^2)}$$

$$\boldsymbol{= 3.4 \times 10^5 Nm^2/C}$$

The symmetry of the sphere plays an essential role in this calculation. We made use of the facts that E has the same value at every point on the surface and that at every point E is perpendicular to the surface.

Example

- What's the total flux on a closed surface with a charge inside?
  - The shape and size don't matter!
  - Just use a sphere



$$\Phi_E = \oint \mathbf{E} \cdot d\mathbf{A}$$
$$= E \oint d\mathbf{A} \cos(0)$$
$$= \frac{1}{4\pi\varepsilon_o} \frac{q}{r^2} \oint dA$$
$$= \frac{1}{4\pi\varepsilon_o} \frac{q}{r^2} 4\pi r^2 = \frac{q}{\varepsilon_o}$$

$$\boxed{\boldsymbol{\Phi_E = \frac{q}{\varepsilon_o}}}$$

The $\oint$ symbol has a little circle to indicate that the integral is over a closed surface

So, what is Gauss's Law? Reminder…

- Gauss's Law does not tell us anything new, it is NOT a new law of physics, but another way of expressing Coulomb's Law, i.e. the relationship between electric charge and electric field

$$\Phi_E = \sum EA\cos\theta = \frac{Q_{encl}}{\varepsilon_o}$$

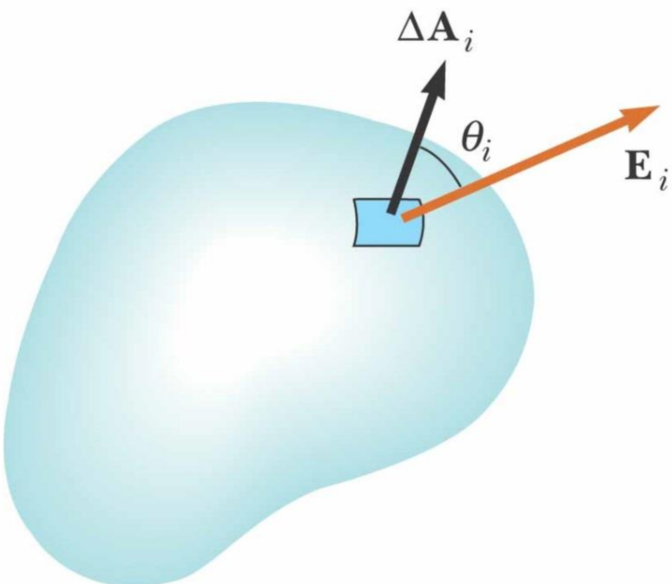
- Gauss's law makes it possible to find the electric field easily in highly symmetric situations

Gauss' Law

- So, as discussed, the precise relationship between flux and the enclosed charge is given by Gauss' Law

$$\Phi_E = \sum EA\cos\theta = \frac{Q_{encl}}{\varepsilon_o}$$

- ε_0 is the permittivity of free space from Coulomb's law



The flux through a closed surface is equal to the total charge contained divided by permittivity of free space

A few important points on Gauss' Law

- The integral is over the value of **E** on a closed surface of our choice in any given situation

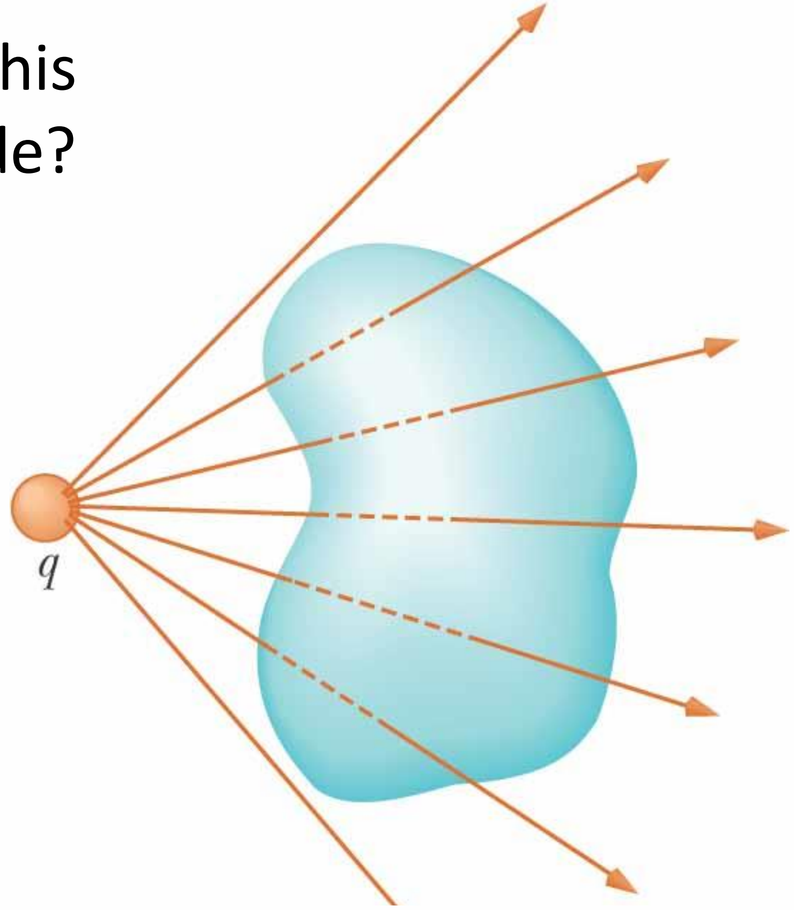
- The charge Q_{encl} is the net charge enclosed by the arbitrary closed surface of our choice.

- It does NOT matter where the charge is distributed inside the surface but depends on the total charge!

- The charge outside the surface does not contribute.

Question

- What's the total flux inside this shape with the charge outside? Why?



Solution:

Zero

Because the surface surrounds no charge i.e. there is no charge inside the surface!

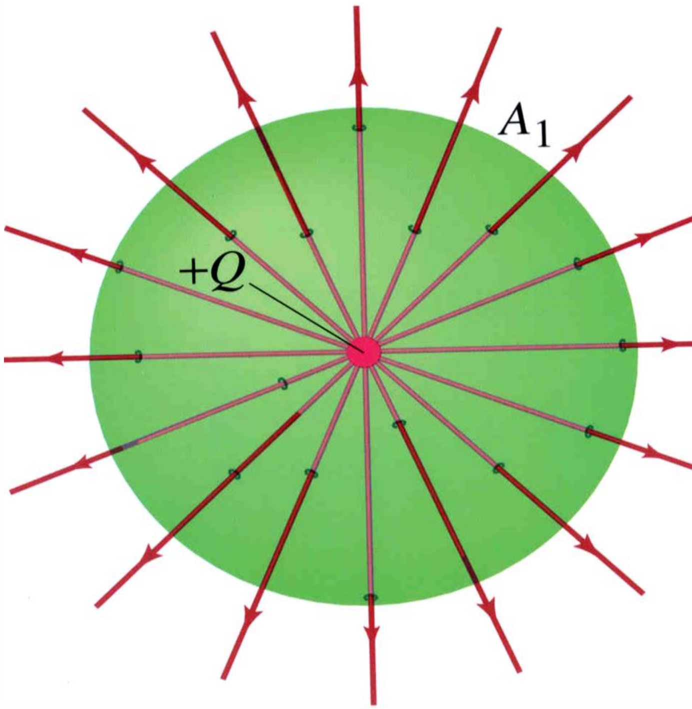
Gauss → Coulomb

- Calculate E of point like (+) charge Q
- Consider sphere radius r centered at the charge
- Spherical symmetry: E is the same everywhere on the sphere, perpendicular to the sphere

$$\Phi = \Sigma \vec{E}.\vec{A} = EA = E(4\pi r^2)$$

$$\Phi = \frac{Q_{en}}{\varepsilon_0}$$

$$E(4\pi r^2) = \frac{Q_{en}}{\varepsilon_0} \quad \Rightarrow \quad E = \frac{Q}{4\pi r^2 \varepsilon_0} = k\frac{Q}{r^2}$$



Application of Gauss's Law

Gauss's law can be used to calculate the electric field if the symmetry of the charge distribution is high. Here we concentrate on three different ways of charge distribution

- A linear charge distribution $\lambda \longrightarrow Q = \lambda L \longrightarrow \boldsymbol{\lambda = \frac{Q}{L}}$
- A surface charge distribution $\sigma \longrightarrow Q = \sigma A \longrightarrow \boldsymbol{\sigma = \frac{Q}{A}}$
- A volume charge distribution $\rho \longrightarrow Q = \rho V \longrightarrow \boldsymbol{\rho = \frac{Q}{V}}$

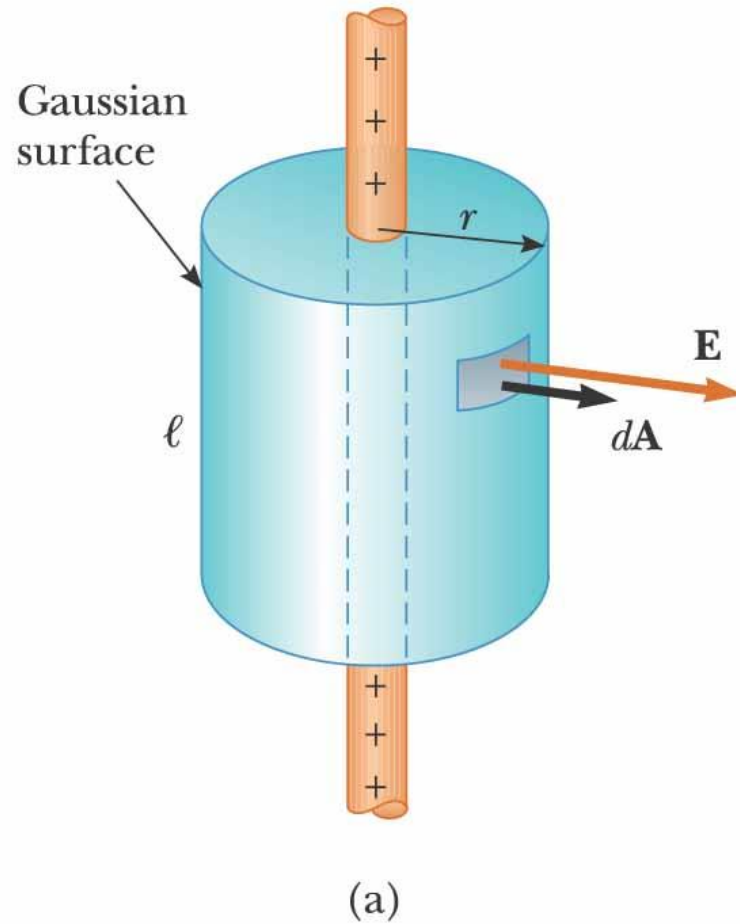
We will analyse a linear charge and volume charge distribution

Linear Charge Distribution

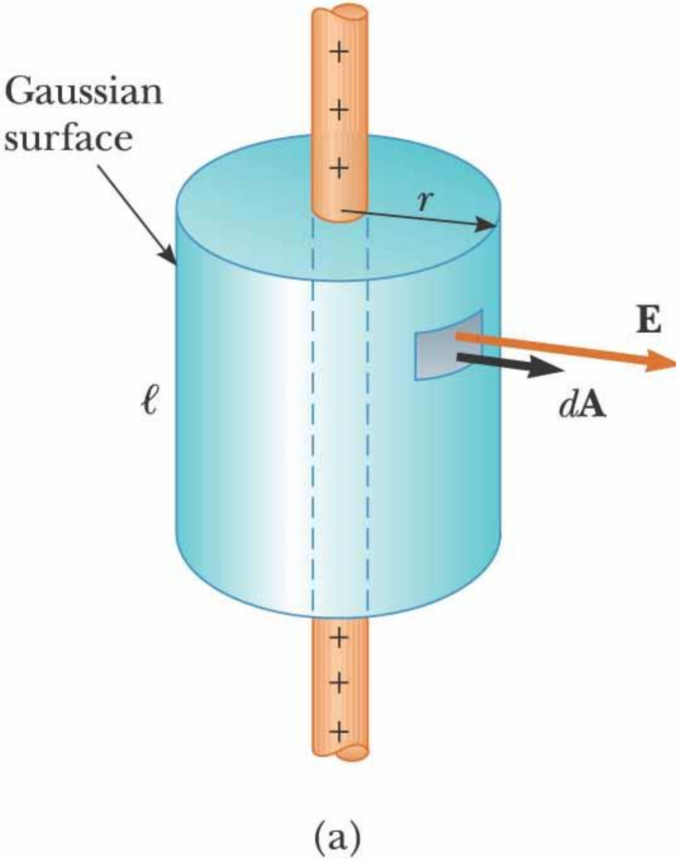
- Let's calculate the electric field from a conducting wire with charge per unit length λ using Gauss' Law

$$\Phi_E = \sum EA\cos\theta = \frac{1}{\varepsilon_o}\sum q$$

- We start by assuming a Gaussian surface in the form of a cylinder with radius *r* and length 'ℓ' placed around the wire such that the wire is along the axis of the cylinder



- From symmetry we can see that the electric field will extend radially from the wire.
- How?
  - If we rotate the wire along its axis, the electric field must look the same
    - Cylindrical symmetry
  - If we imagine a very long wire, the electric field cannot be different anywhere along the length of the wire
    - Translational symmetry
- Thus our assumption of a cylinder as a Gaussian surface is perfectly suited for the calculation of the electric field using Gauss' Law.



- The electric flux through the ends of the cylinder (A_1 & A_2) is zero because the electric field is always parallel to the ends.

- The electric field is always perpendicular to the wall of the cylinder so

$$\Phi = \sum EA\cos\theta = EA_1\cos\theta_1 + EA_2\cos\theta_2 + EA_3\cos\theta_3$$
$$= 0 + 0 + E(2\pi rL) = E(2\pi rL)$$
$$\Rightarrow E(2\pi rL) = q/\varepsilon_0 = \lambda L/\varepsilon_0$$

- … and now solve for the electric field

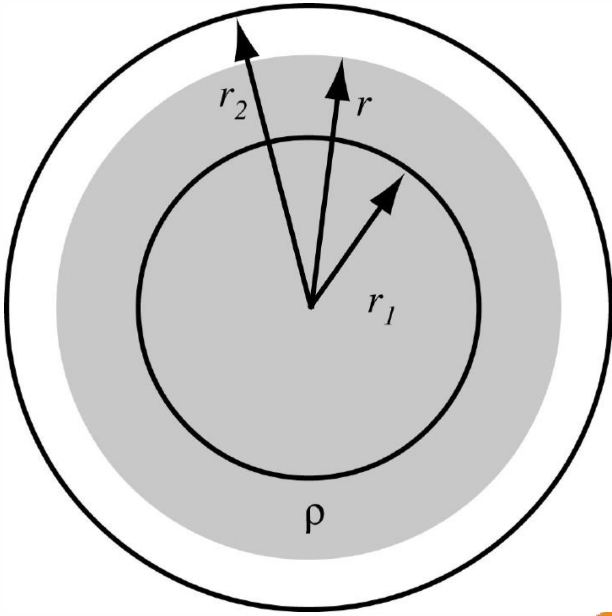
$$E = \frac{\lambda}{2\pi\varepsilon_0 r} = \frac{2k\lambda}{r}$$

Volume Charge Distribution

- Let's calculate the electric field from a charge distributed uniformly throughout a charged sphere.

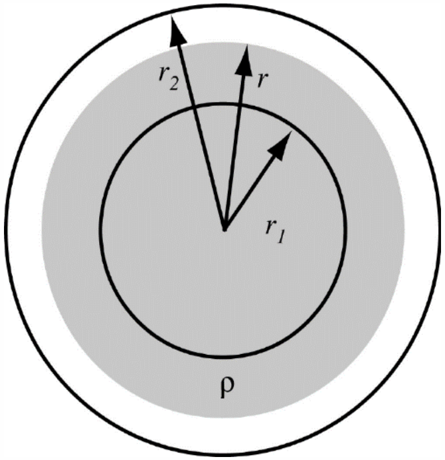
- Assume that we have an insulating solid sphere (NON CONDUCTING) of charge *Q* with radius *r* with constant charge density per unit volume ρ.

- We will assume two different spherical Gaussian surfaces
  - $r_2 > r$ *(outside)*
  - $r_1 < \mathrm{r}$ *(inside)*



- Let's start with a Gaussian surface with $r_1 < r$.
- From spherical symmetry we know that the electric field will be radial and perpendicular to the Gaussian surface.
- For a radius $r_1 < r$, a Gaussian surface will enclose less than the total charge and the electric field will be less. Inside the sphere of charge, the flux is given by

$$\sum EAcos\theta = E(4\pi r_1{}^2) = \frac{q_1}{\varepsilon_o} = \frac{\rho V_1}{\varepsilon_o} = \frac{\rho}{\varepsilon_o}\left(\frac{4}{3}\pi r_1{}^3\right)$$



- Solving for E we find

$$\boldsymbol{E = \frac{\rho r_1}{3\varepsilon_o}}$$

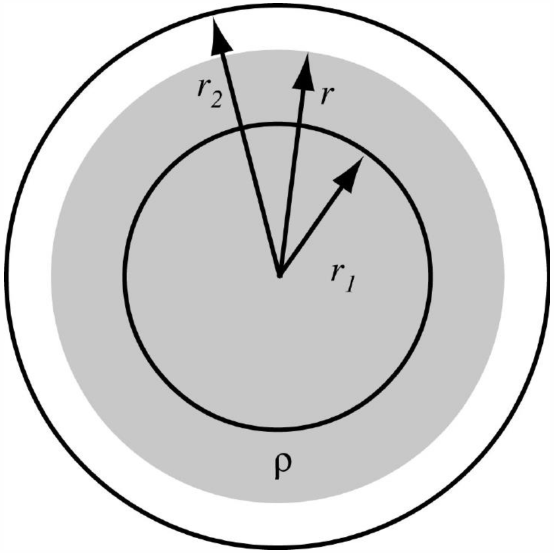
Or…….

In terms of the total charge Q within the sphere…

$$E = \frac{\rho r_1}{3\varepsilon_o} \qquad recall\ \rho = \frac{Q}{V}$$

$$E = \frac{Q r_1}{\frac{4}{3}\pi r^3 3\varepsilon_o}$$

$$E = \frac{Q r_1}{4\pi\varepsilon_o r^3} = k\frac{Q r_1}{r^3}$$

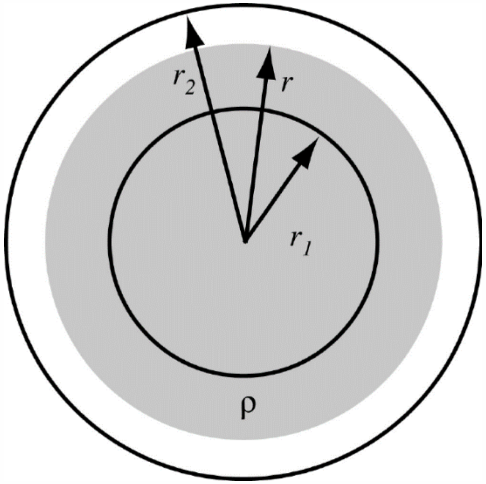


$$E = k\frac{Q r_1}{r^3}$$

Electric field at any radius ‘r_1’ inside a sphere with radius ‘r’ and charge Q

- Now consider a Gaussian surface with radius $r_2 > r$.
- Again by spherical symmetry we know that the electric field will be radial and perpendicular to the Gaussian surface.
- **Since the radius of the Gaussian surface is greater than the radius of the sphere all the charge is enclosed in our Gaussian surface.**
- Gauss' Law gives us

$$\sum EAcos\theta = E\left(4\pi r_2{}^2\right) = \frac{Q}{\varepsilon_o}$$



- Solving for E we find

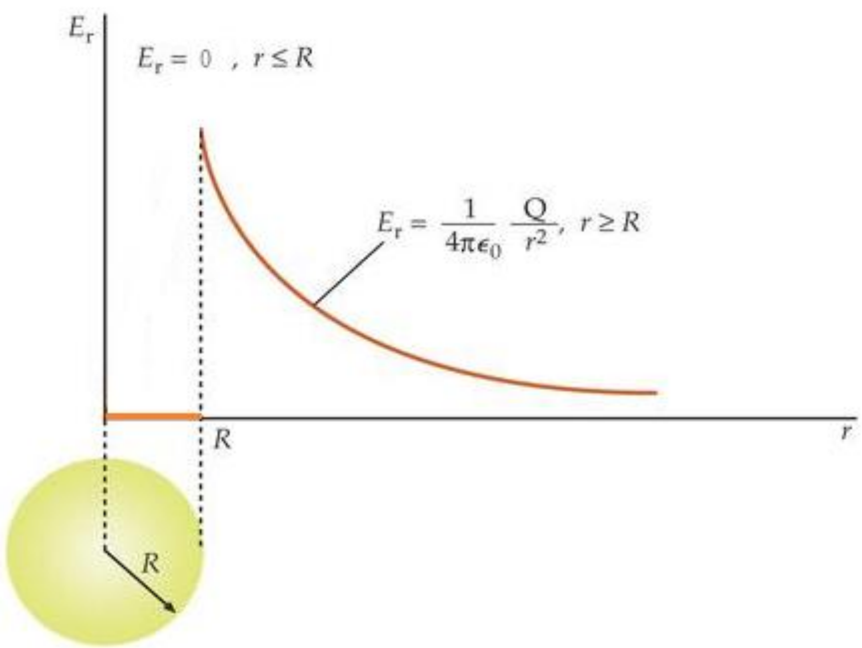
$$E = \frac{kQ}{r_2^2}$$

Electric field any radial distance r_2 from the centre of a non conducting charged sphere

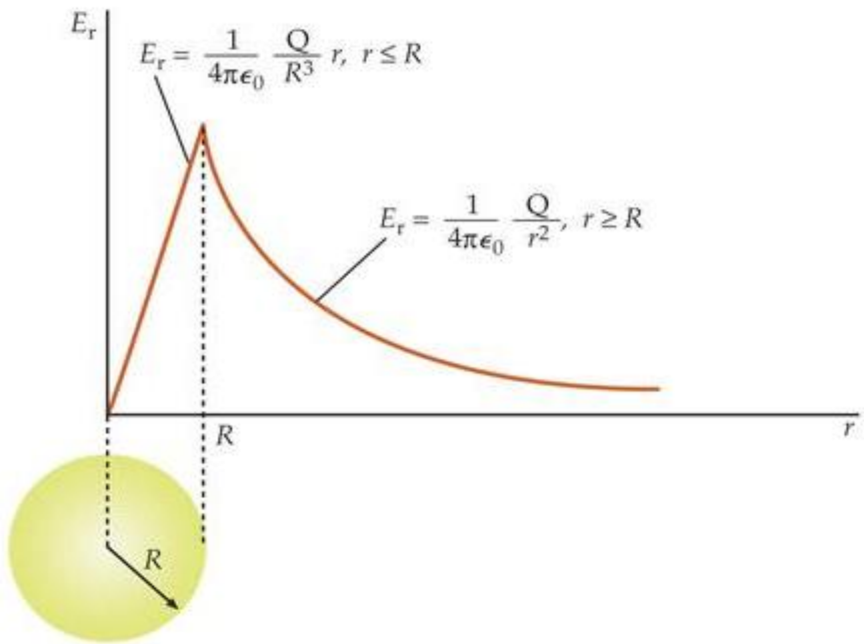
SAME AS A POINT TEST CHARGE!

Electric Field vs. Radius for…

A conducting sphere



A uniformly charged sphere



All the charge will reside on the conducting surface. A Gaussian surface at r< R will enclose no charge, and by its symmetry can be seen to be zero at all points inside the spherical conductor. The field outside the sphere is the same as if all the charges were concentrated at the center of the sphere

The charge is spread evenly inside the charged sphere. A Gaussian surface at r< R will enclose some charge, hence the electric field is directly proportional to a distance 'r' from the centre of the sphere. The field outside the sphere is the same as if all the charges were concentrated at the center of the sphere

Properties of Conductors

E is zero within conductor… WHY?

There are at least two ways to understand this

- The free charge inside the conductor is zero (all charge accumulates on the surface). So the field inside is caused by charges on the surface. Since charges are of the same nature and the distribution is UNIFORM, the electric fields cancel each other. Try drawing the field vectors!

- Consider a Gaussian surface inside the conductor. Charge enclosed by it is zero (charge resides only on surface). Therefore electric flux, $\phi = 0$ Furthermore, $\phi = E * A$. Since area cannot be zero, electric field must zero. Try drawing a diagram and working through the maths!