18 Electric Potential and Capacitance

18.1 Electric Potential Energy

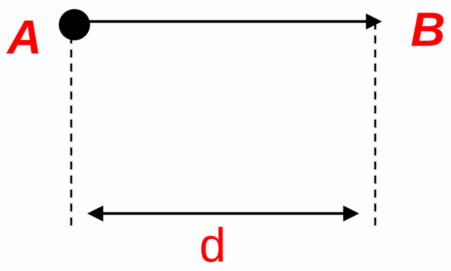
- When a charged particle moves in an *electric field*, the *electric force* does *work* on the particle

- This 'electrical' *work* can be described as a form of energy which we will call, *electric potential energy* or simply, *potential energy*

- In circuits, the *electric potential difference* is related to the *potential energy.* We will refer to it as *potential difference* or simply, *potential* (it is often called *voltage*).

18.1 Electric Potential Energy

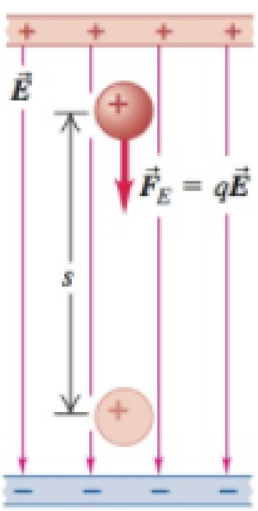
- When a constant Force *F* acts on a particle that moves in a straight line from *A* to *B,* through displacement *d*, the work done by the force is



$$W_{a \to b} = Fd\cos\theta$$

- Work done on a positive charge by moving in an electric field

$$W_{a \to b} = Fd\cos\theta = qEd$$

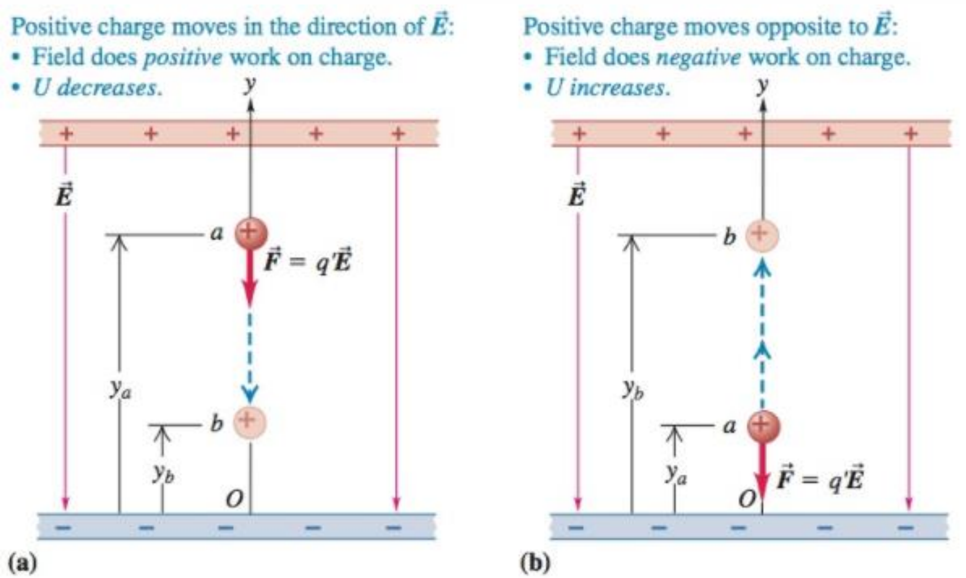


$$recall\ E = \frac{F}{q}$$

- The work done by a conservative force equals the negative of the change in potential energy, $\Delta U = U_b - U_a$

$$\boldsymbol{W_{a\to b} = U_a - U_b = -\Delta U = qEd}$$

This equation is valid only for the case of a **uniform electric field**



If a charged particle moves perpendicular to electric field lines, no work is done

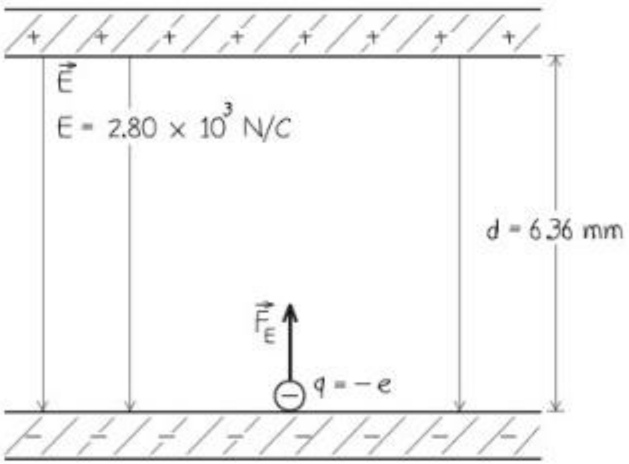
$$\boldsymbol{W = qEd\cos\theta = 0}$$

d ⊥ E

Example

Two large conducting plates separated by 6.36 mm carry charges of equal magnitude and opposite sign, creating a constant electric field with magnitude $2.80 \times 10^3 N/C$ between the plates. An electron moves from the negatively charged plate to the positively charged plate. How much work does the electric field do on the electron?

$$\boldsymbol{W = Fdcos\theta = qEdcos\theta}$$



$$= (-1.60 \times 10^{-19})(2.80 \times 10^3)(6.36 \times 10^{-3})cos180$$

$$= 2.85 \times 10^{-18} J$$

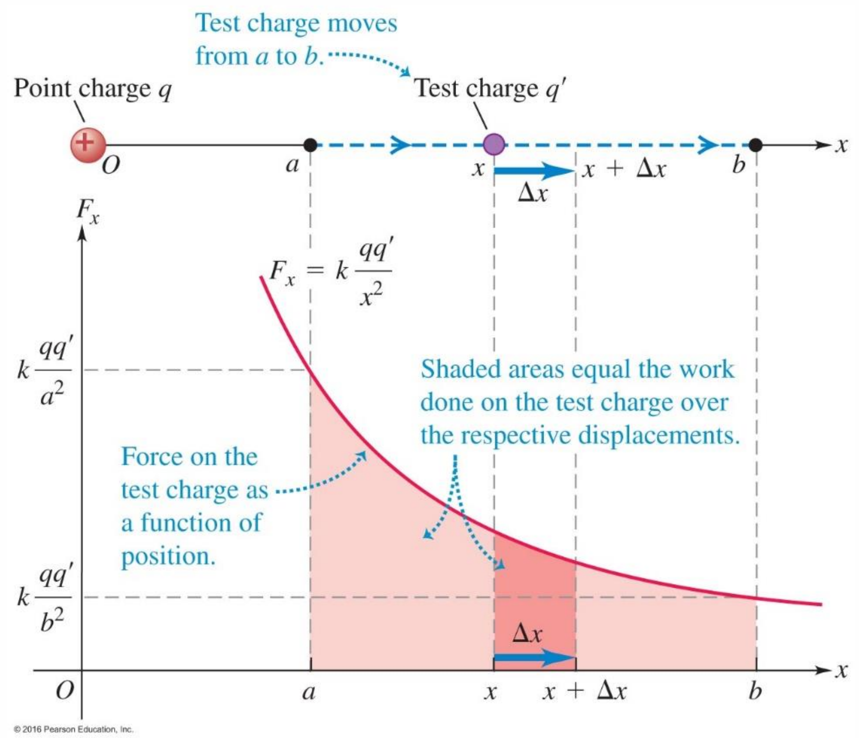
Potential Energy of Point Charges

- Picture a stationary point charge *q* at the origin
- A test charge *q′* is placed at position *a* and moves to *b*

- Work done is the area under the curve

$$W_{a\to b} = kqq'\left(\frac{1}{a} - \frac{1}{b}\right)$$

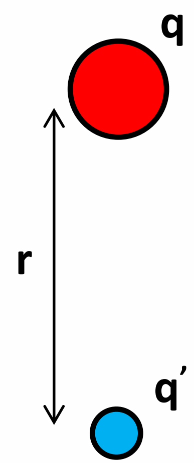
$$W_{a\to b} = U_a - U_b$$



Potential Energy of Point Charges

- The potential energy U of a system consisting of a point charge q' located in the electric field of a stationary charge q, a distance r away is:

$$U = k\frac{qq'}{r}$$

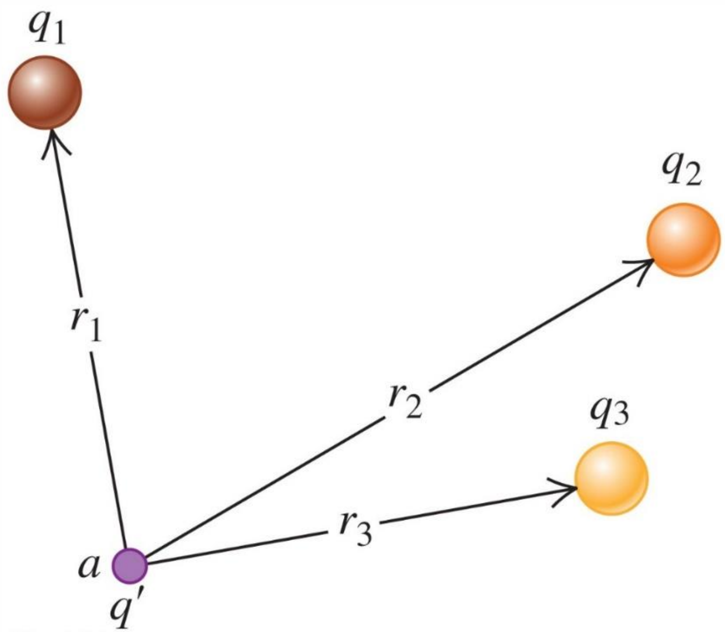


$$a - b = r$$

Potential Energy of Point Charges

- The potential energy associated with a test charge q' due to a collection of charges, q_1, q_2, q_3… at distances r_1, r_2, r_3… is:

$$U = kq'\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} + \cdots\right)$$



18.2 Potential

- The electric potential difference (often called electric potential or simply, *potential*) in an electric field is the electric potential energy per unit charge associated with a test charge *q'*

$$V = \frac{U}{q'} \quad or \quad U = q'V$$

- Electric potential difference is a scalar quantity
- Electric potential difference is a measure of electric energy per unit charge
- Potential is often referred to as "voltage"

Note: If the work done by the electric field is zero, then the electric potential must be constant

- Electric potential difference is the work done to move a charge from a point A to a point B divided by the magnitude of the charge. Thus the SI units of electric potential difference

$$1V = 1\,J/C$$

- In other words, 1 J of work is required to move a 1 C of charge between two points that are at potential difference of 1 V

Example

Here we will calculate the work per unit charge on an electron moving between two potentials. A 9.0 V battery is connected across two large parallel plates that are separated by 4.5 mm of air, creating a potential difference of 9.0 V between the plates.

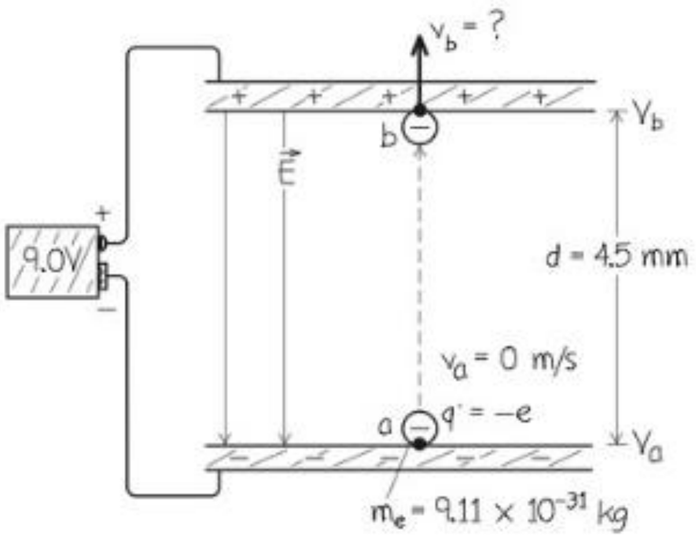
(a) What is the electric field in the region between the plates?

(b) An electron is released from rest at the negative plate. If the only force on the electron is the electric force exerted by the electric field of the plates, what is the velocity of the electron as it reaches the positive plate? $m_e = 9.11 \times 10^{-31} kg$

$$E = \frac{F}{q}$$

$$Ed = \frac{Fd}{q}$$

$$Ed = \frac{W}{q} = V$$



$$(a)\ E = \frac{V_b - V_a}{d} = \frac{9V}{4.5 \times 10^{-3} m} = 2.0 \times 10^{3} V/m$$

Example continued…

(b) Conservation of energy applied to points a and b at the corresponding plates gives…

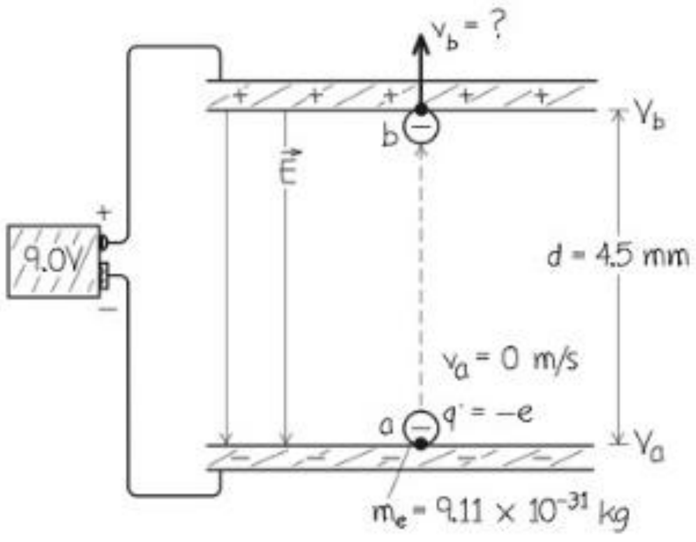
$$\boldsymbol{KE_a + U_a = KE_b + U_b}$$

$$V = \frac{U}{q} \text{ or } U = qV$$

$$KE_a + qV_a = KE_b + qV_b$$

$$KE_a = 0$$

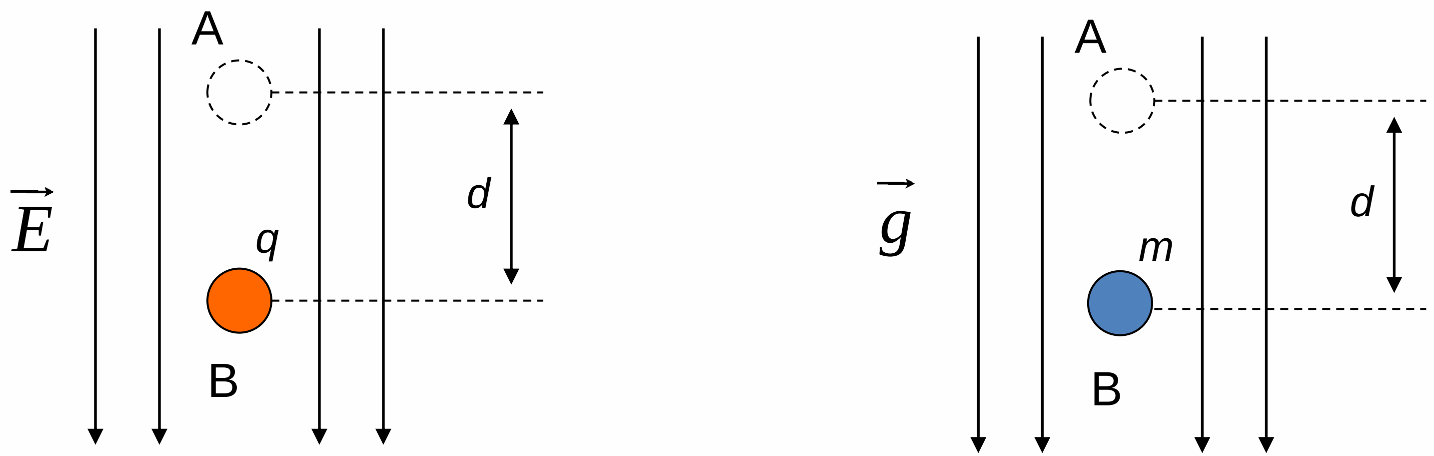
$$KE_b = qV_a - qV_b$$



Solve for final velocity…

Analogy between Electric and Gravitational fields

- The same kinetic-potential energy theorem works here



- If a positive charge is released from A, it accelerates in the direction of electric field, i.e. gains kinetic energy
- If a negative charge is released from A, it accelerates in the direction opposite the electric field i.e. gains kinetic energy

$$KE_a + U_a = KE_b + U_b$$

Example - Motion of an Electron

What is the magnitude of the velocity of an electron (and a proton) accelerated from rest across a potential difference of 100V?

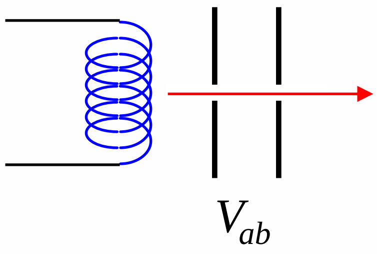
Given:

$\Delta V = 100\ \text{V}$
$m_e = 9.11\times10^{-31}$ kg
$m_p = 1.67\times10^{-27}$ kg
$|e| = 1.60\times10^{-19}$ C

Find:
$v_e = ?$
$v_p = ?$

$$KE_a + U_a = KE_b + U_b$$

$$q\Delta V = \frac{1}{2}m{v_b}^2$$



18.2 Electric Potential due to Point Charges

- Electric circuits: point of zero potential is defined by grounding some point in the circuit
- Electric potential due to a point charge at a point in space: point of zero potential is taken at an infinite distance from the charge
- With this choice, a potential can be found as

$$V_f - V_i = -\int_{i}^{f=\infty} E.dr \Rightarrow V = \frac{kq}{r} \qquad or \ldots \qquad V = \frac{\Delta U}{q'} = k\frac{q}{r}$$

Note: the potential depends on the distance (r) between a test charge (q') and a main charge q

Superposition Principle for Potentials

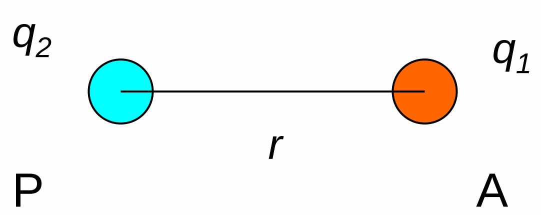
- If more than one point charge is present, their electric potential can be found by applying superposition principle

  The total electric potential at some point P due to several point charges is the algebraic sum of the electric potentials due to the individual charges.

- Remember that potentials are scalar quantities!

Potential Energy of a System of Point Charges

- Consider a system of two particles
- If V_1 is the electric potential due to charge q_1 at a point P, then energy required to bring the charge q_2 from infinity to P without acceleration is q_2V_1. If a distance between P and q_1 is r, then by definition



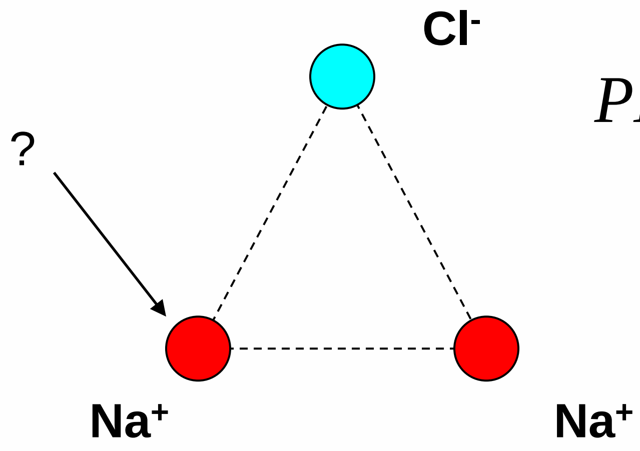
$$U = q_2 V_1 = k_e \frac{q_1 q_2}{r}$$

- Potential energy is positive if charges are of the same sign.

Example

Potential Energy of a group of ions

Three ions, Na^+, Na^+, and Cl^-, located such, that they form corners of an <u>equilateral</u> triangle of side 2 nm in water. What is the electric potential energy of one of the Na^+ ions?



$$PE = k_e \frac{q_{Na} q_{Cl}}{r} + k_e \frac{q_{Na} q_{Na}}{r} = k_e \frac{q_{Na}}{r}\left[q_{Cl} + q_{Na}\right]$$

$$q_{Cl} = -q_{Na}\,!$$

$$PE = k_e \frac{q_{Na}}{r}\left[-q_{Na} + q_{Na}\right] = 0$$

Example: Potential of two point charges

Two electrons are held in place 10.0 cm apart. Point a is midway between the two electrons, and point b is 12.0 cm directly above point a.

(a) Calculate the electric potential at point a and at point b.

(b) A third electron is released from rest at point b. What is the velocity of this electron when it is infinitely far from the other two electrons? The mass of an electron is $m_e = 9.11 \times 10^{-31}$ kg.

Example: Two Point Charges

Suppose two electrons are held in place 10.0 cm apart. Point a is midway between the two electrons, and point b is 12.0 cm directly above point a.

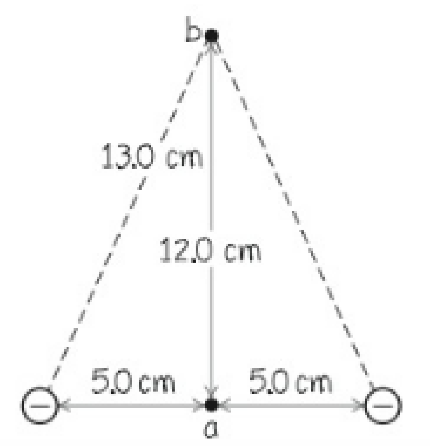
(a) Calculate the electric potential at point ‘a’ and at point ‘b’.

(b) A third electron is released from rest at point ‘b’. What is the velocity of this electron when it is very very far from the other two electrons at position ‘c’? $m_e = 9.11 \times 10^{-31} kg$

$$V = V_1 + V_2 = k\frac{q_1}{r_1} + k\frac{q_2}{r_2}$$

$$\boldsymbol{V_a = -\frac{2ke}{r_a}}$$

$$\boldsymbol{V_b = -\frac{2ke}{r_b}}$$



$$r_1 = r_2 = r_a = 0.050m$$

$$q_1 = q_2 = -e$$

Example continued...

(b) Remember that U = 0 when r = ∞

$$KE_b + U_b = KE_c + U_c$$

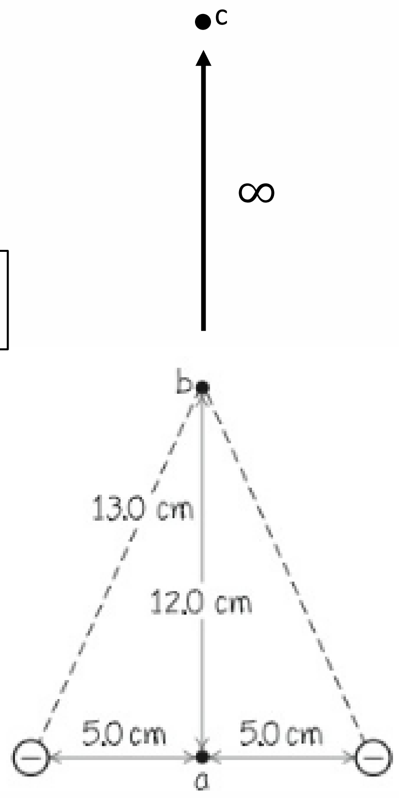
$$0 + U_b = KE_c + 0$$

$$\frac{1}{2}m{v_c}^2 = qV_b = -eV_b$$

$$\downarrow$$

$$v_c = 88\ m/s$$

$$\boxed{V = \frac{PE}{q}\ or\ PE = qV}$$



Additional Information

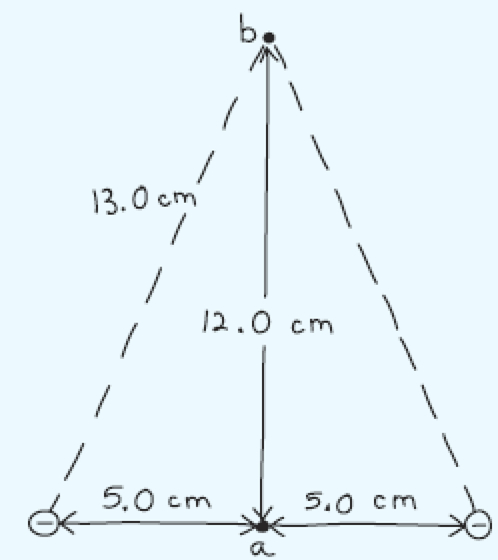
$$r_b = \sqrt{(12.0\text{ cm}^2) + (5.0\text{ cm})^2} = 13.0\text{ cm}$$

SOLVE Part (a): The electric potential V at each point is the sum of the electric potentials of each electron: $V = V_1 + V_2 = k\dfrac{q_1}{r_1} + k\dfrac{q_2}{r_2}$, with $q_1 = q_2 = -e$. At point a, $r_1 = r_2 = r_a = 0.050$ m, so

$$V_a = -\frac{2ke}{r_a} = -\frac{2(8.99 \times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(1.60 \times 10^{-19}\text{ C})}{0.050\text{ m}}$$
$$= -5.8 \times 10^{-8}\text{ V}.$$

At point b, $r_1 = r_2 = r_b = 0.130$ m, so

$$V_b = -\frac{2ke}{r_b} = -\frac{2(8.99 \times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(1.60 \times 10^{-19}\text{ C})}{0.130\text{ m}}$$
$$= -2.2 \times 10^{-8}\text{ V}.$$



Part (b): Remember that our equation for potential assumes that U is zero when $r = \infty$. Thus, when the third electron is far from the other two (at a location we designate c), we can assume that $U = 0$. To find the electron's speed at point c, we use conservation of energy:

$$K_b + U_b = K_c + U_c.$$

We solve for K_c. First we use $U = q'V$ and $q' = -e$ to rewrite the preceding expression as

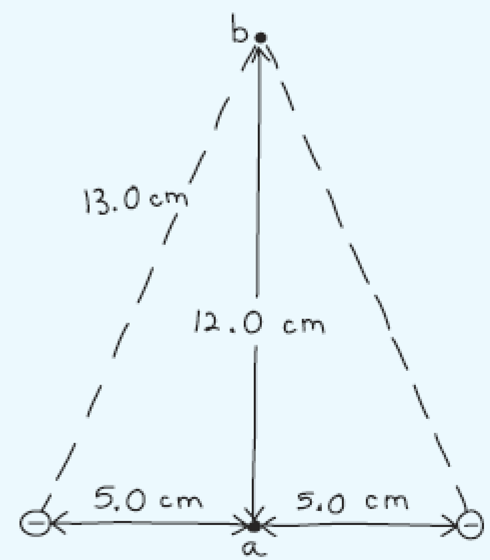
$$K_b - eV_b = K_c - eV_c.$$

We know that $V_c = 0$ because $V_c = \dfrac{kq}{r_c}$ and r_c is very large. Also, $K_b = 0$ because the electron is at rest before it is released. Then

$$\begin{aligned} K_c &= -eV_b = -(1.60 \times 10^{-19}\,\mathrm{C})(-2.2 \times 10^{-8}\,\mathrm{V}) \\ &= +3.52 \times 10^{-27}\,\mathrm{J}, \\ K_c &= \tfrac{1}{2} m_e v_c^2, \end{aligned}$$

so

$$v_c = \sqrt{\frac{2K_c}{m_e}} = \sqrt{\frac{2(3.52 \times 10^{-27}\,\mathrm{J})}{9.11 \times 10^{-31}\,\mathrm{kg}}} = 88\,\mathrm{m/s}.$$



Example: Parallel Plates

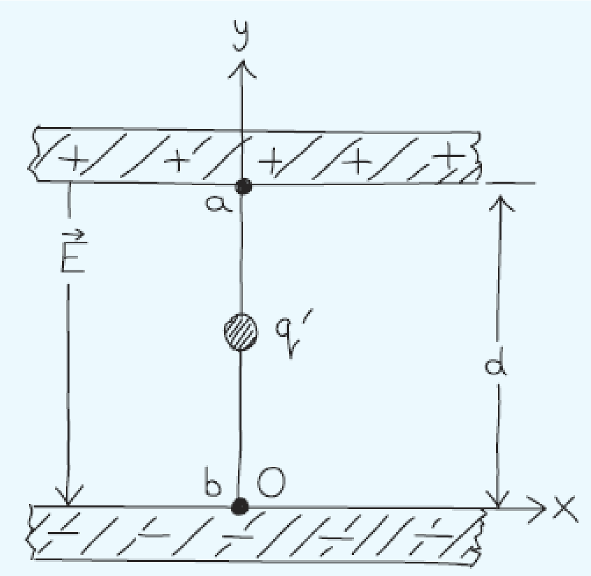
Find the potential at any height '*y*' between the two charged parallel plates.

SOLVE The potential energy U for a test charge q' at a distance y above the bottom plate is given by Equation 18.5, $U = q'Ey$. The potential V at point y is the potential energy per unit charge, $V = U/q'$, so

$$V = Ey.$$

Even if we had chosen a different reference level (at which $V = 0$), it would still be true that $V_y - V_b = Ey$. At point a, where $y = d$ and $V_y = V_a$, $V_a - V_b = Ed$ and

$$E = \frac{V_a - V_b}{d} = \frac{V_{ab}}{d}.$$



Note: this relation is only true for parallel plates i.e. uniform electric fields!

- Units of electric field (N/C) can be expressed in terms of the units of potential (as volts per meter)

$$1\,N/C = 1\,V/m \quad \Rightarrow \quad E = \frac{V}{d}$$

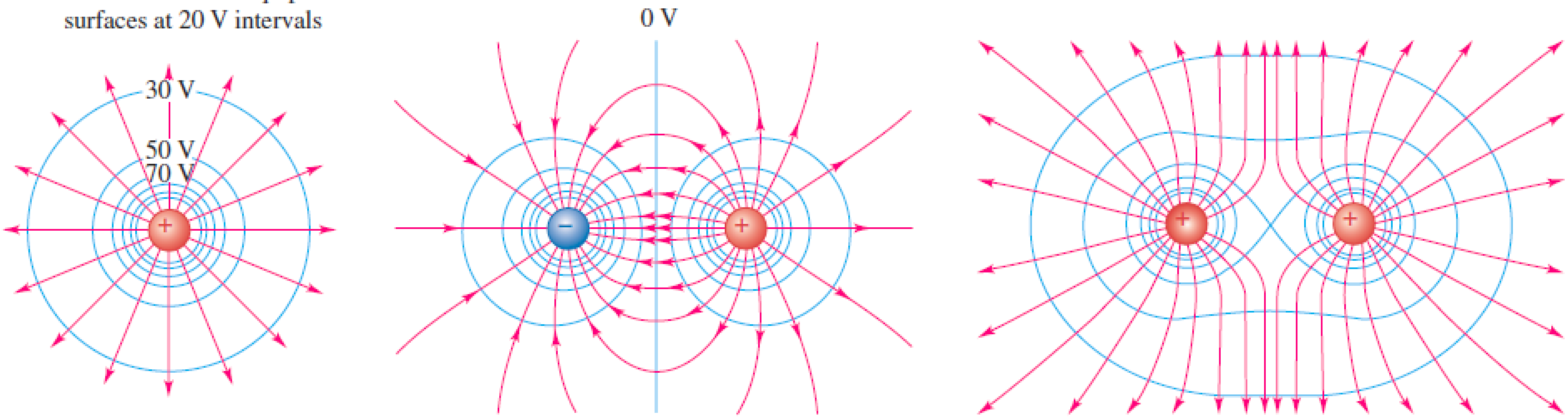
- Because a positive charge tends to move in the direction of the electric field, work must be done on the charge to move it in the direction, opposite the field. Thus,
  - A positive charge gains electrical potential energy when it is moved in a direction opposite the electric field
  - A negative charge loses electrical potential energy when it moves in the direction opposite the electric field

18.3 Equipotential Surfaces

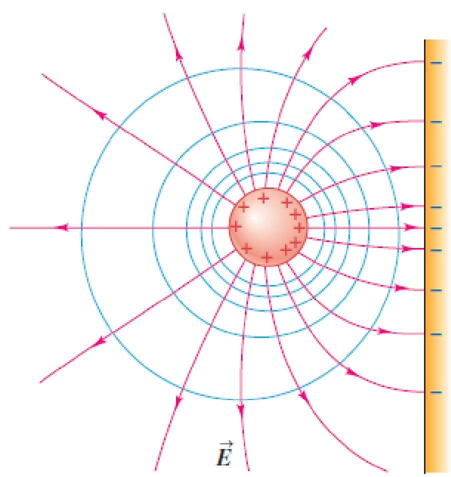
- The potential at various points in an electric field can be represented graphically by equipotential surfaces.
- An equipotential surface is defined as a surface on which the potential is the same at every point.
- Field lines and equipotential surfaces are always mutually perpendicular.
- The electric field at the surface of a conductor must be perpendicular to the surface at every point.

Electric field lines

Cross sections of equipotential surfaces at 20 V intervals



(a) A single positive charge (b) An electric dipole (c) Two equal positive charges



The Electric field and Potential Gradient

- The magnitude of the electric field at any point on an equipotential surface equals the rate of change of potential, ΔV, with respect to distance, as the point moves perpendicularly from the surface to an adjacent one a distance Δs away:

$$W = q'E\Delta s = -q'\Delta V$$

$$E = -\frac{\Delta V}{\Delta s}$$

- Negative sign shows that the potential decreases when a point moves in the direction of the electric field.
- The quantity $\Delta V/\Delta s$ is called the **potential gradient.**

Potentials and Charged Conductors

- Recall that work is the opposite of the change in potential energy,

$$W = -\Delta U = -q(V_B - V_A)$$

- No work is required to move a charge between two points that are at the same potential. That is, W=0 if $V_B=V_A$
- Recall:
  - all charge of a charged conductor is located on its surface
  - electric field, E, is always perpendicular to its surface, i.e. no work is done if charges are moved along the surface
- Thus: potential is constant everywhere on the surface of a charged conductor in equilibrium

… but that's not all!

- Because the electric field is zero inside the conductor, no work is required to move charges between any two points, i.e.

$$W = -q\left[V_B - V_A\right] = 0$$

- If work is zero, any two points inside the conductor have the same potential, i.e. potential is constant everywhere inside a conductor
- Finally, since one of the points can be arbitrarily close to the surface of the conductor, the electric potential is constant everywhere inside a conductor and equal to its value at the surface!
    - Note that the potential inside a conductor is not necessarily zero, even though the interior electric field is always zero!

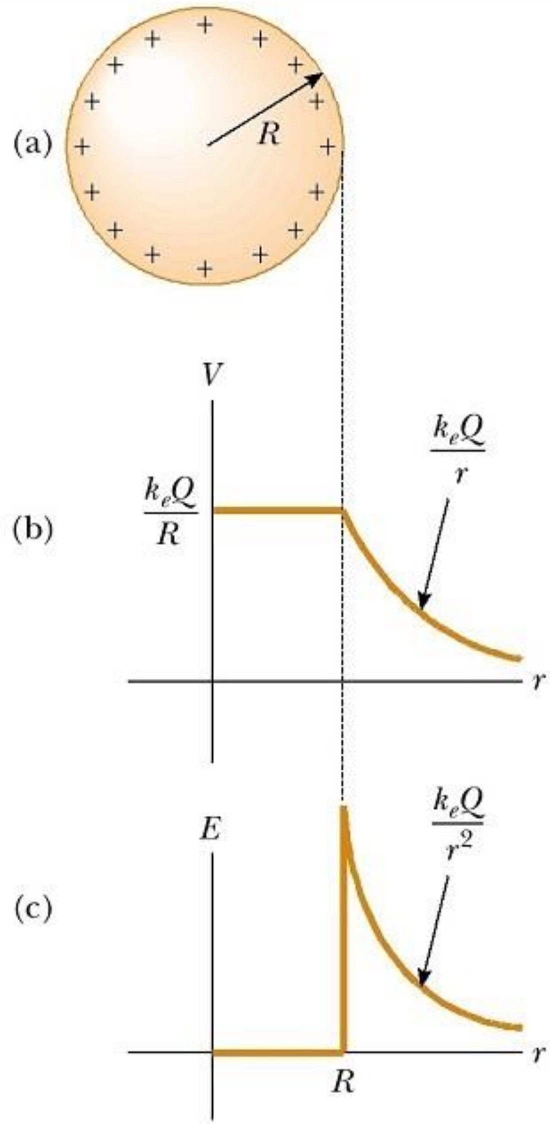


Figure 25.21 (a) The excess charge on a conducting sphere of radius R is uniformly distributed on its surface. (b) Electric potential versus distance r from the center of the charged conducting sphere. (c) Electric field magnitude versus distance r from the center of the charged conducting sphere.

The electron Volt (eV)

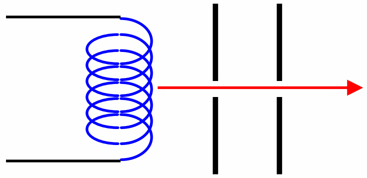
- A unit of energy commonly used in atomic, nuclear and particle physics is electron volt (eV)

The electron volt is defined as the energy that an electron (or proton) gains when moving through a potential difference of 1 V

$$\Delta U = q(V_b - V_a) = q\Delta V$$
$$= (1.6 \times 10^{-19})(1) = 1.6 \times 10^{-19} J$$

- Relation to SI:

1 eV = 1.6×10^{-19} C·V = 1.6×10^{-19} J



$V_{ab}=1\ V$

18.5 Capacitance and Capacitors

https://phet.colorado.edu/en/simulation/legacy/capacitor-lab

Capacitors

- Capacitors are devices that store electric potential energy and electric charge.
- Capacitors are used in many every-day applications
  - Heart defibrillators
  - Camera flash units
  - Touch screens
- Capacitors are an essential part of electronics.
  - Capacitors can be micro-sized on computer chips or super-sized for high power circuits such as FM radio transmitters.

Definition of Capacitance

- The Capaitance C is the ratio of the magnitude of the charge Q on *either* plate to the magnitude of the potential difference between them

$$C = \frac{Q}{V}$$

- The units of capacitance are coulombs per volt or… **Farads**
- The unit of capacitance has been given the name Farad (abbreviated F) named after British physicist Michael Faraday (1791 - 1867)

$$1\,\mathrm{F} = \frac{1\,\mathrm{C}}{1\,\mathrm{V}}$$

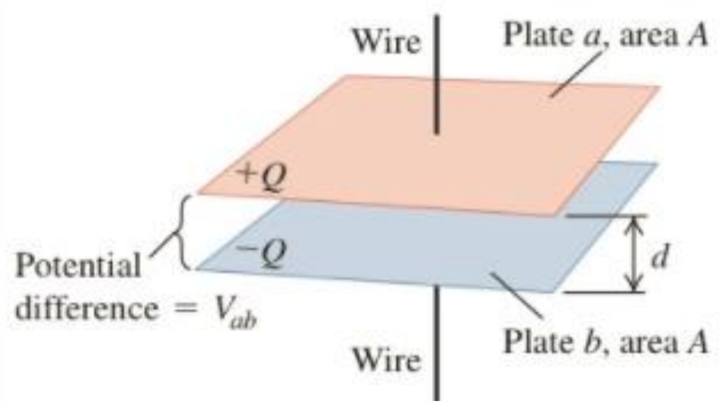
$$Capacitance\ (F) = \frac{Charge\ (C)}{Volt\ (V)}$$

- A farad is a very large capacitance
  - Typically we deal with μF (10^{-6} F), nF (10^{-9} F), or pF (10^{-12} F)

The Parallel-Plate Capacitor

- The capacitance of a device is directly proportional to the area of each plate and inversely proportional to their separation distance

$$C = \frac{Q}{V} = \varepsilon_0 \frac{A}{d}$$



- where A is the area of **one** of the plates, d is the separation, ε_0 is a constant (permittivity of free space)

$\varepsilon_0 = 8.85\times10^{-12}\ C^2/N\cdot m^2$

Example

Example: A parallel plate capacitor has plates 2.00 m^2 in area, separated by a distance of 5.00 mm. A potential difference of 10000 V is applied across the capacitor. Determine:

- the capacitance
- the charge on each plate
- the electric field between the plates

Solution:

Given:

ΔV=10,000 V
A = 2.00 m^2
d = 5.00 mm

Find:

C=?
Q=?
E=?

Since we are dealing with the parallel-plate capacitor, the capacitance can be found as

$$C = \varepsilon_0 \frac{A}{d} = \left(8.85\times10^{-12}\, C^2/N\cdot m^2\right)\frac{2.00\, m^2}{5.00\times10^{-3}\, m}$$

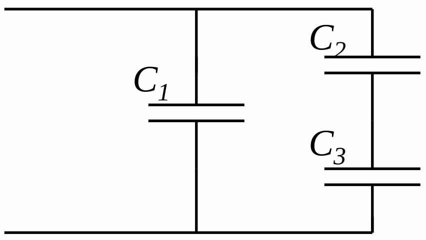
$$= 3.54\times10^{-9}\, F = 3.54\, nF$$

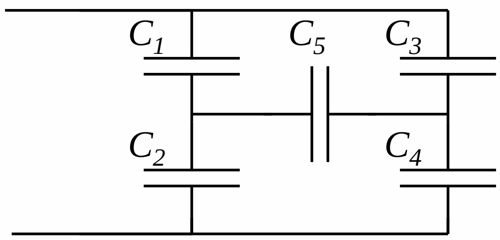
Once the capacitance is known, the charge can be found from the definition of a capacitance via charge and potential difference:

$$Q = C\Delta V = \left(3.54\times10^{-9}\, F\right)\left(10000V\right) = 3.54\times10^{-5}\, C$$

Combinations of capacitors

- It is very often that more than one capacitor is used in an electric circuit
- We would have to learn how to compute the equivalent capacitance of certain combinations of capacitors





Capacitors in Series

Connecting a battery to the serial combination of capacitors is equivalent to introducing the same charge for both capacitors, $Q_1 = Q_2 = Q$

A voltage induced in the system from the battery is the sum of potential differences across the individual capacitors, $V = V_1 + V_2$

$$\text{By definition,}$$

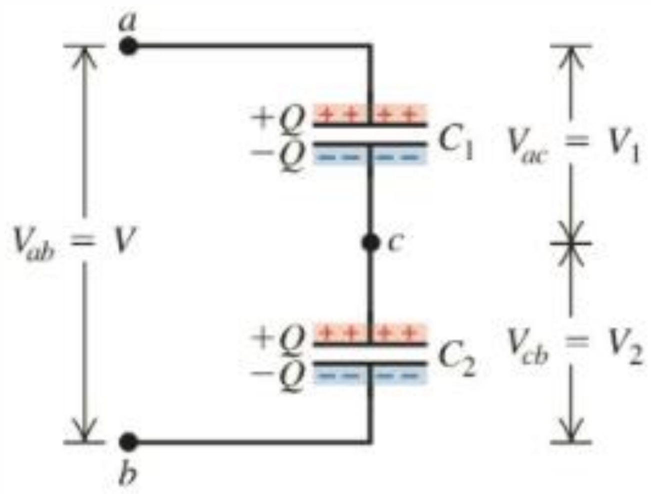
$$V_1 = \frac{Q_1}{C_1} \;\; \& \;\; V_2 = \frac{Q_2}{C_2}$$

$$\text{Thus, } C_{eq} \text{ would be}$$

$$V = V_1 + V_2$$

$$\frac{Q}{C_{eq}} = \frac{Q_1}{C_1} + \frac{Q_2}{C_2}$$

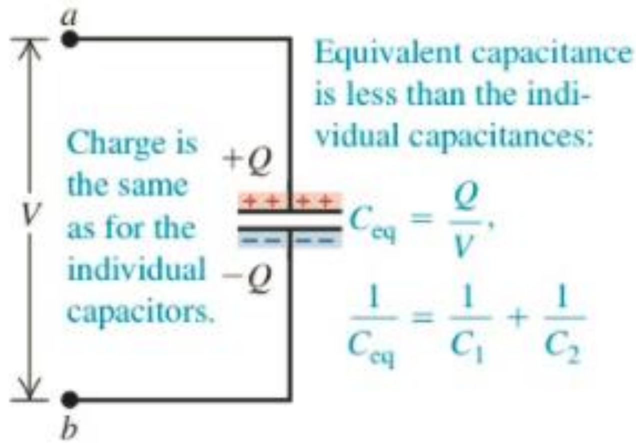
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}, \quad \text{because } Q = Q_1 = Q_2$$



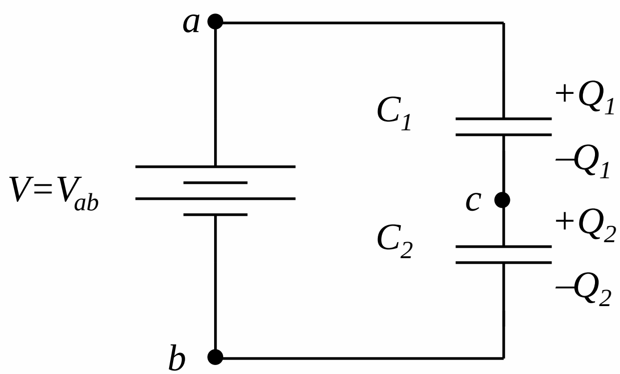
- Analogous formula is true for any number of capacitors,

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots \qquad \text{(series combination)}$$

- It follows that the equivalent capacitance of a series combination of capacitors is always less than any of the individual capacitance in the combination



Example: A 3 μF capacitor and a 6 μF capacitor are connected in series across an 18 V battery. Determine the equivalent capacitance and total charge deposited.



Given:

$V = 18\ V$
$C_1 = 3\ \mu F$
$C_2 = 6\ \mu F$

Find:

C_{eq}=?
Q=?

First determine equivalent capacitance of C_1 and C_2:

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{2 \times 10^{-6}} \quad \Rightarrow \quad C_{eq} = 2\mu F$$

Next, determine the charge

$$Q = C\Delta V = \left(2\times10^{-6}F\right)\left(18V\right) = 3.6\times10^{-5}C$$

Capacitors in Parallel

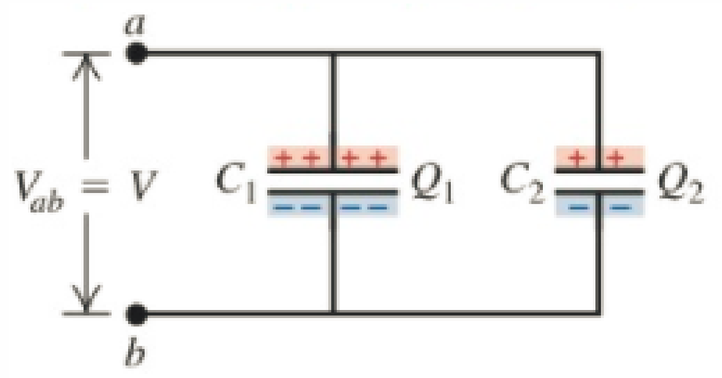
Connecting a battery to a parallel combination of capacitors is equivalent to introducing the same potential difference for both capacitors, $V_1 = V_2 = V$

A total charge transferred to the system from the battery is the sum of charges of the two capacitors, $Q_1 + Q_2 = Q$

$$By \quad definition,$$

$$Q_1 = C_1 V_1 \quad \& \quad Q_2 = C_2 V_2$$

$$Thus, \quad C_{eq} \quad would \quad be$$



$$Q = Q_1 + Q_2$$

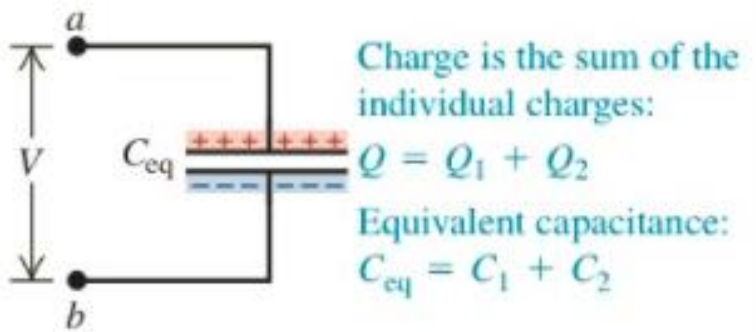
$$C_{eq} V = C_1 V_1 + C_2 V_2$$

$$C_{eq} = C_1 + C_2, \quad \text{since} \quad V = V_1 = V_2$$

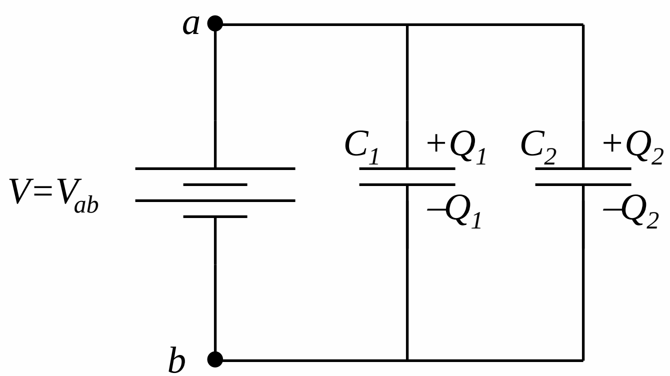
- Analogous formula is true for any number of capacitors,

$$C_{eq} = C_1 + C_2 + C_3 + \ldots \qquad \text{(parallel combination)}$$

- It follows that the equivalent capacitance of a parallel combination of capacitors is greater than any of the individual capacitors



Example: A 3 μF capacitor and a 6 μF capacitor are connected in parallel across an 18 V battery. Determine the equivalent capacitance and total charge deposited.



Given:

$V = 18$ V
$C_1 = 3\ \mu F$
$C_2 = 6\ \mu F$

Find:

C_{eq}=?
Q=?

First determine equivalent capacitance of C_1 and C_2:

$$C_{12} = C_1 + C_2 = 9\ \mu F$$

Next, determine the charge

$$Q = C\Delta V = \left(9\times10^{-6} F\right)\left(18V\right) = 1.6\times10^{-4} C$$

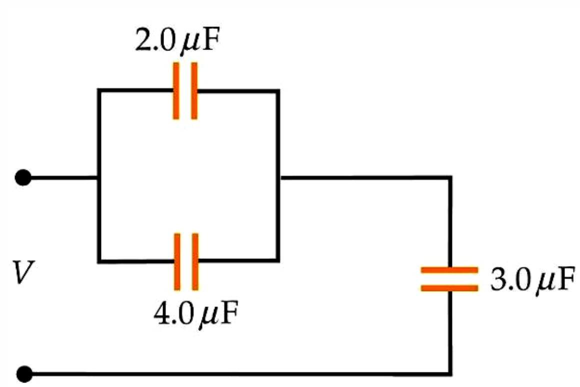
Example

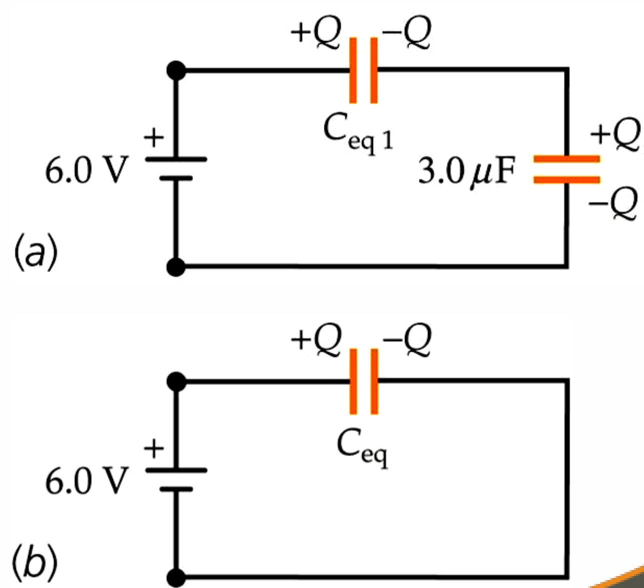
Capacitors in Series and Parallel

Three capacitors are connected as shown

(a) Find the equivalent capacitance of the 3-capacitor combination

(b) The capacitors, initially uncharged, are connected across a 6.0 V battery. Find the charge and voltage drop for each capacitor

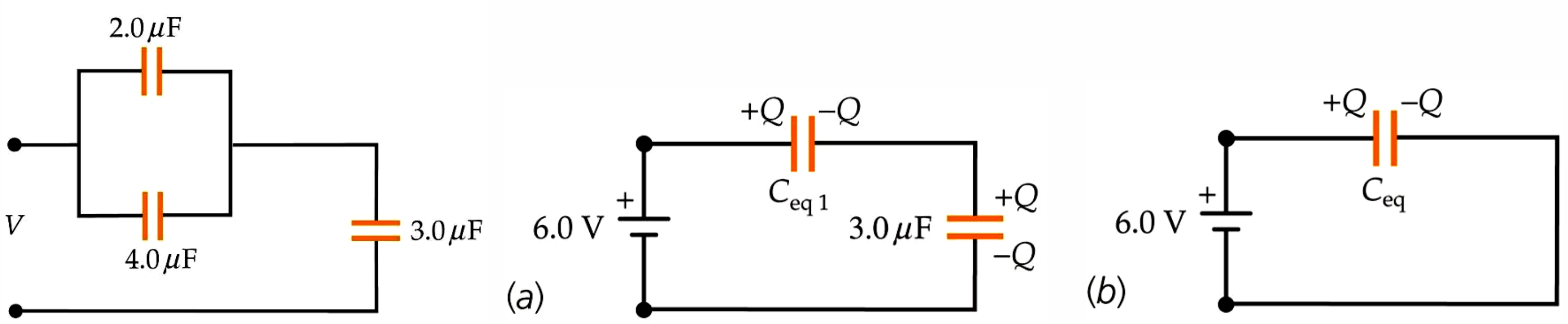




(a) Find the equivalent capacitance of the 3-capacitor combination

$$C_{eq1} = C_1 + C_2 = 6.0\ \mu\text{F}$$

$$C_{eq} = 1/(1/C_{eq1} + 1/C_3) = 1/[1/(6.0\ \mu\text{F}) + 1/(3.0\ \mu\text{F})] = 2.0\ \mu\text{F}$$



(b) The capacitors, initially uncharged, are connected across a 6.0 V battery. Find the charge and voltage drop for each capacitor.

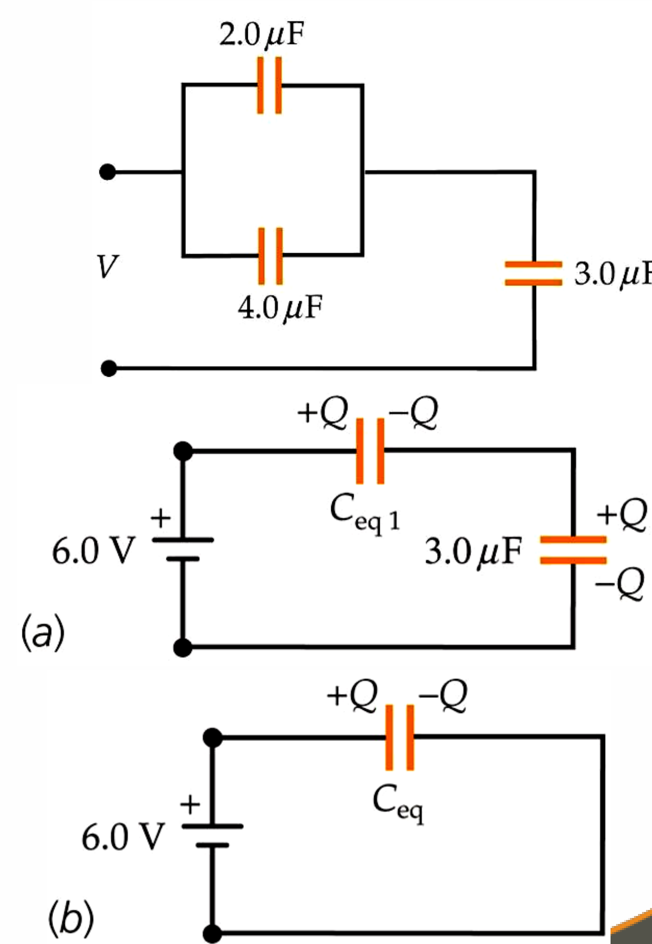
$$Q = C_{eq}V = (2\mu F)(6V) = 12\mu C$$

$$V_3 = \frac{Q}{C_3} = \frac{12\mu C}{3\mu F} = 4V$$

$$V_{24} = V - V_3 = 6 - 4 = 2.0V$$

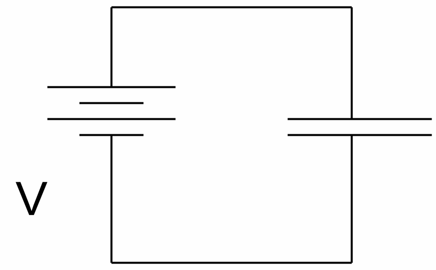
$$Q_2 = C_2 V_{24} = (2\mu F)(2.0V) = 4\mu C$$

$$Q_4 = C_4 V_{24} = (4.0\mu F)(2.0V) = 8.0\mu C$$



18.7 Electric Field Energy

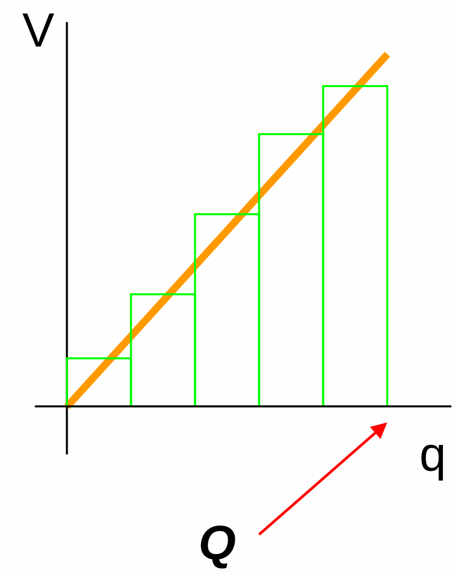
Energy stored in a charged capacitor



- Consider a battery connected to a capacitor
- A battery must do work to move charges from one plate to the other. The work done to move a small additional charge q across a voltage ΔV is

$\Delta W = q\Delta V$

- As the charge increases, V increases so the work to bring q increases. Using calculus methods we find that the energy (U) stored in a capacitor is given by:



$$U = \frac{1}{2}QV = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2$$

$recall \quad Q = CV$

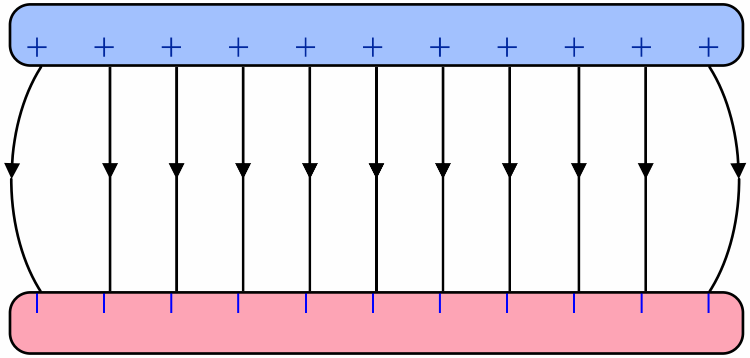
Example

Electric field energy in parallel-plate capacitor

Find <u>electric field energy density</u> (energy per unit volume) in a parallel-plate capacitor

Recall $$U = \frac{1}{2} C V^2$$

$$C = \frac{\varepsilon_0 A}{d} \qquad V = Ed$$



Thus, $$u \equiv U / volume = \text{energy density}$$

$$= \frac{1}{2} \frac{\varepsilon_0 A}{d} (Ed)^2 / (Ad)$$

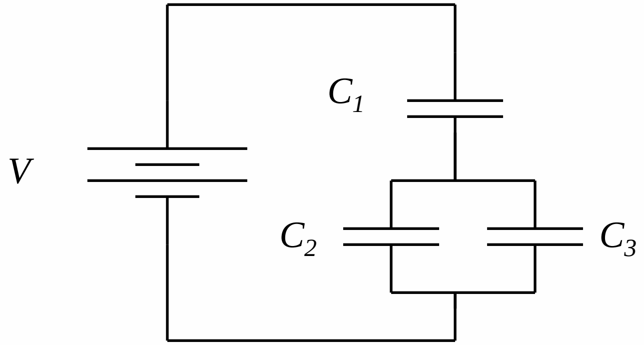
and so, the energy density is $$u = \frac{1}{2} \varepsilon_0 E^2$$

Example: In the circuit shown V = 48V, C_1 = 9μF, C_2 = 4μF and C_3 = 8μF.

(a) determine the equivalent capacitance of the circuit,

(b) determine the energy stored in the combination by calculating the energy stored in the equivalent capacitance,

$$C_{eq} = 5.14\mu F$$



The energy stored in the capacitor C_{123} is then

$$U = \frac{1}{2}CV^2 = \frac{1}{2}\left(5.14\times10^{-6}F\right)\left(48V\right)^2 = 5.9\times10^{-3}J$$