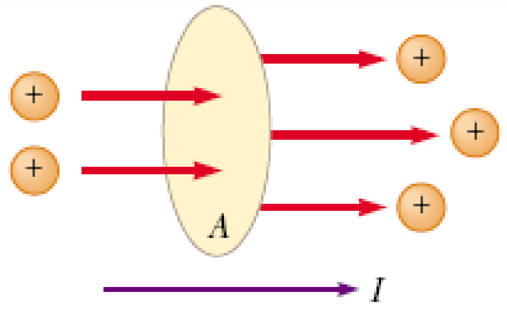
19 Current, Resistance, and Direct-Current Circuits

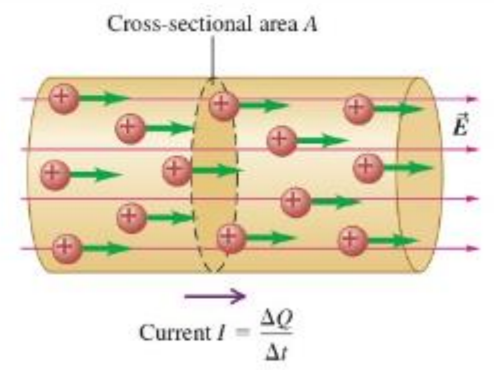
19.1 Electric Current

Definition: the current is the rate at which charge flows through this surface.



When a net charge ΔQ passes through a cross section of conductor during time Δt, the current is

$$I = \frac{\Delta Q}{\Delta t}$$



The SI units of current is **the ampere (A)**.

- 1 A = 1 C/s
- 1 A of current is equivalent to 1 C of charge passing through the area in a time interval of 1 s.

Example: The amount of charge that passes through the filament of a certain light bulb in 2.00s is 1.67C. Find the current in the light bulb.

$$I = \frac{\Delta Q}{\Delta t} = \frac{1.67C}{2.00s} = 0.835A$$

0.835C every second!

What is the number of fundamental units of charge?

$$q = ne$$

$$so\ n = \frac{q}{e}$$

Extra example:

The current which passes through the filament of a certain light bulb in 3.6 minutes is 8.4μA. Calculate the number of fundamental units of charge which pass during this time

19.2 Resistance and Ohm's Law

When a voltage (potential difference) is applied across the ends of a metallic conductor, the current is found to be proportional to the applied voltage.

$$I \propto \Delta V$$

In situations where the proportionality is exact, one can write.

$$\Delta V = IR$$

The proportionality constant R is called resistance of the conductor.

The resistance is defined as the ratio

$$R = \frac{V}{I} \qquad\qquad 1\Omega = \frac{1V}{1A}$$

In SI units, resistance is expressed in volts per ampere.
A special name is given: ohms (Ω)

Example: if a potential difference of 10 V applied across a conductor produces a 0.2 A current

then one concludes the conductors has a resistance of

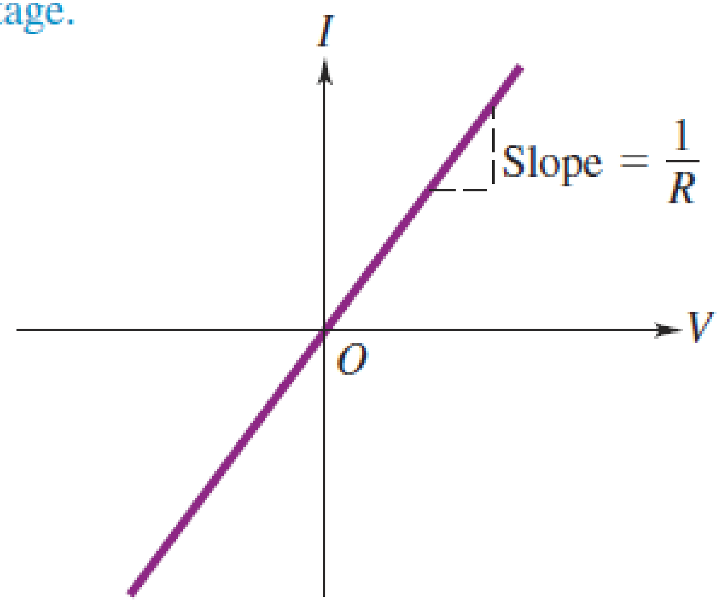
$$R = \frac{V}{I} = \frac{10V}{0.2A} = 50\Omega$$

Ohm's Law

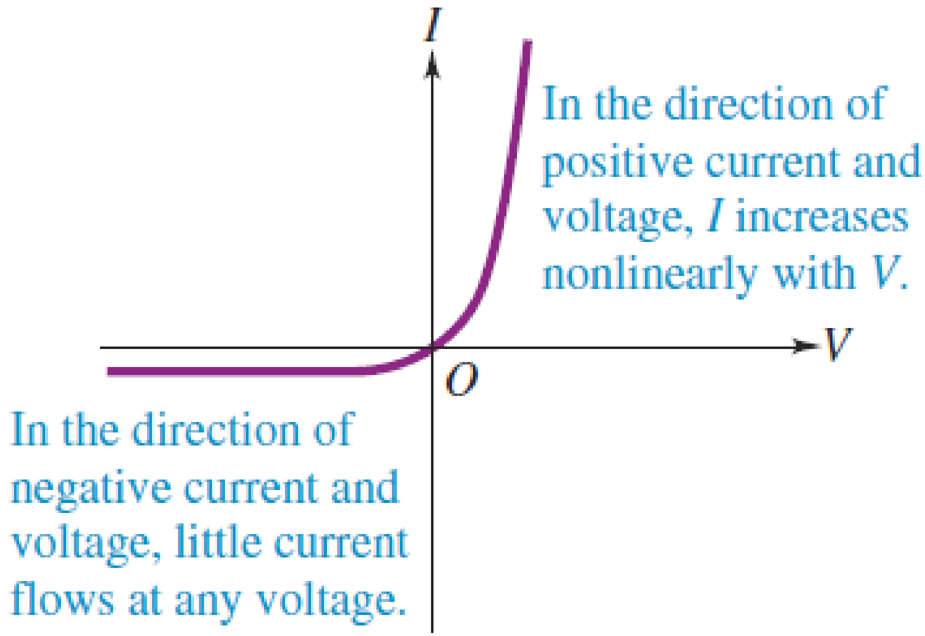
- Resistance in a conductor arises because of collisions between electrons/moving charges and fixed charges within the material.
- In many materials, including most metals, if the physical properties (e.g. length, width) are constant, the resistance is constant over a wide range of applied voltages.
- This is a statement of Ohm's law.

Ohm's Law $$\Delta V = IR$$

Ohmic resistor (e.g., typical metal wire): At a given temperature, current is proportional to voltage.



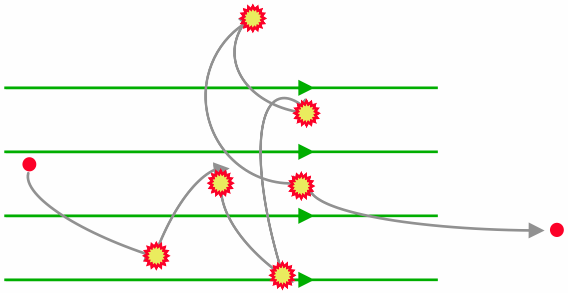
Semiconductor diode: a non-ohmic resistor



- ❑ Ohmic materials: the I-V curve is linear. *This device does obey Ohm's law.*
- ❑ Non-Ohmic materials: the I-V curve is nonlinear for a diode. *This device does not obey Ohm's law*.

Resistivity

- Electrons/charges moving inside a conductor subject to an external potential (V) constantly collide with atoms of the conductor.
- They lose energy and are repeated re-accelerated by the electric field produced by the external potential.
- The collision process is equivalent to an internal friction.
- This is the origin of a material's resistance.



- The resistance R of an ohmic conductor is proportional to the its length, *l*, and inversely proportional to the cross sectional area, *A*, of the conductor.

$$R = \rho \frac{l}{A}$$

The constant of proportionality ρ is called the resistivity of the material.

- Every material has a characteristic resistivity that depends on its electronic structure and the temperature.
- Good conductors have low resistivity.
- Insulators have high resistivity.

Resistivity - Units

$$R = \rho \frac{l}{A} \quad \Longrightarrow \quad \rho = \frac{RA}{l}$$

- Resistance expressed in Ohms
- Length in meter
- Area are m^2

So resistivity has units of Ω m

Resistivity of various materials at room temperature

Substance	$\rho\ (\Omega \cdot m)$	Substance	$\rho\ (\Omega \cdot m)$
Conductors:		Mercury	95×10^{-8}
Silver	1.47×10^{-8}	Nichrome alloy	100×10^{-8}
Copper	1.72×10^{-8}	Insulators:	
Gold	2.44×10^{-8}	Glass	$10^{10} - 10^{14}$
Aluminum	2.63×10^{-8}	Lucite	$> 10^{13}$
Tungsten	5.51×10^{-8}	Quartz (fused)	75×10^{16}
Steel	20×10^{-8}	Teflon®	$> 10^{13}$
Lead	22×10^{-8}	Wood	$10^{8} - 10^{11}$

Example…

Nichrome resistivity = 1.5 x 10^{-6} Ωm

(a) Calculate the resistance per unit length of a nichrome wire of radius 0.321 mm.

Cross section: $A = \pi r^2 = \pi \left(0.321\times10^{-3}\,m\right)^2 = 3.24\times10^{-7}\,m^2$

Resistivity (Table): 1.5 x 10^{-6} Ωm.

Resistance/unit length: $\frac{R}{l} = \frac{\rho}{A} = \frac{1.5\times10^{-6}\,\Omega m}{3.24\times10^{-7}\,m^2} = 4.6\,\Omega/m$

(b) If a potential difference of 10.0 V is maintained across a 1.0 m length of the nichrome wire, what is the current?

$$I = \frac{\Delta V}{R} = \frac{10.0V}{4.6\Omega} = 2.2A$$

Additional Questions…

- Taking the resistivity of platinoid as $3.3 \times 10^{-7}\ \Omega m$, find the resistance of 7.0 m of platinoid wire which has a diameter 0.14 cm.

- The resistance of one ohm is approximated to a column of mercury 1.06 m long and of uniform cross-section of one hundredth of a cm^2. Find the resistivity of mercury.

- The maximum allowable resistance for an underwater cable is one hundredth of an ohm per metre. If the resistivity of copper is $1.54 \times 10^{-8}\ \Omega m$, find the minimum diameter of copper cable that could be used.

The reciprocal of the resistivity is called the **conductivity**,

$$\sigma = \frac{1}{\rho}$$

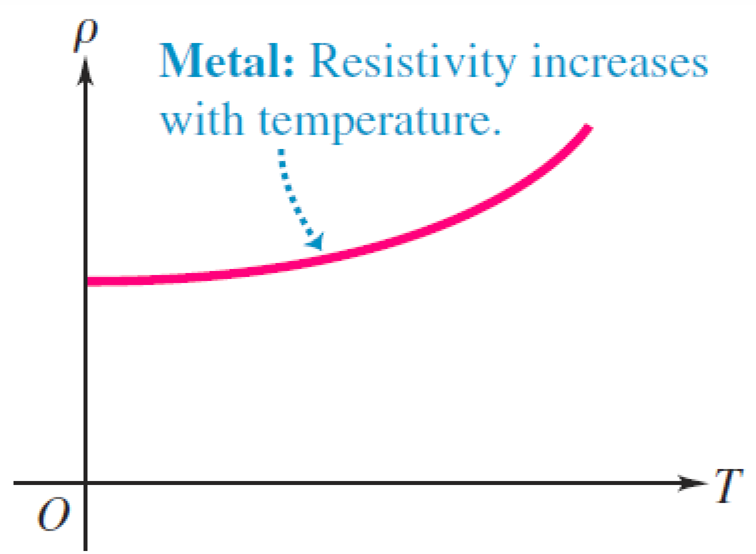
[Q] Stretching changes resistance: A wire of resistance R is stretched uniformly until it is twice its original length. What happens to its resistance?

[Q] Speaker wires: Suppose you want to connect your stereo to remote speakers. (a) If each wire must be 20m long, what diameter copper wire should you use to keep the resistance less than 0.1Ω per wire? (b) If the current on each speaker is 4.0A, what is the voltage drop across each wire?

[Q] A 2.4m length of wire that is 0.031cm^2 in cross section has a measured resistance of 0.24Ω. Calculate the conductivity of the material.

Temperature Dependence of Resistance

- The resistivity of a metal depends on many (environmental) factors.
- The most important factor is the temperature.
- For most metals, the resistivity increases with increasing temperature.
- The increased resistivity arises because of larger friction caused by the more violent motion of the atoms of the metal.



For most metals, resistivity increases approximately linearly with temperature

$$\rho = \rho_o\left[1+\alpha\left(T-T_o\right)\right]$$

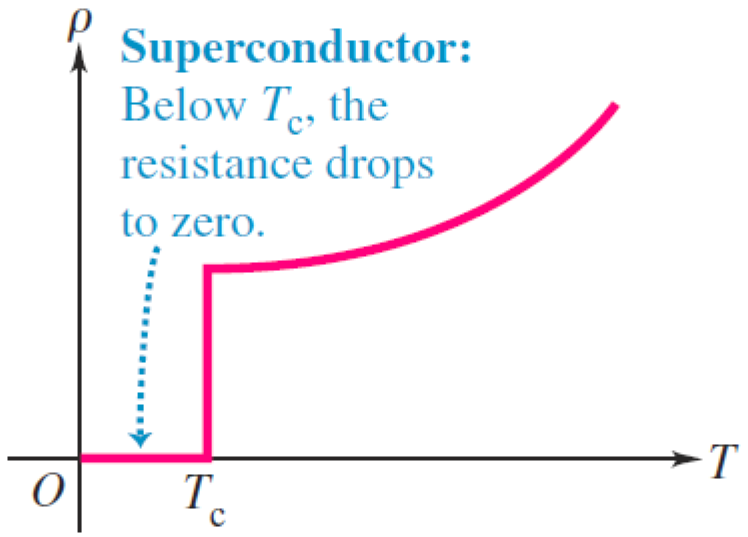
- ρ is the resistivity at temperature T (measured in Celsius).
- ρ_o is the reference resistivity at the reference temperature T_o (usually taken to be 20 ^oC).
- α is a parameter called **temperature coefficient of resistivity**.

If the resistivity increases linearly with temperature, so does the resistance

$$R = R_o\left[1+\alpha\left(T-T_o\right)\right]$$

Superconductivity

- In some materials as the temperature decreases, the resistivity at first decreases smoothly, like that of any metal. But then, at a certain critical transition temperature T_c the resistivity suddenly drops to zero!
- The established current in a superconducting wire continues indefinitely without the presence of any driving field.



Example

A length of 18 gauge copper wire with a diameter of 1.02 mm and a cross-sectional area of 8.20×10^{-7} m^2 has a resistance of 1.02 Ω at a temperature of 20°C. Find the resistance at 0°C and at 100°C. The temperature coefficient of resistivity of copper is 0.0039 1 $C°^{-1}$.

$$\begin{aligned} R &= R_0[1 + \alpha(T - T_0)] \\ &= (1.02\ \Omega)(1 + [0.0039(\text{C}°)^{-1}][0°\text{C} - 20°\text{C}]) \\ &= 0.94\ \Omega. \end{aligned}$$

At $T = 100°\text{C}$,

$$\begin{aligned} R &= (1.02\ \Omega)(1 + [0.0039(\text{C}°)^{-1}][100°\text{C} - 20°\text{C}]) \\ &= 1.34\ \Omega. \end{aligned}$$

19.3 Electromotive Force and Circuits

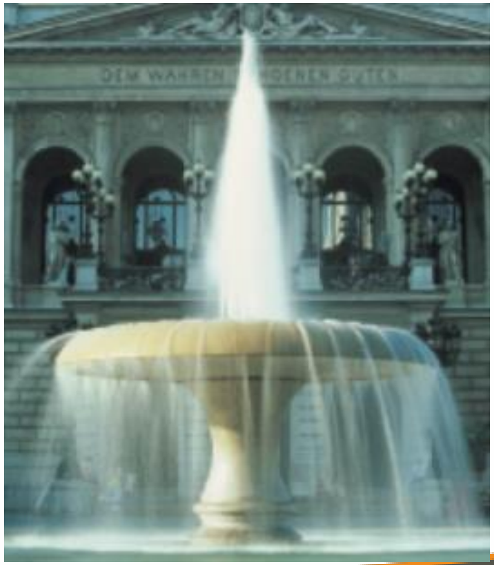
In a closed circuit, charge always moves in the direction of decreasing potential energy. There must be some part of the circuit where the potential energy ***increases***!

Electromotive force (emf): The influence that moves charge from lower to higher potential

A source of emf works as "charge pump" that forces charge to move in a direction opposite the electrostatic field inside the source.

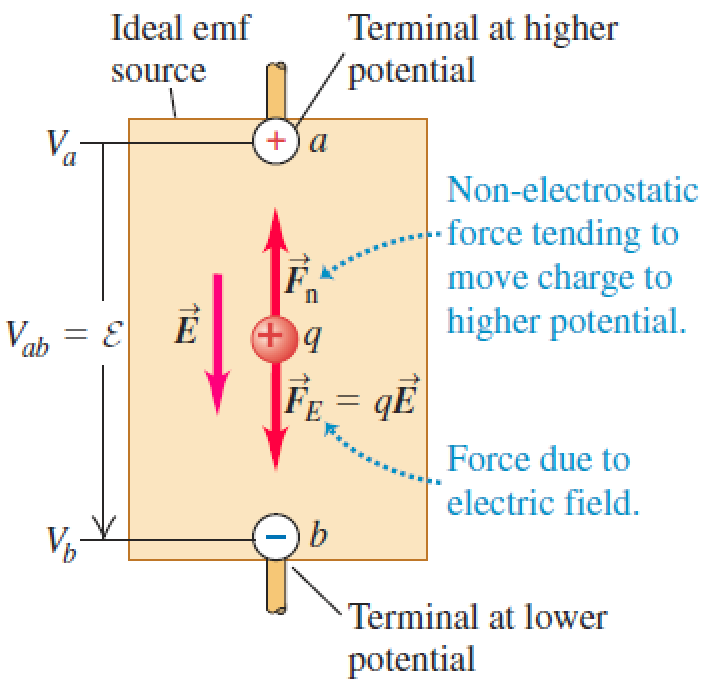
Examples of such sources are:

- **Batteries**
- **Generators**
- **Thermocouples**
- **Photo-voltaic cells**



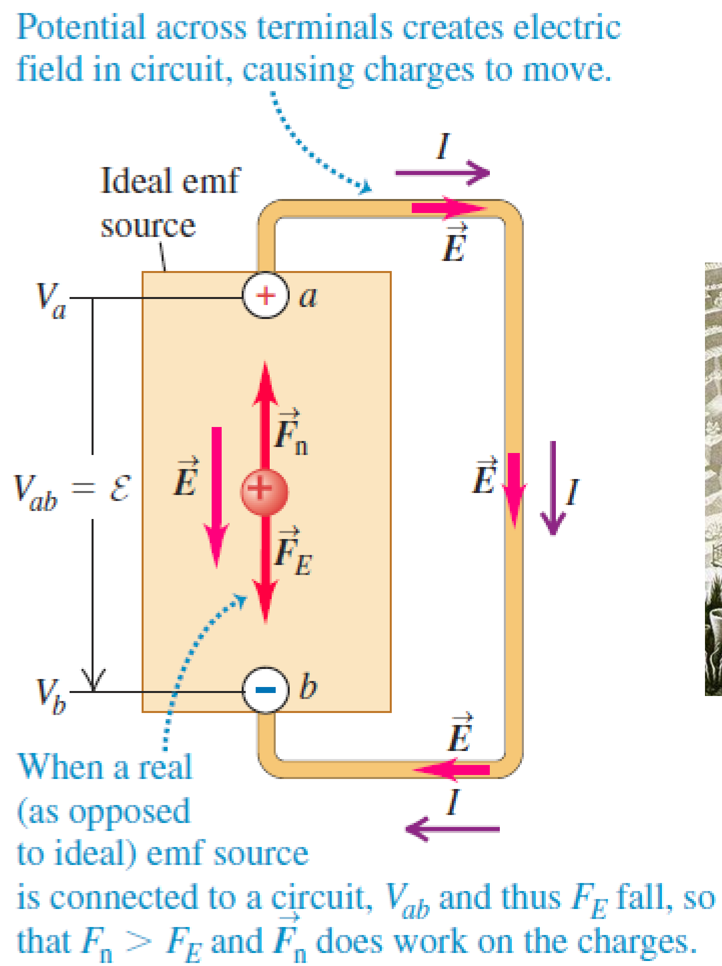
$$1V = \frac{1J}{1C}$$

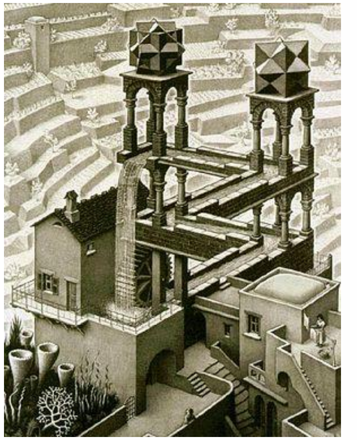
Ideal emf Source



When the emf source is not part of a closed circuit, $F_n = F_E$ and there is no net motion of charge between the terminals.

$$V_{ab} = \varepsilon$$

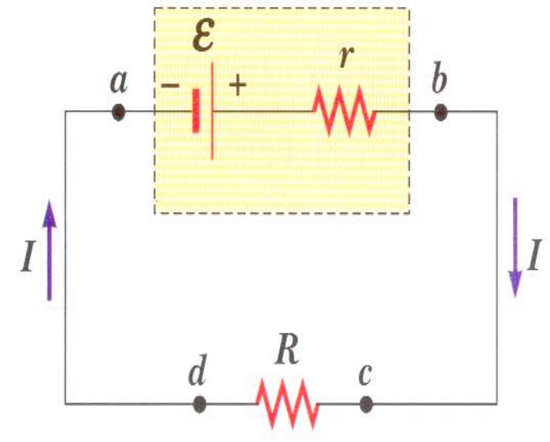




$$\varepsilon = V_{ab} = IR$$

Nothing is Perfect?

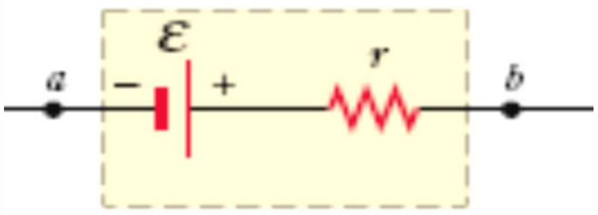
- Each **real** battery has some internal resistance



- 'a' to 'b': potential increases by the source of EMF, then decreases by *Ir* (because of the internal resistance – drop in potential)

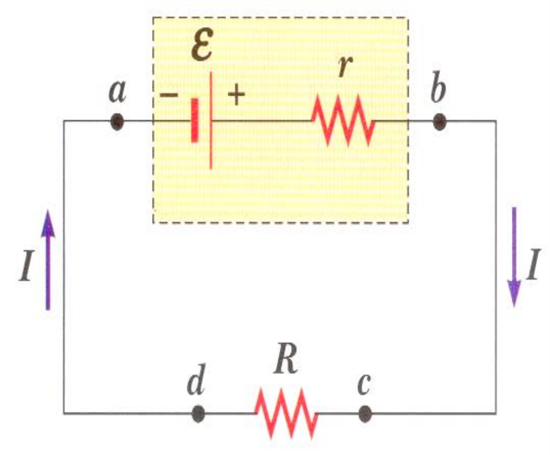
- Thus, terminal voltage on the battery V_{ab} is

$$V_{ab} = \varepsilon - Ir$$



Note: EMF is the same as the terminal voltage when the current is zero (open circuit)

- Now add a load resistance R
- Since it is connected by a conducting wire to the battery the terminal voltage is the same as the potential difference across the load resistance



$$V_{a \to a} = \varepsilon - Ir - IR = 0$$

$$or$$

$$\varepsilon = IR + Ir$$

- Thus, the current in the circuit is: $I = \dfrac{\varepsilon}{R + r}$

Example

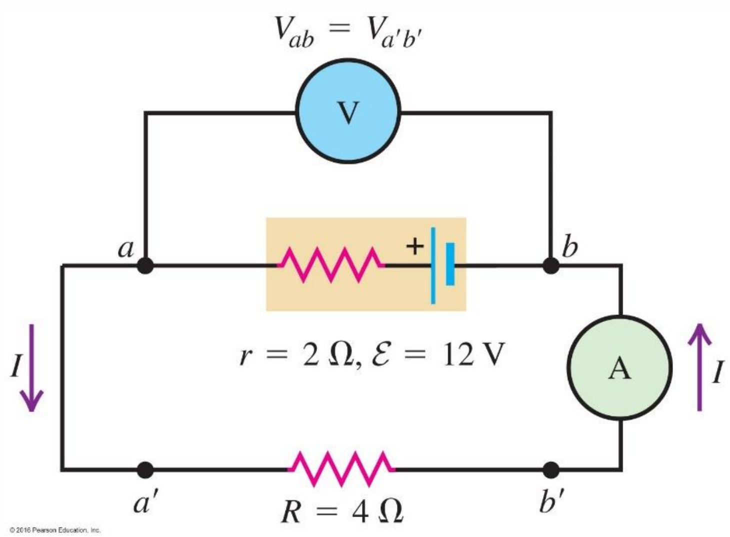
- What are the voltmeter and ammeter readings in this circuit?

$$V = \varepsilon - Ir$$

$$IR = \varepsilon - Ir$$

$$IR + Ir = \varepsilon$$

$$I(R + r) = \varepsilon$$

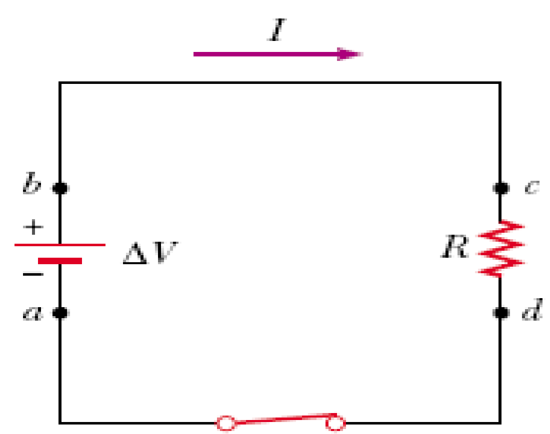


$$I = \frac{\varepsilon}{(R + r)}$$

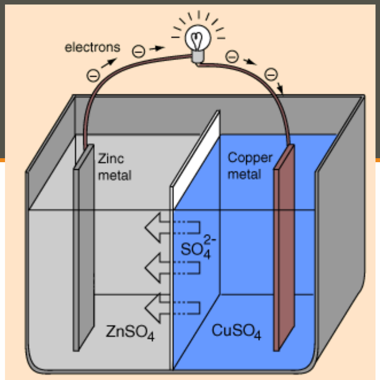
$$V = \varepsilon - Ir$$

19.4 Electrical Energy and Power

- In any circuit, a battery is used to induce electrical current
- Chemical energy of the battery is transformed into kinetic energy of mobile charge carriers (electrical energy gain)
- Any device that has resistance present in the circuit will transform this electrical energy into heat
- Kinetic energy of charge carriers is transformed into heat via collisions with atoms in a conductor (electrical potential energy loss)



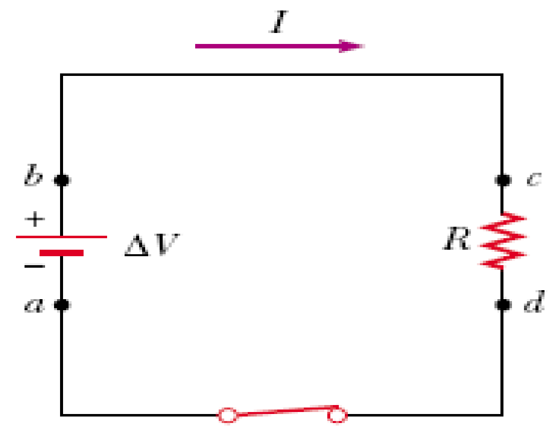
Electrical energy



$Zn(s) \rightarrow Zn^{2+}(aq) + 2e^-$

$Cu^{2+}(aq) + 2e^- \rightarrow Cu(s)$

- Consider circuit on the right
- Between ‘a’ and ‘b’: charge gains electrical energy from the battery and can be represented as work done on the charges



$$\Delta W = \Delta Q \Delta V$$
$$= I \Delta t \Delta V$$

- During this time, *battery loses chemical energy*

  - Between ‘c’ and ‘d’: electrical energy lost (transferred into heat)
  - Back to ‘a’: same potential energy (zero) as before
  - **Electrical energy gained** (from battery) = **Electrical energy lost** (on the resistor)

Power

- Power is defined as the amount of energy dissipated (consumed) or delivered in a certain time:

$$P = \frac{Energy}{Time} = \frac{\Delta W}{\Delta t} = \frac{I \Delta t \Delta V}{\Delta t} = I \Delta V$$

- The power supplied to a circuit by a battery or dissipated in a circuit component (e.g. a resistor):

$$P = I\Delta V = I^2 R = \frac{(\Delta V)^2}{R}$$

- Units of power: watt (W)

Energy delivered: kilowatt-hours (kWh) or Joules (J)

$$1\,\text{kWh} = \left(10^3 W\right)\left(3600\ s\right) = 3.60 \times 10^6\ J$$

Examples

- The bulb for an interior light in a car is rated at 15.0W and operates from the car battery voltage of 12.6V. What is the resistance of the bulb?

- A 1200W floor heater, a 360W TV and a hand iron operating at 900W are all plugged into the same 12V circuit in a house (in parallel!). What is the total current through this circuit?

Example

A high-voltage transmission line with resistance per kilometre of 0.31 Ω/km carries 1000A for a distance of 160 km. What is the power loss due to resistance in the wire?

Observations:

1. *Given resistance/length, compute total resistance*
2. *Given resistance and current, compute power loss*

$$R = \frac{R}{l}\mathrm{L} = \frac{0.31\Omega}{1km}(160km) = 49.6\Omega$$

Now compute power

$$P = I^2 R = (1000\ A)^2 (49.6\ \Omega) = 49.6 \times 10^6\ W$$

19.5 Resistors in Series

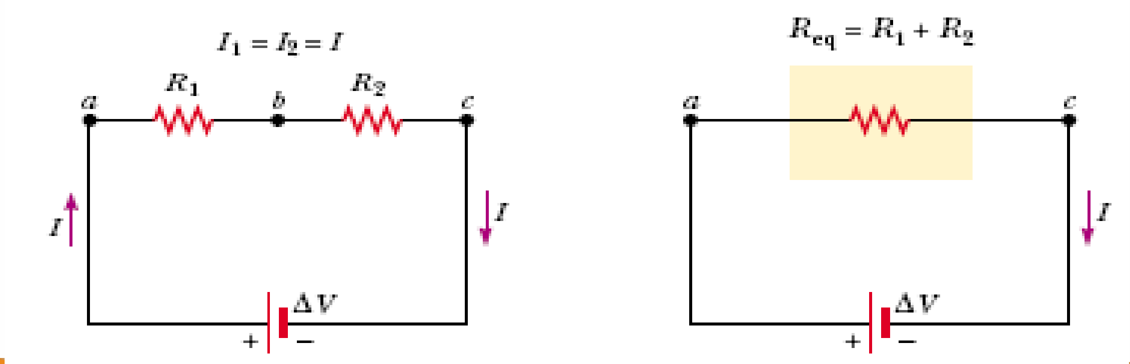
1. Consider the circuit below. Due to <u>charge conservation</u>, all charges going through the resistor R_2 will also go through resistor R_1. Thus, currents in R_1 and R_2 are the same,

$$I_1 = I_2 = I$$

2. Because of the <u>energy conservation</u>, total potential drop (between 'a' and 'c') equals the <u>sum</u> of potential drops between 'a' and 'b' and 'b' and 'c',

By definition, $\Delta V = IR_{eq}$

Thus, R_{eq} would be $R_{eq} \equiv \frac{\Delta V}{I} = \frac{IR_1 + IR_2}{I} = R_1 + R_2$



- Analogous formula is true for any number of resistors,

$$R_{eq} = R_1 + R_2 + R_3 + \ldots \qquad \text{(series combination)}$$

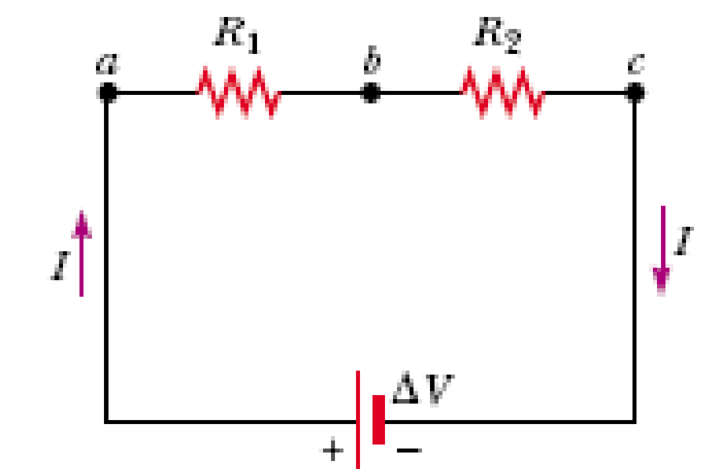
- It follows that the equivalent resistance of a series combination of resistors is greater than any of the individual resistors

[Q] How would you connect resistors so that the equivalent resistance is **larger** than the individual resistance?

[Q] When different resistors are connected in **series,** which of the following would be the **same** for each resistor: potential difference, current, power?

Example

In the electrical circuit below, find voltage across the resistor R_1 in terms of the resistances R_1, R_2 and potential difference between the battery's terminals V.



Energy conservation implies:

$$V = V_1 + V_2$$

with $V_1 = IR_1$ and $V_2 = IR_2$

Then, $V = I\left(R_1 + R_2\right)$, so $I = \dfrac{V}{R_1 + R_2}$

Thus,

$$V_1 = V\frac{R_1}{R_1 + R_2}$$

This circuit is known as ***voltage divider***.

Resistors in Parallel

1. Since both R_1 and R_2 are connected to the same battery, potential differences across R_1 and R_2 are the same,

$$V_1 = V_2 = V$$

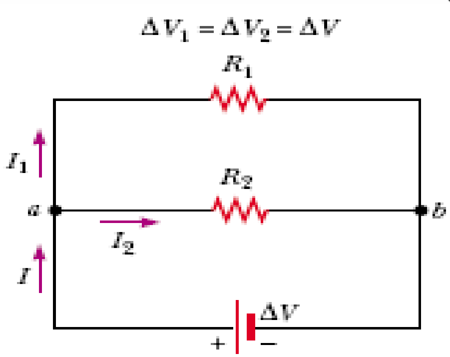
2. Because of the charge conservation, current, entering the junction A, must equal the current leaving this junction,

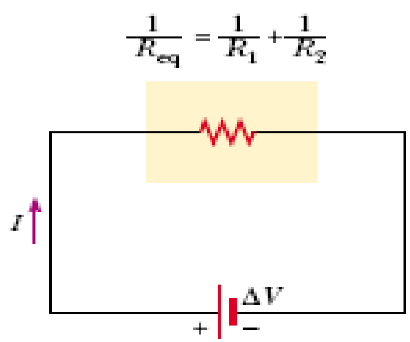
By definition,

$$I = \frac{V}{R_{eq}}$$

Thus, R_{eq} would be

$$I = \frac{V}{R_{eq}} = \frac{V_1}{R_1} + \frac{V_2}{R_2} = \frac{V}{R_1} + \frac{V}{R_2}$$





$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

or

- Analogous formula is true for any number of resistors,

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots \qquad \text{(parallel combination)}$$

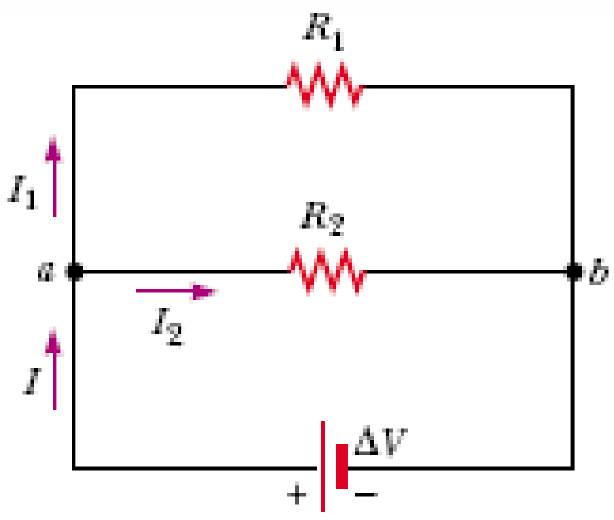
- It follows that the equivalent resistance of a parallel combination of resistors is always less than any of the individual resistors

[Q] How would you connect resistors so that the equivalent resistance is **smaller** than the individual resistance?

[Q] When resistors are connected in **parallel,** which of the following would be the **same** for each resistor: potential difference, current, power?

Example

In the electrical circuit below, find current through the resistor R_1 in terms of the resistances R_1, R_2 and total current I induced by the battery.



Charge conservation implies:

with $$I_1 = \frac{V}{R_1}, \text{ and } I_2 = \frac{V}{R_2}$$

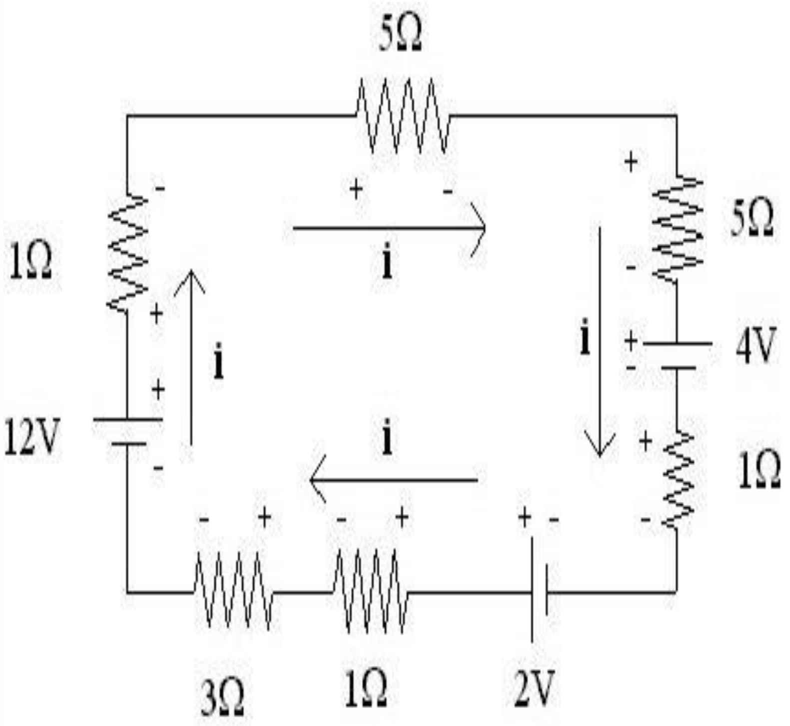
Then, $$I_1 = \frac{IR_{eq}}{R_1}, \text{ with } R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

Thus, $$I_1 = I\frac{R_2}{R_1 + R_2}$$

This circuit is known as ***current divider***.

Example

Find the current in the circuit shown



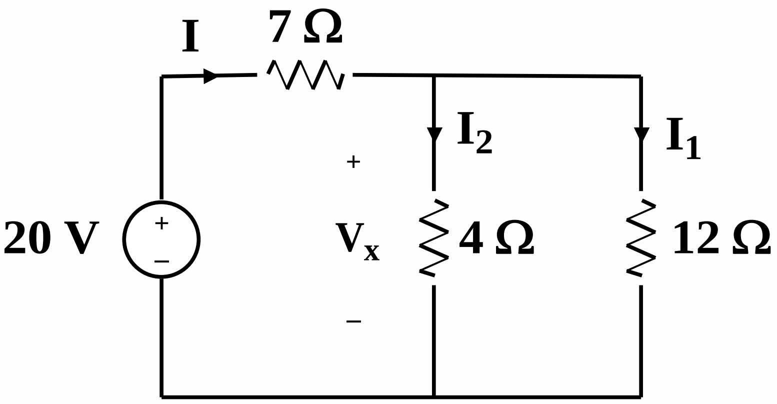
$$\sum_{i=1}^{3} V_{total} = 12V - 4V + 2V = 10V$$

$$\sum_{i=1}^{6} R_i = 16\,\Omega$$

$$I = \frac{V_{total}}{R_{eq.}} = \frac{10}{16} A$$

Example

Find the currents I_1 and I_2 and the voltage V_x in the circuit shown below.



First find the equivalent resistance seen by the 20 V source:

$$R_{eq} = 7\Omega + \frac{4\Omega(12\Omega)}{12\Omega + 4\Omega} = 10\ \Omega$$

$$I = \frac{20V}{R_{eq}} = \frac{20V}{10\Omega} = 2\,A$$

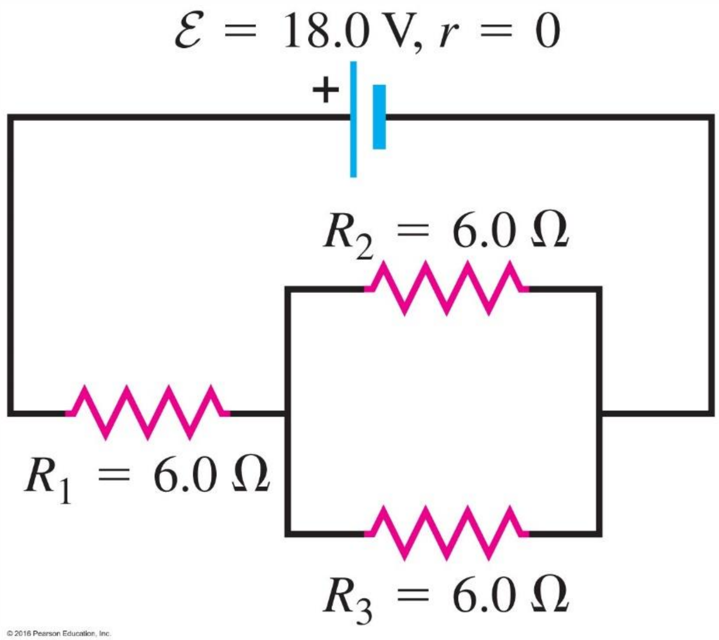
We now find I_1 and I_2 directly from the current division rule:

$$I_1 = \frac{2A(4\Omega)}{12\Omega + 4\Omega} = 0.5\,A, \text{ and } I_2 = I - I_1 = 1.5A$$

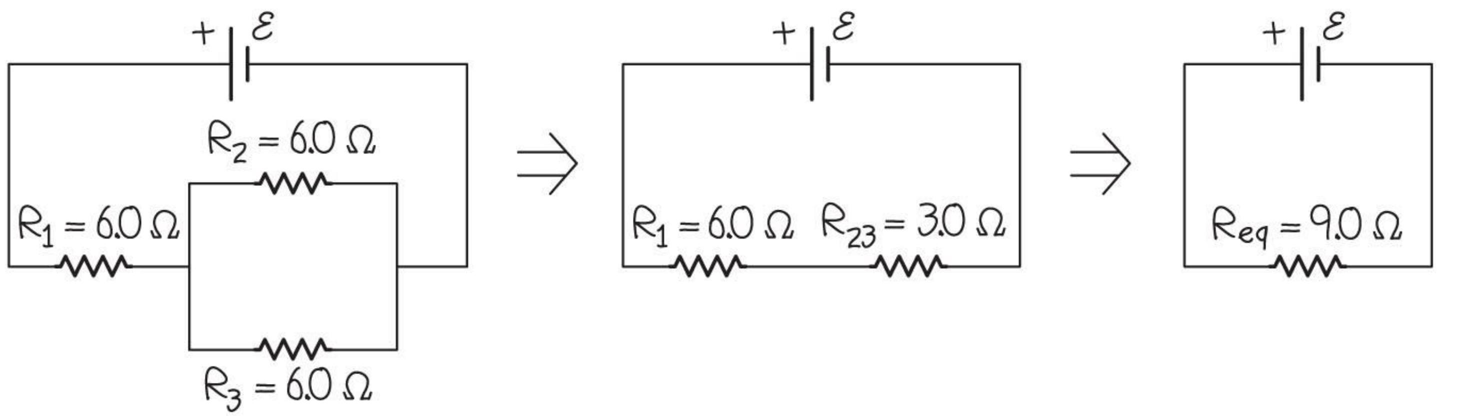
Finally, voltage V_x is $V_x = I_2\left(4\Omega\right) = 1.5A\left(4\Omega\right) = 6V$

Example

Find the equivalent resistance of the circuit below and the current in each resistor



Simplify Your Circuit…



(a) Original circuit **(b)** Parallel resistors combined **(c)** Equivalent resistor

(parallel combination)

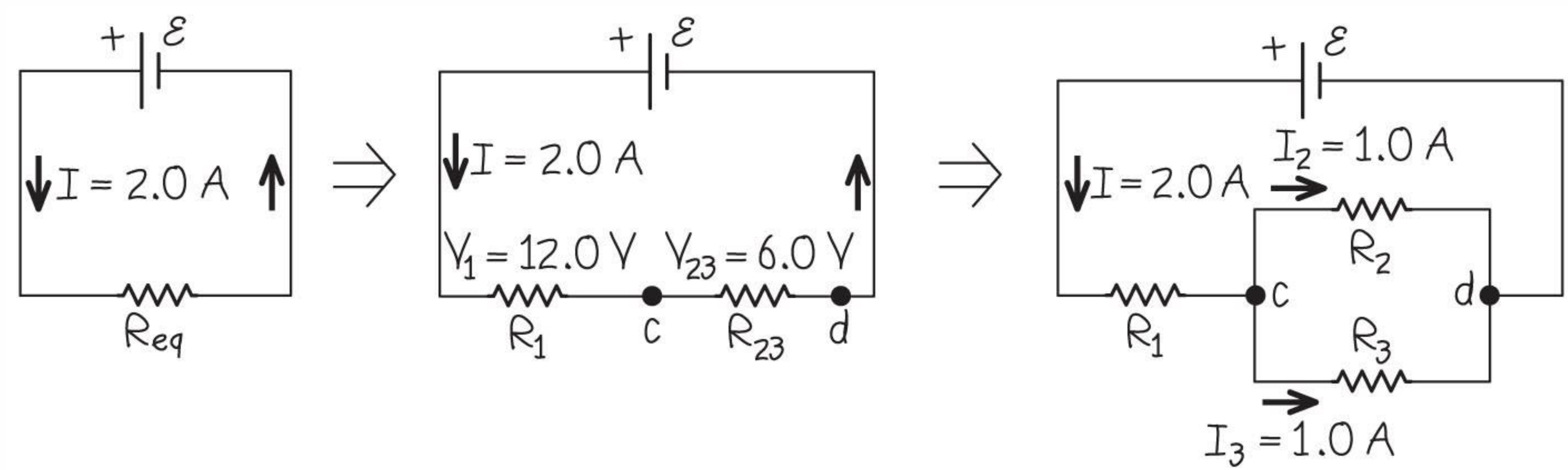
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots$$

(series combination)

$$R_{eq} = R_1 + R_2 + R_3 + \ldots$$

Work Backwards!

We start with our simplified circuit and use our understanding of resistors in series and in parallel



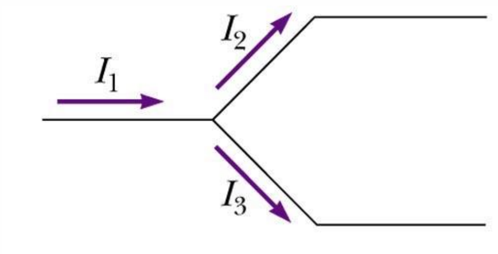
$$I_T = \frac{V_T}{R_T}$$

19.6 Kirchhoff's Rules

1. The sum of currents entering any junction must equal the sum of the currents leaving that junction (current or junction rule) .

$$\sum I = 0$$

Charge conservation

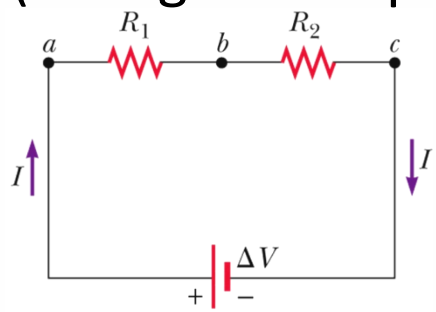


$$-I_1 + I_2 + I_3 = 0 \Rightarrow I_1 = I_2 + I_3$$

2. The sum of the potential differences across all the elements around any closed-circuit loop must be zero (voltage or loop rule).

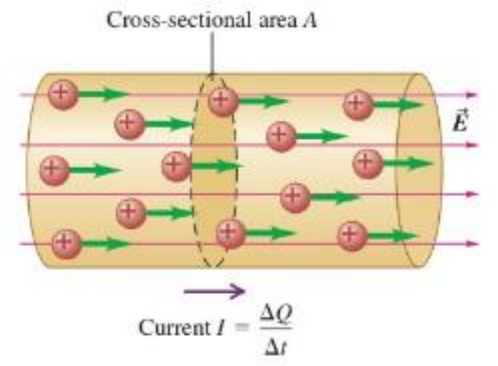
$$\sum_{\text{loop}} V = 0$$

Energy conservation

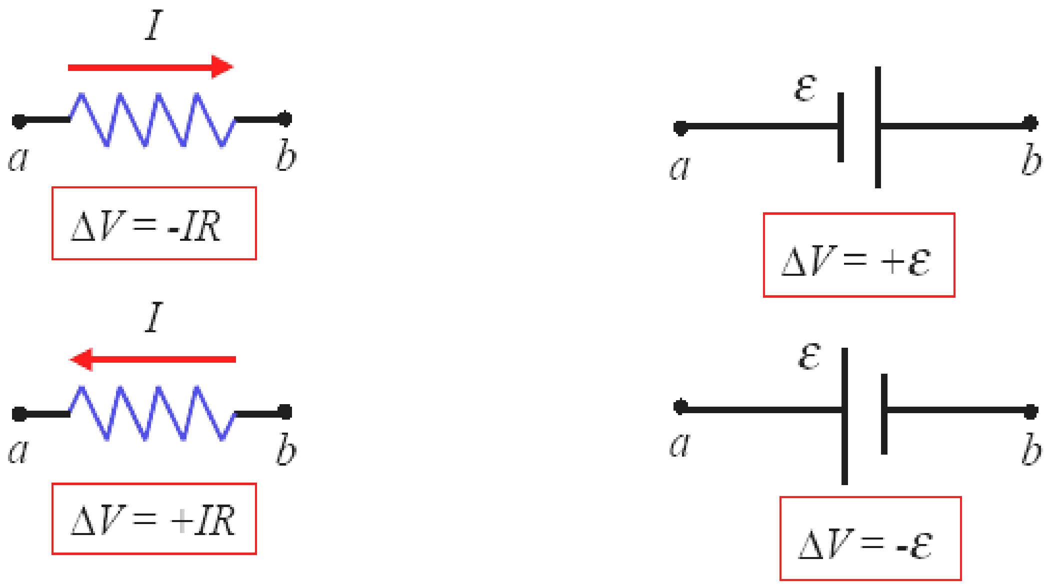


$$V - IR_1 - IR_2 = 0 \Rightarrow V = I(R_1 + R_2)$$

❑ Rules for Kirchhoff's loop rule



- If a resistor is traversed in the direction of the current, the change in potential across the resistor is ***–IR***
- If a resistor is traversed in the direction ***opposite*** the current, the change in potential across the resistor is ***+IR***
- If an emf is traversed from the – to the + terminal, the change in potential is +ε
- If an emf is traversed from the + to the – terminal, the change in potential is -ε



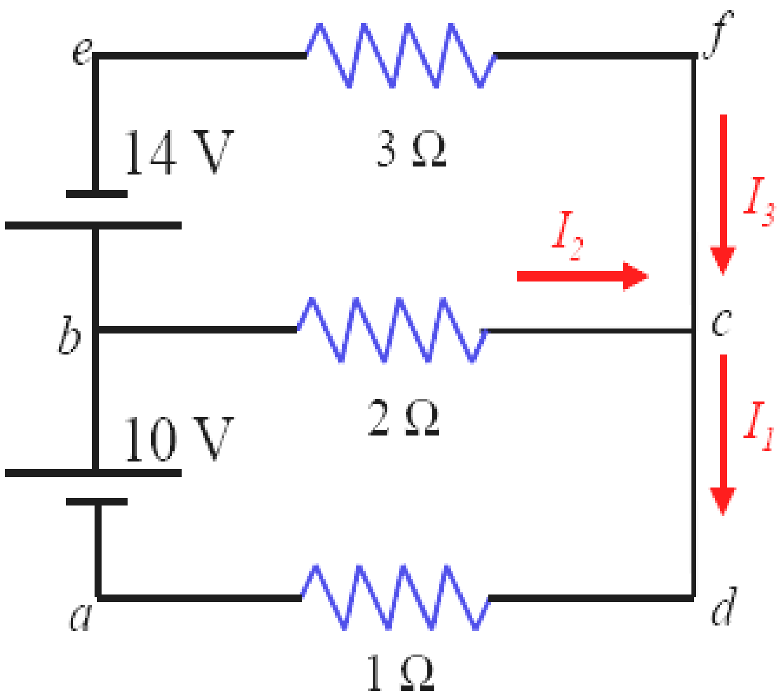
For all these we are travelling (traversing a loop) from *a* to *b*

❑ Solving problems using Kirchhoff's rules

- Draw the circuit diagram and label all known and unknown quantities.
- Assign a direction to the currents in each part of the circuit. If guess a wrong direction – don't panic -- the result will be negative, but will have the correct magnitude.
- Once a current direction is chosen you must rigorously follow Kirchhoff's rules.
- Apply the junction rule to any junction that provides a relationship between the various currents.
- Apply the loop rule to as many loops as needed to solve for all the unknowns. Careful with the signs!
- Algebra: solve equations for unknown quantities.

Example

Find currents I_1, I_2, and I_3.



1. Apply the junction rule at *c*.

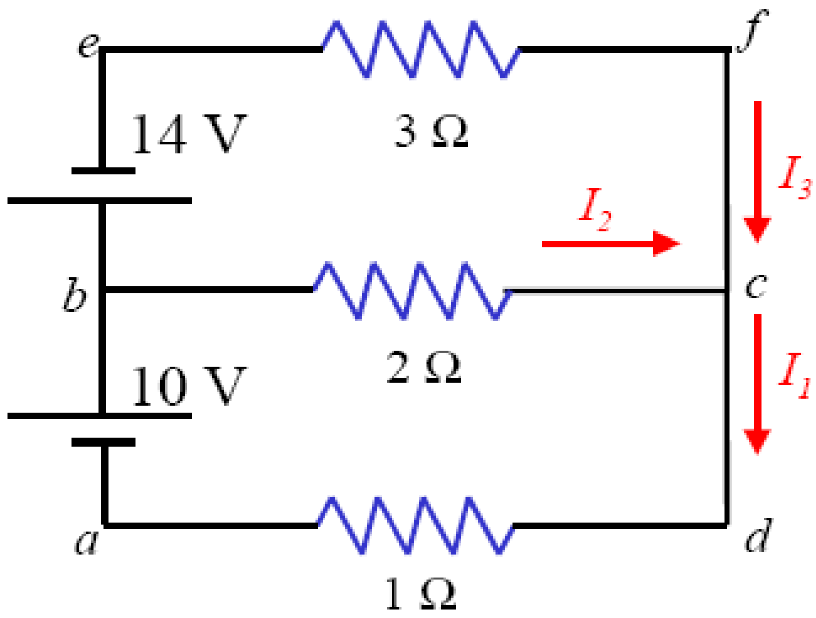
$$I_2 + I_3 = I_1$$

2. Apply the loop rule in the **clockwise** direction for loop *abcda*.

$$10\ \text{V} - (2\Omega)I_2 - (1\Omega)I_1 = 0$$

3. Apply the loop rule in the **clockwise** direction for loop *befcb*.

$$-14\text{V} - (3\Omega)I_3 + (2\Omega)I_2 = 0$$



❶ $$I_1 = I_2 + I_3$$

❷ $$10\text{ V} - (2\Omega)I_2 - (1\Omega)I_1 = 0$$

❸ $$-14\text{ V} - (3\Omega)I_3 + (2\Omega)I_2 = 0$$

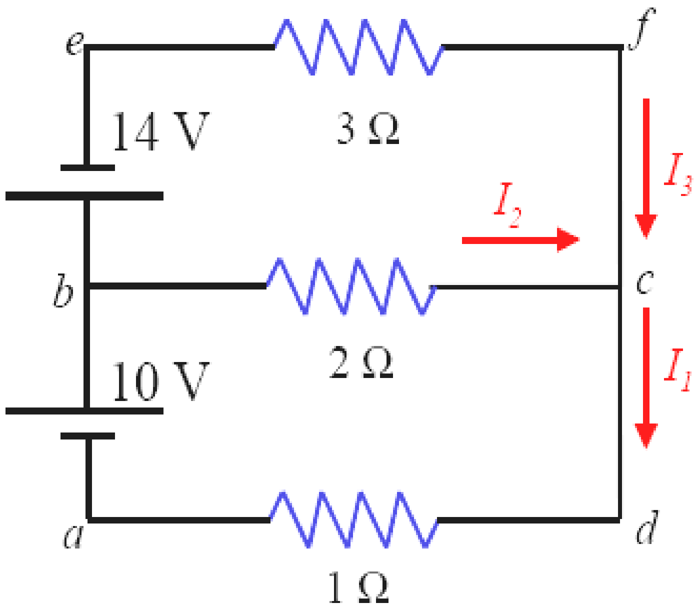
Substituting ❶ into ❷ gives

$$10 - 2I_2 - \left(I_2 + I_3\right) = 0$$

❹ $$10 = 3I_2 + I_3$$

Rearranging ❸ gives

❺ $$14 = 2I_2 - 3I_3$$



❶ $I_1 = I_2 + I_3$

❹ $10 = 3I_2 + I_3$

❺ $14 = 2I_2 - 3I_3$

Multiplying ❹ by 3 and adding to ❺ gives

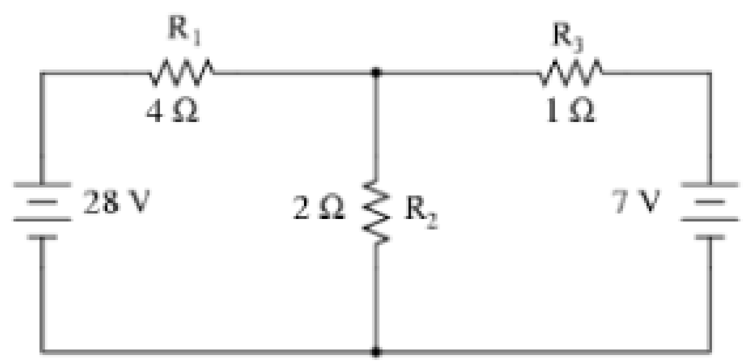
$$44 = 11I_2$$
$$I_2 = 4\text{A}$$

Using this in ❺ gives

$$I_3 = -2\text{A}$$

Finally ❶ gives $I_1 = 2\text{A}$

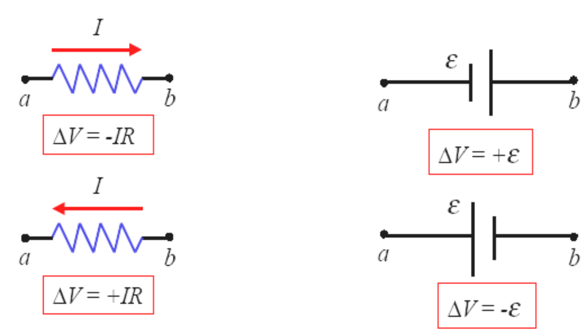
Example



$$\sum I = 0$$ *Charge conservation*

$$\sum_{\text{loop}} V = 0$$ *Energy conservation*

- Calculate all currents in this system
- Calculate voltages across each resistor



For all these we are traversing a loop in the direction from point a to point b.

Example

- Find all three currents
  - Need three equations for three unknowns
  - Note that current *directions* are already picked for us (sometimes have to pick for yourself)
  - Use the junction rule first

$$I_1 + I_2 = I_3$$

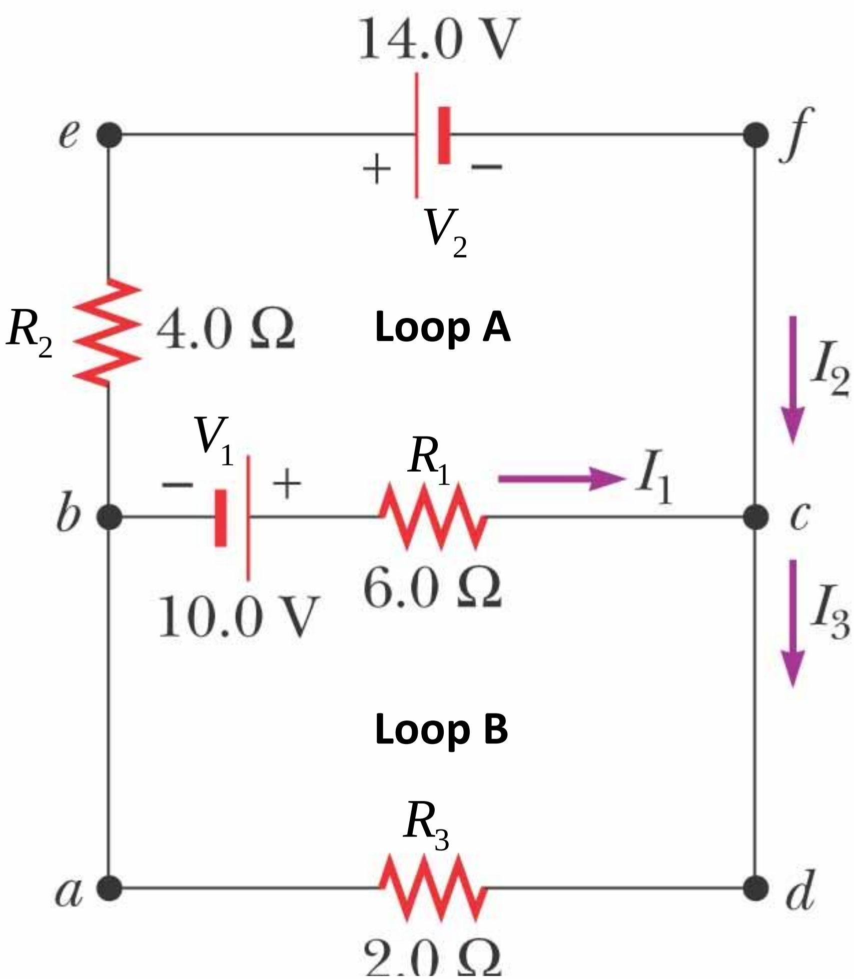
  - Alternative two loops

A: $V_1 - I_1R_1 + V_2 + I_2R_2 = 0$

$$10 - 6I_1 + 14 + 4I_2 = 0$$

B: $V_1 - I_1R_1 - I_3R_3 = 0$

$$10 - 6I_1 - 2I_3 = 0$$



Example

Loop A

$$10 - 6I_1 + 14 + 4I_2 = 0$$
$$4I_2 = -24 + 6I_1$$
$$I_2 = -6 + 1.5I_1$$

Loop B

$$10 - 6I_1 - 2I_3 = 0$$
$$2I_3 = 10 - 6I_1$$
$$I_3 = 5 - 3I_1$$

Substitution

Substitution

Junction Rule

$$I_1 + I_2 = I_3$$

$$I_1 + (-6 + 1.5I_1) = (5 - 3I_1)$$

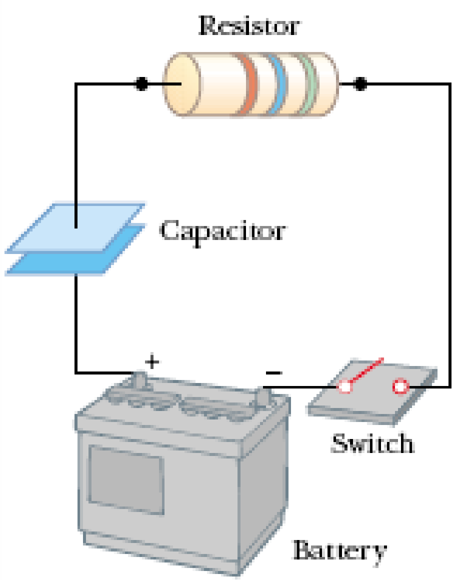
$$-11 = -5.5I_1$$

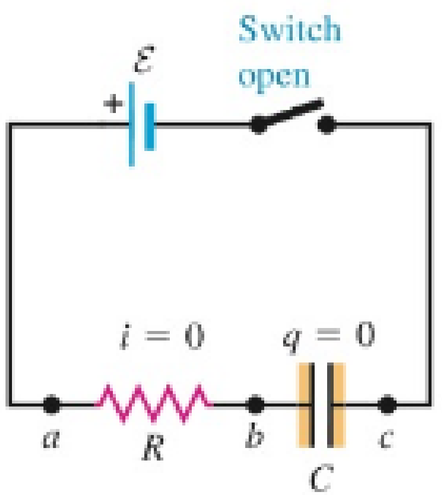
$$I_1 = 2A$$

Now substitute $I_1 = 2A$ into **Loop A** and **Loop B** equations to calculate I_2 (*-3A*) and I_3 (*-1A*)

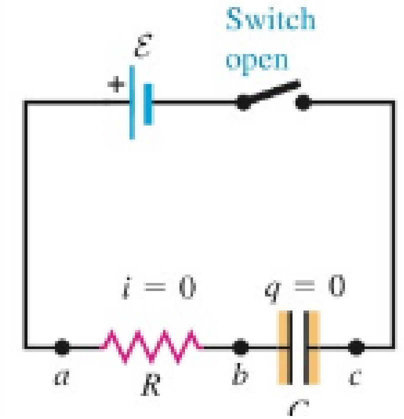
RC Circuits

- A direct current circuit may contain capacitors and resistors, the current will vary with time
- When the circuit is completed (t=0, switch closes), current begins to flow and the capacitor starts to charge
- The capacitor continues to charge until it reaches its maximum charge ($Q = C\varepsilon$)
- Once the capacitor is fully charged, the current in the circuit is zero
- Development of the capacitor charging relationship requires calculus methods and involves a differential equation





Charging Capacitor in an RC Circuit



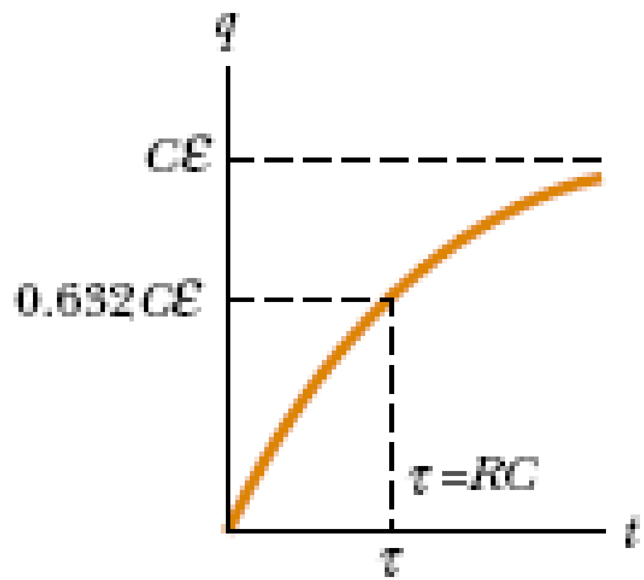
- When the switch is closed, the charge on the capacitor varies with time

$$q = Q_T(1 - e^{-t/RC})$$

Where *time constant*, τ = RC

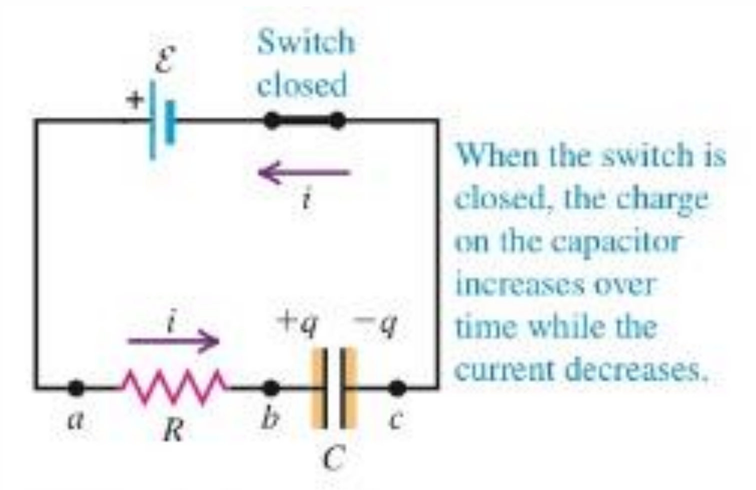
- When the switch is closed, the current I is

$$I = \frac{dq}{dt} = \frac{V_T}{R} e^{-t/RC} = I_T e^{-t/RC}$$

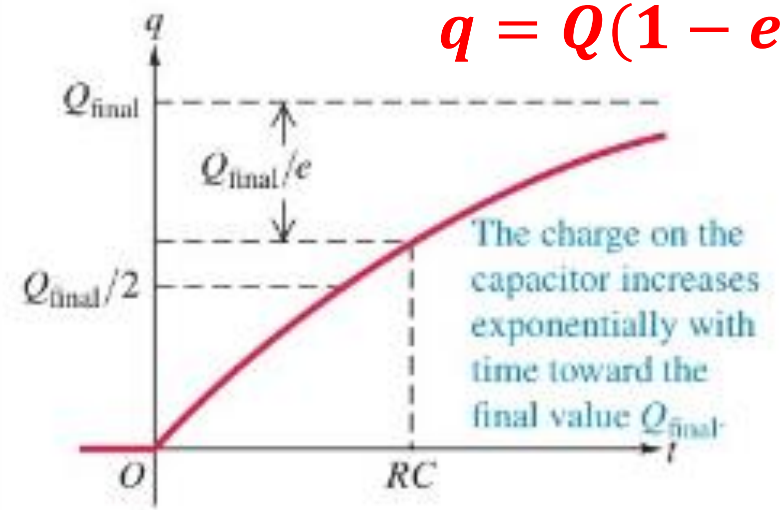


- The *time constant* (τ) represents the time required for the charge to increase from zero to 63.2% of its maximum

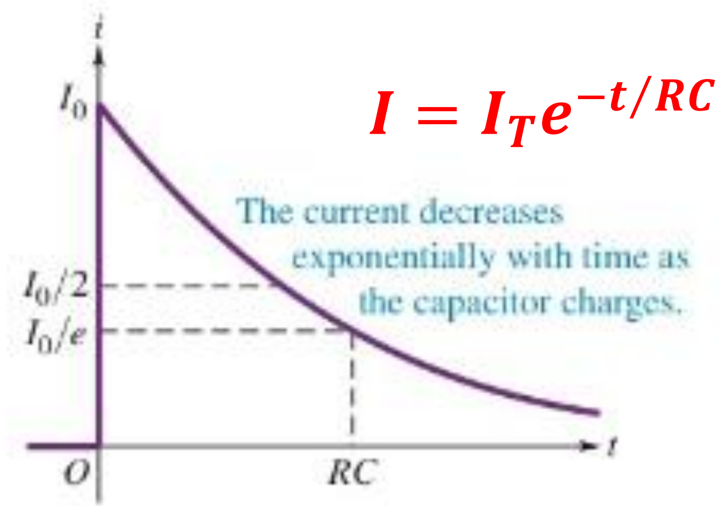
Charging



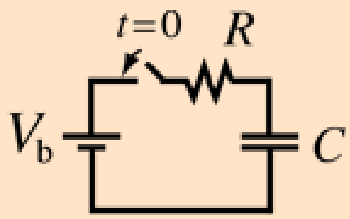
$$q = Q(1 - e^{-t/RC})$$



$$I = I_T e^{-t/RC}$$



Charging Summary



$$V_b = V_R + V_C$$

$$V_b = IR + \frac{Q}{C}$$

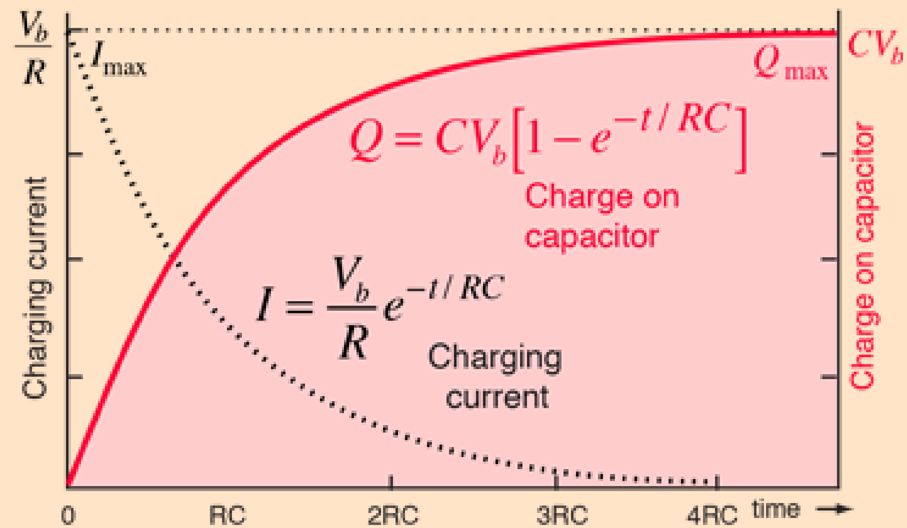
As charging progresses,

$$V_b = IR + \frac{Q}{C}$$

current decreases and charge increases.

$$V_b = 0 + \frac{Q}{C}$$

$$V_b = \frac{Q}{C} = V_C$$



At $t = 0$
$Q = 0$
$V_C = 0$
$I = \frac{V_b}{R}$

As $t \to \infty$
$Q \to CV_b$
$V_C \to V_b$
$I \to 0$

Notes on Time Constant

- In a circuit with a large time constant, the capacitor charges very slowly
- The capacitor charges very quickly if there is a small time constant
- After $t = 10\tau$, the capacitor is over 99.99% charged

Discharging Capacitor in an RC Circuit

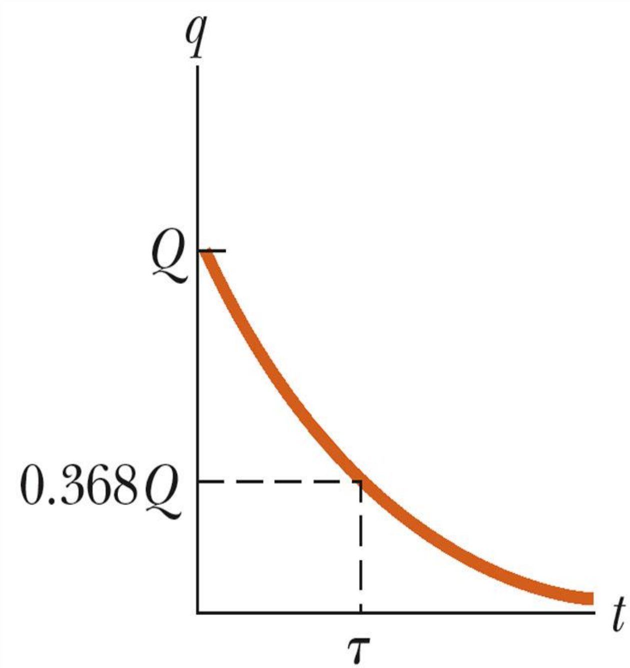
- When a charged capacitor is placed in the circuit, it can be discharged

$$q = Q_T(e^{-t/RC})$$

- The charge decreases exponentially

- The current I is

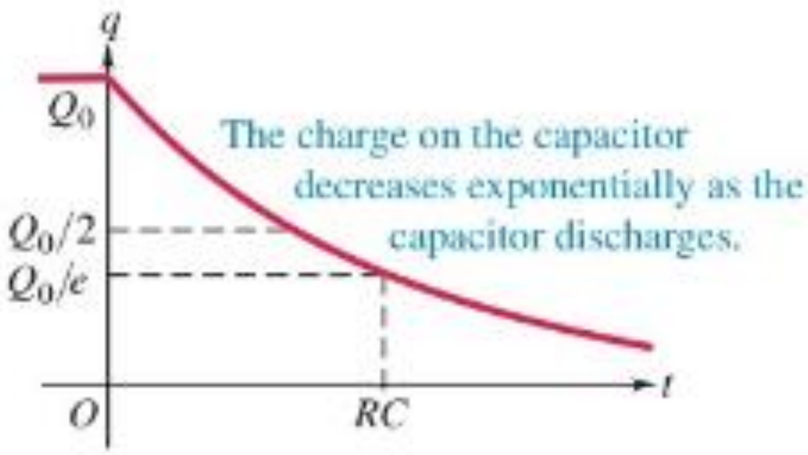
$$I = \frac{dq}{dt} = -\frac{Q}{RC}e^{-t/RC} = I_T e^{-t/RC} \quad or \quad I = \frac{V_T}{R}e^{-t/RC}$$



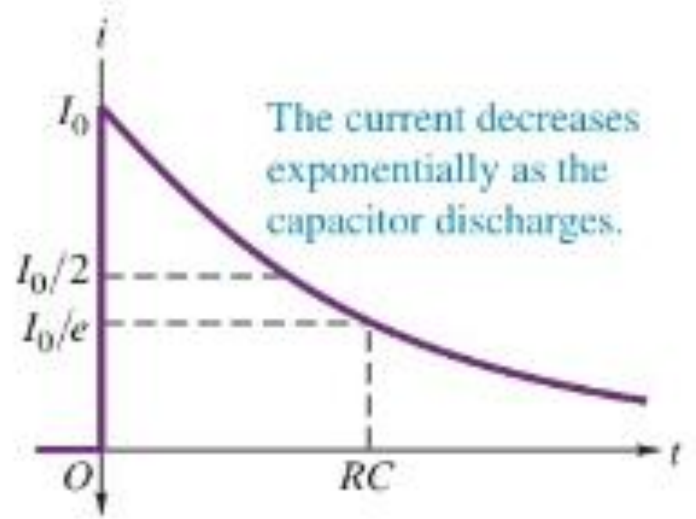
© 2006 Brooks/Cole - Thomson

- At t = τ = RC, the charge decreases to 0.368 Q_{max}
  - In other words, in one time constant, the capacitor loses 63.2% of its initial charge

Discharging



$$q = Q(e^{-t/RC})$$



$$I = I_T e^{-t/RC}$$

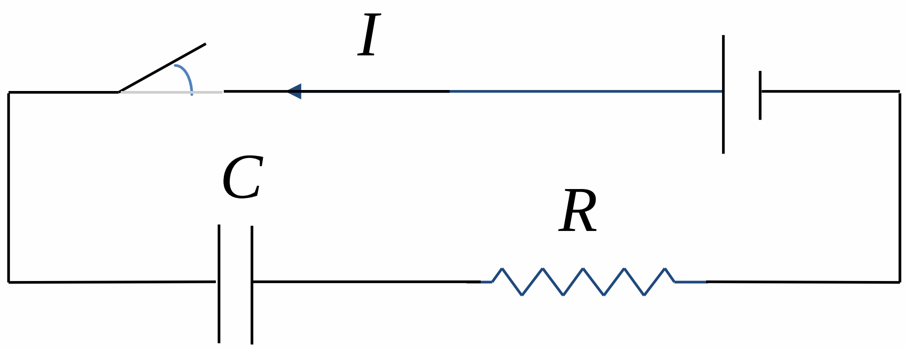
The current has the same magnitude

https://www.youtube.com/watch?v=X5bzjs3ByBU
https://www.youtube.com/watch?v=f_MZNsEqyQw

Example

Charging the unknown capacitor

A series combination of a 12 kΩ resistor and an unknown capacitor is connected to a 12 V battery. One second after the circuit is completed, the voltage across the capacitor is 10 V. Determine the capacitance of the capacitor.



Recall that the charge is building up according to

$$q = Q\left(1 - e^{-t/RC}\right)$$

Thus the voltage across the capacitor changes as

$$V = \frac{q}{C} = \frac{Q}{C}\left(1 - e^{-t/RC}\right) = V_T\left(1 - e^{-t/RC}\right)$$

This is also true for voltage at t = 1s after the switch is closed,

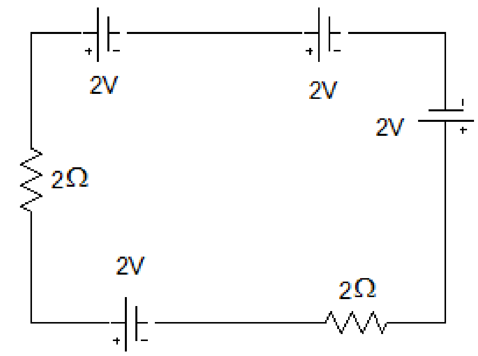
$$\frac{V}{V_T} = 1 - e^{-t/RC} \Rightarrow e^{-t/RC} = 1 - \frac{V}{V_T} \Rightarrow \frac{-t}{RC} = \ln\left[1 - \frac{V}{V_T}\right]$$

Solve for C…

Discussion

[1] What is the magnitude of the current flowing in the circuit shown in Fig. ?

The net voltage drop due to the batteries is 0V so no current flows. I=0A



[2] A copper wire has resistance 5 Ohms. Given that the resistivity of silver is 85 percent of the resistivity of copper, what is the resistance of a silver wire three times as long with twice the diameter?

$$Given,\ 5\Omega = \rho_{Cu} l_{Cu} / A_{Cu}, \quad \rho_{Ag} = 0.85\rho_{Cu}, \quad l_{Ag} = 3l_{Cu}, and \quad d_{Ag} = 2d_{Cu}. \Rightarrow$$

$$R = \frac{\rho_{Ag} l_{Ag}}{\pi r_{Ag}^2} = \frac{(0.85\rho_{Cu})(3l_{Cu})}{\pi (2r_{Cu})^2} = \frac{(0.6375)\rho_{Cu} l_{Cu}}{\pi r_{Cu}^2} = (0.6375)(5\Omega) = 3.2\Omega$$

[3] A resistor draws a current of 1A when connected across an ideal 3V battery. Another resistor draws a current of 2A when connected across an ideal 3V battery. What current do the two resistors draw when they are connected in series across an ideal 3V battery?

- The second resistor has a resistance of

$$R_2 = \frac{V}{I} = \frac{3}{2} = 1.5\Omega$$

- The first resistor has a resistance of

$$R_1 = \frac{V}{I} = \frac{3}{1} = 3\Omega$$

- The series combination of the two resistors is

$$R_1 + R_2 = 4.5\Omega$$

- Which when connected across a 3V battery will draw a current of

$$I = \frac{V}{R} = \frac{3}{4.5} = 0.67A$$

[4] Consider an RC circuit in which the capacitor is being charged by a battery connected in the circuit. In five time constants, what percentage of final charge is on the capacitor?

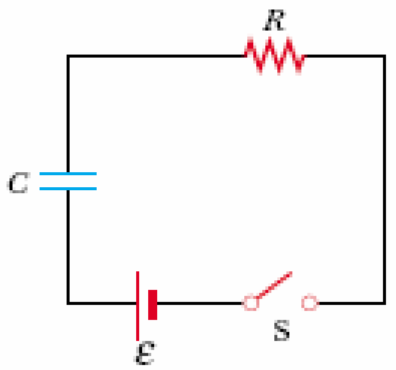
$$q = Q(1 - e^{-t/RC})$$

$$t = 5RC$$

$$\frac{q}{Q} = 1 - e^{-t/RC}$$

$$\frac{q}{Q} = 1 - e^{-5RC/RC} = 1 - e^{-5} = 0.993 = 99.3\%$$

[5] In fig. (a) find the time constant of the circuit and the charge in the capacitor after the switch is closed. (b) find the current in the resistor R at time 10 sec after the switch is closed. Assume $R=1\times10^{6}\,\Omega$, emf $=30$ V and $C=5\times10^{-6}$F



(a) The time constant $\tau = RC = (1\times10^{6})(5\times10^{-6}) = 5s$

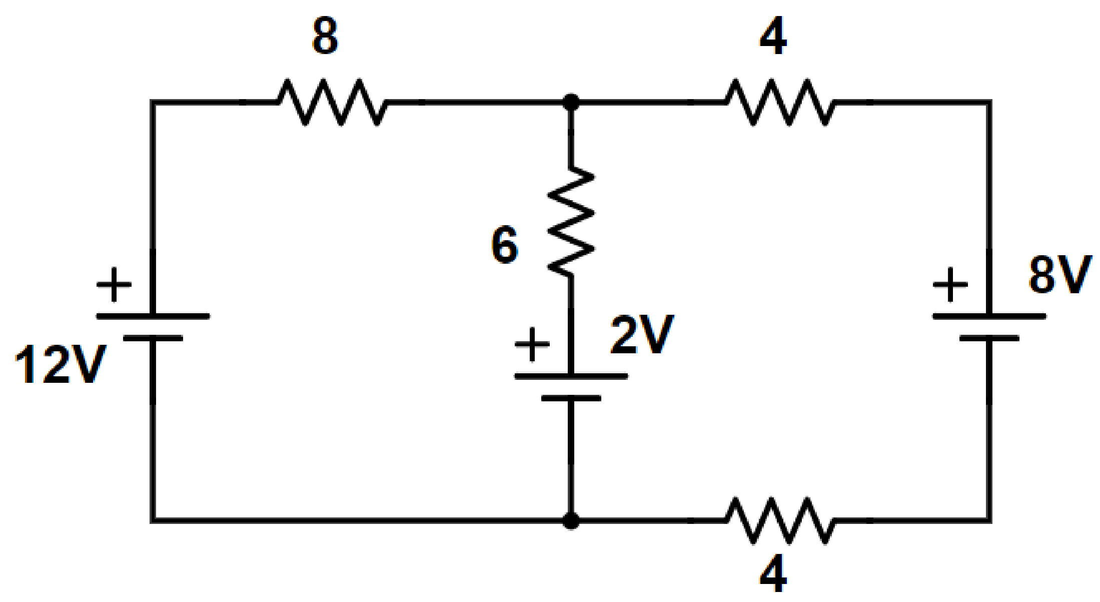
The charge on the capacitor $Q = C\varepsilon = (5\times10^{-6})(30) = 150\mu C$

(b) The current related to charging of the capacitor is given by

$$I = \frac{\varepsilon}{R} e^{-t/RC}$$

$$I = \frac{30}{1\times10^{6}} e^{-10/(1\times10^{6})(5\times10^{-6})} = 4.06\times10^{-6} A$$

[6] In the circuit shown below, calculate the current everywhere.



[7] A certain wire has resistance R. What is the resistance of a second wire, made of the same material, which is half as long and has 1/3 the diameter?

The resistance is proportional to the length of the wire and inversely proportional to the area. Since area is proportional to the diameter squared, the resistance is

$$R_{new} = R/(2\times\frac{1}{9}) = 9R/2$$

[8] Suppose that you wish to fabricate a uniform wire out of 1.00 g of copper. If the wire is to have a resistance of R = 0.500 Ω, and if all of the copper is to be used, what will be (a) the length and (b) the diameter of this wire? The resistivity and density of copper are 1.7×10^{-8} Ωm and 8.92×10^{3} kg/m^{3} respectively.

$$[a] \quad \rho_d (density) = \frac{m}{V} \Rightarrow V = \frac{m}{\rho_d} \Rightarrow Al = \frac{m}{\rho_d} \Rightarrow A = \frac{m}{l\rho_d}$$

$$Now, \quad R = \rho_{Cu}\frac{l}{A} \Rightarrow R = \frac{\rho_{Cu} l}{\frac{m}{l\rho_d}} \Rightarrow R = \frac{\rho_{Cu}\rho_d l^2}{m}$$

$$l = \sqrt{\frac{mR}{\rho_{Cu}\rho_d}} = \sqrt{\frac{(0.5)(1\times10^{-3})}{(1.7\times10^{-8})(8.92\times10^{3})}} = 1.82m$$

$[b] \quad find \quad r$