20 Magnetic Field and Magnetic Forces

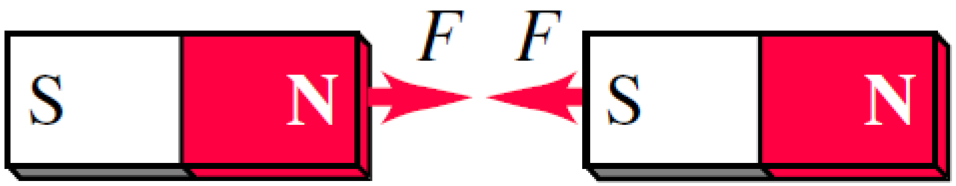
In industrial settings, electromagnets are often used to pick up and move iron-containing material, such as this shredded scrap. How can electric currents cause magnetic forces? We'll learn in this chapter.

20.1 Magnetism

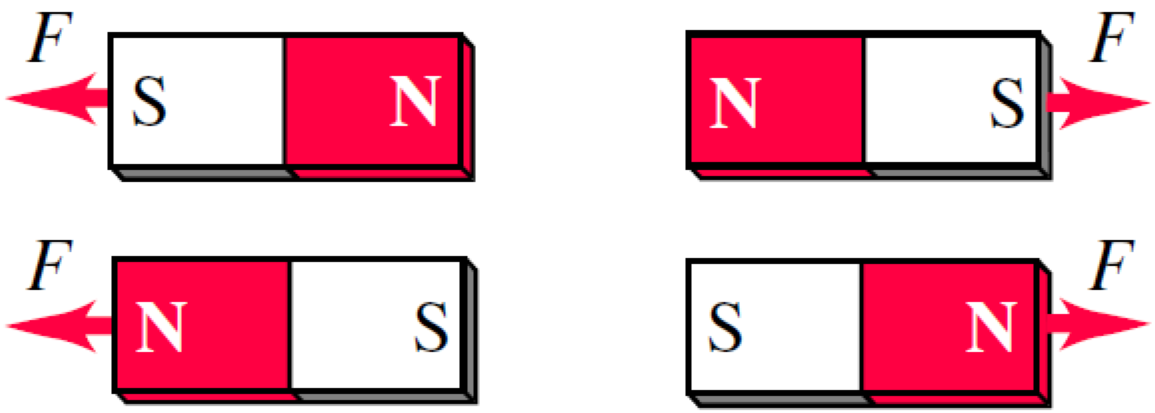
Magnets:

- two poles: N and S

Unlike poles attract.



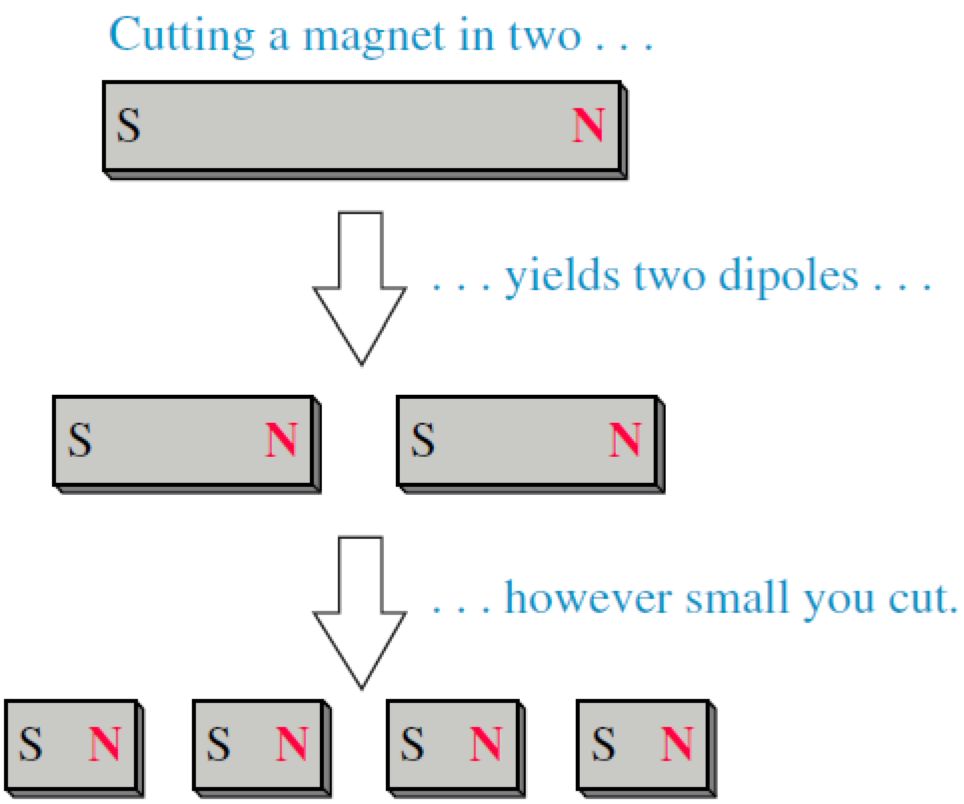
Like poles repel.



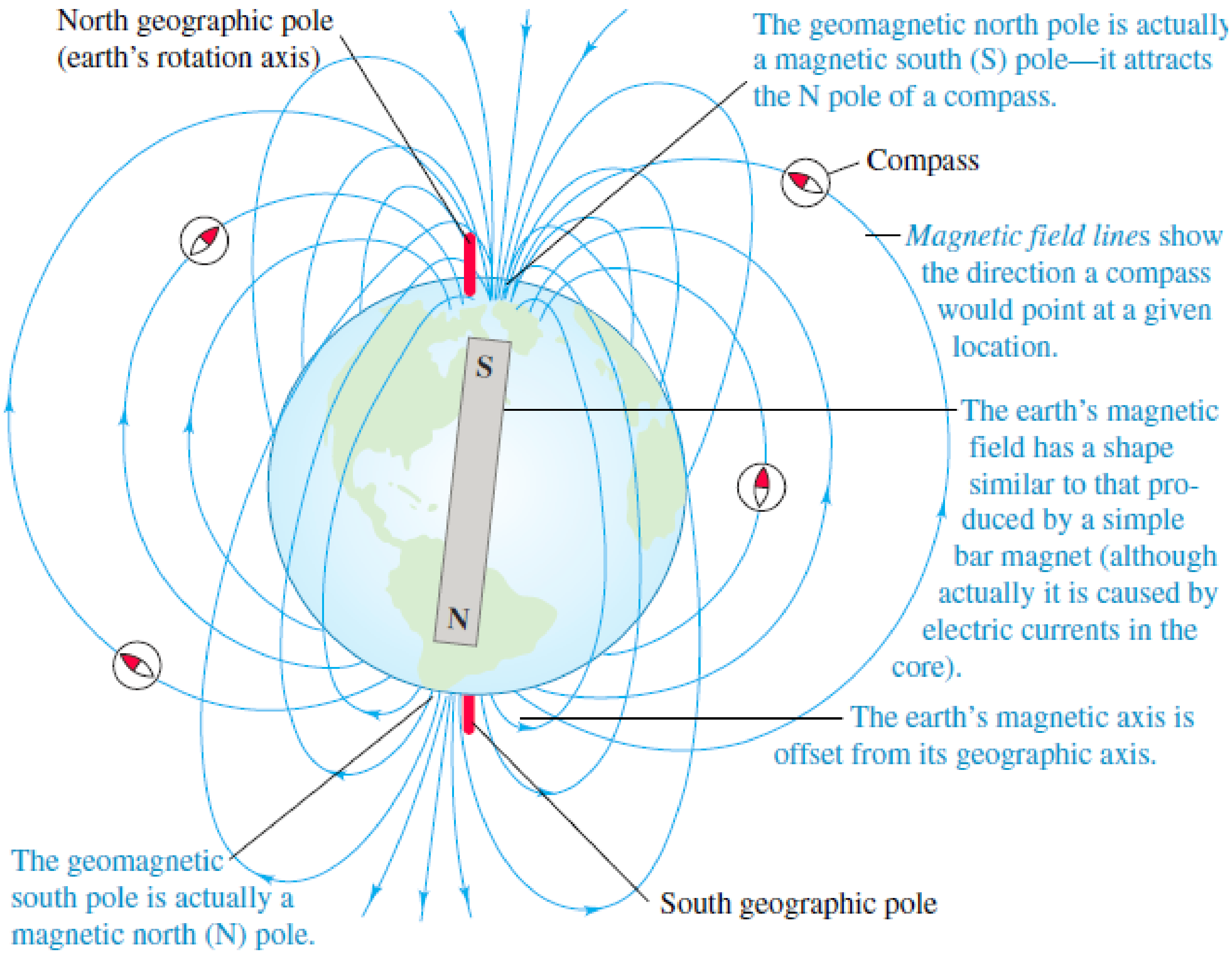
Magnets: North and South Poles

- If we break a permanent magnet in half, we do not get a separate north pole and south pole.
- When we break a bar magnet in half, we always get two new magnets, each with its own north and south pole.

Differing from electric charges, magnetic poles always come paired and can't be isolated.

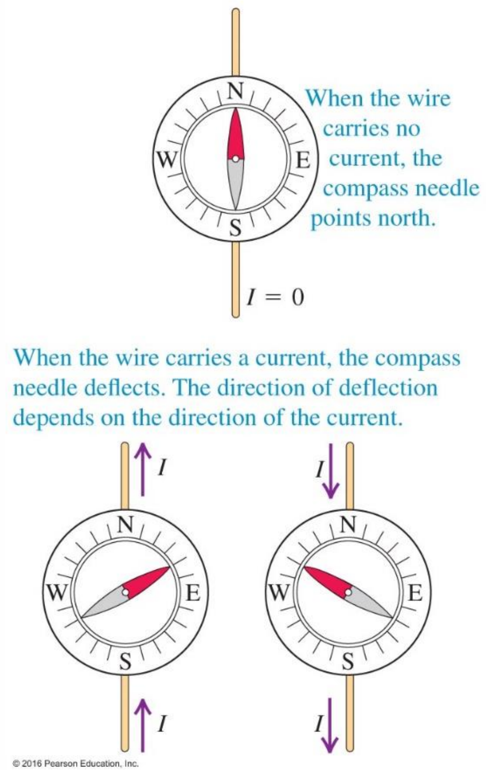


Magnetic Field of the Earth



Observations of Current Carrying Wire Deflecting a Compass Needle

- In 1819, Hans Christian Oersted observed a compass needle being deflected by a current-carrying wire!



- Moving a magnet near a conducting loop can cause a current in the loop

- A changing current in a conductor can cause a current in another conductor

Magnetic Field analogy with Electric Field

Electric Field:

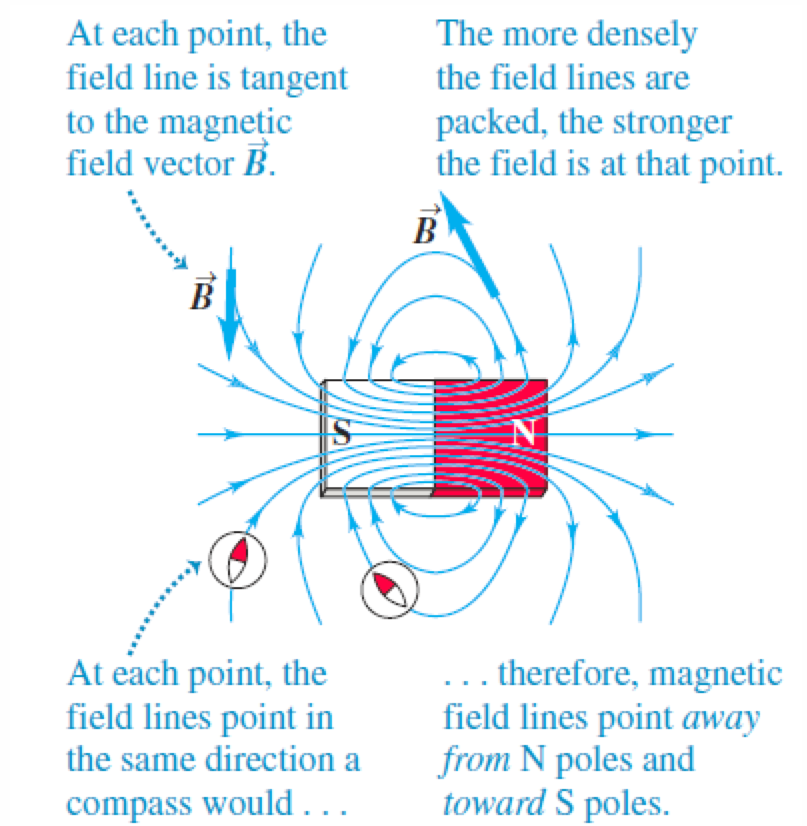
- Electric charge creates an electric field ***E*** in the surrounding space.
- The electric field exerts a force $\mathbf{F} = q\,\mathbf{E}$ on a charge q present in the field

Magnetic Field:

- A permanent magnet, a moving charge or a current creates a magnetic field ***B*** in the surrounding space.
- The magnetic field exerts a force ***F*** on a moving charge q or current present in the field

Magnetic Field lines

Defined in same way as electric field lines, direction and density



PERMANENT MAGNETS

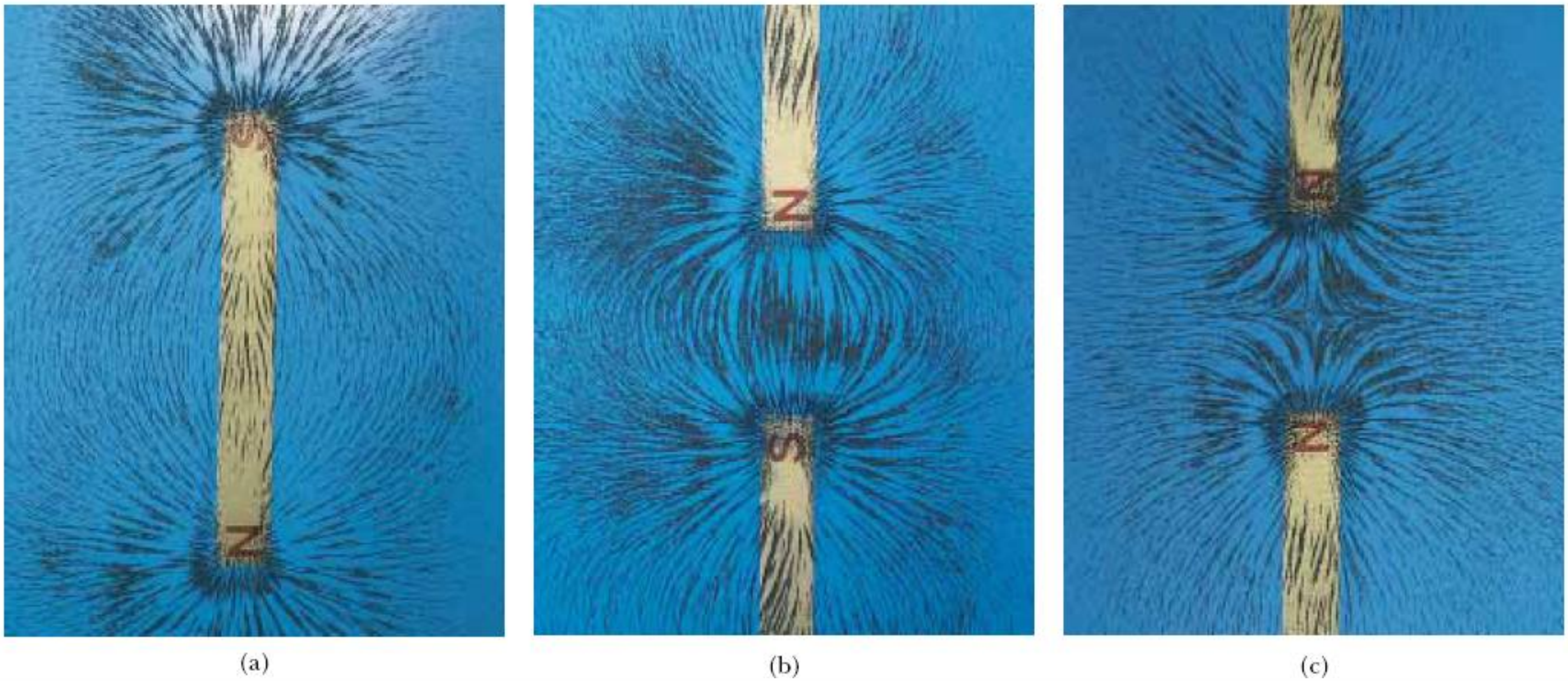


Figure (a) Magnetic field pattern surrounding a bar magnet as displayed with iron filings. (b) Magnetic field pattern between *unlike* poles of two bar magnets. (c) Magnetic field pattern between like poles of two bar magnets

20.2 Magnetic Field and Magnetic Force

Observations show that the force is proportional to

- The magnetic field
- The charge
- The velocity of the particle
- The sine of the angle between the field and the direction of the particle's motion.

$$F = |q|vB\sin\phi$$

Magnetic Force on a charge in motion

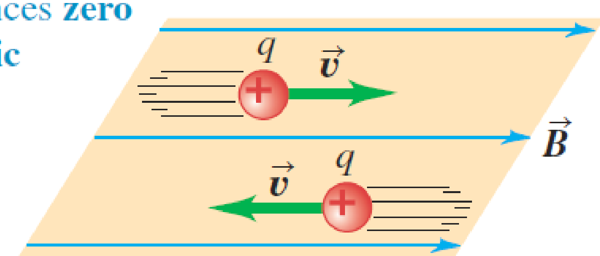
Magnitude of the magnetic force

When a charged particle moves with velocity $\vec{v}$ in a magnetic field $\vec{B}$, the magnitude F of the force exerted on it is

$$F = |q|v_{\perp}B = |q|vB\sin\phi,$$

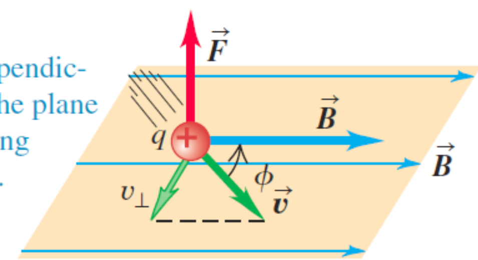
where $|q|$ is the magnitude of the charge and ϕ is the angle measured from the direction of $\vec{v}$ to the direction of $\vec{B}$, as shown in Figure 20.9.

A charge moving **parallel** to a magnetic field experiences **zero magnetic force.**

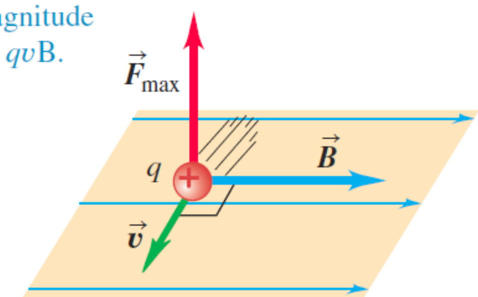


A charge moving at an angle ϕ to a magnetic field experiences a magnetic force with magnitude $F = |q|v_{\perp}B = |q|vB\sin\phi$.

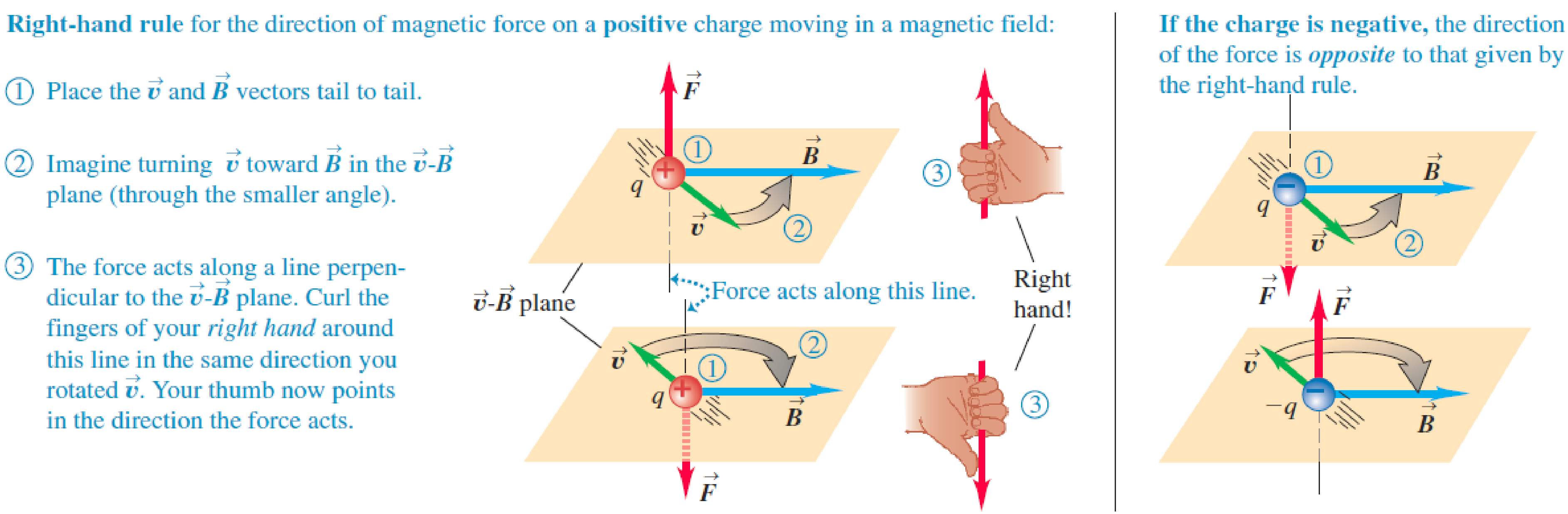
$\vec{F}$ is perpendicular to the plane containing $\vec{v}$ and $\vec{B}$.



A charge moving **perpendicular** to a magnetic field experiences a maximal magnetic force with magnitude $F_{max} = qvB$.



Right Hand Rule



▲ **FIGURE 20.10** The right-hand rule for the direction of the magnetic force on a charge moving in a magnetic field.

Magnetic Field Units

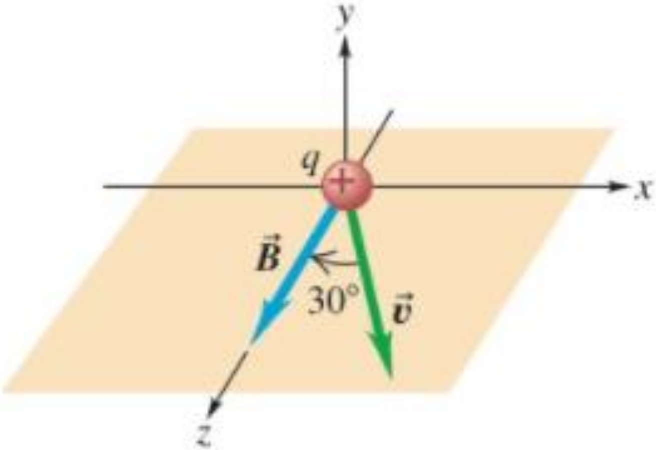
• Force (F)	=	Newton (N)
• Veolcity (v)	=	metres per second (m/s)
• Charge (q)	=	Coloumb (C)
• Magentic field (B)	=	tesla (T)

- Also called weber (Wb) per square meter.
- $1\ T = 1\ Wb/m^2$
- $1\ T = 1\ N\ s\ m^{-1}\ C^{-1}$
- **$1\ T = 1\ N\ A^{-1}\ m^{-1}$**

- CGS unit is the Gauss (G)
  - $1\ G = 10^{-4}\ T$

Example…

In this example we will calculate the force on a proton moving in a magnetic field. In Figure 20.13, a beam of protons moves through a uniform magnetic field with magnitude 2.0 T, directed along the positive z axis. The protons have a velocity of magnitude 3.0×10^5 m/s in the x-z plane at an angle of 30° to the positive z axis. Find the force on a proton. The charge of the proton is $q = +1.6 \times 10^{-19}$ C.

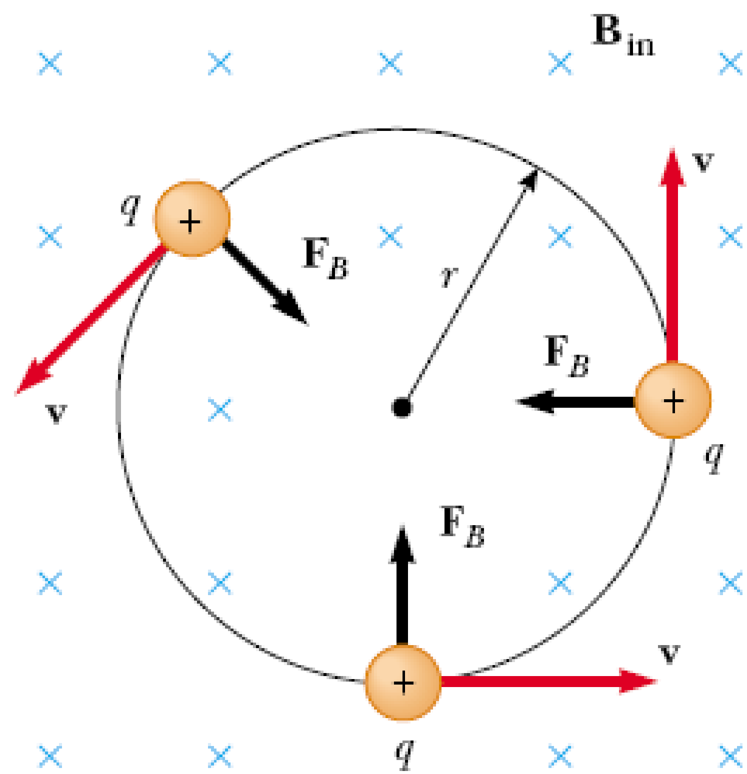


SET UP We use the right-hand rule to find the direction of the force. The force acts along the y axis, so we curl the fingers of our right hand around this axis in the direction from $\vec{v}$ toward $\vec{B}$. We find that the force acts in the $-y$ direction.

$$\begin{aligned} F &= qvB\sin\phi \\ &= (1.6 \times 10^{-19}\ \text{C})(3.0 \times 10^5\ \text{m/s})(2.0\ \text{T})(\sin 30°) \\ &= 4.8 \times 10^{-14}\ \text{N}. \end{aligned}$$

20.3 Motion of Charged Particle in magnetic field

- Consider a positively charged particle moving in a uniform magnetic field.
- Suppose the initial velocity of the particle is perpendicular to the direction of the field.
- Then a magnetic force will be exerted on the particle and make it follow a circular path.



Some Expressions Associated with the Particle…

The magnetic force produces a centripetal acceleration:

$$F = qvB = \frac{mv^2}{r}$$

The particle travels on a circular trajectory with a radius:

$$r = \frac{mv}{qB}$$

What is the period of revolution of the motion?

$$Period = T = \frac{2\pi r}{v} = \frac{2\pi m}{qB}$$

The frequency: $f = 1/T$

$$f = \frac{qB}{2\pi m}$$

Some Expressions Associated with the Particle…

The angular velocity ω of the particle is defined by

$$\omega = \frac{v}{r}$$

Combining this with relationship with the expression for radius, we get

$$\omega = \frac{v}{r} = v\frac{qB}{mv} = \frac{qB}{m}$$

$$r = \frac{mv}{qB}$$

Example : Proton moving in uniform magnetic field

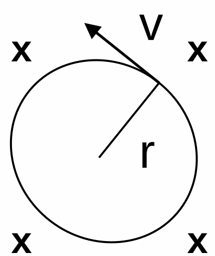
A proton is moving in a circular orbit of radius 14 cm in a uniform magnetic field of magnitude 0.35 T, directed perpendicular to the velocity of the proton. Find the orbital speed of the proton. What is the angular velocity of the particle?

$$v = \frac{qBr}{m}$$

$$= \frac{\left(1.6\times10^{-19}C\right)\left(0.35T\right)\left(14\times10^{-2}m\right)}{\left(1.67\times10^{-27}kg\right)}$$

$$= 4.7\times10^{6}\ {}^{m}\!/_{s}$$

Example: If a proton moves in a circle of radius 21 cm perpendicular to a B field of 0.4 T, what is the speed of the proton and the frequency of motion?



$$f = \frac{qB}{2\pi m}$$

$$f = \frac{1.6\times10^{-19}C\cdot0.4T}{2\pi\cdot1.67\times10^{-27}kg}$$

$$f = \frac{1.6\cdot0.4}{6.28\cdot1.67}\times10^{8}Hz = 6.1\times10^{6}Hz$$

$$\boxed{f = 6.1\times10^{6}Hz}$$

$$v = \frac{qBr}{m}$$

$$v = \frac{1.6\times10^{-19}C\cdot0.4T\cdot0.21m}{1.67\times10^{-27}kg}$$

$$v = \frac{1.6\cdot0.4\cdot0.21}{1.67}\times10^{8}\tfrac{m}{s} = 8.1\times10^{6}\tfrac{m}{s}$$

$$\boxed{v = 8.1\times10^{6}\tfrac{m}{s}}$$

Example

In a cyclotron suppose that deuterons moving in a circular trajectory have a frequency f = 12 MHz and radius of r = 53cm. What is the kinetic energy of the deuterons in this cyclotron (m=3.34 10^{-27}kg, B=1.57 T, q=+e)?

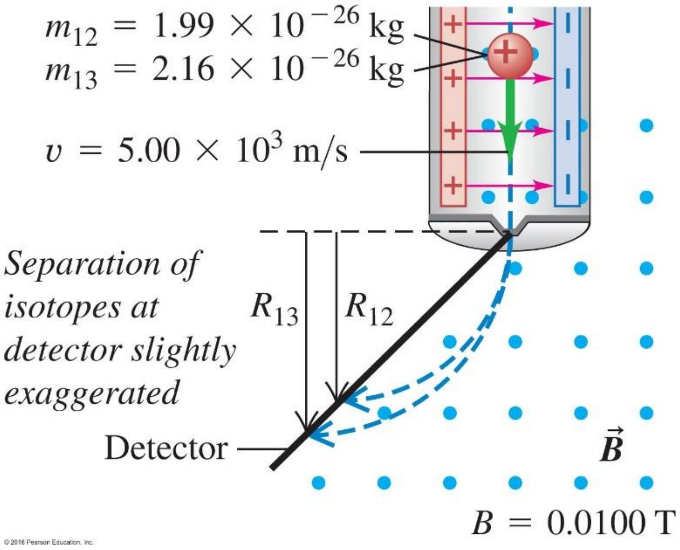
$$r = \frac{mv}{qB} \quad \text{implies} \quad v = \frac{RqB}{m} = 3.99\times 10^{7} \text{ m/s}$$

$$K = \tfrac{1}{2}mv^2 = 2.7\times 10^{-12} \text{ J}$$

Example

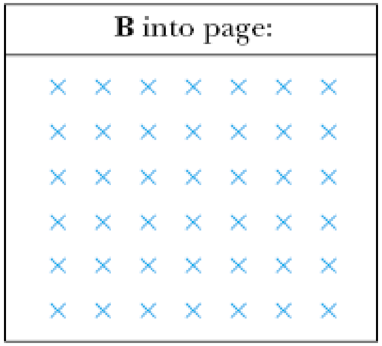
- A mass spectrometer is designed to collect carbon samples from Mars, C^{12} and C^{13}. The instrument has a magnetic field of 0.0100T and selects carbon ions which have a speed of 5.00×10^3 m/s and are singly ionized (+e). What are the size of the radial orbits for each carbon ion?

$$r = \frac{mv}{qB}$$

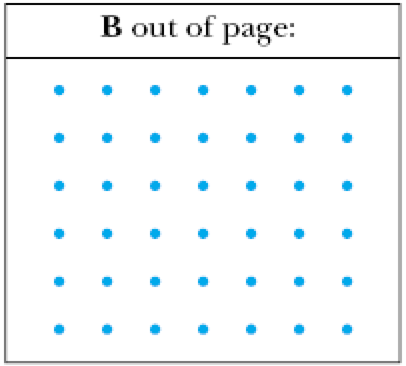


Magnetic Force on Current

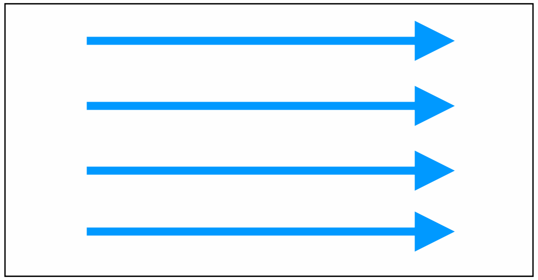
- If B is directed into the page we use blue crosses representing the tail of arrows indicating the direction of the field.



- If B is directed out of the page, we use dots.



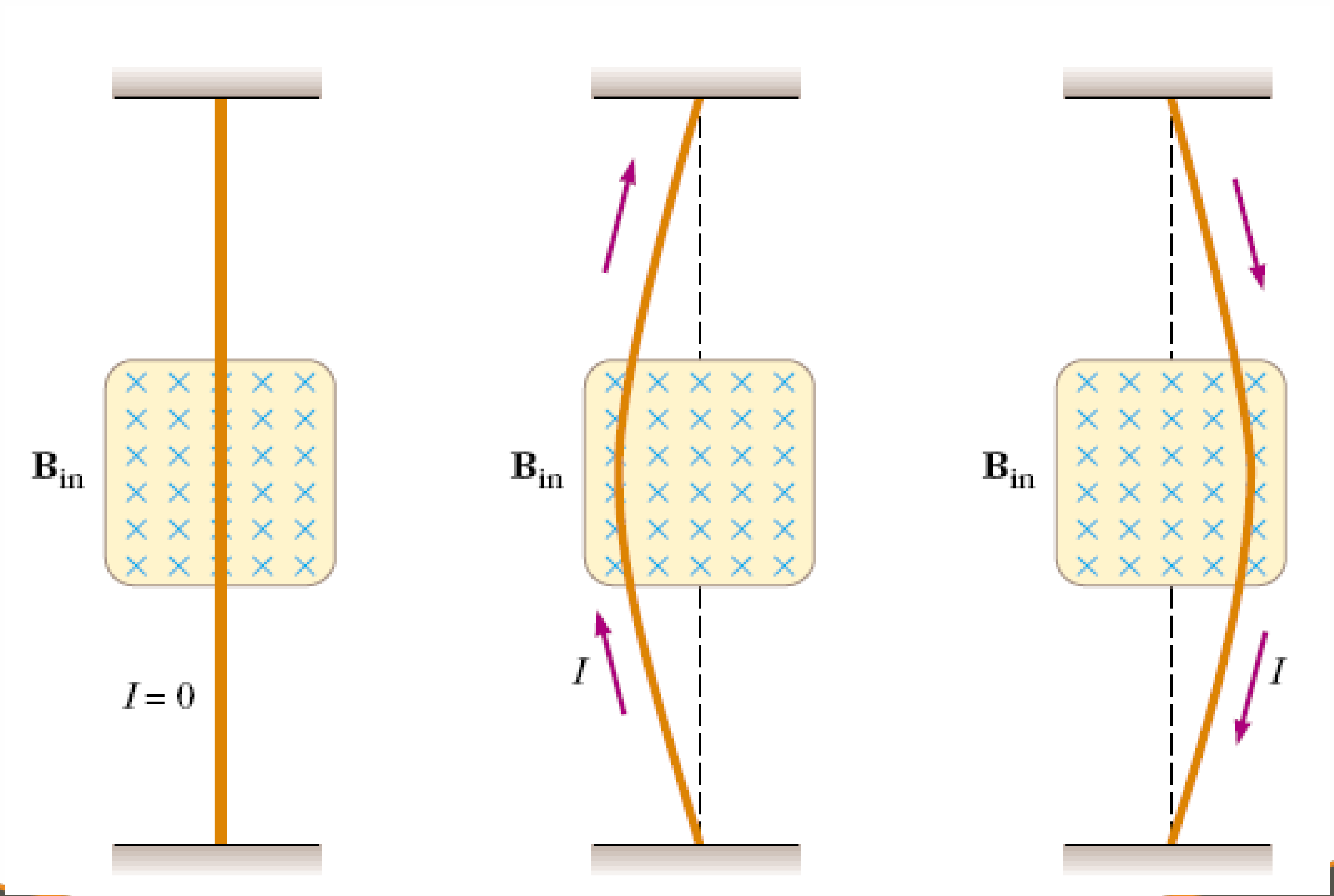
- If B is in the page, we use lines with arrow heads.

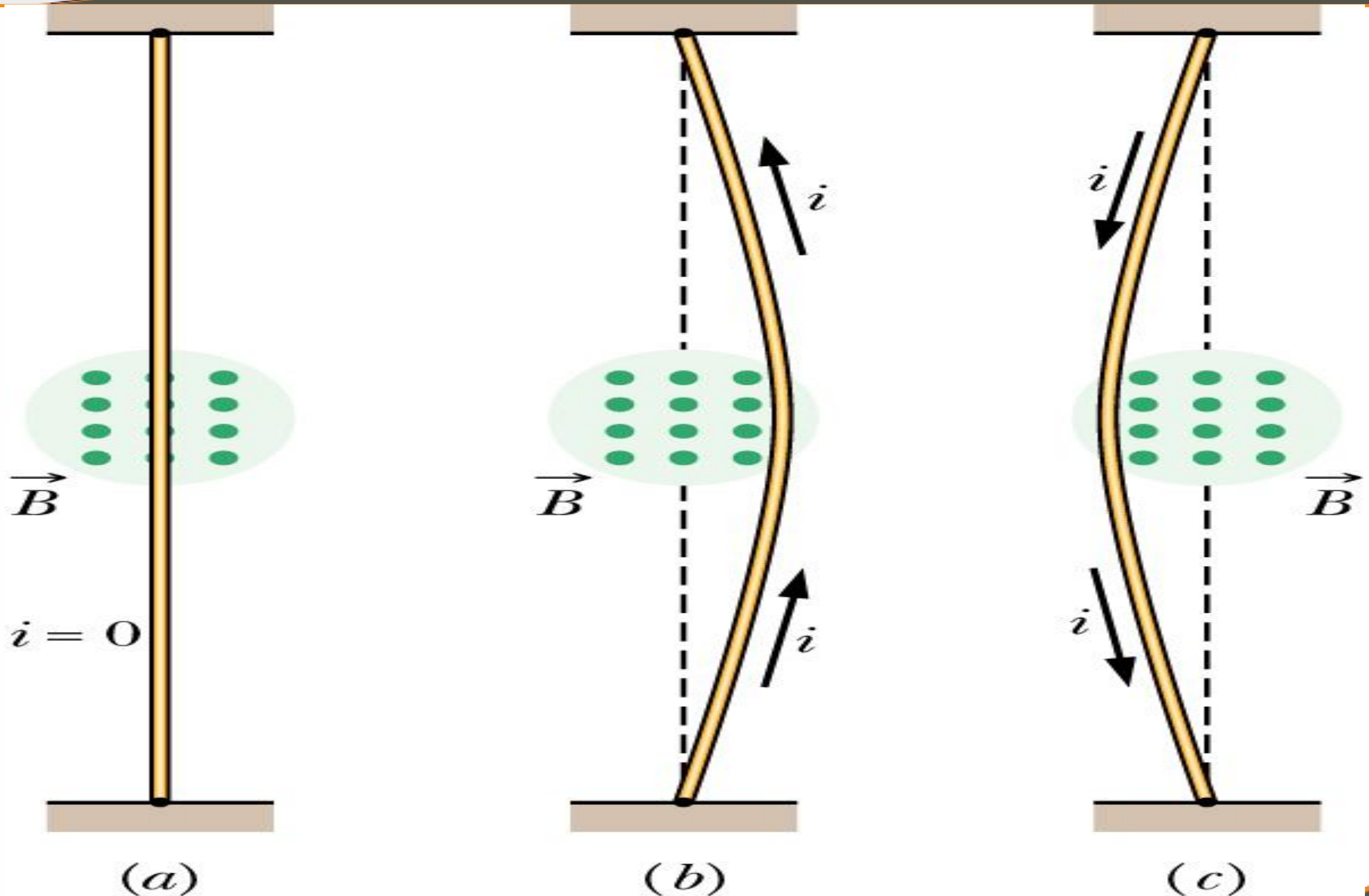


20.5 Magnetic Force on Current Carrying Conductor

- A magnetic force is exerted on a single charge in motion through a magnetic field.
- That implies a force should also be exerted on a collection of charges in motion through a conductor i.e. a current.
- The force on a current is the sum of all elementary forces exerted on all charge carriers in motion.

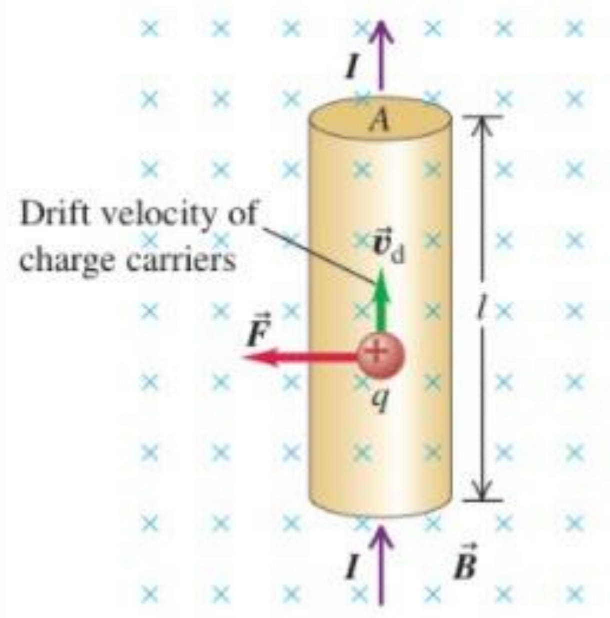
Force on a wire carrying current in a magnetic field.





$\vec{B}$
$i = 0$
i
i
$\vec{B}$
i
i
$\vec{B}$
(a)
(b)
(c)

20.5 Magnetic Force on Current Carrying Wire



$$F = qvBsin\theta$$
$$= Qv_dB_\perp$$

Substitute:

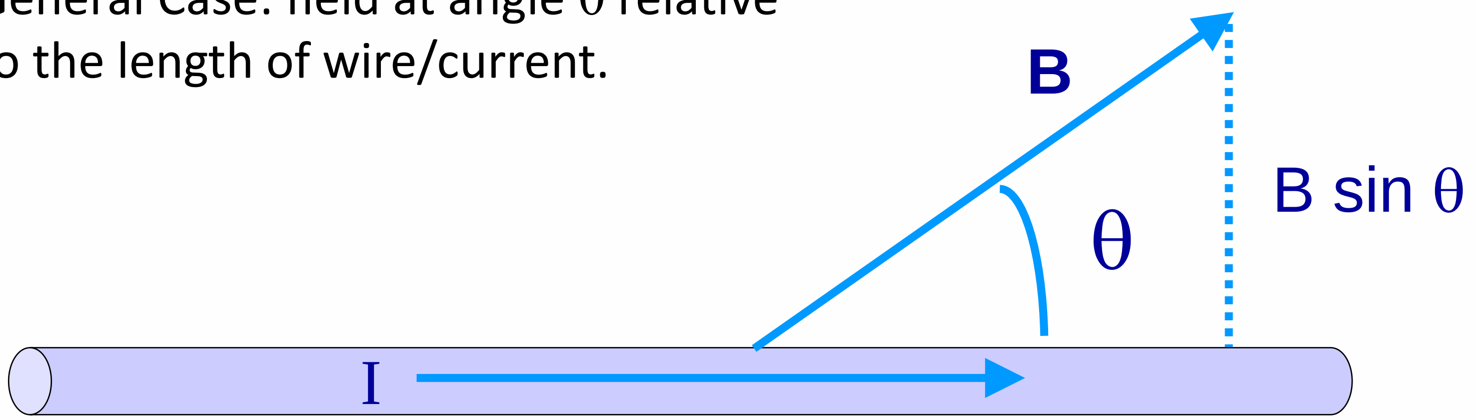
$Q = I\Delta t$ and $\Delta t = \frac{l}{v_d}$

$$F = (I\Delta t)\left(\frac{l}{\Delta t}\right)B_\perp$$
$$= IlB \quad or \quad BIl \quad or \quad BIlsin\theta$$

$$F_{\max} = BIl \rightarrow F_{\max} = BIl\sin\theta$$

Force on a Current Carrying Wire in a Magnetic Field

- General Case: field at angle θ relative to the length of wire/current.



$$F_{\max} = BIl\sin\theta$$

Note: If wire is not straight, compute force on differential elements and integrate:

$$d\vec{F} = i\,d\vec{L} \times \vec{B}$$

Example

Wire in Earth's B Field

A wire carries a current of 22 A from east to west. Assume that at this location the magnetic field of the earth is vertical and directed from north to south, and has a magnitude of 0.50 x 10^{-4} T. Find the magnitude and direction of the magnetic force on a 36m length of wire. What happens if the direction of the current is reversed?

$$F_{max} = BIl$$

$$= \left(0.50 \times 10^{-4} T\right)\left(22 A\right)\left(36 m\right)$$

$$= 4.0 \times 10^{-2} N$$

Example

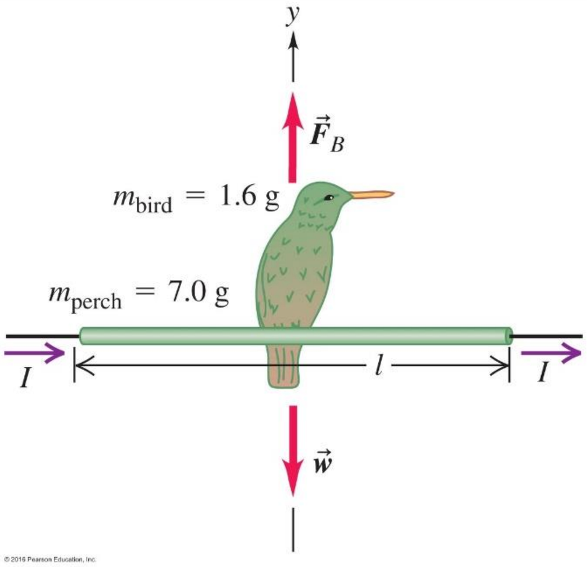
- A bird stands supported on a current carrying conductor (a perch) as shown. The perch is 10 cm long and carries a current of 4.0 A. The perch is orientated at right angles to a uniform magnetic field that is just strong enough to support the perch and the bird. What are the direction and magnitude of the magnetic field?

$$\sum F_y = 0$$

$$F_B - w = 0$$

$$BIl - m_T g = 0$$

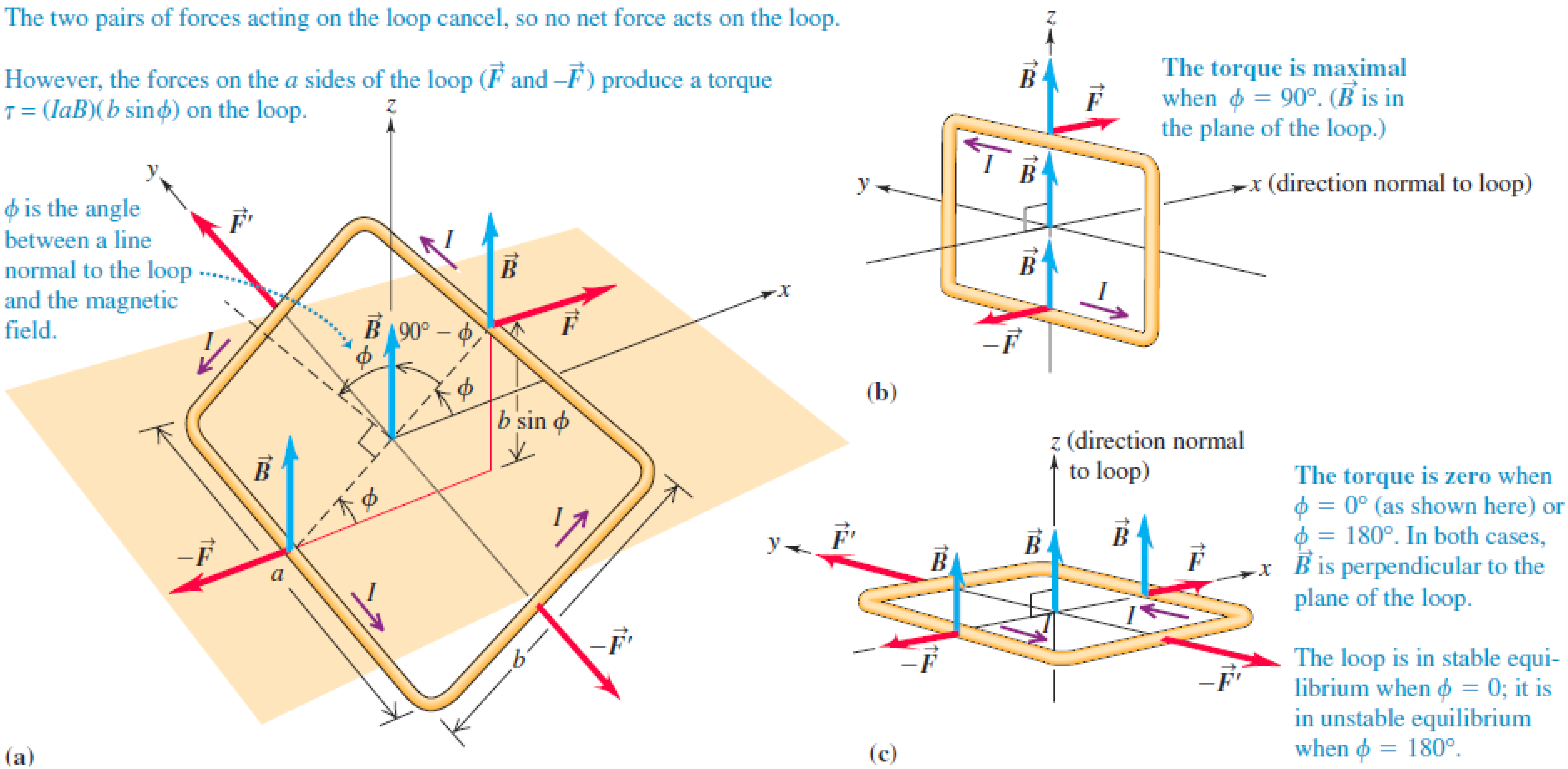
$$B = \frac{m_T g}{Il}$$



20.6 Force and Torque on a Current Loop

- Let's consider a rectangular loop carrying current in a uniform magnetic field
- We can represent this loop as a series of straight line segments
- We will find that the total force on the loop is zero but there is a net torque acting on the loop!

20.6 Force and Torque on a Current Loop – ELECTRIC MOTORS



▲ FIGURE 20.27 (a) Forces on the sides of a current-carrying loop rotating in a magnetic field. (b), (c) orientations at which the torque on the loop is maximal and zero, respectively.

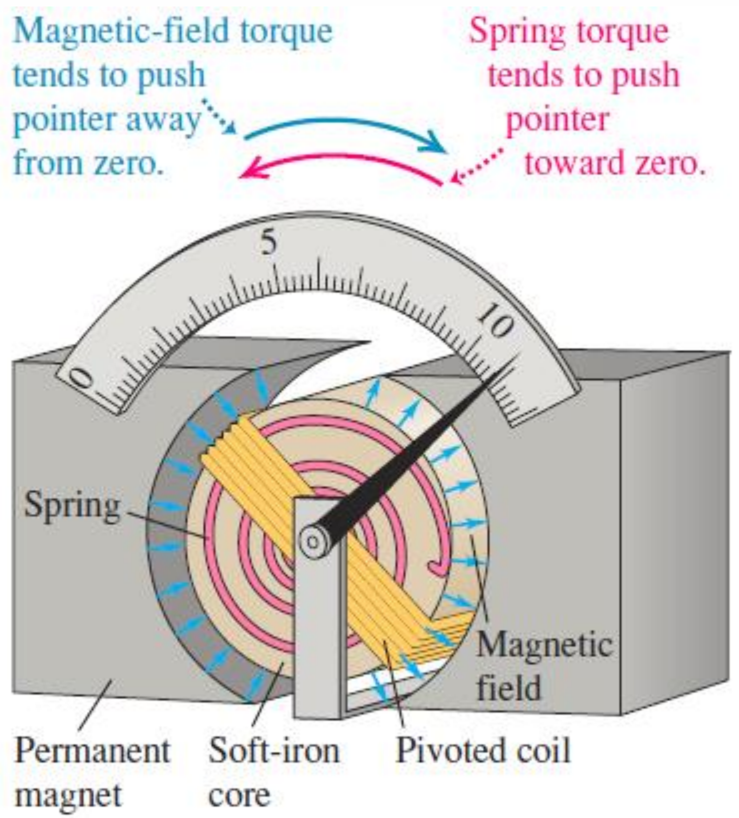
$$F = IaB \qquad \tau = (IaB)(b\sin\phi) \qquad \tau = IAB\sin\phi$$

Galvanometer

Device used in the construction of ammeters and voltmeters.

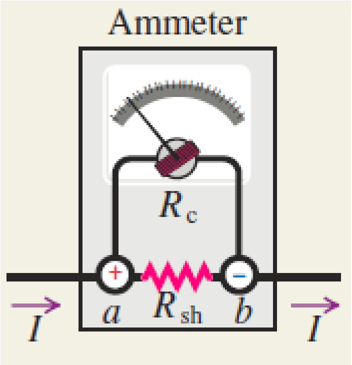
- The deflection in galvanometer is proportional to the current in the coil.

- Typical galvanometer have an internal resistance of the order of 60 Ω - that could significantly reduce a current measurement.

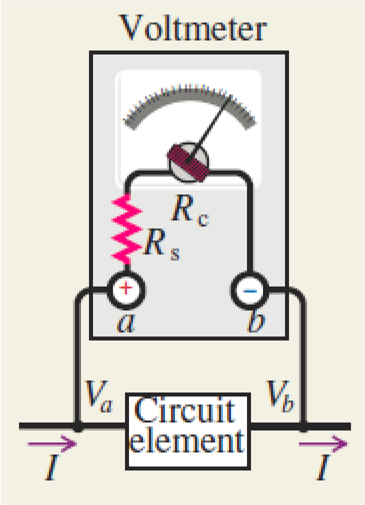


Electrical Measuring Instruments

- **Ammeter:** is a galvanometer mounted in parallel with a small resistor. (in circuits connected in series)



- **Voltmeter:** is a galvanometer mounted in series with a large resistance. (in circuits connected in parallel)



20.7 Magnetic Field of a Long, Straight Conductor

The passage of a steady current in a wire produces a magnetic field around the wire.

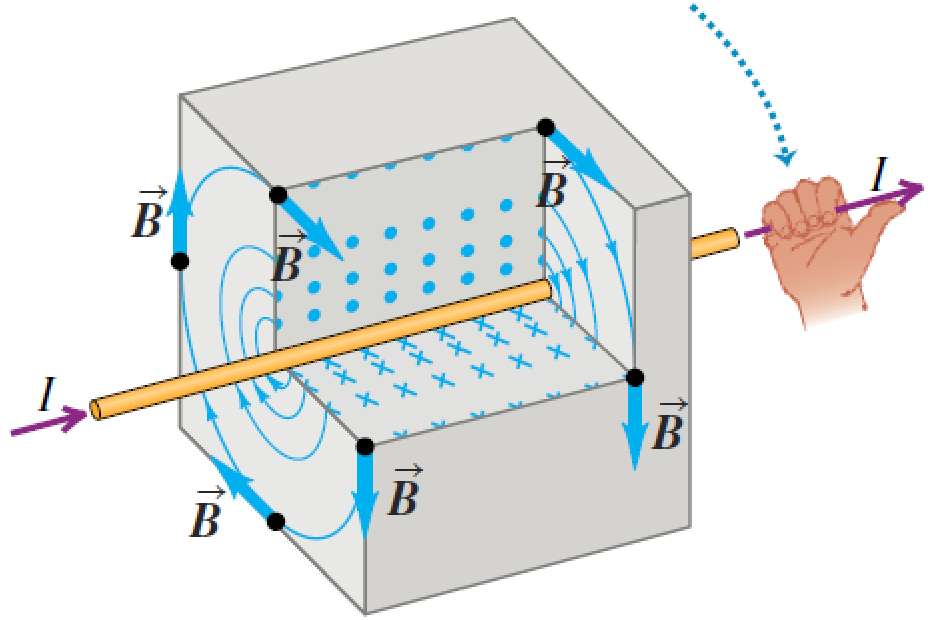
- Field forms concentric lines around the wire
- Direction of the field given by the right hand rule.

  If the wire is grasped in the right hand with the thumb in the direction of the current, the fingers will curl in the direction of the field.

- Magnitude of the field

$$B = \frac{\mu_o I}{2\pi r}$$

Right-hand rule for the magnetic field around a current-carrying wire: Point the thumb of your right hand in the direction of the current. Your fingers now curl around the wire in the direction of the magnetic field lines.



Constant, permeability of vacuum:

$$\mu_o = 4\pi \times 10^{-7}\, Tm/A$$

Example…

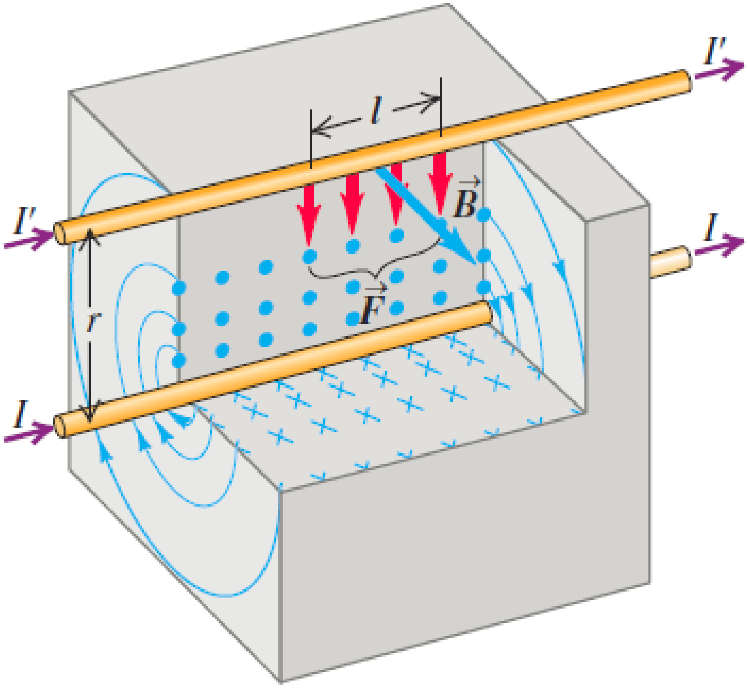
A long straight DC power cable carries a current of 220 A. At what distance from the power line is the magnitude of the magnetic field from the current equal to the magnitude of the Earth's magnetic field, approximately 5.0×10^{-5} T.

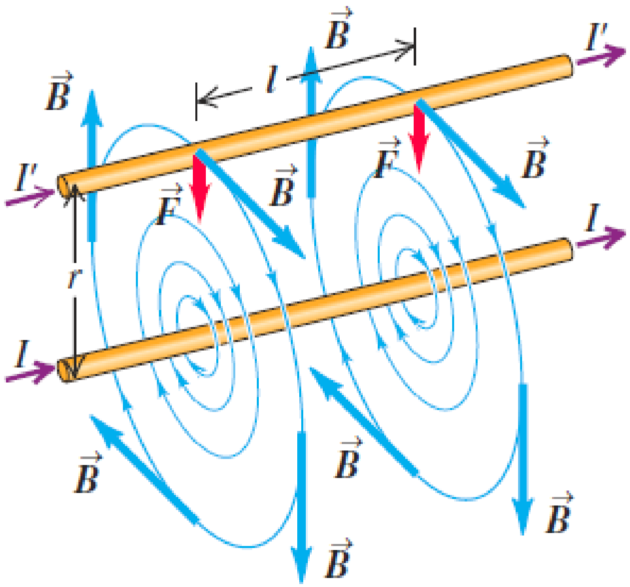
$$B = \frac{\mu_o I}{2\pi r}$$

20.8 Force between two Parallel Conductors

The magnetic field of the lower wire exerts an attractive force on the upper wire. By the same token, the upper wire attracts the lower one.

If the wires had currents in *opposite* directions, they would *repel* each other.



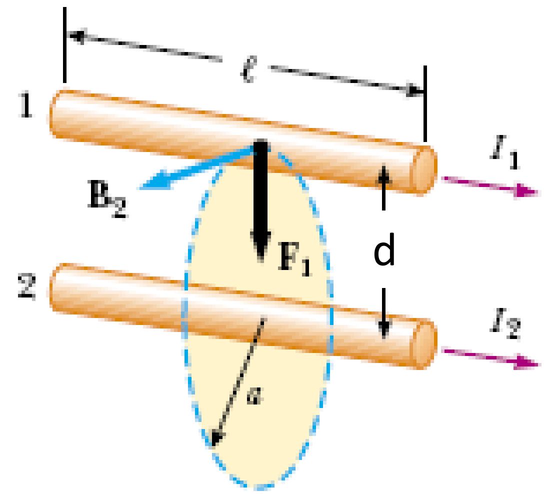


Force per Unit Length

$$B_2 = \frac{\mu_o I_2}{2\pi d}$$

$$F_1 = B_2 I_1 l = \left[\frac{\mu_o I_2}{2\pi d}\right] I_1 l = \frac{\mu_o I_1 I_2 l}{2\pi d}$$

$$\frac{F_1}{l} = \frac{\mu_o I_1 I_2}{2\pi d}$$



Definition of the SI unit Ampere

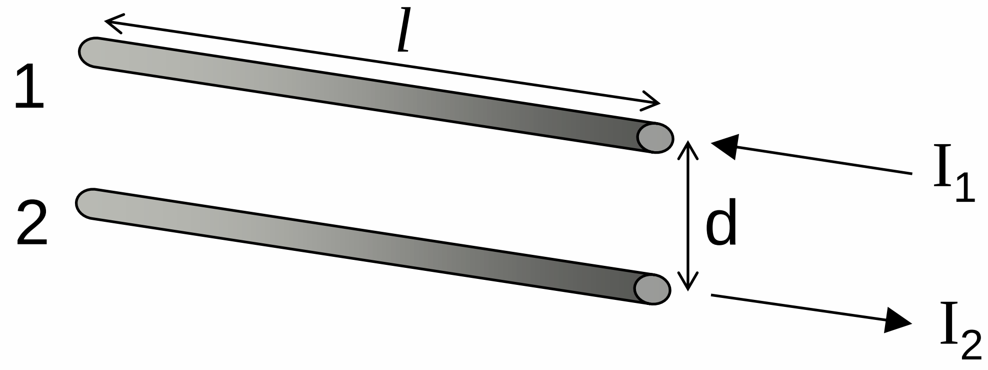
Used to define the SI unit of *current called **Ampere**.*

$$\frac{F_1}{l} = \frac{\mu_o I_1 I_2}{2\pi d}$$

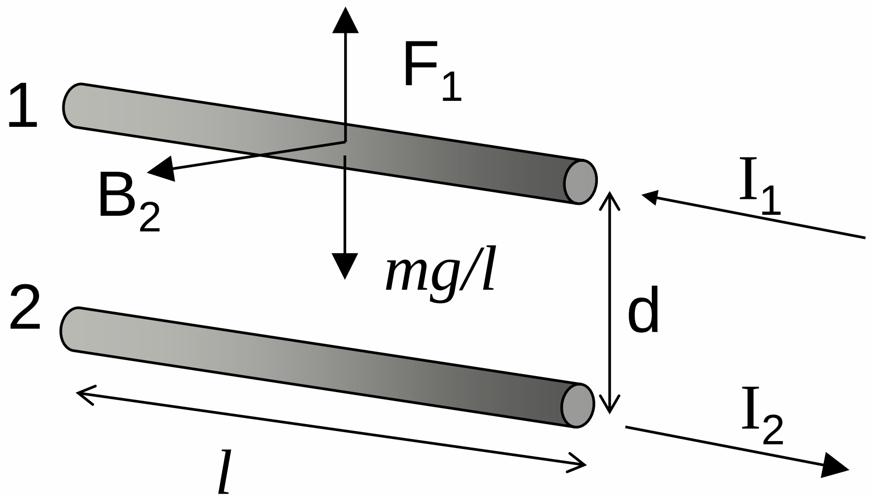
If two infinitely long, parallel wires 1 m apart carry the same current, induce a magnetic force per unit length on each wire of $2x10^{-7}$ N/m, then the current is defined to be 1 A.

Example

Two wires, each having a weight per unit length of 1.0×10^{-4} N/m, are strung parallel to one another above the surface of the Earth, one directly above the other. The wires are aligned as shown below. When their distance of separation is 0.10 mm what must be the current in each in order for the lower wire to push the upper wire north. (Assume the two wires carry the same current).



Example continued…



$$\frac{F_1}{l} = \frac{W}{l} = \frac{\mu_0 I^2}{2\pi d}$$

$$I^2 = \frac{2\pi d W}{\mu_0 l}$$

$$I = 0.22A$$

Magnetic Field of a Current Loop

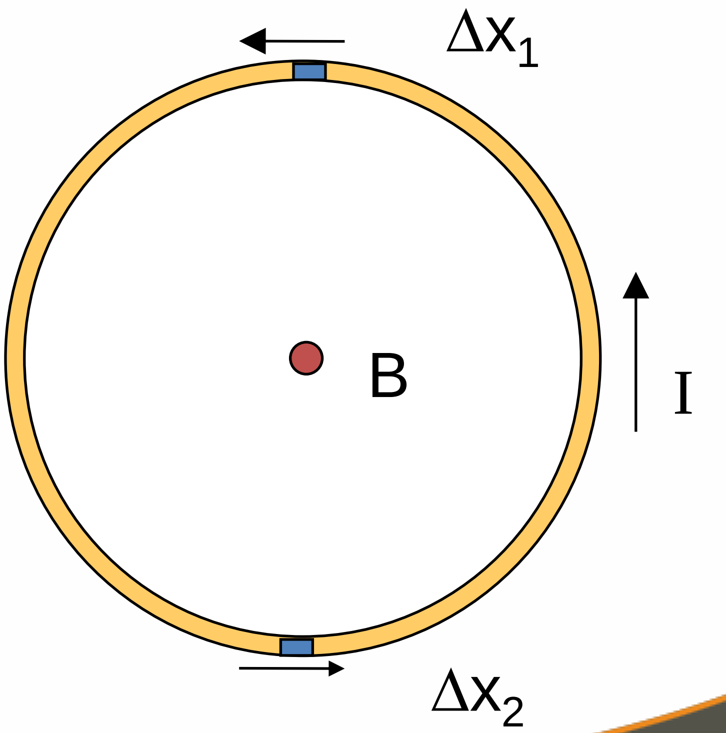
Magnetic field produced by a wire can be enhanced by having the wire in a loop with radius *R*.

1* loop Current → *I

$$B = \frac{\mu_0 I}{2R}$$

N* loops Current → *NI

$$B = \frac{N\mu_0 I}{2R}$$

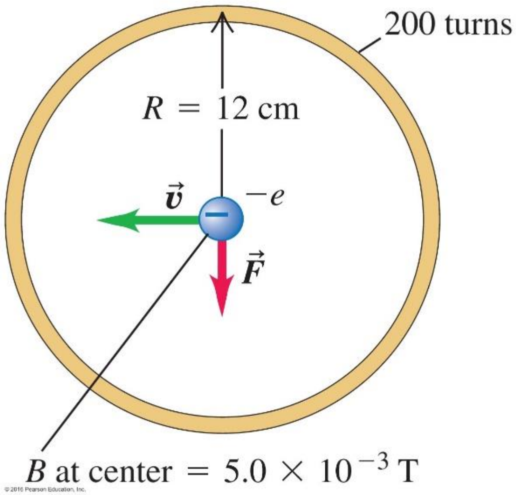


Example

- A coil used to produce a magnetic field for an electron beam experiment has 200 turns and a radius of 12cm

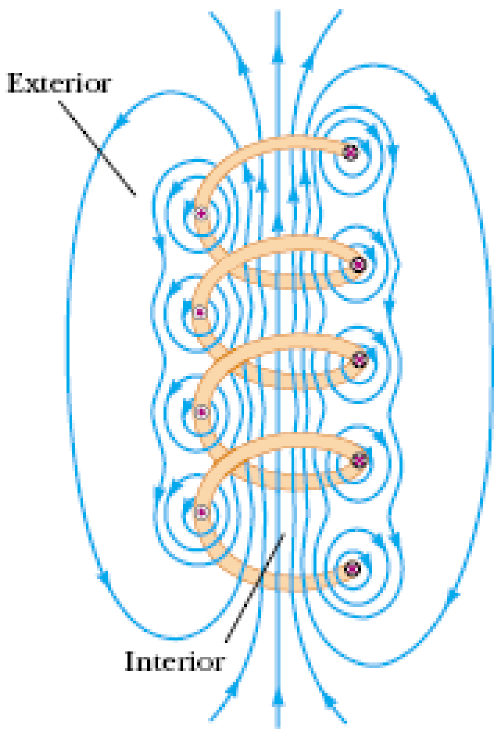
  (a) What current is required to produce a magnetic field with a magnitude of 5.0×10^{-3} T at the centre of the coil?

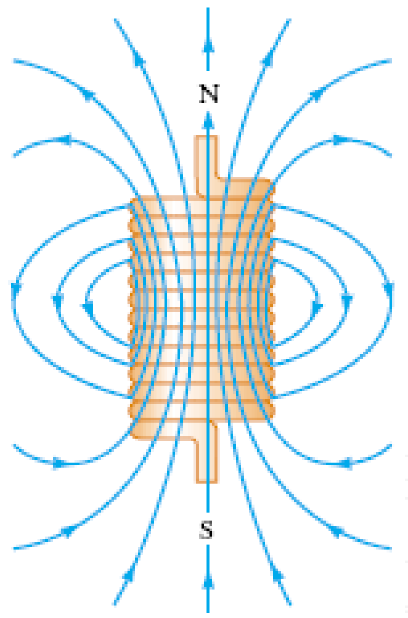
  (b) The diagram shows an electron being deflected as it moves through the coil. What is the direction of the current in the coil?



Magnetic Field of a Solenoid

- Solenoid magnet consists of a wire coil with multiple loops
- It is often called an electromagnet

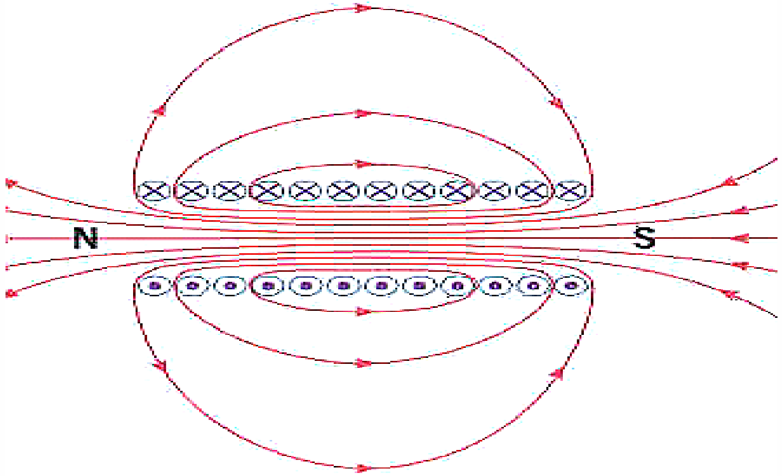




Solenoid Magnet

- Field lines inside a solenoid magnet are parallel, uniformly spaced and close together.
- The field inside is uniform and strong.
- The field outside is non uniform and much weaker.
- One end of the solenoid acts as a north pole, the other as a south pole.
- For a long and tightly looped solenoid, the field inside has a value:

$$B = \mu_o n I$$



Solenoid Magnet

The field inside has a value:

$$B = \mu_o n I$$

n = N/*L* : number of (loops) turns per unit length.

I : current in the solenoid.

$$\mu_o = 4\pi \times 10^{-7} \, Tm/A$$

Example:

Consider a solenoid consisting of 100 turns of wire and length of 10.0 cm. Find the magnetic field inside when it carries a current of 0.500 A.

$$n = \frac{N}{l} = \frac{100turns}{0.10m} = 1000turns/m$$

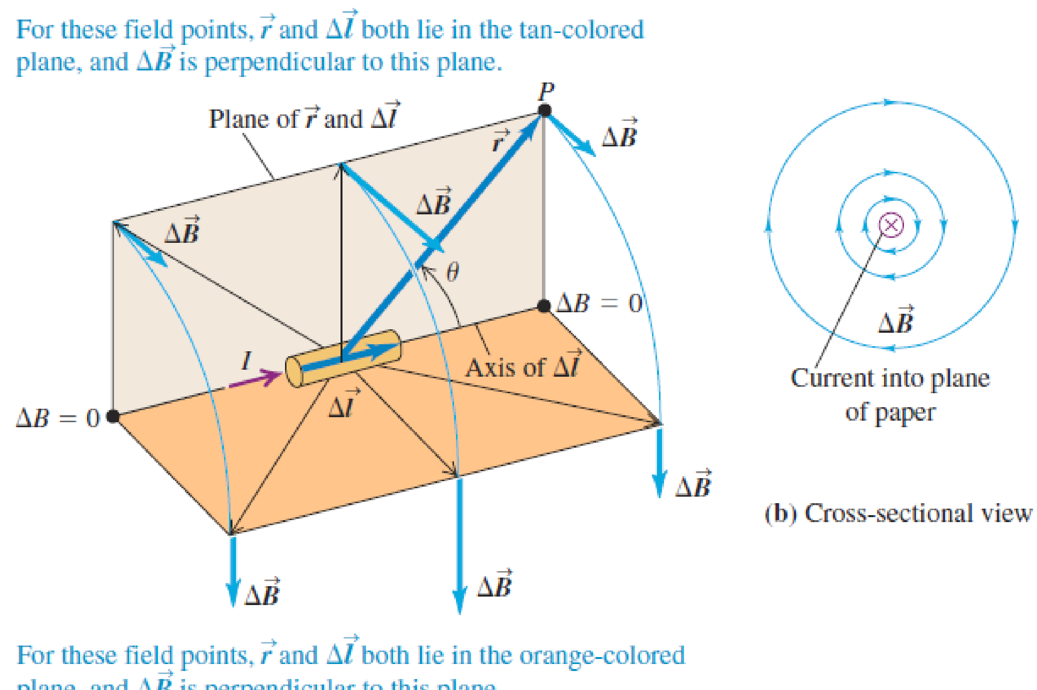
$$B = \mu_o nI = \left(4\pi \times 10^{-7}\,Tm/A\right)\left(1000turns/m\right)\left(0.500A\right)$$

$$B = 6.28 \times 10^{-4}\,T$$

20.10 Biot and Savart's Law

- The magnetic field produced at some point *P* a distance *r* from a current carrying wire, *I*

$$\Delta B = \frac{\mu_0}{4\pi}\frac{I\Delta l \sin\theta}{r^2}$$



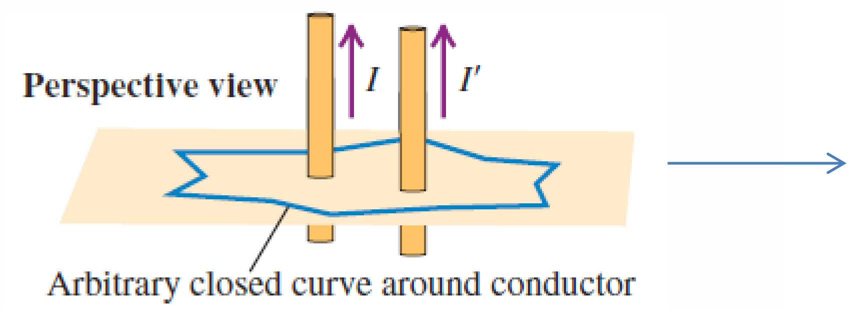
(a) Perspective view

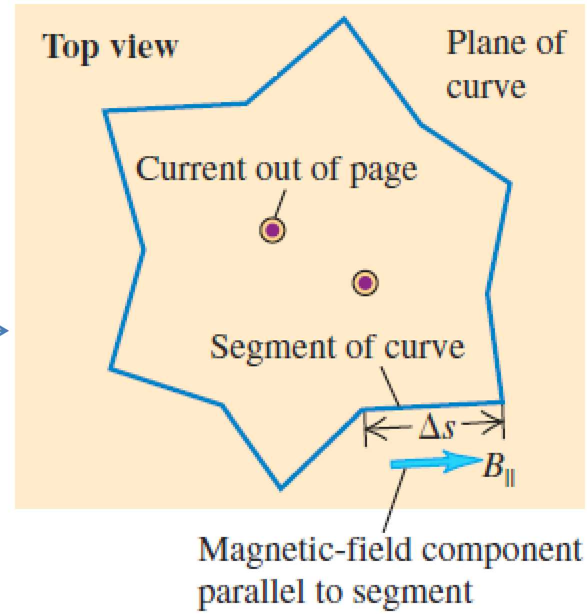
(b) Cross-sectional view

Ampère's law

When a path is made up of a series of segments Δs, and when that path links conductors carrying total current I_{encl},

$$\sum B_{\parallel}\,\Delta s = \mu_0 I_{\text{encl}}.$$





Ampère's law: If we take the products $B_{\parallel}\Delta s$ for all segments around the curve, their sum equals μ_0 times the total enclosed current:

$$\sum B_{\parallel}\Delta s = \mu_0 I_{\text{encl}}.$$

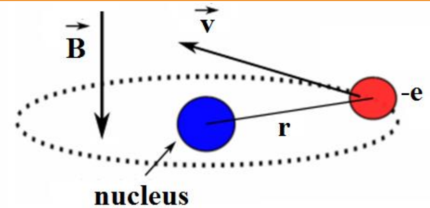
Example

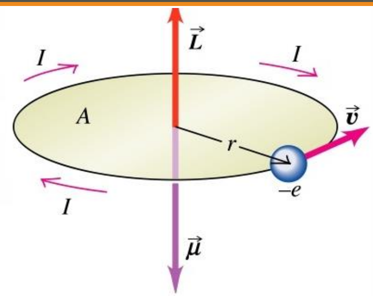
- Three long straight electrical cables are tightly enclosed in an insulating sheath. One of the cables carries 23.0 A current towards the south; the other two carry currents of 17.5 A and 11.3 A north. Use Ampere's law to calculate the magnitude of the magnetic field at a distance of 10.0 m from the cables.

Note – the currents are directed in different directions!

20.11 Magnetic Materials

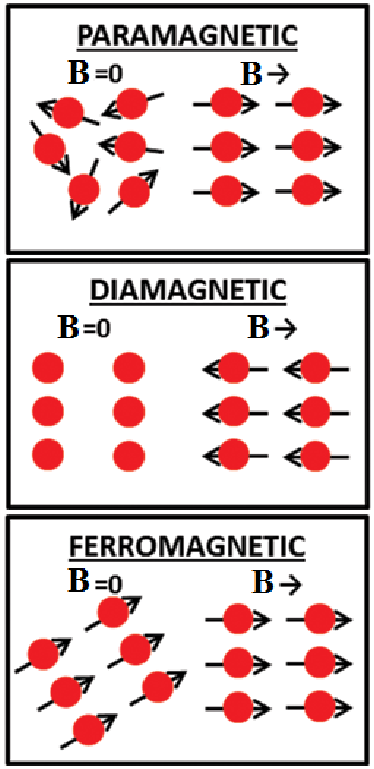
- **Materials can be classified by how they respond to an applied magnetic field, B_{app}.**





- **Paramagnetic (e.g. aluminum, tungsten, oxygen)**
  - **Atomic magnetic dipoles (~atomic bar magnets) tend to line up with the applied field, *increasing* it. But thermal motion randomizes their directions, so only a small effect persists: $B_{ind} \sim B_{app} \bullet 10^{-5}$**
- **Diamagnetic (e.g. gold, copper, water)**
  - **The applied field induces an *opposing* field; again, this is usually very weak; $B_{ind} \sim B_{app} \bullet 10^{-5}$** [Exception: Superconductors exhibit perfect diamagnetism → they exclude all magnetic fields]
- **Ferromagnetic (e.g. iron, cobalt, nickel)**
  - **Somewhat like paramagnetic, the dipoles prefer to line up with the applied field but in DOMAINS (these domains exist randomly in the absence of a magnetic field!).** However, there is a complicated collective effect due to strong interactions between neighboring dipoles → **they tend to all line up the same way.**

***Very* strong enhancement. $B_{ind} \sim B_{app} \bullet 10^{+5}$**



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