18 Electric Potential and Capacitance

18.1 Electric Potential Energy

Remember the equations from mechanics and the fact that the work done is change in energy

$$W_{a\rightarrow b} = F\, s \cos(\emptyset) = U_a - U_b = W_{net} = \Delta KE$$

$$W_{a\to b} = F\, s \cos(\emptyset) = U_a - U_b = W_{net} = \Delta KE$$

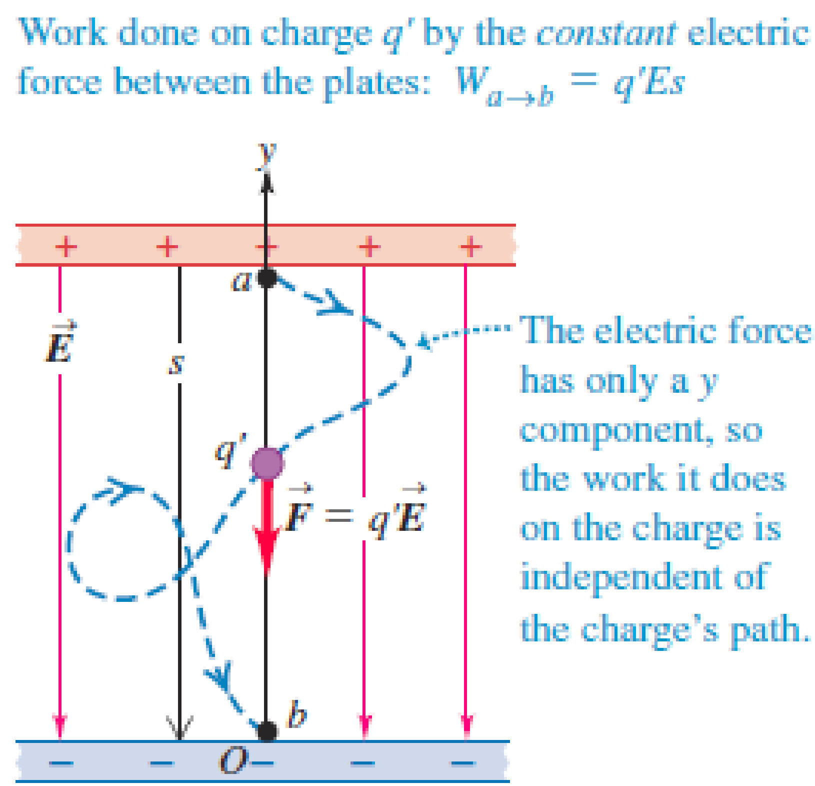
Apply this to electric fields

Test charge

$$W_{a\to b} = F\, s = q'\, E\, s$$

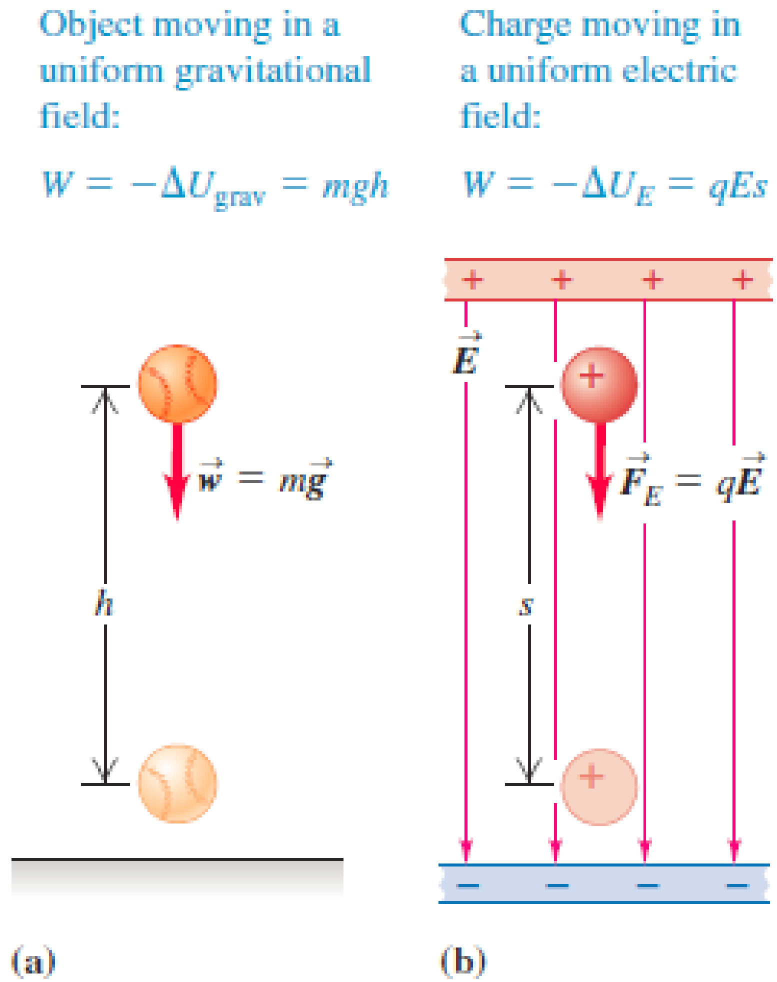
Electric potential energy

$$W_{a\to b} = U_a - U_b$$
$$= q'\, E\,(y_a - y_b)$$

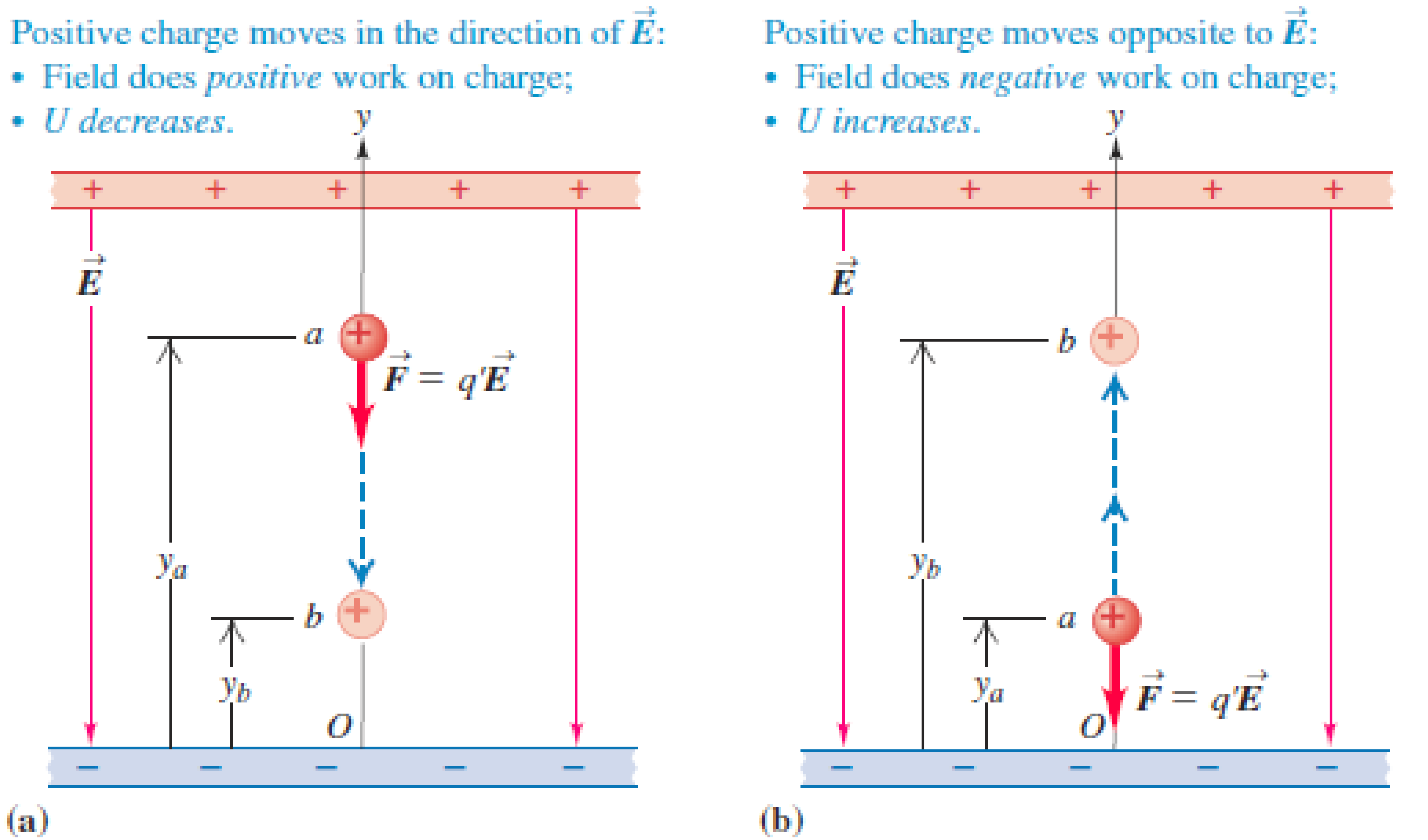


▲ **FIGURE 18.2** A test charge q' moves from point a to point b in a uniform electric field.

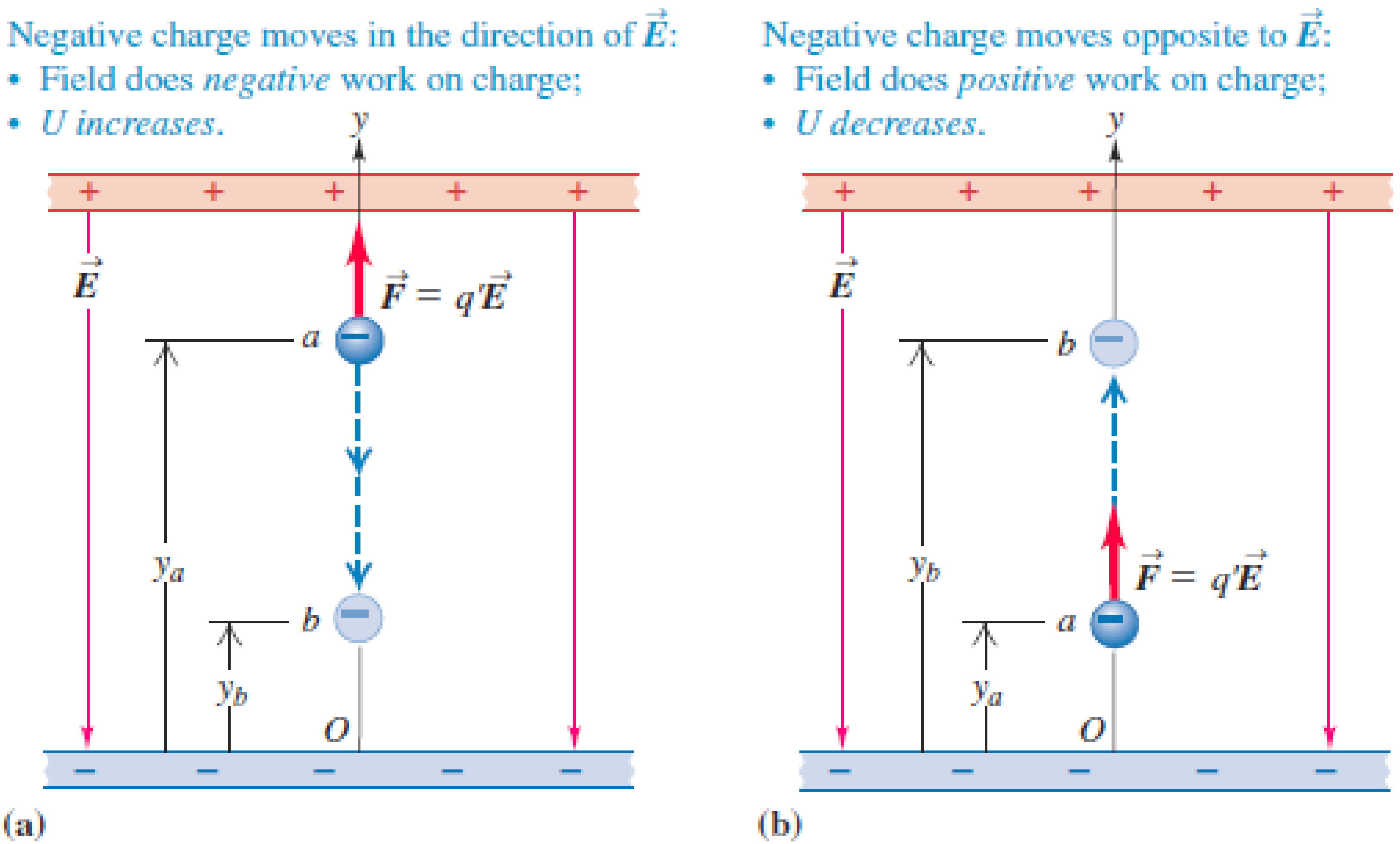
Comparing gravitational and electrical conservative forces



▲ **FIGURE 18.1** Because electric and gravitational forces are conservative, work done by either can be expressed in terms of a potential energy.



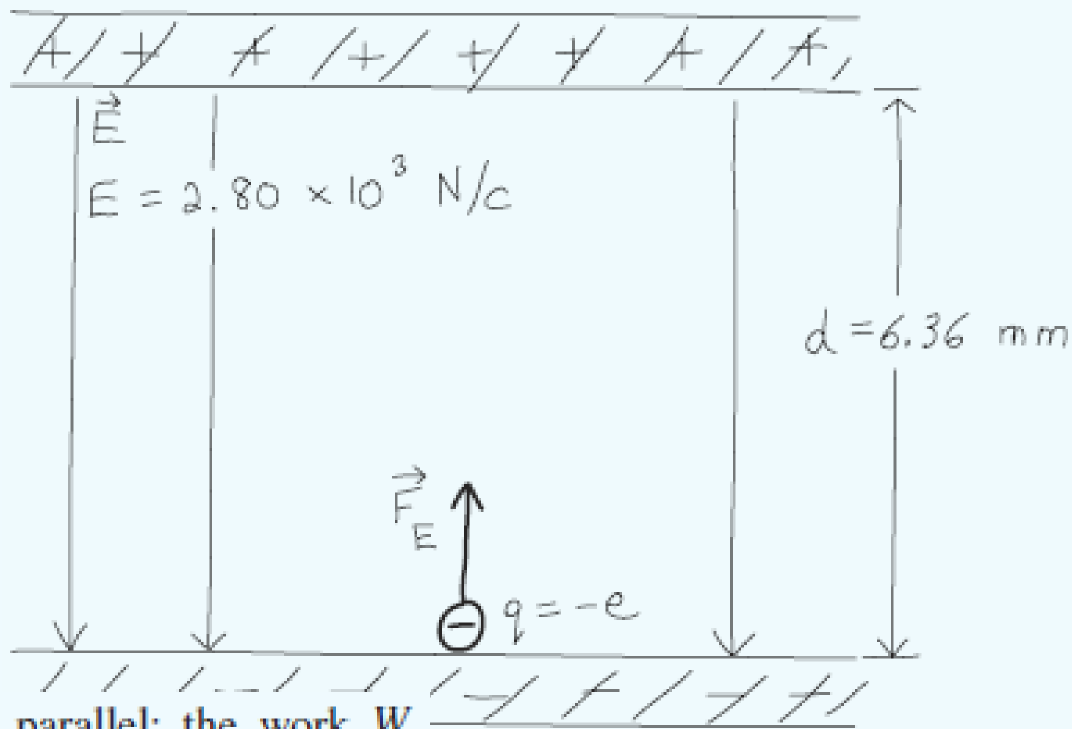
▲ **FIGURE 18.3** The work done by an electric field on a positive charge moving (a) in the direction of and (b) opposite to the electric field.



▲ **FIGURE 18.4** The work done by an electric field on a negative charge moving (a) in the direction of and (b) opposite to the electric field.

EXAMPLE 18.1 **Work in a uniform electric field**

Two large conducting plates separated by 6.36 mm carry charges of equal magnitude and opposite sign, creating a uniform electric field with magnitude 2.80×10^3 N/C between the plates. An electron moves from the negatively charged plate to the positively charged plate. How much work does the electric field do on the electron?

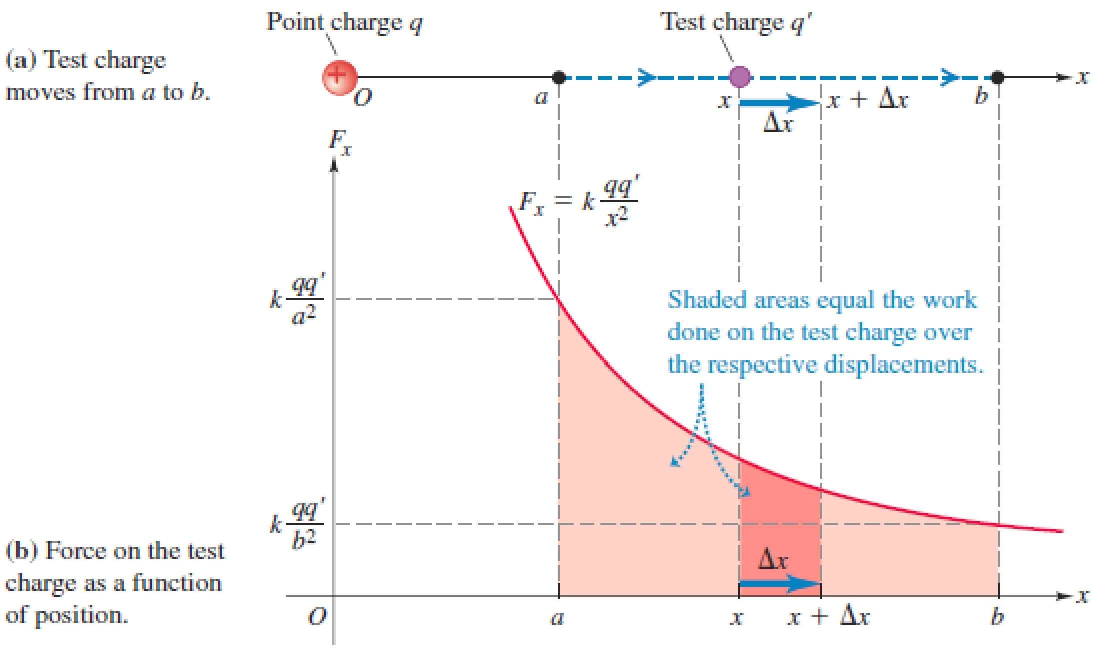


SOLVE The force and displacement are parallel; the work W done by the electric-field force during a displacement of magnitude d is $W = F_e d \cos\phi$ with $\phi = 0$, so

$$\begin{aligned} W &= F_e d = eEd \\ &= (1.60 \times 10^{-19}\ \text{C})(2.80 \times 10^3\ \text{N/C})(6.36 \times 10^{-3}\ \text{m}) \\ &= 2.85 \times 10^{-18}\ \text{J}. \end{aligned}$$

Potential Energy of Point Charges

It's useful to calculate the work done on a test charge q' when it moves in the electric field caused by a single stationary point charge q.



▲ **FIGURE 18.6** A test charge q' moves radially along a straight line extending from charge q. As it does so, the electric force on it decreases in magnitude.

$$W_{a\to b} = \frac{k\,q\,q'}{x} = k\,q\,q'\left(\frac{1}{a} - \frac{1}{b}\right)$$

Potential energy of point charges

The potential energy U of a system consisting of a point charge q' located in the field produced by a stationary point charge q, at a distance r from the charge, is

$$U = k\frac{qq'}{r}. \tag{18.8}$$

Quantitative Analysis 18.1

Change in potential energy

Consider two positive point charges q_1 and q_2. Their potential energy is defined as zero when they are infinitely far apart, and it increases as they move closer. If q_2 starts at an initial distance r_i from q_1 and moves toward q_1 to a final distance $r_i - \Delta r$ (where Δr is positive), by how much does the system's potential energy change?

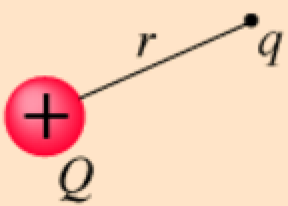
SOLUTION The electric potential energy of the two charges depends on the distance r between them: $U = k(q_1 q_2)/r$. Initially, the distance between them is r_i. After q_2 moves a distance Δr toward q_1, the distance is $r_i - \Delta r$. The change in potential energy depends on the reciprocal of these distances, so C must be the answer. More formally, the change in potential energy is

$$\Delta U = U_f - U_i = \frac{kq_1q_2}{r_i - \Delta r} - \frac{kq_1q_2}{r_i}.$$

Electric Potential Energy

Potential energy can be defined as the capacity for doing work which arises from position or configuration. In the electrical case, a charge will exert a force on any other charge and potential energy arises from any collection of charges. For example, if a positive charge Q is fixed at some point in space, any other positive charge which is brought close to it will experience a repulsive force and will therefore have potential energy. The potential energy of a test charge q in the vicinity of this source charge will be:

$$U = \frac{kQq}{r}$$



Show

where k is Coulomb's constant.

In electricity, it is usually more convenient to use the electric potential energy per unit charge, just called electric potential or voltage.

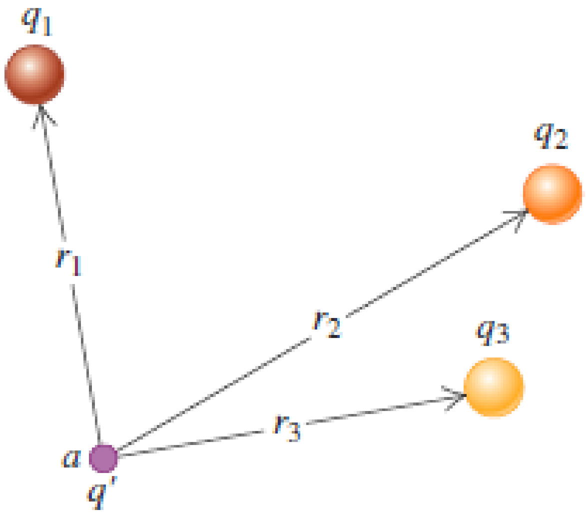
Application: Coulomb barrier for nuclear fusion

Energy in electron volts

Zero Potential

The nature of potential is that the zero point is arbitrary; it can be set like the origin of a coordinate system. That is not to say that it is insignificant; once the zero of potential is set, then every value of potential is measured with respect to that zero. Another way of saying it is that it is the change in potential which has physical significance. The zero of electric potential (voltage) is set for convenience, but there is usually some physical or geometric logic to the choice of the zero point. For a single point charge or localized collection of charges, it is logical to set the zero point at infinity. But for an infinite line charge, that is not a logical choice, since the local values of potential would go to infinity. For practical electrical circuits, the earth or ground potential is usually taken to be zero and everything is referenced to the earth.

Zero of potential at infinity
Zero of mechanical potential energy



▲ **FIGURE 18.7** Potential energy associated with a charge q' at point a depends on charges q_1, q_2, and q_3 and on their respective distances r_1, r_2, and r_3 from point a.

$$U = kq'\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} + \cdots\right).$$

Making U = 0 at infinity is a convenient reference level for electrostatic problems, but in circuit analysis other reference levels are often more convenient.

18.2 Potential

▲ Application ***Really*** **high voltage.**
A lightning bolt occurs when the electric potential difference between cloud and ground becomes so great that the air between them ionizes and allows a current to flow. A typical bolt discharges about 10^9 J of energy across a potential difference of about 10^7 V. In a major electrical storm, the total potential energy accumulated and discharged is enormous.

Definition of potential

The electric potential V at any point in an electric field is the electric potential energy U per unit charge associated with a test charge q' at that point:

$$V = \frac{U}{q'}, \qquad \text{or} \qquad U = q'V. \tag{18.10}$$

Potential energy and charge are both scalars, so potential is a scalar quantity.

Unit: From Equation 18.10, the units of potential are energy divided by charge. The SI unit of potential, 1 J/C, is called one **volt** (1 V), in honor of the Italian scientist Alessandro Volta (1745–1827):

$$1\ \text{V} = 1\ \text{volt} = 1\ \text{J/C} = 1\ \text{joule/coulomb}.$$

In the context of electric circuits, potential is often called voltage.
For instance, a 9 V battery has a difference in electric potential (potential difference) of 9 V between its two terminals. A 20,000 V power line has a potential difference of 20,000 V between itself and the ground.

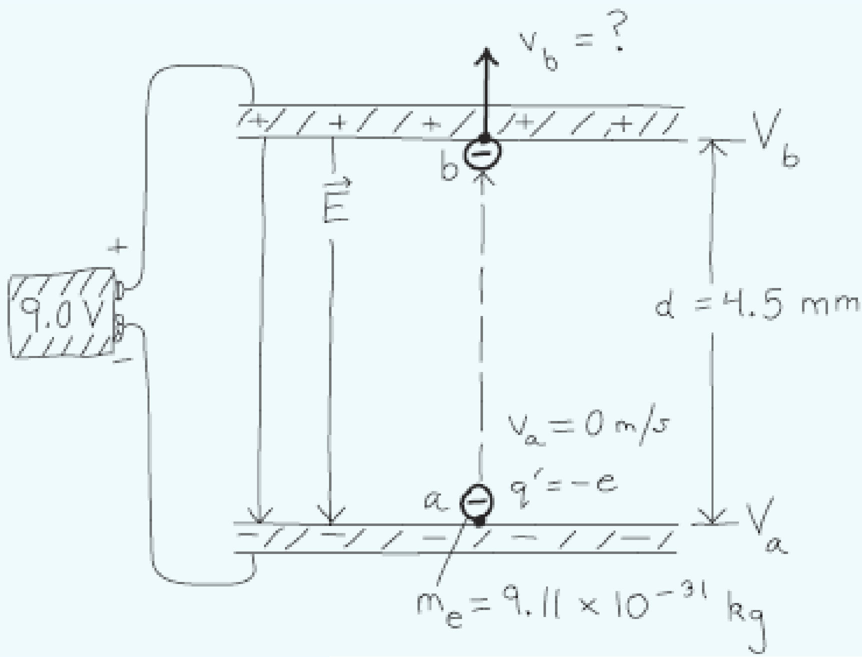
To put Equation 18.2 on a "work per unit charge" basis, we divide both sides by q', obtaining

$$\frac{W_{a\to b}}{q'} = \frac{U_a}{q'} - \frac{U_b}{q'} = V_a - V_b, \tag{18.11}$$

where $V_a = U_a/q'$ is the potential energy per unit charge at point a and V_b is that at b. We call V_a and V_b the *potential at point a* and *potential at point b*, respectively. The potential difference $V_a - V_b$ is called *the potential of a with respect to b*.

EXAMPLE 18.2 **Parallel plates and conservation of energy**

A 9.0 V battery is connected across two large parallel plates that are separated by 4.5 mm of air, creating a potential difference of 9.0 V between the plates. (**a**) What is the electric field in the region between the plates? (**b**) An electron is released from rest at the negative plate. If the only force on the electron is the electric force exerted by the electric field of the plates, what is the speed of the electron as it reaches the positive plate? The mass of an electron is $m_e = 9.11 \times 10^{-31}$ kg.



$$E = \frac{V_b - V_a}{d} = \frac{9.0\ \text{V}}{4.5 \times 10^{-3}\ \text{m}} = 2.0 \times 10^3\ \text{V/m}.$$

Part (b): Conservation of energy applied to points a and b at the corresponding plates gives

$$K_a + U_a = K_b + U_b.$$

Also, $U = q'V$, where $q' = -e$, the charge of an electron. Using this expression to replace U in the conservation-of-energy equation gives

$$K_a + q'V_a = K_b + q'V_b.$$

The electron is released from rest from point a, so $K_a = 0$. We next solve for K_b:

$$\begin{aligned} K_b &= q'(V_a - V_b) = -e(V_a - V_b) = +e(V_b - V_a) \\ &= (1.60 \times 10^{-19}\,\text{C})(9.0\,\text{V}) \\ &= 1.44 \times 10^{-18}\,\text{J}. \end{aligned}$$

Then $K_b = \frac{1}{2} m_e v_b^2$ gives

$$v_b = \sqrt{\frac{2K_b}{m_e}} = \sqrt{\frac{2(1.44 \times 10^{-18}\,\text{J})}{9.11 \times 10^{-31}\,\text{kg}}} = 1.8 \times 10^6\,\text{m/s}.$$

Potential of a point charge

When a test charge q' is a distance r from a point charge q, the potential V is

$$V = \frac{U}{q'} = k\frac{q}{r}, \tag{18.12}$$

where k is the same constant as in Coulomb's law (Equation 17.1).

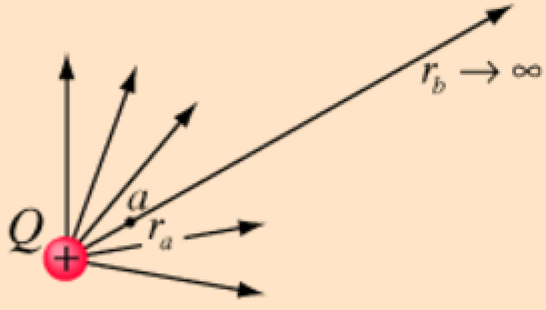
Similarly, to find the potential V at a point due to any collection of point charges $q_1, q_2, q_3, \ldots$ at distances $r_1, r_2, r_3, \ldots$, respectively, from q', we divide Equation 18.9 by q':

$$V = \frac{U}{q'} = k\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} + \cdots\right). \tag{18.13}$$

Potential Reference at Infinity

The general expression for the electric potential as a result of a point charge Q can be obtained by referencing to a zero of potential at infinity. The expression for the potential difference is:

$$V_a - V_b = kQ\left[\frac{1}{r_a} - \frac{1}{r_b}\right]$$



Taking the limit as $r_b \rightarrow \infty$ gives simply

$$V = \frac{kQ}{r} = \frac{Q}{4\pi\varepsilon_0 r}$$

for any arbitrary value of r. The choice of potential equal to zero at infinity is an arbitrary one, but is logical in this case because the electric field and force approach zero there. The electric potential energy for a charge q at r is then

$$U = \frac{kQq}{r}$$

EXAMPLE 18.3 Potential of two point charges

Two electrons are held in place 10.0 cm apart. Point *a* is midway between the two electrons, and point *b* is 12.0 cm directly above point *a*.

Calculate the electric potential at point *a* and at point *b*.

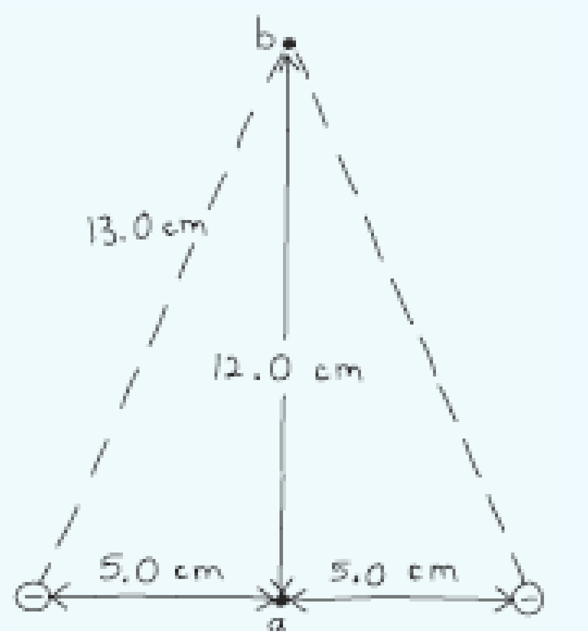
SET UP Figure 18.9 shows our sketch. Point b is a distance $r_b = \sqrt{(12.0\ \text{cm}^2) + (5.0\ \text{cm})^2} = 13.0$ cm from each electron.

SOLVE Part (a): The electric potential V at each point is the sum of the electric potentials of each electron: $V = V_1 + V_2 = k\frac{q_1}{r_1} + k\frac{q_2}{r_2}$, with $q_1 = q_2 = -e$. At point a, $r_1 = r_2 = r_a = 0.050$ m, so

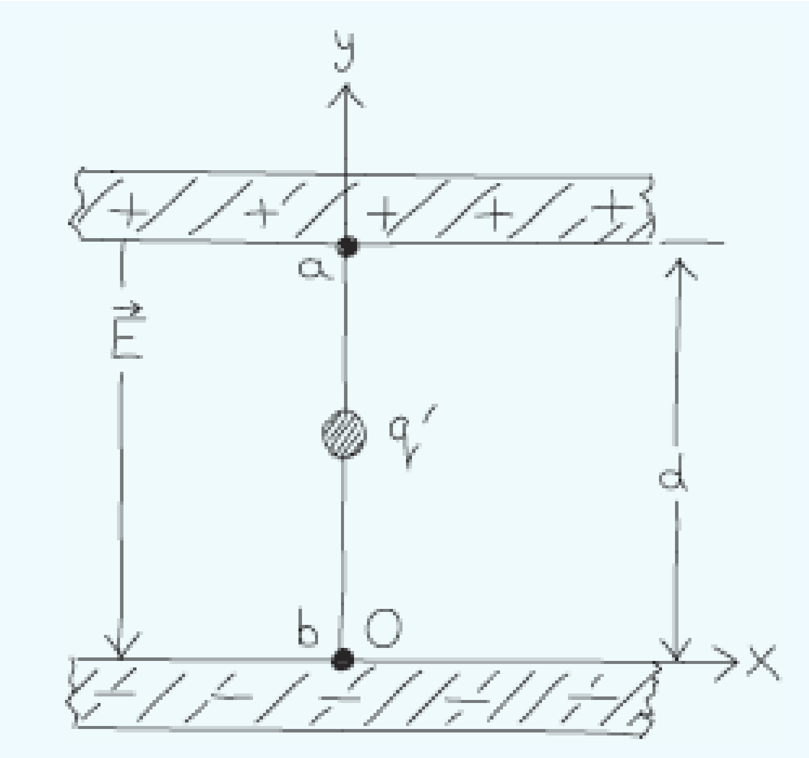
$$V_a = -\frac{2ke}{r_a} = -\frac{2(8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.60 \times 10^{-19}\ \text{C})}{0.050\ \text{m}} = -5.8 \times 10^{-8}\ \text{V}.$$

At point b, $r_1 = r_2 = r_b = 0.130$ m, so

$$V_b = -\frac{2ke}{r_b} = -\frac{2(8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.60 \times 10^{-19}\ \text{C})}{0.130\ \text{m}} = -2.2 \times 10^{-8}\ \text{V}.$$



Parallel plates



Remember that potential is simply *potential energy per unit charge.*

We **choose** the potential *V* to be zero at y = 0 (point *b* in our sketch).

SOLVE The potential energy U for a test charge q' at a distance y above the bottom plate is given by Equation 18.5, $U = q'Ey$. The potential V at point y is the potential energy per unit charge, $V = U/q'$, so

$$V = Ey.$$

Even if we had chosen a different reference level (at which $V = 0$), it would still be true that $V_y - V_b = Ey$. At point a, where $y = d$ and $V_y = V_a$, $V_a - V_b = Ed$ and

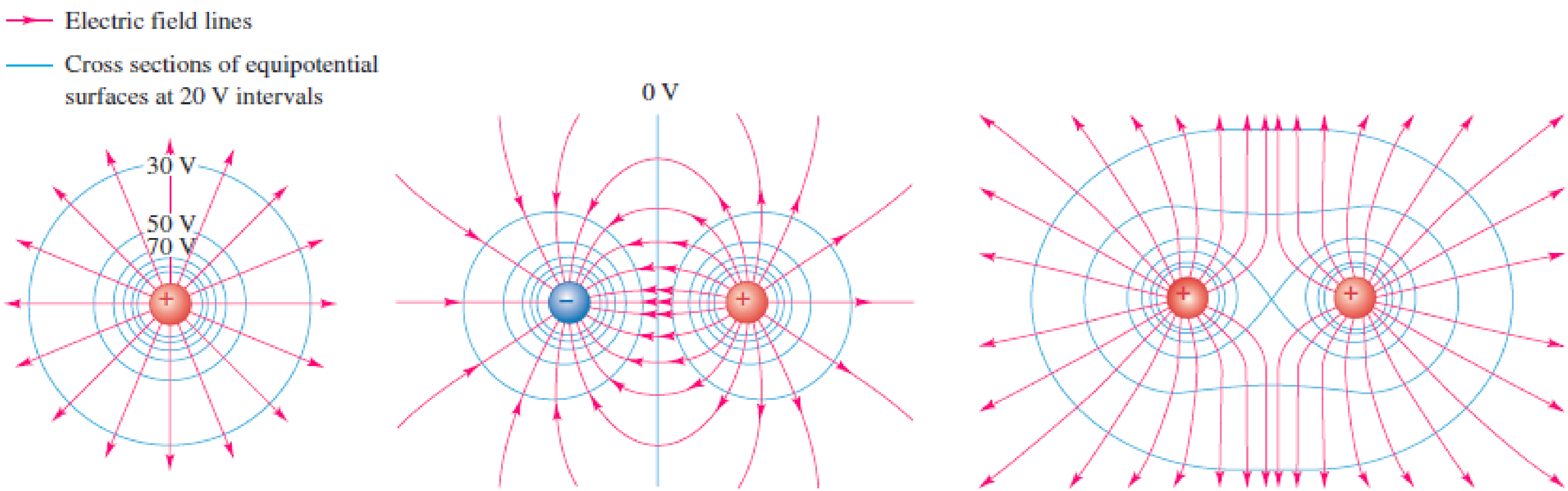
$$E = \frac{V_a - V_b}{d} = \frac{V_{ab}}{d}.$$

18.3 Equipotential Surfaces

An equipotential surface is defined as a surface on which the potential is the same at every point.

No point can be at two different potentials, so equipotential surfaces for different potentials can never touch or intersect.

The potential energy for a test charge is the same at every point on a given equipotential surface, so the field does no work on a test charge when it moves from point to point on such a surface.

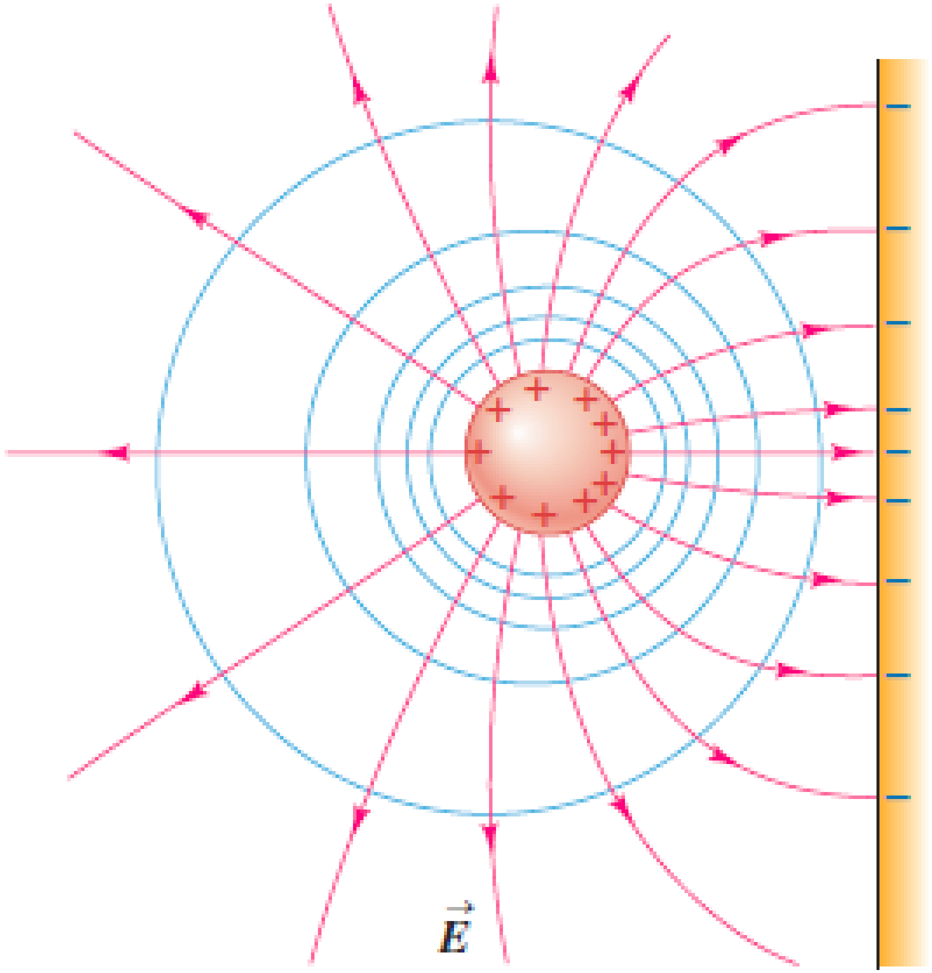


(a) A single positive charge

(b) An electric dipole

(c) Two equal positive charges

▲ **FIGURE 18.11** Equipotential surfaces and electric field lines for assemblies of point charges. How would the diagrams change if the charges were reversed?



▲ **FIGURE 18.13** When charges are at rest, a conducting surface is always an equipotential surface. Field lines are perpendicular to a conducting surface.

E must be perpendicular to the surface at every point. Field lines and equipotential surfaces are always mutually perpendicular.

We can prove that when all charges are at rest, the electric field just outside a conductor must be perpendicular to the surface at every point.

It follows that, in an electrostatic situation, a conducting surface is always an equipotential surface.

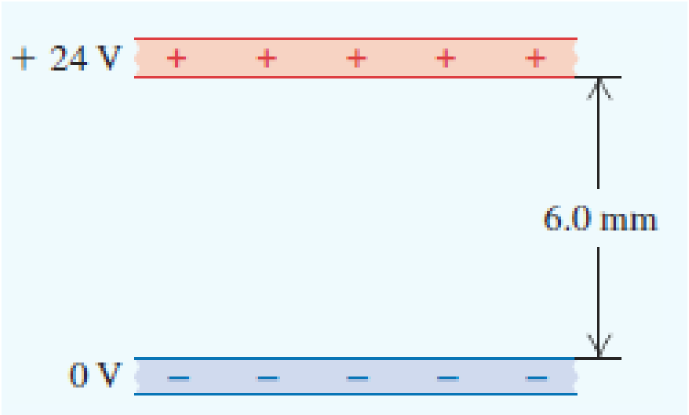
Electric field represented as potential gradient

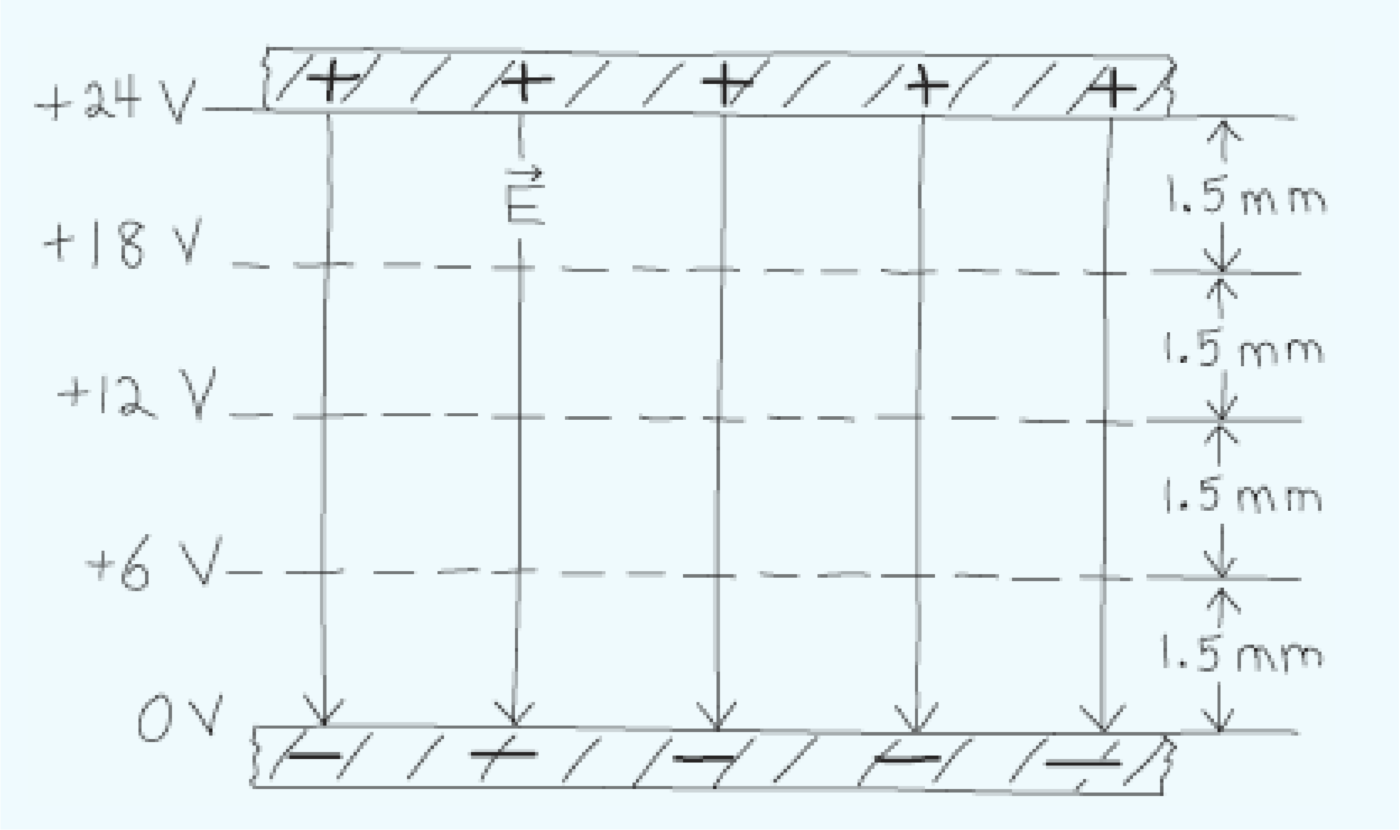
The magnitude of the electric field at any point on an equipotential surface equals the rate of change of potential, ΔV, with distance Δs as the point moves perpendicularly from the surface to an adjacent one a distance Δs away:

$$E = -\frac{\Delta V}{\Delta s}. \tag{18.14}$$

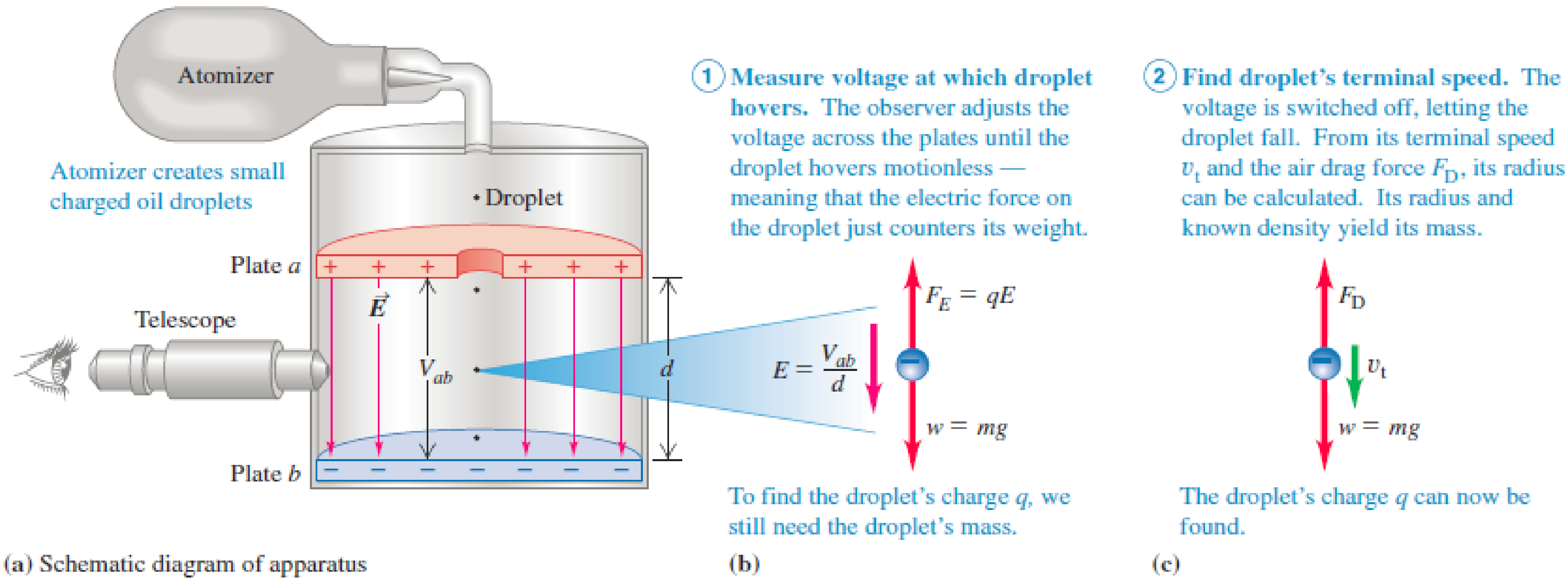
The negative sign shows that when a point moves in the direction of the electric field, the potential decreases. The quantity $\Delta V/\Delta s$, representing a rate of change of V with distance, is called the **potential gradient.** We see that this is an alternative name for electric field.

Equipotential surfaces within a capacitor



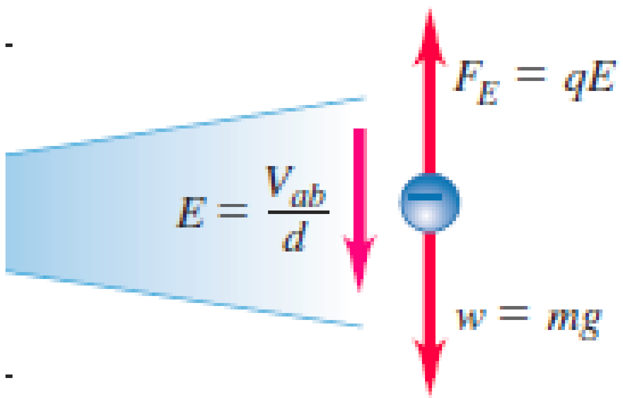


18.4 The Millikan Oil-Drop Experiment



▲ **FIGURE 18.16** The Millikan oil-drop experiment, which demonstrated that charge is quantized and provided the first determination of e.

① **Measure voltage at which droplet hovers.** The observer adjusts the voltage across the plates until the droplet hovers motionless — meaning that the electric force on the droplet just counters its weight.



To find the droplet's charge q, we still need the droplet's mass.

Droplet stationary

$$qE = mg$$

So

$$q = \frac{mg}{E}$$

we can find E from

$$E = \frac{V}{d}$$

and find m from

$$m = \rho V = \rho \frac{4\pi r^2}{3}$$

and r from terminal velocity

Now we can measure the charge on a droplet.
Each droplet will have a different charge (+ or -).

So with MANY measurements and knowing that

$$q = \pm n\, e$$

We can determine e.

(Where n is an integer and e is the charge on an electron)

An electron has a charge of

$$1.602\ x\ 10^{-19}\ C$$

Electrovolt

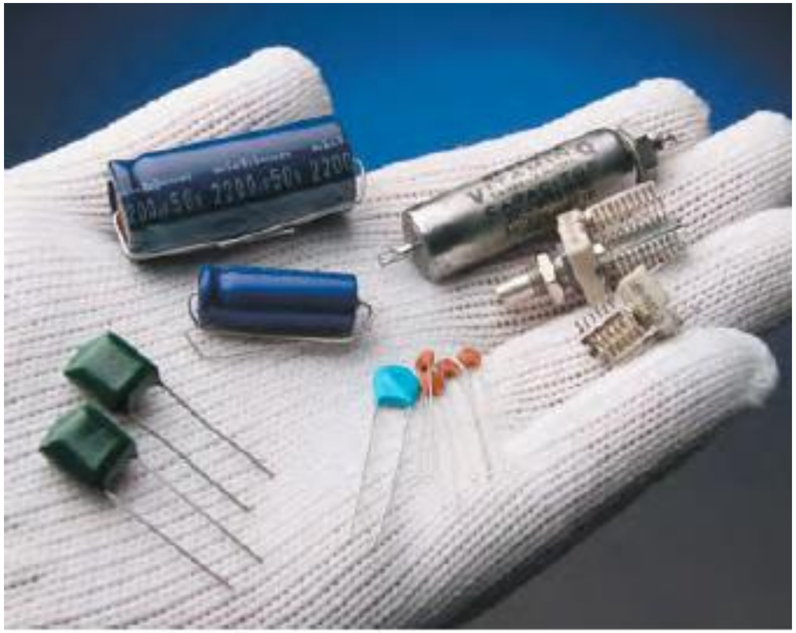
An electrovolt is a unit of energy

If we move an electron through a potential difference of 1V

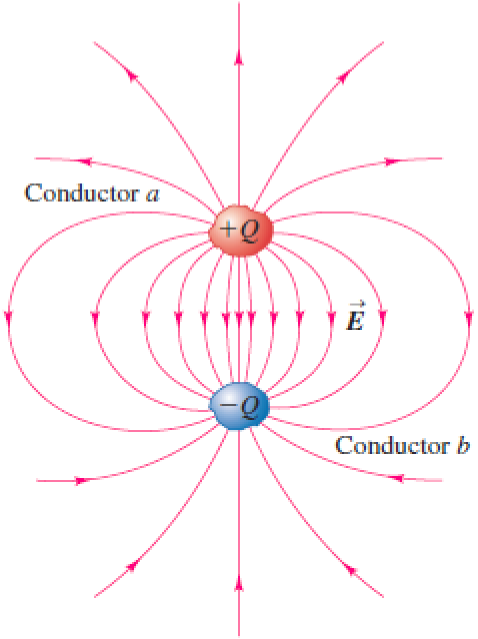
$\Delta U = qV = 1.602\ x\ 10^{-19}$ x 1

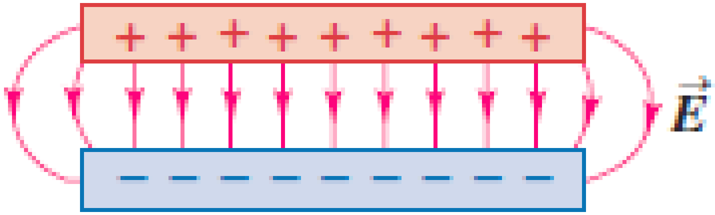
$$1\ eV = 1.602\ x\ 10^{-19}\ \text{J}$$

18.5 Capacitors



▲ **FIGURE 18.17** An assortment of practical capacitors.





Definition of capacitance

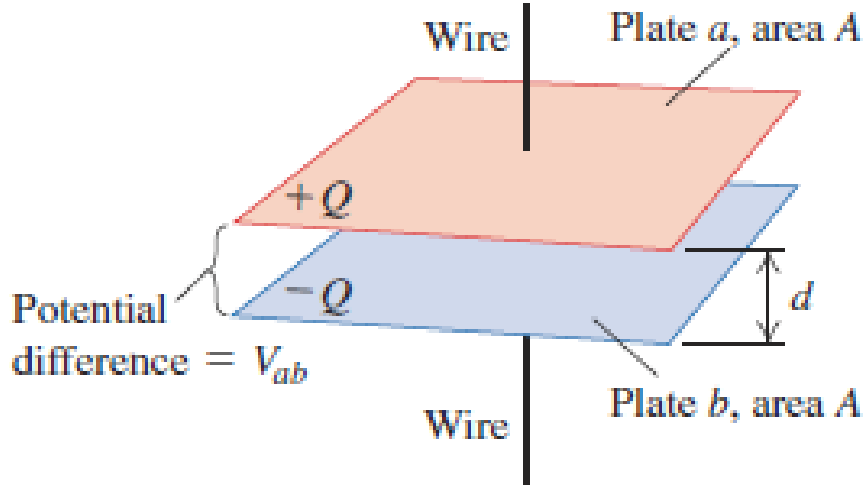
The capacitance C of a capacitor is the ratio of the magnitude of the charge Q on *either* conductor to the magnitude of the potential difference V_{ab} between the conductors:

$$C = \frac{Q}{V_{ab}}. \qquad (18.15)$$

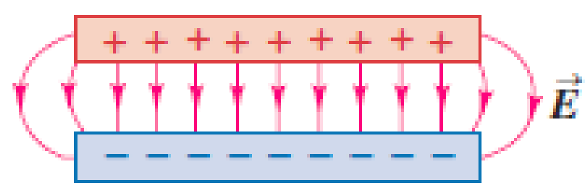
Unit: The SI unit of capacitance is called 1 **farad** (1 F), in honor of Michael Faraday. From Equation 18.15, 1 farad is equal to 1 *coulomb per volt* $(1\ \mathrm{C/V})$: $1\ \mathrm{F} = 1\ \mathrm{C/V}$.

In circuit diagrams, a capacitor is represented by either of these symbols:

Parallel plate capacitors



(a) A basic parallel-plate capacitor



(b) Electric field due to a parallel-plate capacitor

▲ **FIGURE 18.19** The elements of a parallel-plate capacitor.

We can define the surface charge density as

$$\sigma = \frac{Q}{A}$$

Where Q is the charge on the plates and A is the area of the plates

$$E = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 A} = \frac{V}{d}$$

$k = 1/4\pi\epsilon_0$, where $\epsilon_0 = 8.854 \times 10^{-12}\ C^2/N \cdot m^2$.

Capacitance of a parallel-plate capacitor

The capacitance C of a parallel-plate capacitor in vacuum is directly proportional to the area A of each plate and inversely proportional to their separation d:

$$C = \frac{Q}{V_{ab}} = \epsilon_0 \frac{A}{d}. \qquad (18.16)$$

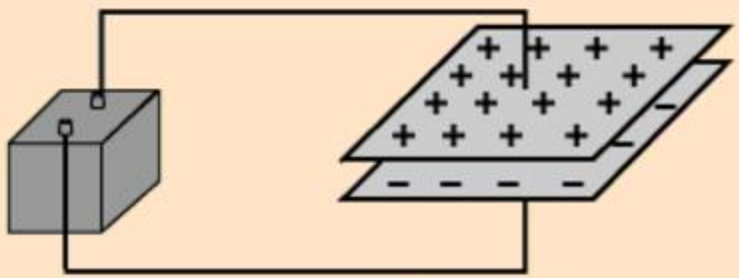
This gives a value of free space permittivity

$$\varepsilon_0 = 8.854187817 \times 10^{-12} F/m \approx 8.85 \times 10^{-12} F/m$$

which in practice is often used in the form

$$k = \frac{1}{4\pi\varepsilon_0} = 8.987552 \times 10^{9} Nm^2/C^2$$ = Coulomb's constant

Capacitors



Capacitor

A battery will transport charge from one plate to the other until the voltage produced by the charge buildup is equal to the battery voltage.

Capacitance is typified by a parallel plate arrangement and is defined in terms of charge storage:

$$C = \frac{Q}{V}$$

$$\text{Unit} = \frac{\text{coulomb}}{\text{volt}} = \text{Farad}$$

where

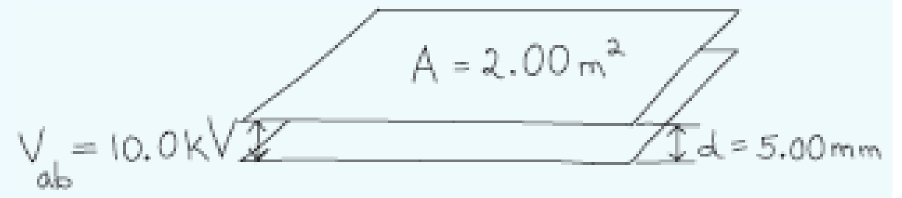
- Q = magnitude of charge stored on each plate.
- V = voltage applied to the plates.

EXAMPLE 18.7 **Properties of a parallel-plate capacitor**

The plates of a parallel-plate capacitor are 5.00 mm apart and 2.00 m^2 in area. A potential difference of 10.0 kV is applied across the capacitor. Compute (a) the capacitance, (b) the charge on each plate, and (c) the magnitude of the electric field in the region between the plates.

a

$$C = \frac{\epsilon_0 A}{d} = \frac{(8.85 \times 10^{-12}\ \text{F/m})(2.00\ \text{m}^2)}{5.00 \times 10^{-3}\ \text{m}}$$
$$= 3.54 \times 10^{-9}\ \text{F} = 0.00354\ \mu\text{F}.$$



b

$$Q = CV_{ab} = (3.54 \times 10^{-9}\ \text{F})(1.00 \times 10^{4}\ \text{V})$$
$$= 3.54 \times 10^{-5}\ \text{C} = 35.4\ \mu\text{C}.$$

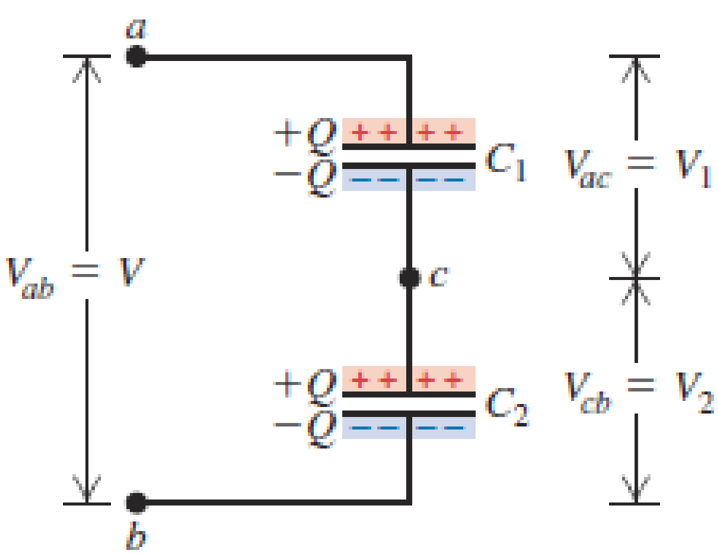
c

$$E = \frac{V_{ab}}{d} = \frac{1.00 \times 10^{4}\ \text{V}}{5.00 \times 10^{-3}\ \text{m}} = 2.00 \times 10^{6}\ \text{V/m}.$$

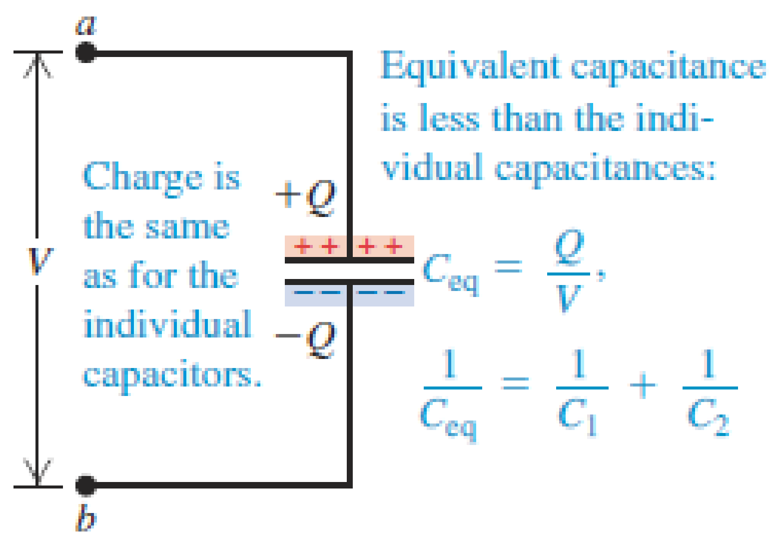
18.6 Capacitors in Series and in Parallel

Capacitors in series:

- The capacitors have the same charge Q.
- Their potential differences add: $V_{ac} + V_{cb} = V_{ab}$.



(a) Two capacitors in series



(b) The equivalent single capacitor

Equivalent capacitance of capacitors in series

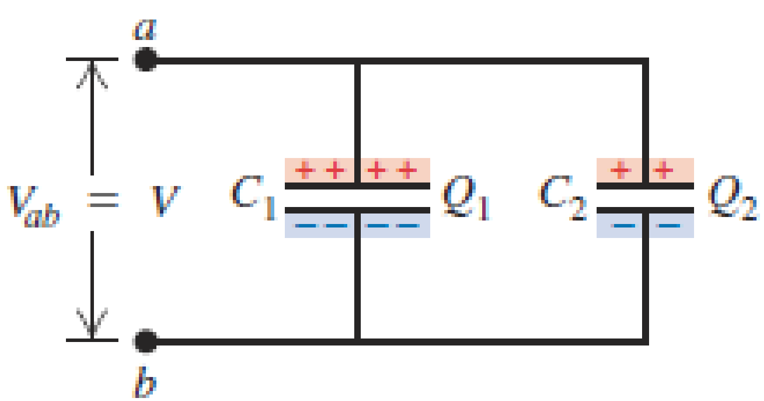
When capacitors are connected in series, the reciprocal of the equivalent capacitance of a series combination equals the sum of the reciprocals of the individual capacitances:

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots. \qquad \text{(capacitors in series)} \qquad (18.17)$$

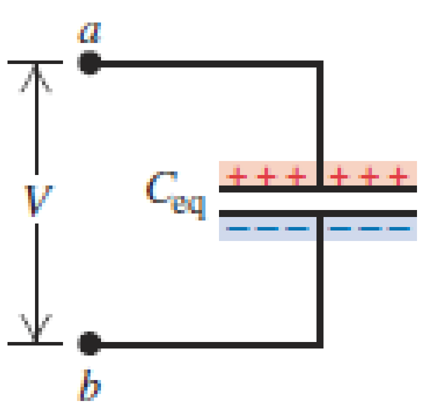
The magnitude of charge is the same on all of the plates of all of the capacitors, but the potential differences across individual capacitors are, in general, different.

Capacitors in parallel:

- The capacitors have the same potential V.
- The charge on each capacitor depends on its capacitance: $Q_1 = C_1V$, $Q_2 = C_2V$.



(a) Capacitors connected in parallel



Charge is the sum of the individual charges:
$Q = Q_1 + Q_2$

Equivalent capacitance:
$C_{eq} = C_1 + C_2$.

(b) The equivalent single capacitor

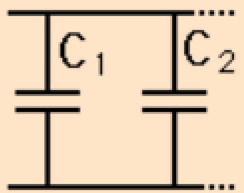
Equivalent capacitance of capacitors in parallel

When capacitors are connected in parallel, the equivalent capacitance of the combination equals the *sum* of the individual capacitances:

$$C_{eq} = C_1 + C_2 + C_3 + \cdots \qquad \text{(capacitors in parallel)} \qquad (18.18)$$

Capacitor Combinations

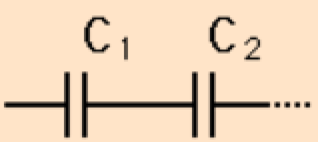
Capacitors in parallel add ...



If $C_1 =$ ____ μF, $C_2 =$ ____ μF

then $C_{eq} = C_1 + C_2 + \ldots =$ ____ μF

Capacitors in series combine as reciprocals ...



$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots$$

$C_{eq} =$ ____ μF

EXAMPLE 18.8 **Capacitors in series and in parallel**

Two capacitors, one with $C_1 = 6.0\ \mu F$ and the other with $C_2 = 3.0\ \mu F$, are connected to a potential difference of $V_{ab} = 18$ V. Find the equivalent capacitance, and find the charge and potential difference for each capacitor when the two capacitors are connected (a) in series and (b) in parallel.

SOLUTION

SET UP Figure 18.24 shows our sketches of the two situations. We remember that when capacitors are connected in series, the charges are the same on the two capacitors and the potential differences add. When they are connected in parallel, the potential differences are the same and the charges add.

SOLVE **Part (a):** The equivalent capacitance for the capacitors in series is given by Equation 18.17:

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{C_1 + C_2}{C_1 C_2}.$$

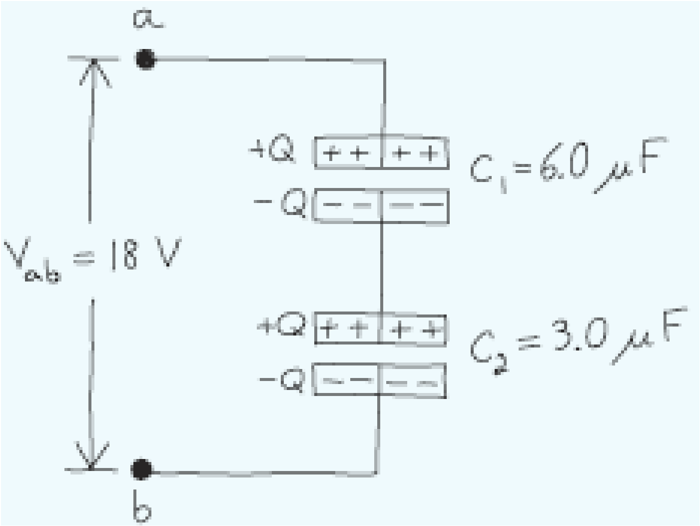
Thus,

$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2} = \frac{(6.0\ \mu F)(3.0\ \mu F)}{6.0\ \mu F + 3.0\ \mu F} = 2.0\ \mu F.$$

The charge is $Q = C_{eq}V = (2.0\ \mu F)(18\ V) = 36\ \mu C$, the same for both capacitors. The voltages are

$$V_1 = \frac{Q}{C_1} = \frac{36\ \mu C}{6.0\ \mu F} = 6.0\ V \quad \text{and} \quad V_2 = \frac{Q}{C_2} = \frac{36\ \mu C}{3.0\ \mu F} = 12.0\ V.$$

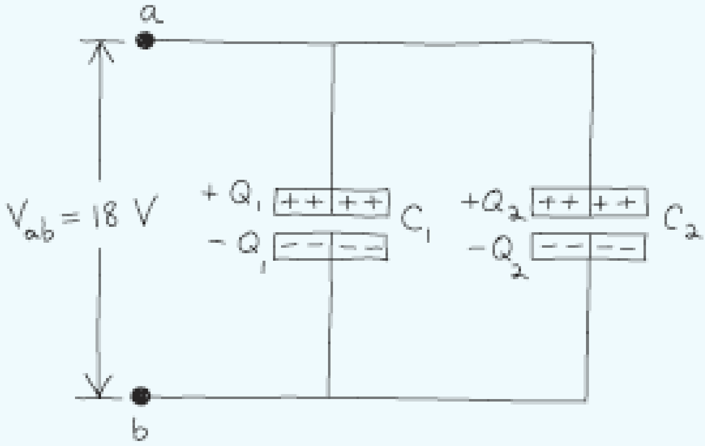
Note that $V_1 + V_2 = V_{ab}$ (i.e., 6.0 V + 12 V = 18 V).



Part (b): When capacitors are connected in parallel, the potential differences are the same and the charges add. The equivalent capacitance is given by Equation 18.18:

$$C_{\text{eq}} = C_1 + C_2 = 6.0\ \mu\text{F} + 3.0\ \mu\text{F} = 9.0\ \mu\text{F}.$$

The potential difference for the equivalent capacitor is equal to the potential difference for each capacitor:



$$V_1 = V_2 = V_{ab} = 18\ \text{V}.$$

The charges of the capacitors are

$$Q_1 = C_1 V = (6.0\ \mu\text{F})(18\ \text{V}) = 108\ \mu\text{C},$$
$$Q_2 = C_2 V = (3.0\ \mu\text{F})(18\ \text{V}) = 54\ \mu\text{C}.$$

The total charge is $Q_1 + Q_2 = Q$, so the charge on the equivalent capacitor is $Q = C_{\text{eq}} V = (9.0\ \mu\text{F})(18\ \text{V}) = 162\ \mu\text{C}$.

18.7 Electric Field Energy

Many of the most important applications of capacitors depend on their ability to store energy.
The capacitor plates, with opposite charges, separated and attracted toward each other, are analogous to a stretched spring or an object lifted in the earth’s gravitational field.
The potential energy corresponds to the energy input required to charge the capacitor and to the work done by the electrical forces when it discharges. This work is analogous to the work done by a spring or the earth’s gravity when the system returns from its displaced position to the reference position.

Energy in a capacitor

$$V = \frac{\Delta W}{\Delta q}$$

$$\Delta W = V\,\Delta q = \frac{q}{C}\,\Delta q$$

$$U = W_{total} = \left(\frac{V}{2}\right) Q = \left(\frac{V}{2}\right) CV = \frac{1}{2} CV^2$$

Where V/2 is the average potential difference during the charging process.

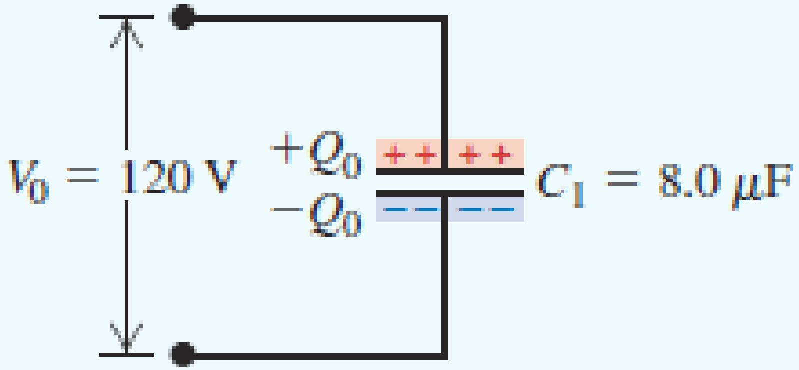
Energy density in an Electric field

Since $C = \frac{\epsilon_o A}{d}$ and $V = Ed$

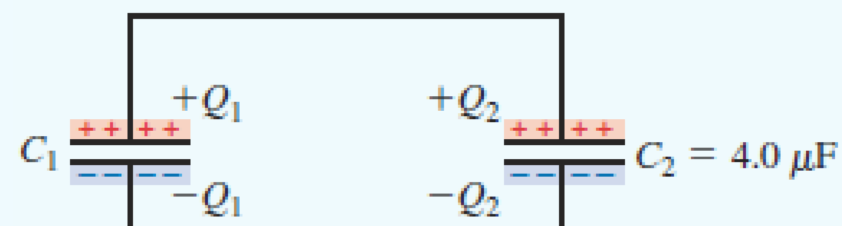
$$u = energy\ density = \frac{Energy}{Volume} = \frac{\frac{1}{2}CV^2}{Ad} = \frac{1}{2}\epsilon_o E^2$$

EXAMPLE 18.9 **Stored energy**

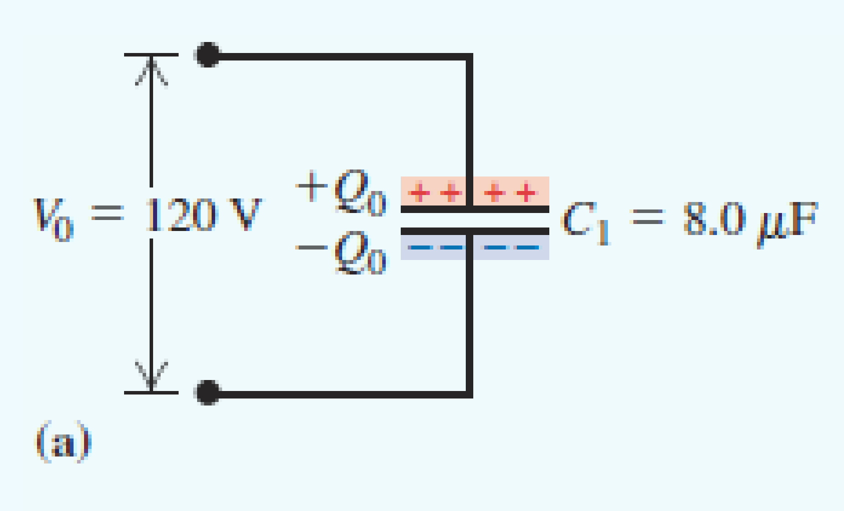
A capacitor with $C_1 = 8.0\ \mu F$ is connected to a potential difference $V_0 = 120$ V, as shown in Figure 18.25a. (a) Find the magnitude of charge Q_0 and the total energy stored after the capacitor has become fully charged. (b) Without any charge being lost from the plates, the capacitor is disconnected from the source of potential difference and connected to a second capacitor $C_2 = 4.0\ \mu F$ that is initially uncharged (Figure 18.25b). After the charge has finished redistributing between the two capacitors, find the charge and potential difference for each capacitor, and find the total stored energy.



(a)

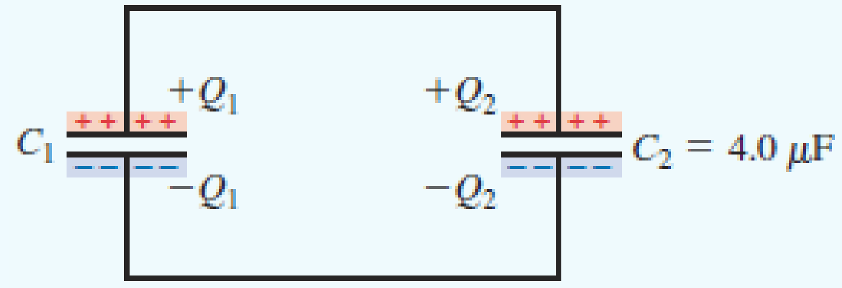


(b)



SOLVE **Part (a):** For the original capacitor, we use the potential difference and the capacitance to find the charge: $Q_0 = C_1V_0 = (8.0\ \mu\text{F})(120\ \text{V}) = 960\ \mu\text{C}$. To find the stored energy, we use Equation 18.19:

$$U = \frac{1}{2}Q_0V_0 = \frac{1}{2}(960 \times 10^{-6}\ \text{C})(120\ \text{V}) = 0.058\ \text{J}.$$



(b)

Part (b): From conservation of charge, $Q_1 + Q_2 = Q_0$. Since V is the same for both capacitors, $Q_1 = C_1V$ and $Q_2 = C_2V$. When we substitute these equations into the conservation-of-charge equation, we find that $C_1V + C_2V = Q_0$ and

$$V = \frac{Q_0}{C_1 + C_2} = \frac{960\ \mu C}{12\ \mu F} = 80\ V.$$

Then $Q_1 = C_1V = 640\ \mu C$ and $Q_2 = C_2V = 320\ \mu C$.

The final total stored energy is the sum of the energies stored by each capacitor:

$$\frac{1}{2}Q_1V + \frac{1}{2}Q_2V = \frac{1}{2}(Q_1 + Q_2)V = \frac{1}{2}Q_0V$$
$$= \frac{1}{2}(960 \times 10^{-6}\ C)(80\ V) = 0.038\ J.$$

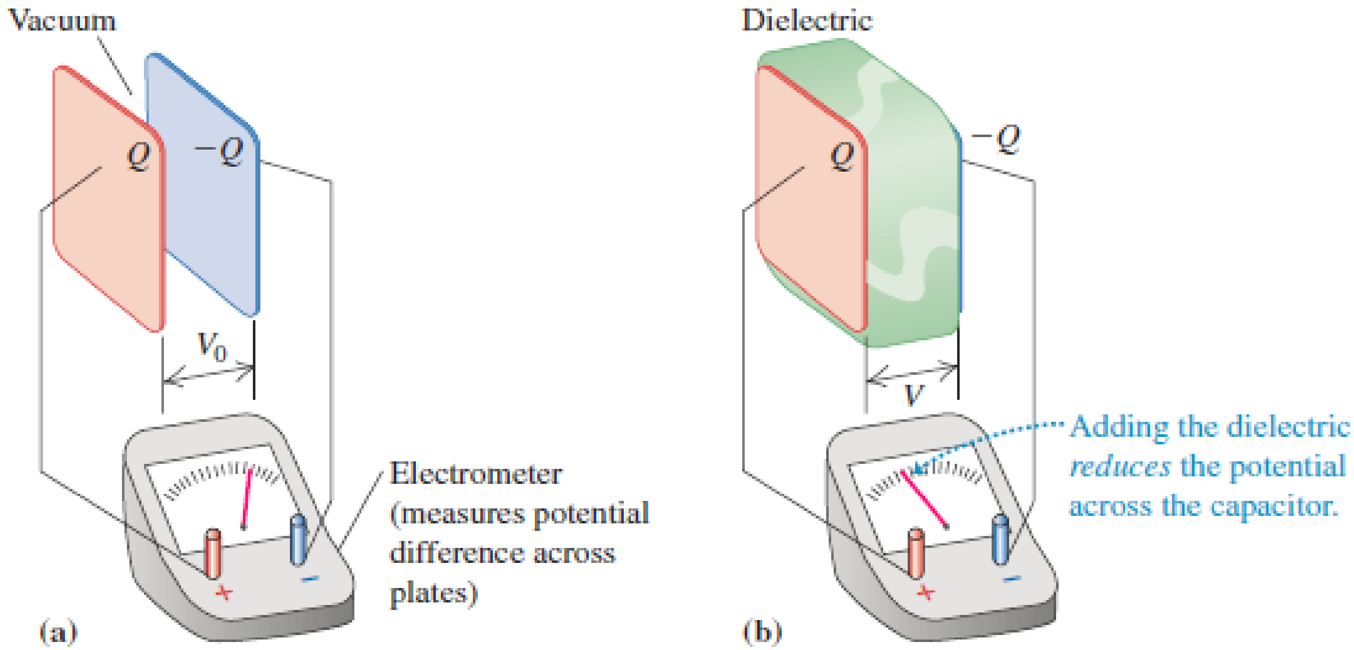
18.8 Dielectrics

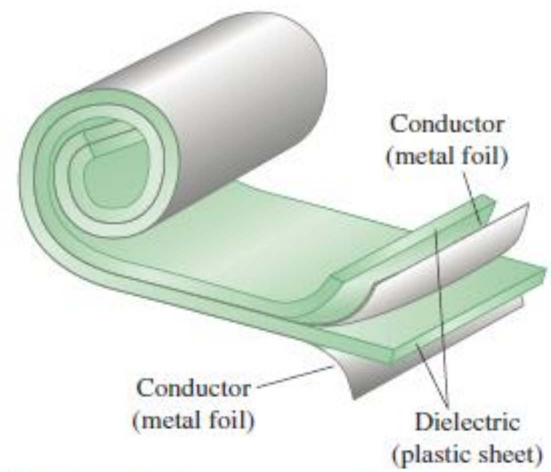
Placing a solid dielectric between the plates of a capacitor serves three functions.

First, it solves the mechanical problem of maintaining two large metal sheets at a very small separation without actual contact.

Second, many insulating materials can tolerate stronger electric fields without breakdown than can air.

Third, the capacitance of a capacitor of given dimensions is *greater* when there is a dielectric material between the plates than when there is air or vacuum.





▲ **FIGURE 18.26** A common type of parallel-plate capacitor is made from a rolled-up sandwich of metal foil and plastic film.

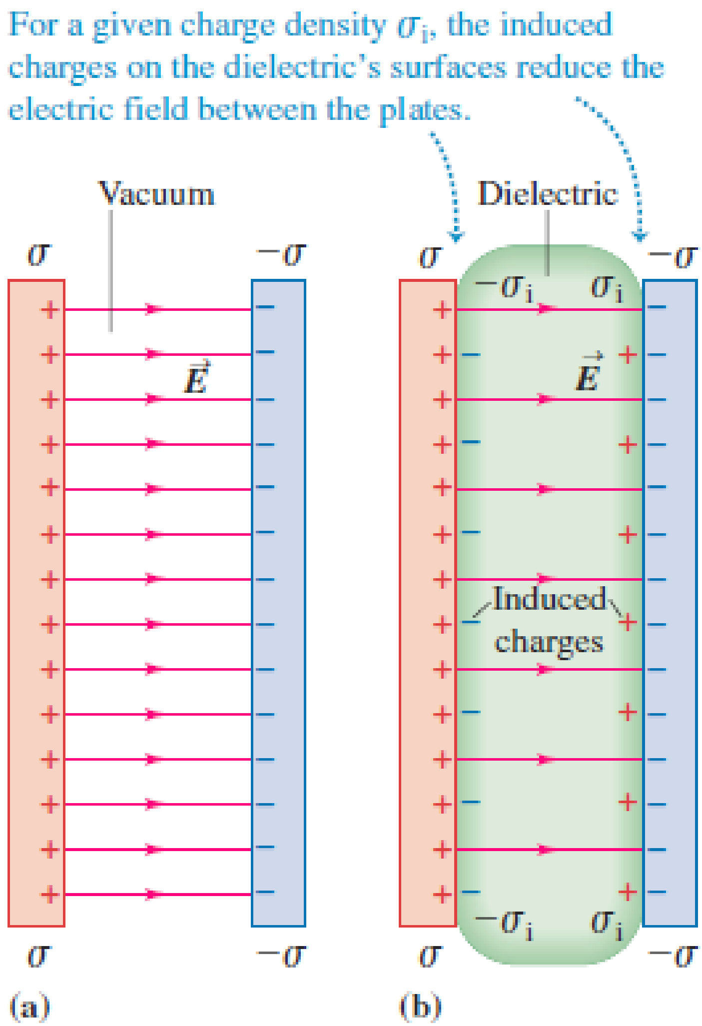
Dielectric constant of the material, *K*

$$K = \frac{C}{C_o}$$

TABLE 18.1 Values of dielectric constant *K* at 20°C

Material	K	Material	K
Vacuum	1	Polyvinyl chloride	3.18
Air (1 atm)	1.00059	Plexiglas®	3.40
Air (100 atm)	1.0548	Glass	5–10
Teflon®	2.1	Neoprene	6.70
Polyethylene	2.25	Germanium	16
Benzene	2.28	Glycerin	42.5
Mica	3.–6	Water	80.4
Mylar®	3.1	Strontium titanate	310

Where C_0 is the capacitance in a vacuum



▲ **FIGURE 18.28** The effect of a dielectric on the electric field between the plates of a capacitor.

$$K = \frac{C}{C_o}$$

$$C = K\,C_o$$

$$V = \frac{V_o}{K}$$

$$E = \frac{E_o}{K}$$

SOLVE **Part (a):** The presence of the dielectric increases the capacitance. Without the dielectric, the capacitance is

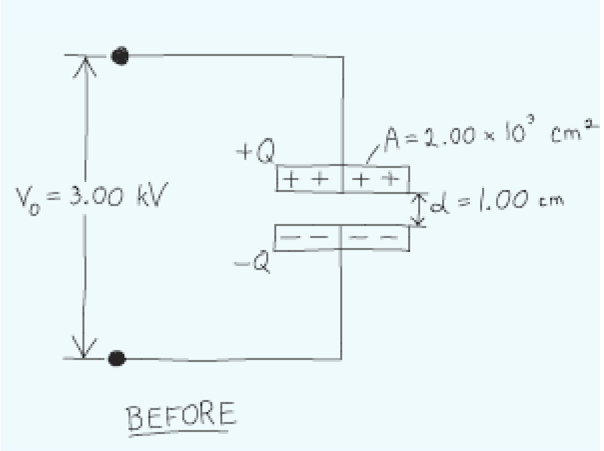
$$C_0 = \frac{\epsilon_0 A}{d} = \frac{(8.85 \times 10^{-12}\ \mathrm{F/m})(0.200\ \mathrm{m^2})}{0.010\ \mathrm{m}}$$
$$= 1.77 \times 10^{-10}\ \mathrm{F} = 177\ \mathrm{pF}.$$

The original charge on the capacitor is

$$Q = C_0 V_0 = (1.77 \times 10^{-10}\ \mathrm{F})(3.00 \times 10^3\ \mathrm{V})$$
$$= 5.31 \times 10^{-7}\ \mathrm{C} = 0.531\ \mu\mathrm{C}.$$

After the dielectric is inserted, the charge is still $Q = 0.531\ \mu\mathrm{C}$, but now $V = 1.00\ \mathrm{kV}$, so

$$C = \frac{Q}{V} = \frac{0.531 \times 10^{-6}\ \mathrm{C}}{1.00 \times 10^3\ \mathrm{V}} = 5.31 \times 10^{-10}\ \mathrm{F} = 531\ \mathrm{pF}.$$

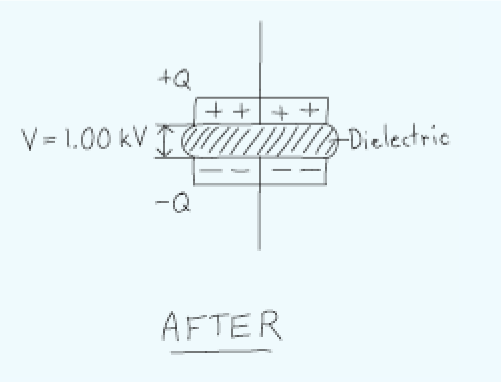


Part (b): By definition, the dielectric constant is

$$K = C/C_0 = (531\ \mathrm{pF})/(177\ \mathrm{pF}) = 3.00.$$

Note that this is also

$$K = V_0/V = (3.00\ \mathrm{kV})/(1.00\ \mathrm{kV}) = 3.00.$$

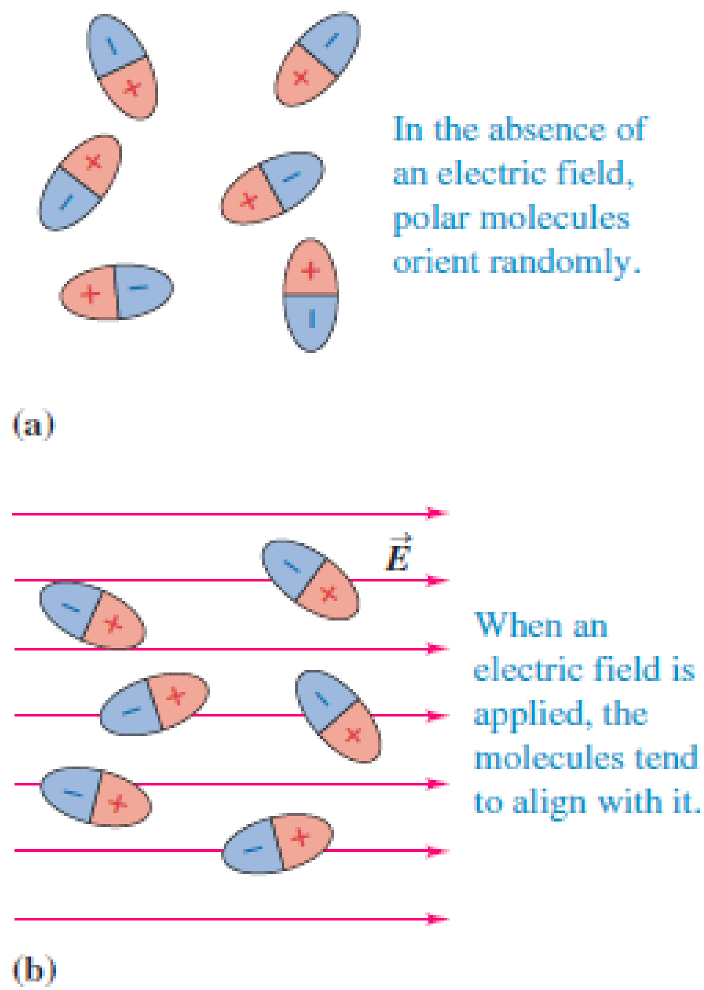


Dielectric Breakdown

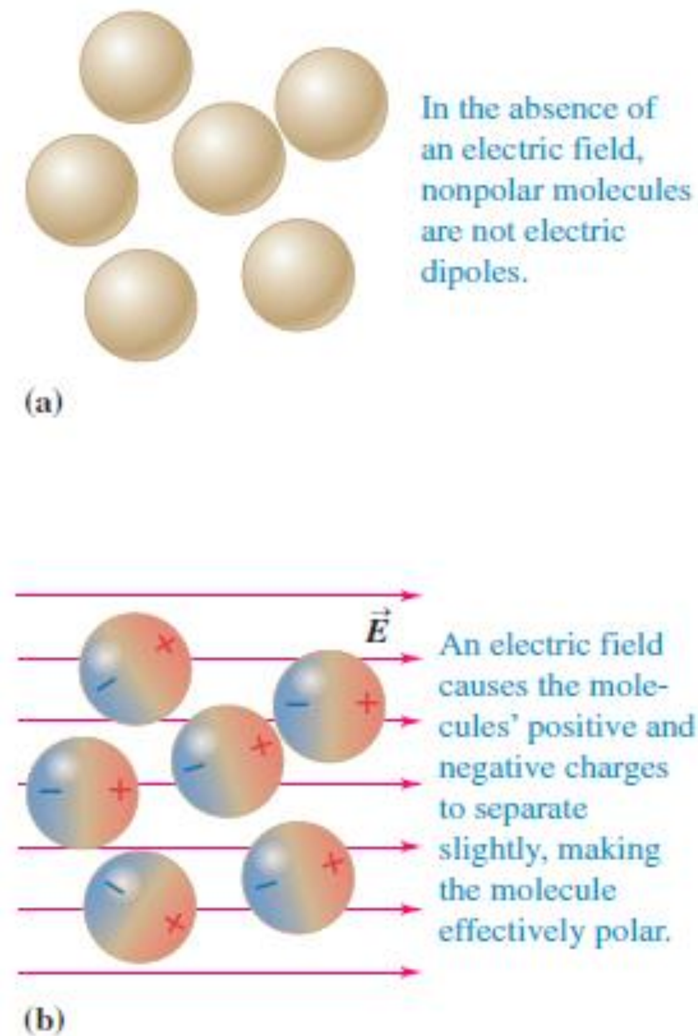
The maximum electric field a material can withstand without the occurrence of breakdown is called its dielectric strength.

▲ **FIGURE 18.30** Dielectric breakdown in the laboratory and in nature. The left-hand photo shows a block of Plexiglas® subjected to a very strong electric field; the pattern was etched by flowing charge.

18.9 Molecular Model of Induced Charge

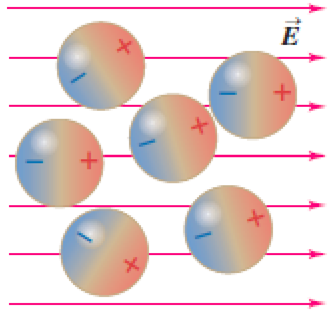


▲ **FIGURE 18.31** The effect of an electric field on a group of polar molecules.

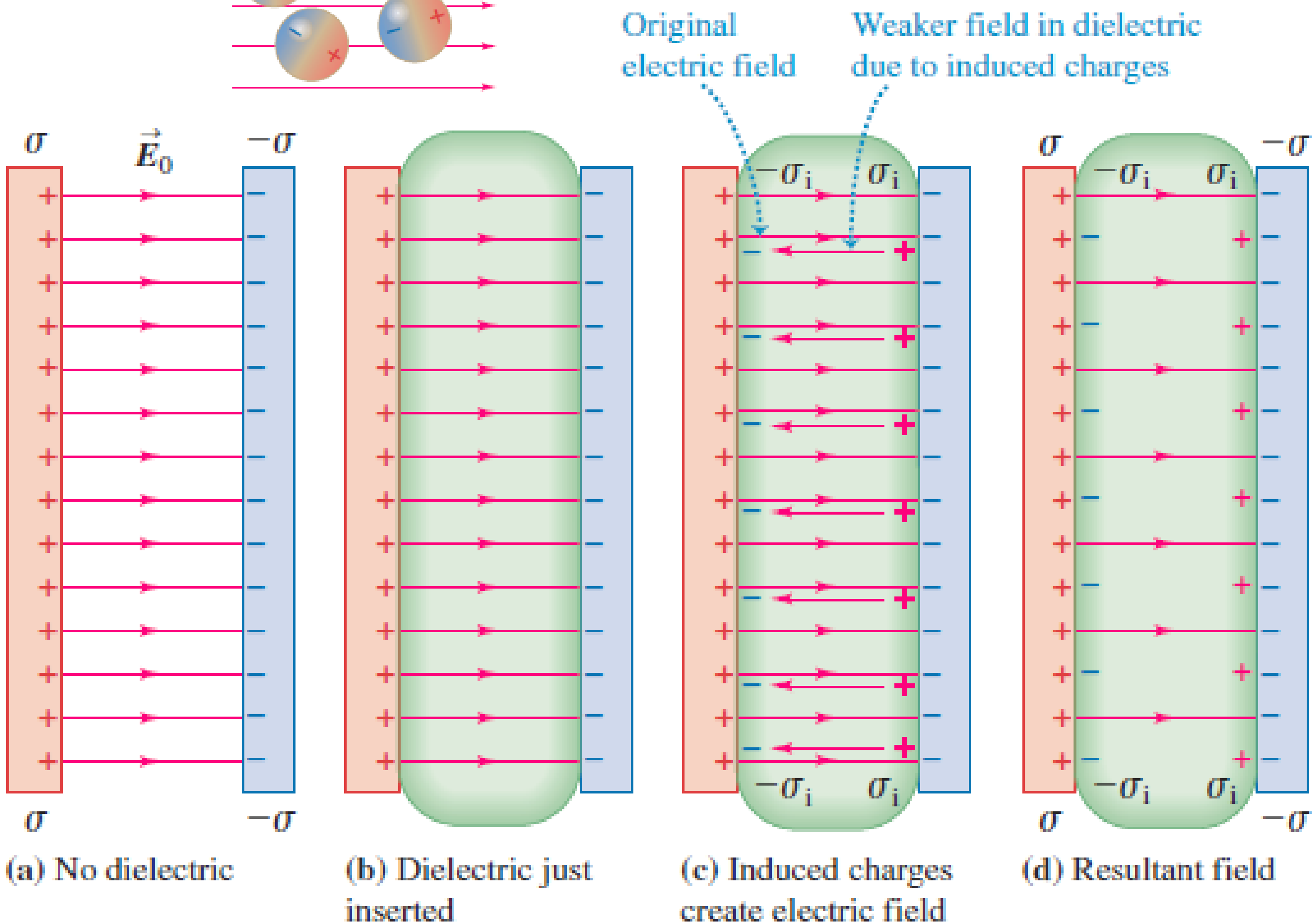


▲ **FIGURE 18.32** The effect of an electric field on a group of nonpolar molecules.

Induced negative



Induced positive



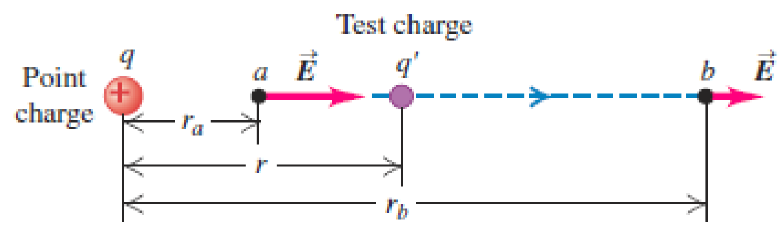
▲ **FIGURE 18.33** How a dielectric reduces the electric field between capacitor plates.

SUMMARY

Electric Potential Energy

(Section 18.1) The work W done by the electric-field force on a charged particle moving in a field can be represented in terms of potential energy U: $W_{a \to b} = U_a - U_b$ (Equation 18.2). For a charge q' that undergoes a displacement $\vec{s}$ parallel to a uniform electric field, the change in potential energy is $U_a - U_b = q'Es$ (Equation 18.5). The potential energy for a point charge q' moving in the field produced by a point charge q at a distance r from q' is

$$U = k\frac{qq'}{r}. \qquad (18.8)$$



Potential

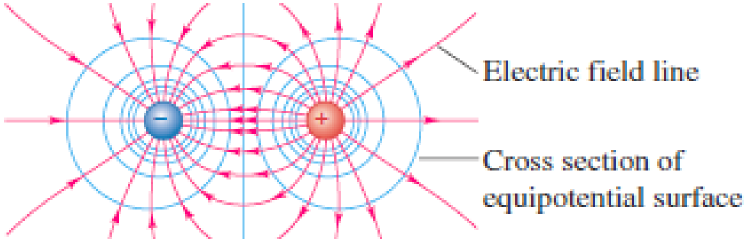
(Section 18.2) Potential, a scalar quantity denoted by V, is potential energy per unit charge. The potential at any point due to a point charge is

$$V = \frac{U}{q'} = k\frac{q}{r}. \qquad (18.12)$$

A positive test charge tends to “fall” from a high-potential region to a low-potential region.

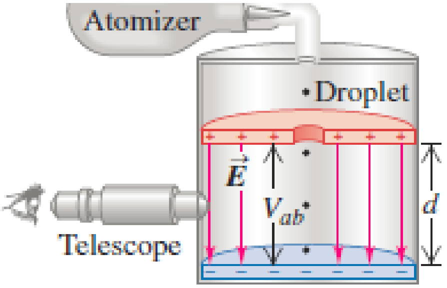
Equipotential Surfaces

(Section 18.3) An **equipotential surface** is a surface on which the potential has the same value at every point. At a point where a field line crosses an equipotential surface, the two are perpendicular. When all charges are at rest, the surface of a conductor is always an equipotential surface, and all points in the interior of a conductor are at the same potential.



The Millikan Oil-Drop Experiment

(Section 18.4) The Millikan oil-drop experiment determined the electric charge of individual electrons by measuring the motion of electrically charged oil drops in an electric field. The size of a drop is determined by measuring its terminal speed of fall under gravity and the drag force of air.



Capacitors

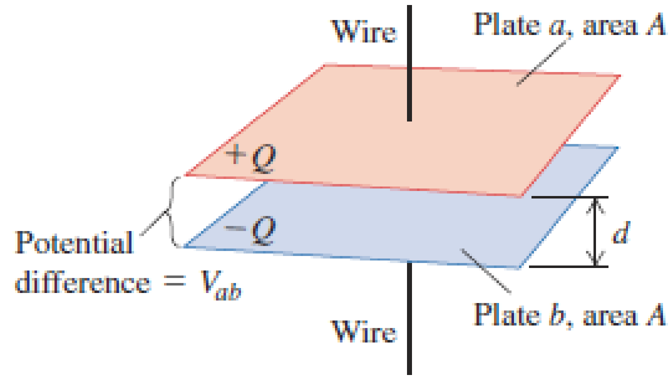
(Sections 18.5 and 18.6) A **capacitor** consists of any pair of conductors separated by vacuum or an insulating material. The **capacitance** C is defined as $C = Q/V_{ab}$ (Equation 18.14). A **parallel-plate capacitor** is made with two parallel plates, each with area A, separated by a distance d. If they are separated by vacuum, the capacitance is $C = \epsilon_0(A/d)$ (Equation 18.16).

When capacitors with capacitances C_1, C_2, C_3, . . . are connected in series, the equivalent capacitance C_{eq} is given by

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots. \qquad (18.17)$$

When they are connected in parallel, the equivalent capacitance is

$$C_{eq} = C_1 + C_2 + C_3 + \cdots. \qquad (18.18)$$

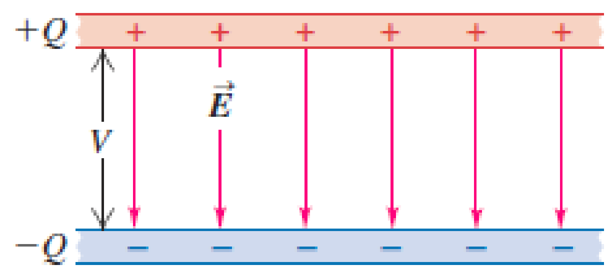


Electric Field Energy

(Section 18.7) The energy U required to charge a capacitor C to a potential difference V and a charge Q is equal to the energy stored in the capacitor and is given by

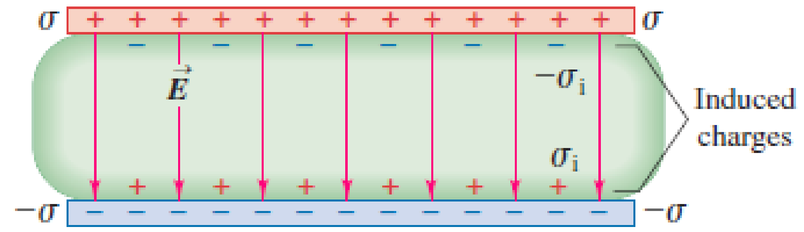
$$U = W_{total} = \left(\frac{V}{2}\right)Q = \frac{Q^2}{2C} = \frac{1}{2}CV^2. \qquad (18.19)$$

This energy can be thought of as residing in the electric field between the conductors; the energy density u (energy per unit volume) is $u = \frac{1}{2}\epsilon_0 E^2$ (Equation 18.20).



Dielectrics

(Section 18.8) When the space between the conductors is filled with a dielectric material, the capacitance *increases* by a factor K called the dielectric constant of the material. When the charges $\pm Q$ on the plates remain constant, charges induced on the surface of the dielectric *decrease* the electric field and potential difference between conductors by the same factor K. Under sufficiently strong fields, dielectrics become conductors, a phenomenon called dielectric breakdown. The maximum field that a material can withstand without breakdown is called its dielectric strength.



Molecular Model of Induced Charge

(Section 18.9) A *polar molecule* has equal amounts of positive and negative charge, but a lopsided distribution, with excess positive charge concentrated on one side of the molecule and negative charge on the other. When placed in an electric field, polar molecules tend to partially align with the field. For a material containing polar molecules, this microscopic alignment appears as an induced surface charge density. Even a molecule that is not ordinarily polar attains a lopsided charge distribution when it is placed in an electric field: The field pushes the positive charges in the molecule in the direction of the field and pushes the negative charges in the opposite direction.