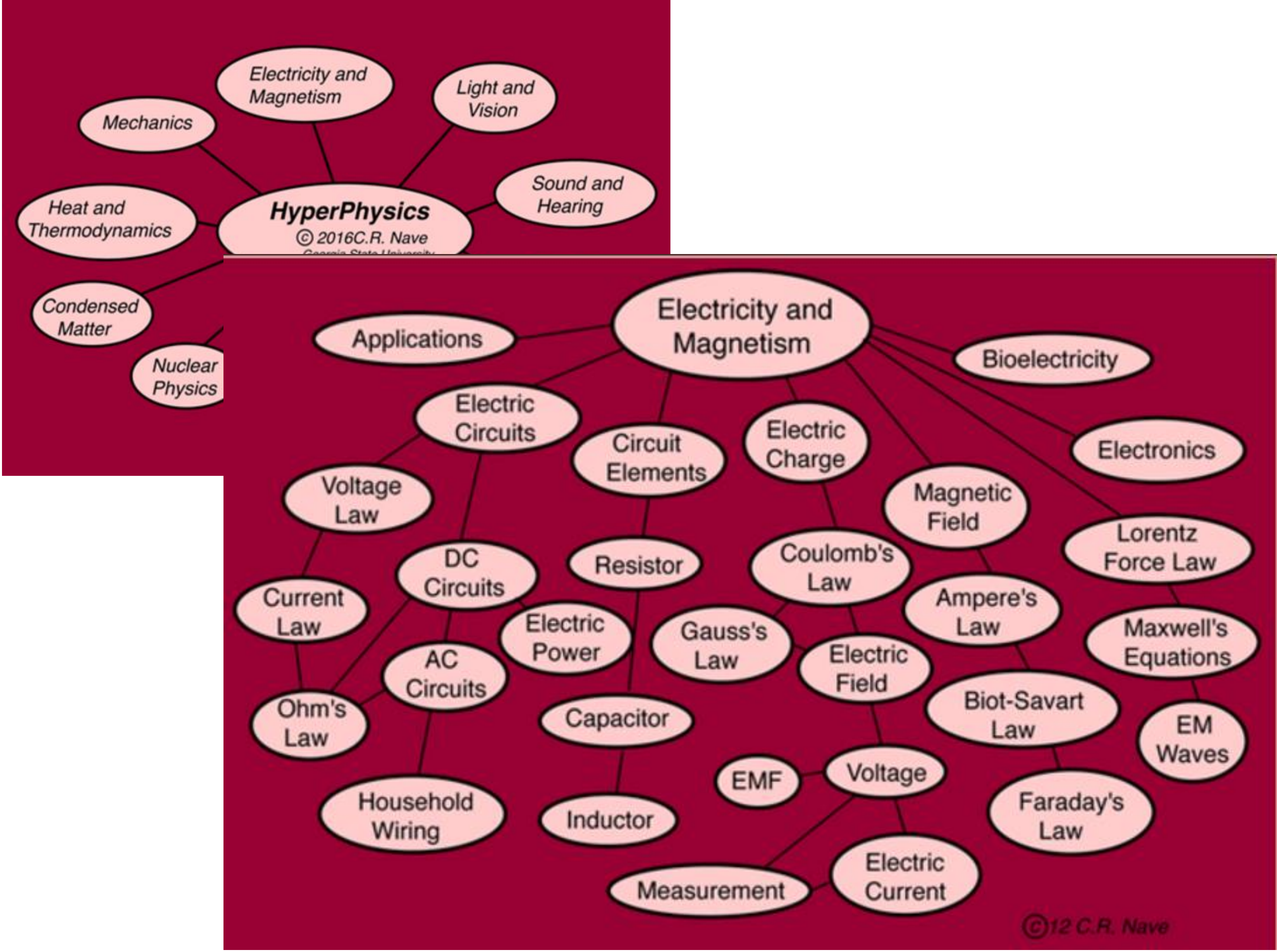
Week	Topic	Content	Text reference [CLO][PLO]
1	Electric Charge	• Electric charge (Section 17.1) • Conductor and insulators (Section 17.2)	College Physics: Chapter 17
2	Electric Charge	• Conservation and quantization of charge (Section 17.3) • Coulomb's Law (Section 17.4)	College Physics: Chapter 17
3 & 4	Electric Field	• Electric Fields and Electric Forces (Section 17.5) • Calculating Electric Fields (Section 17.6) • Electric Field Lines (Section 17.7) • Gauss's Law and Field Calculations (Section 17.8)	College Physics: Chapter 17
5 & 6	Electric Potential	• Electric Potential Energy (Section 18.1) • Potential (Section 18.2) • Equipotential Surfaces (Section 18.3)	College Physics: Chapter 18
7	Capacitance	• Vacuum & Dielectric Capacitors (Sections 18.5 and 18.8) • Capacitors in series and in parallel (Section 18.6)	College Physics: Chapter 18
	MIDTERM		
8	Current, Resistance, and Dielectric Current Circuits	• Current, Resistance & Ohm's Law (sections 19.1 & 2) • Electromotive force and Circuits (section 19.3) • Energy and Power in Electric Circuits (section 19.4)	College Physics: Chapter 19
9		• Resistors in Series and in Parallel (section 19.5) • Kirchhoff's Rules (section 19.6)	
10 & 11	Magnetic Field and Magnetic Forces	• Magnetism (Section 20.1) • Magnetic Field and Magnetic Force (Section 20.2) • Motion of Charged Particles in a Magnetic Field (Section 20.3) • Magnetic force on a current-Carrying Conductor (section 20.5) • Force & Torque on a Current Loop; Direct-Current Motors (section 20.6)	College Physics: Chapter 20
12		• Magnetic Field of a Long, Straight Conductor & forces between Parallel Conductors (sections 20.7-8) • Solenoid Magnetic Field (section 20.9) • Biot-Savart and Ampere's laws (section 20.10)	
13 & 14	Electromagnetic Induction	• Electromagnetic Induction & Faraday's Law (sections 21.1-3) • Lenz's Law (section 21.4) • Motional Electromotive Force (section 21.5)	College Physics: Chapter 21
15	Electromagnetic Induction	• Mutual Inductance and Self-Inductance (section 21.7-8) • Transformers (section 21.9)	College Physics: Chapter 21
16		EXAMS	


Mechanics
Electricity and Magnetism
Light and Vision
Sound and Hearing
Heat and Thermodynamics
HyperPhysics
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Condensed Matter
Nuclear Physics
Applications
Electricity and Magnetism
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Current Law
DC Circuits
Resistor
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Ampere's Law
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Electric Power
Gauss's Law
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AC Circuits
Ohm's Law
Biot-Savart Law
EM Waves
Capacitor
EMF
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Faraday's Law
Household Wiring
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17 Electric Charge and Electric Field

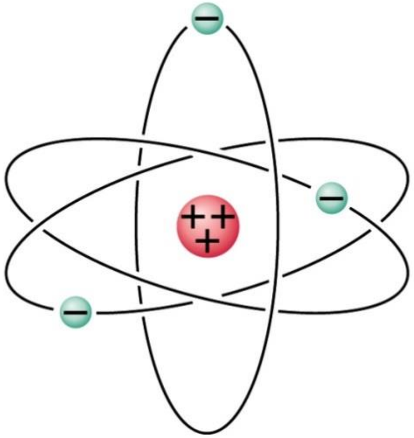
- Materials/substances may be classified according to their capacity to carry or ***conduct*** electric charge:

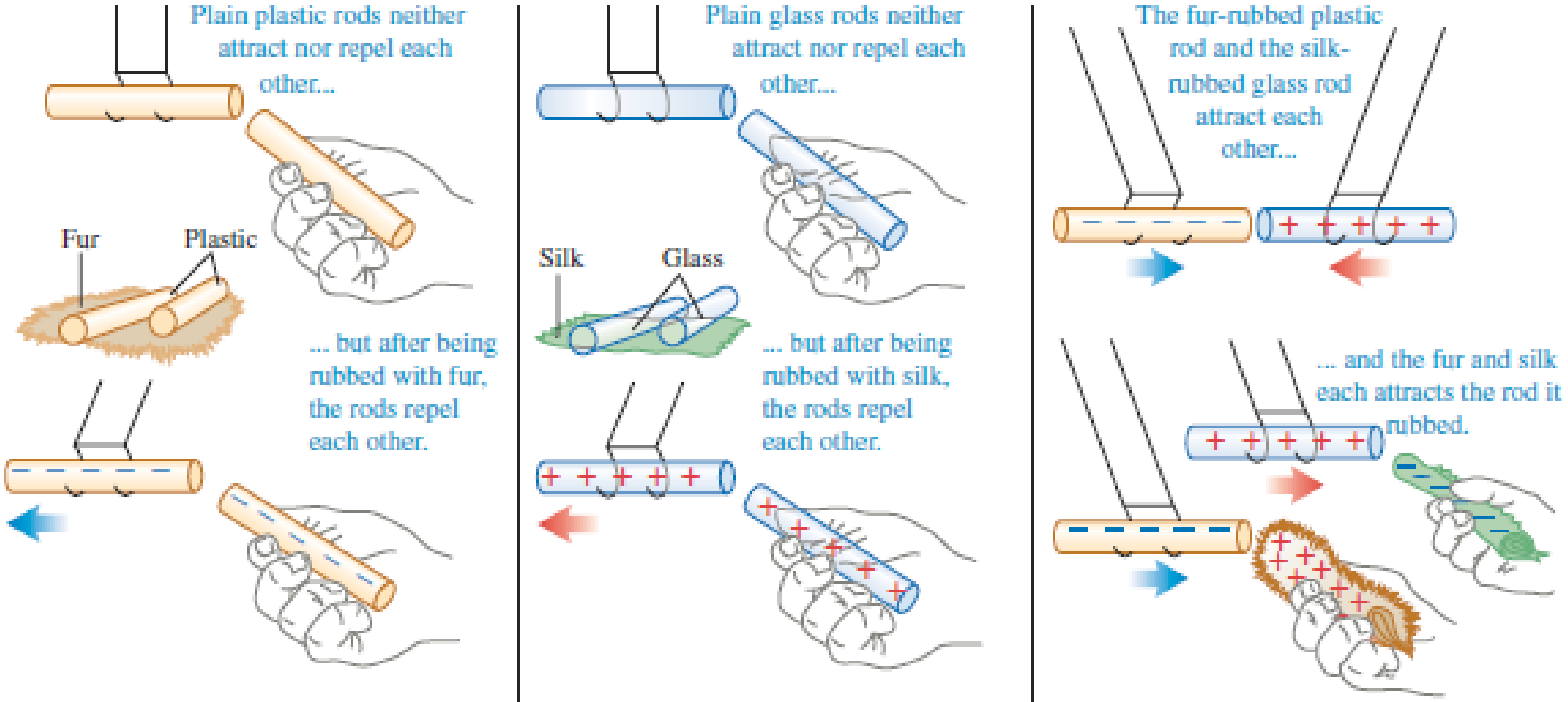
- Conductors are material in which electric charges move freely.
  - Metals are good conductors: Copper, aluminum, and silver.

- Insulator are materials in which electrical charge do not move freely.
  - Most nonmetals are insulator: Glass, Rubber are good insulators.

- Semiconductors are a third class of materials with electrical properties somewhere between those of insulators and conductors.
  - Silicon and germanium are semiconductors used widely in the fabrication of electronic devices.

Objects that exert electric forces are said to have charge. Charge is the source of electrical force. There are two kinds of electrical charges, positive and negative. Same charges (+ and +, or - and -) repel and opposite charges (+ and -) attract each other.





(a) Interaction between plastic rods rubbed on fur

(b) Interaction between glass rods rubbed on silk

(c) Interaction between objects with opposite charges

▲ **FIGURE 17.1** Experiments illustrating the nature of electric charge.

Like and unlike charges

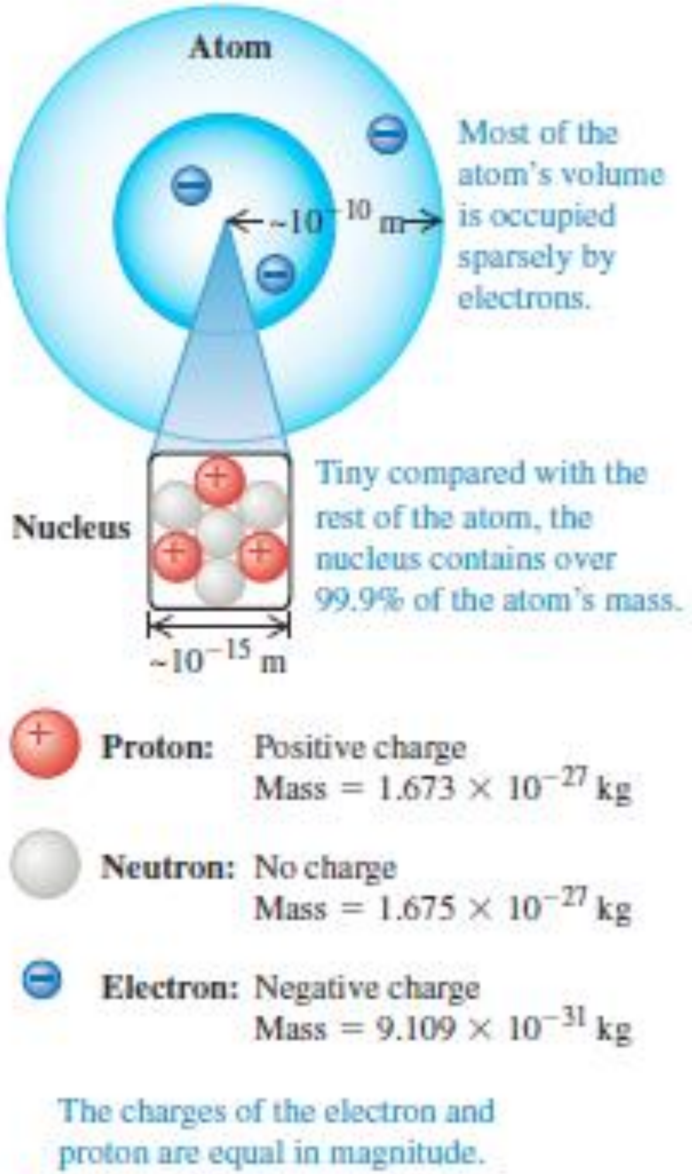
Two positive charges or two negative charges repel each other; a positive and a negative charge attract each other.

In the preceding discussion, the plastic rod and the silk have negative charge; the glass rod and the fur have positive charge.

The person in this snapshot was amused to find her hair standing on end.
Luckily, she and her companion left before the area was hit by lightning.

Just before lightning strikes, strong charges build up in the ground and in the clouds overhead. If you’re standing on charged ground, the charge will spread onto your body.

Because like charges repel, all your hairs tend to get as far from each other as they can.



Most of the atom's volume is occupied sparsely by electrons.

Tiny compared with the rest of the atom, the nucleus contains over 99.9% of the atom's mass.

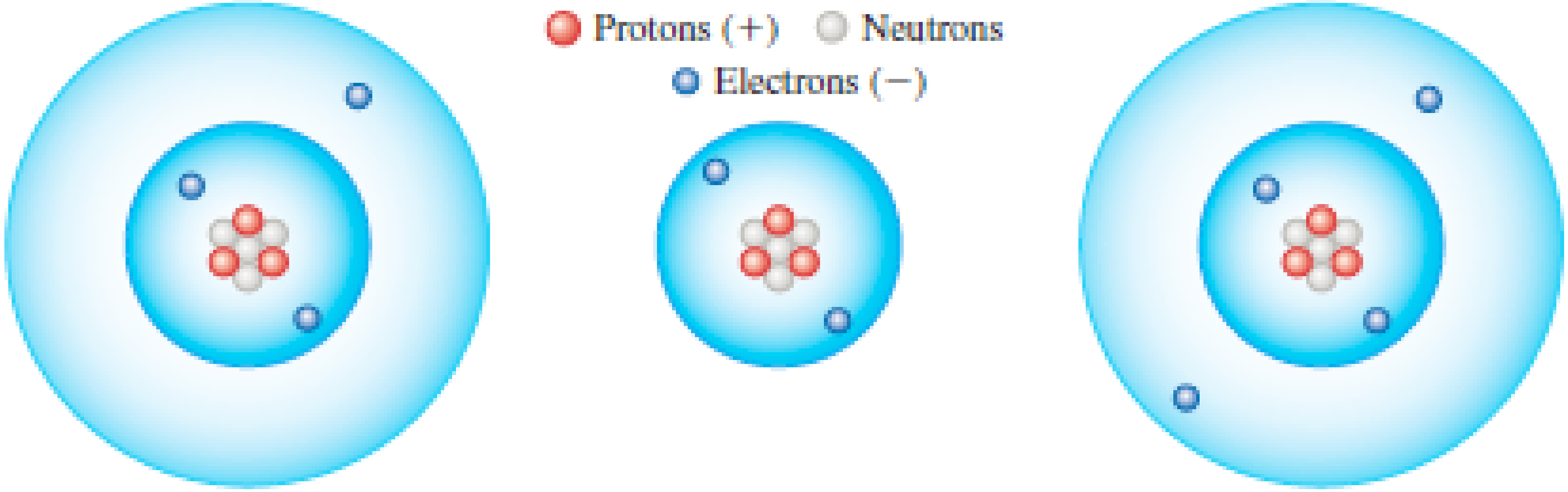
Proton: Positive charge
Mass = 1.673×10^{-27} kg

Neutron: No charge
Mass = 1.675×10^{-27} kg

Electron: Negative charge
Mass = 9.109×10^{-31} kg

The charges of the electron and proton are equal in magnitude.

▲ **FIGURE 17.2** Schematic depiction of the structure and components of an atom.



(a) Neutral lithium atom (Li):

3 protons (3+)
4 neutrons
3 electrons (3−)

Electrons equal protons:
Zero net charge

(b) Positive lithium ion (Li^+):

3 protons (3+)
4 neutrons
2 electrons (2−)

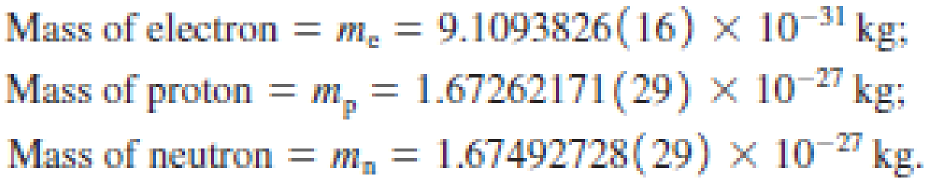
Fewer electrons than protons:
Positive net charge

(c) Negative lithium ion (Li^-):

3 protons (3+)
4 neutrons
4 electrons (4−)

More electrons than protons:
Negative net charge

▲ **FIGURE 17.3** The neutral lithium (Li) atom and positive and negative lithium ions.

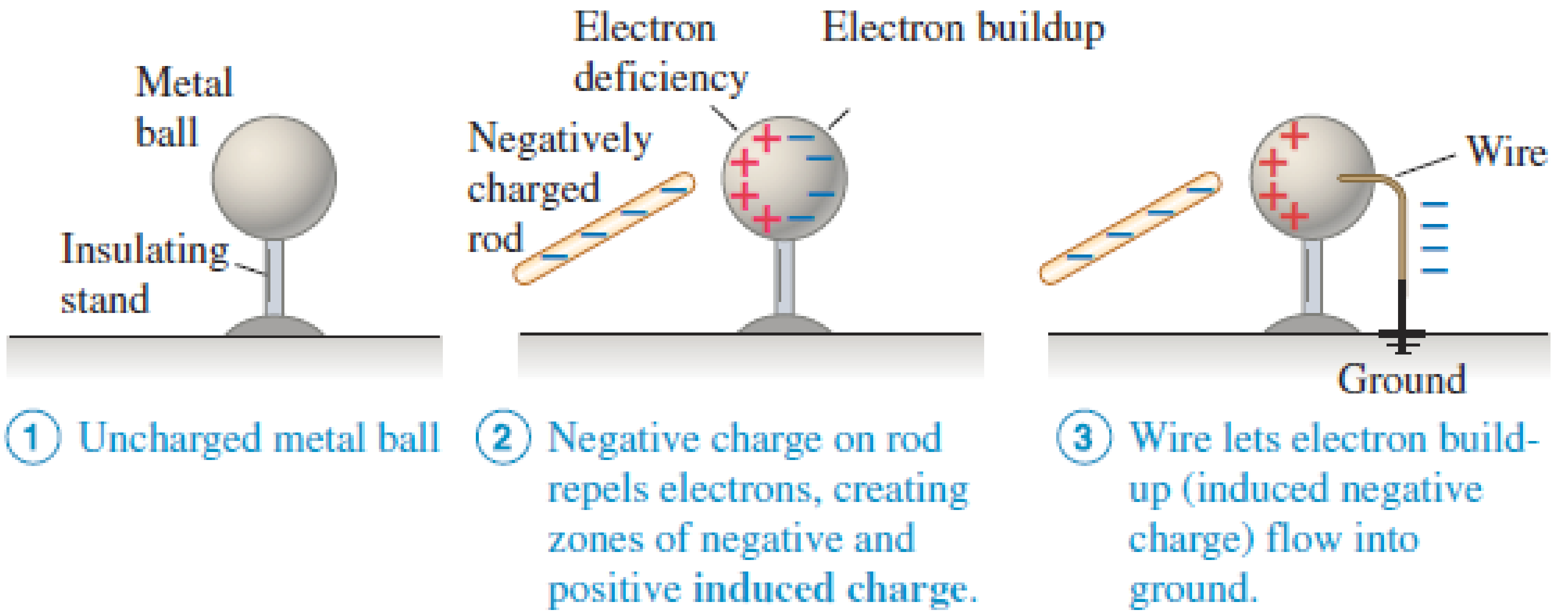


$$\text{Mass of electron} = m_e = 9.1093826(16) \times 10^{-31}\ \text{kg};$$
$$\text{Mass of proton} = m_p = 1.67262171(29) \times 10^{-27}\ \text{kg};$$
$$\text{Mass of neutron} = m_n = 1.67492728(29) \times 10^{-27}\ \text{kg}.$$

An **ion** is an atom that has lost or gained one or more electrons.

Ordinarily, when an ion is formed, the structure of the nucleus is unchanged. In a solid object such as a carpet or a copper wire, the nuclei of the atoms are not free to move about, so a net charge is due to an excess or deficit of electrons.

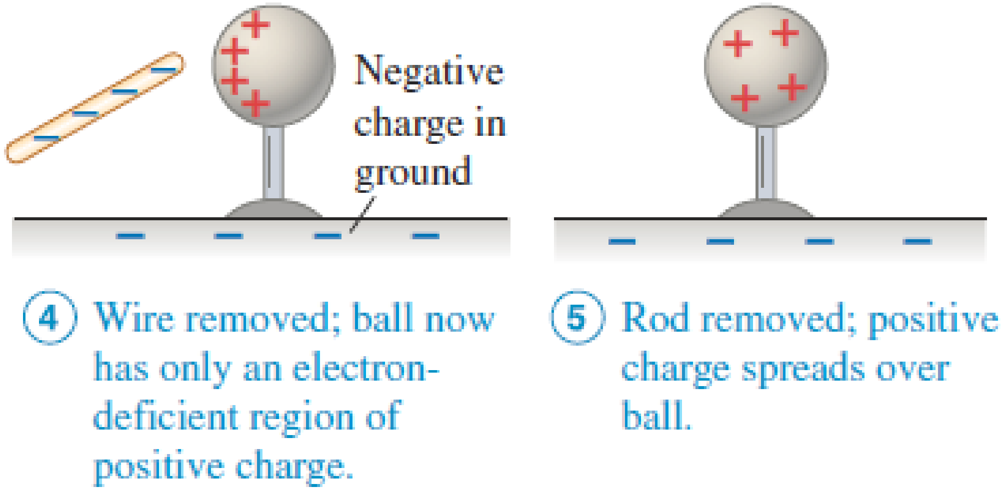
Induction



① Uncharged metal ball

② Negative charge on rod repels electrons, creating zones of negative and positive induced charge.

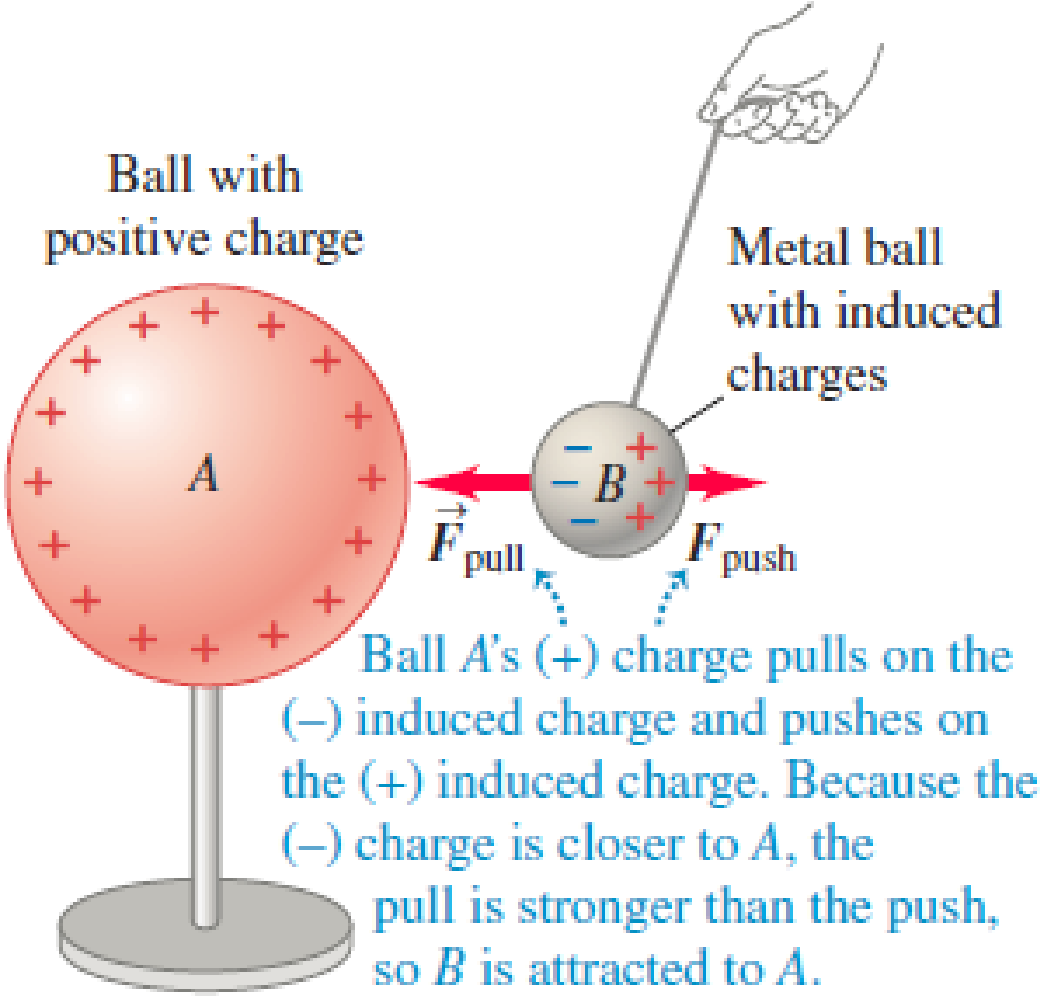
③ Wire lets electron buildup (induced negative charge) flow into ground.



④ Wire removed; ball now has only an electron-deficient region of positive charge.

⑤ Rod removed; positive charge spreads over ball.

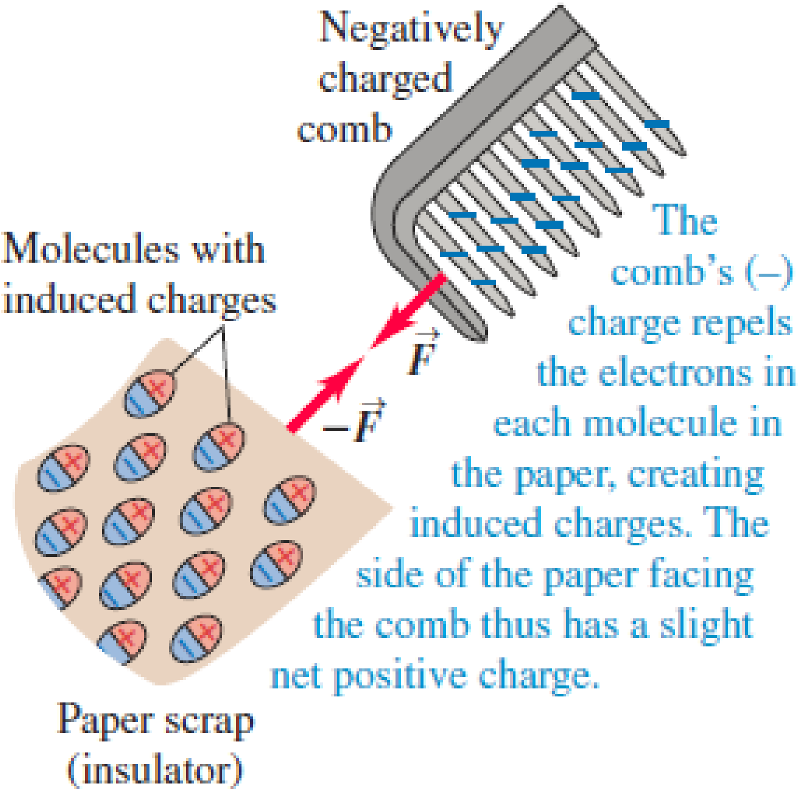
Induced Polarisation



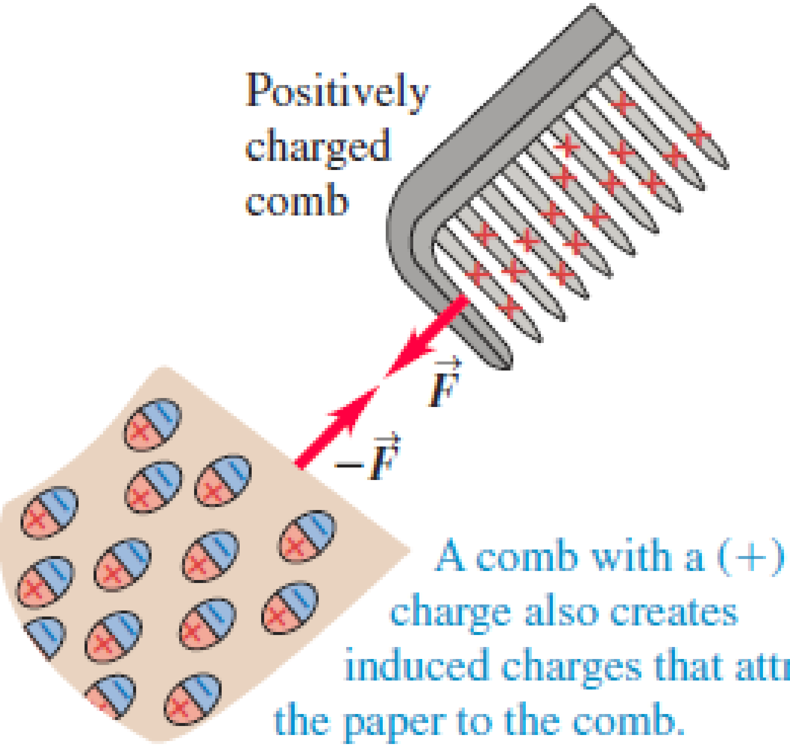
Ball *A*'s (+) charge pulls on the (–) induced charge and pushes on the (+) induced charge. Because the (–) charge is closer to *A*, the pull is stronger than the push, so *B* is attracted to *A*.

▲ **FIGURE 17.7** The charge on ball *A* induces charges in ball *B*, resulting in a net attractive force between the balls.

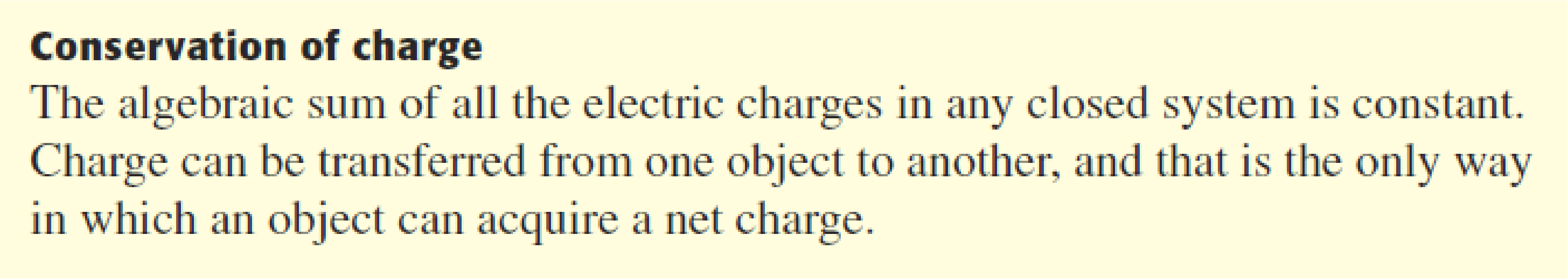
A charged plastic comb picks up uncharged bits of paper.



The comb's (–) charge repels the electrons in each molecule in the paper, creating induced charges. The side of the paper facing the comb thus has a slight net positive charge.

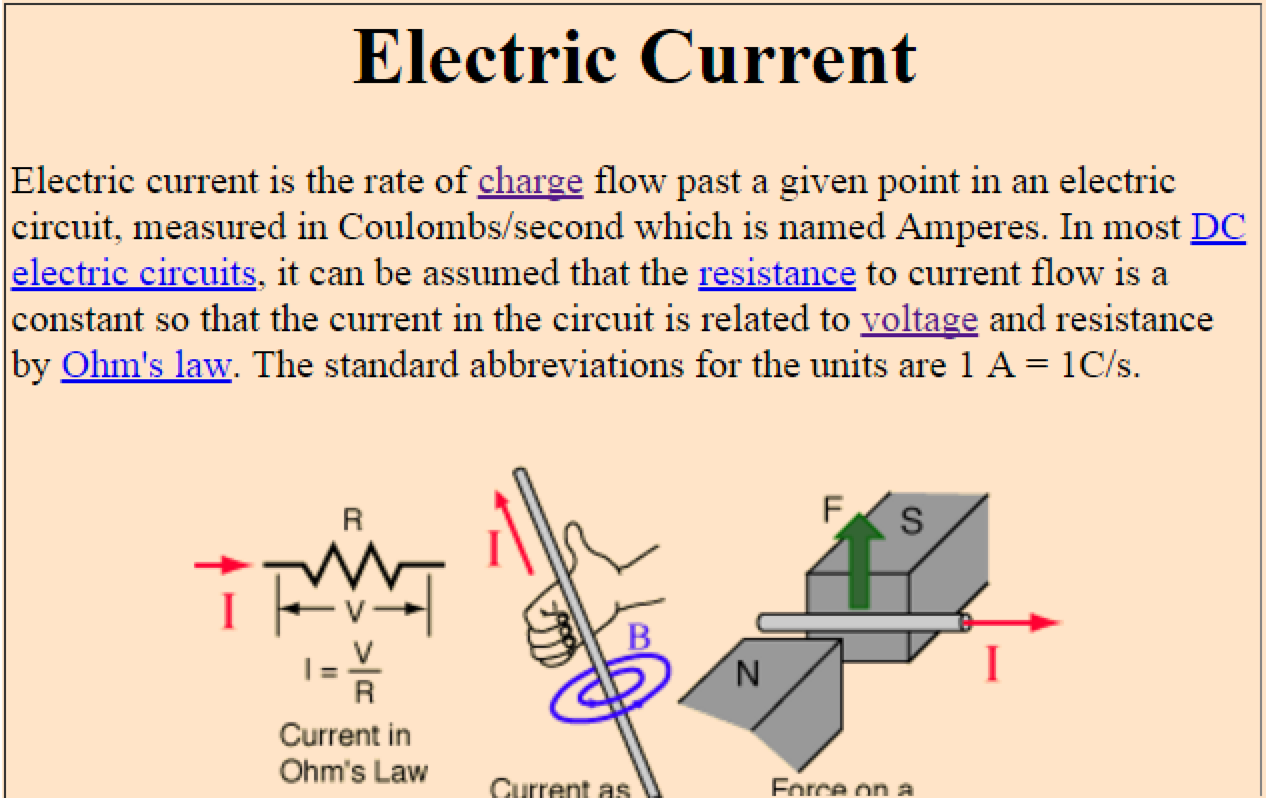


A comb with a (+) charge also creates induced charges that attract the paper to the comb.



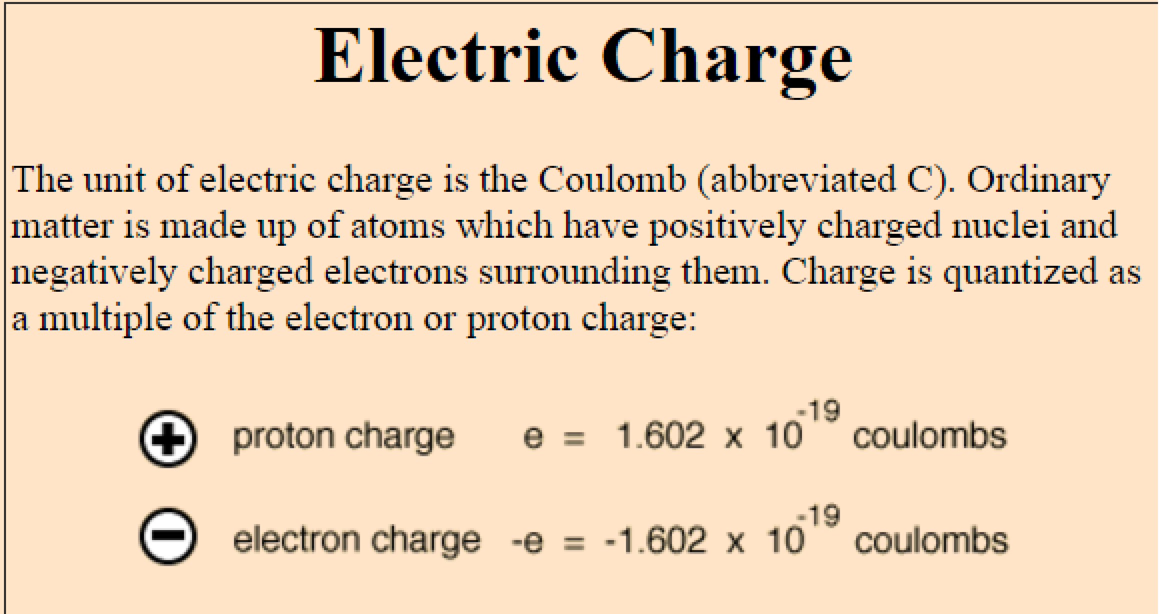
Conservation of charge

The algebraic sum of all the electric charges in any closed system is constant. Charge can be transferred from one object to another, and that is the only way in which an object can acquire a net charge.



Electric Current

Electric current is the rate of charge flow past a given point in an electric circuit, measured in Coulombs/second which is named Amperes. In most DC electric circuits, it can be assumed that the resistance to current flow is a constant so that the current in the circuit is related to voltage and resistance by Ohm's law. The standard abbreviations for the units are 1 A = 1C/s.

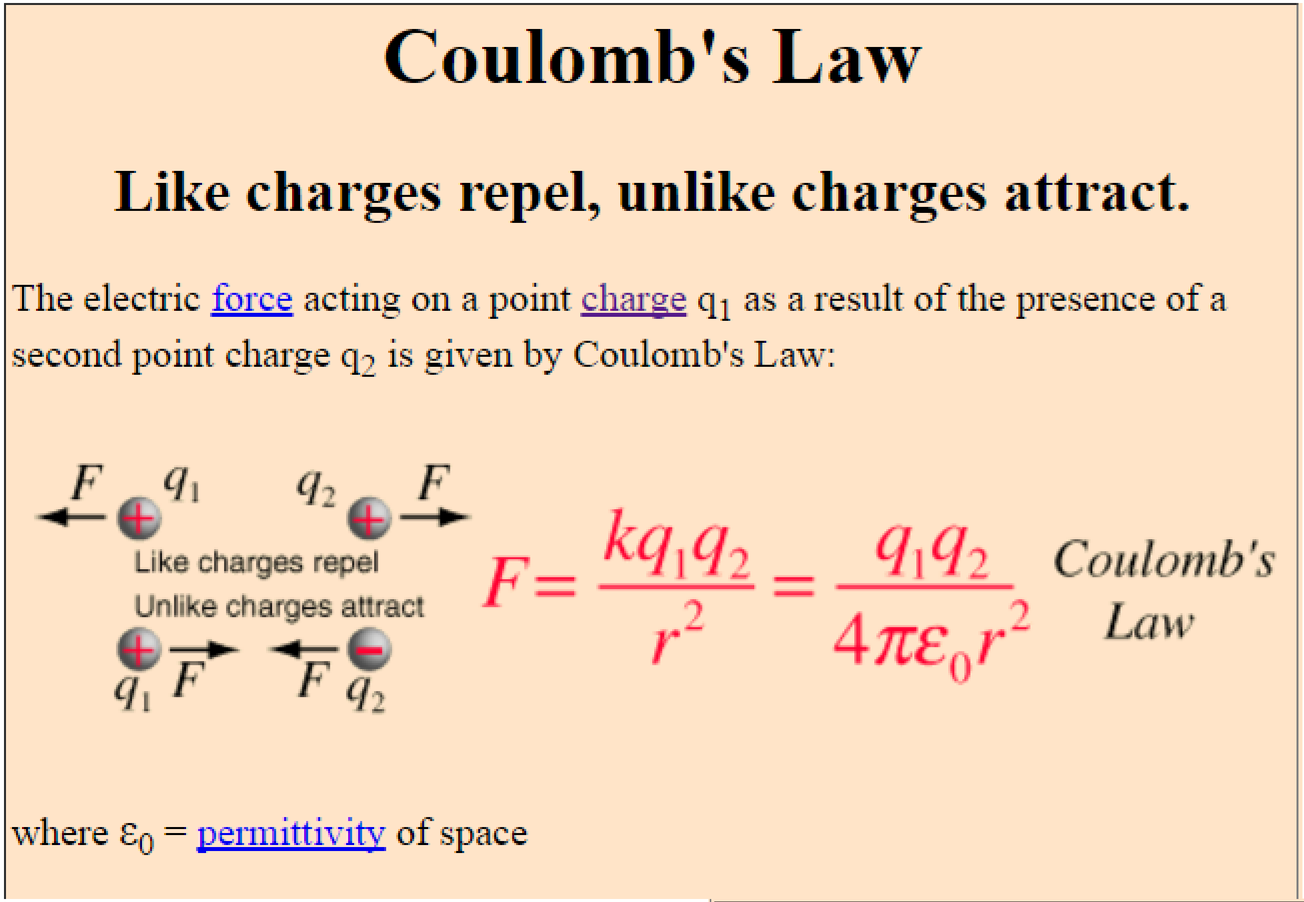


Electric Charge

The unit of electric charge is the Coulomb (abbreviated C). Ordinary matter is made up of atoms which have positively charged nuclei and negatively charged electrons surrounding them. Charge is quantized as a multiple of the electron or proton charge:

⊕ proton charge $e = 1.602 \times 10^{-19}$ coulombs

⊖ electron charge $-e = -1.602 \times 10^{-19}$ coulombs



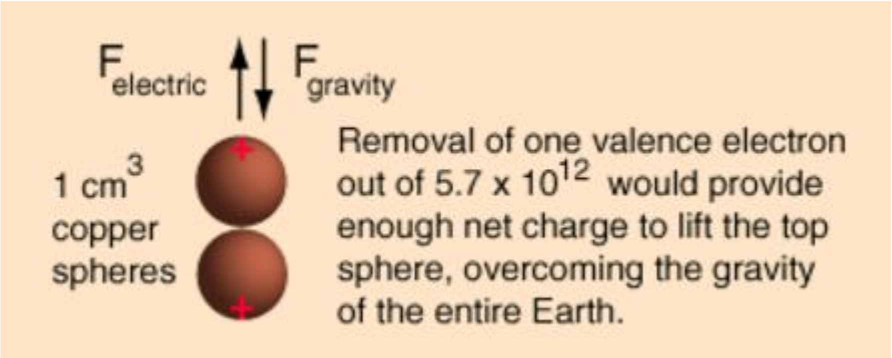
Coulomb's Law

Like charges repel, unlike charges attract.

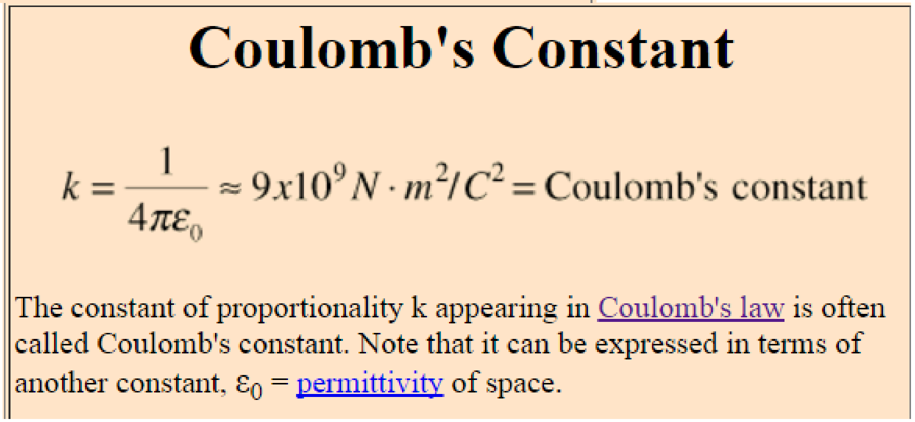
The electric force acting on a point charge q_1 as a result of the presence of a second point charge q_2 is given by Coulomb's Law:

$$F = \frac{kq_1q_2}{r^2} = \frac{q_1q_2}{4\pi\varepsilon_0 r^2}$$ Coulomb's Law

where ε_0 = permittivity of space



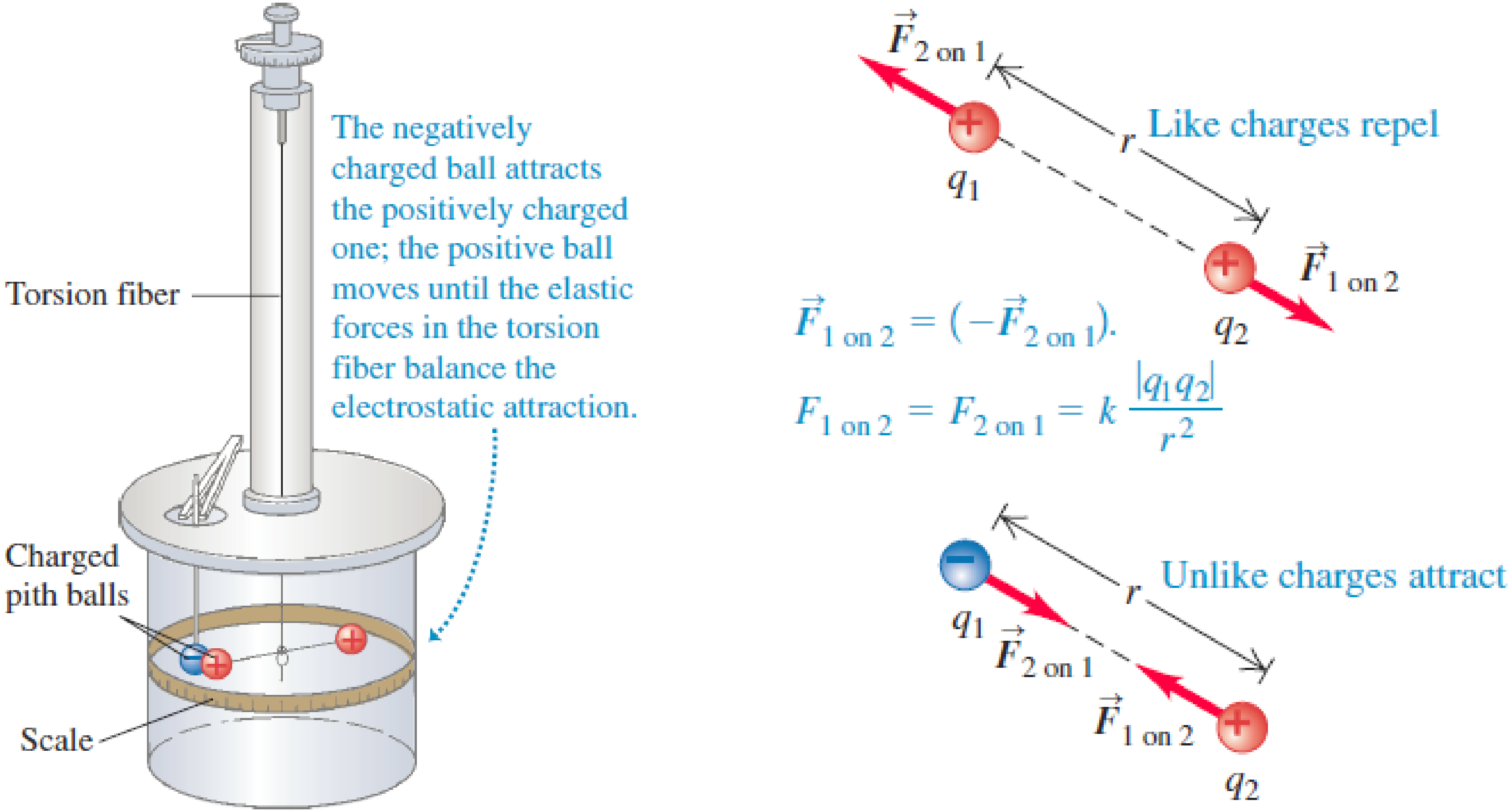
Removal of one valence electron out of 5.7 x 10^{12} would provide enough net charge to lift the top sphere, overcoming the gravity of the entire Earth.



Coulomb's Constant

$$k = \frac{1}{4\pi\varepsilon_0} \approx 9x10^9\, N \cdot m^2/C^2 = \text{Coulomb's constant}$$

The constant of proportionality k appearing in Coulomb's law is often called Coulomb's constant. Note that it can be expressed in terms of another constant, ε_0 = permittivity of space.

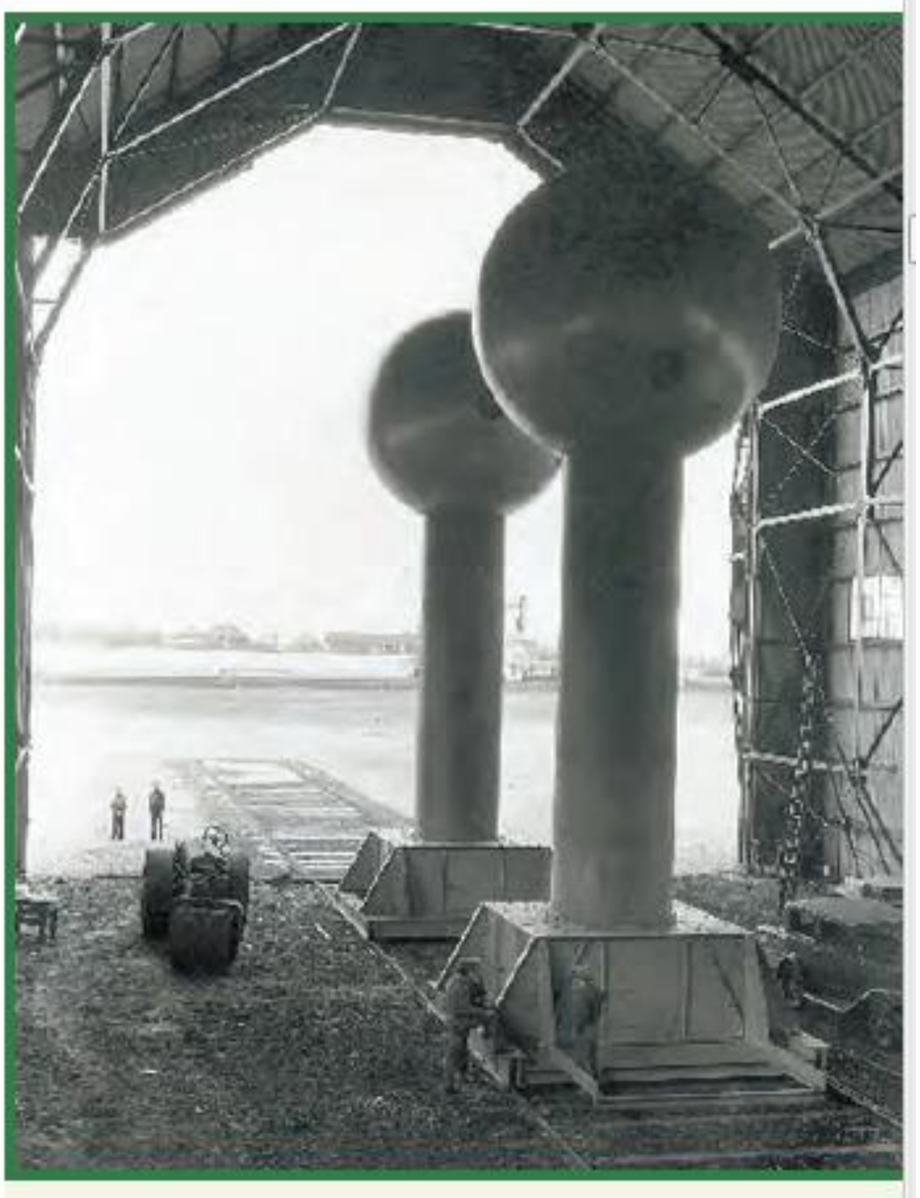


(a) A torsion balance of the type used by Coulomb to measure the electric force

(b) Interaction of like and unlike charges

Schematic depiction of the apparatus Coulomb used to determine the forces between charged objects that can be treated as point charges.

The forces that two charges exert on each other always act along the line joining the charges. The two forces are always equal in magnitude and opposite in direction, even when the charges are not equal. *The forces obey Newton’s third law.*



Generators, like the huge Van de Graaff generators shown here, can accumulate either positive or negative charges on the surface of a metal sphere, thus generating immense electric fields.

EXAMPLE 17.2 Gravity in the hydrogen atom

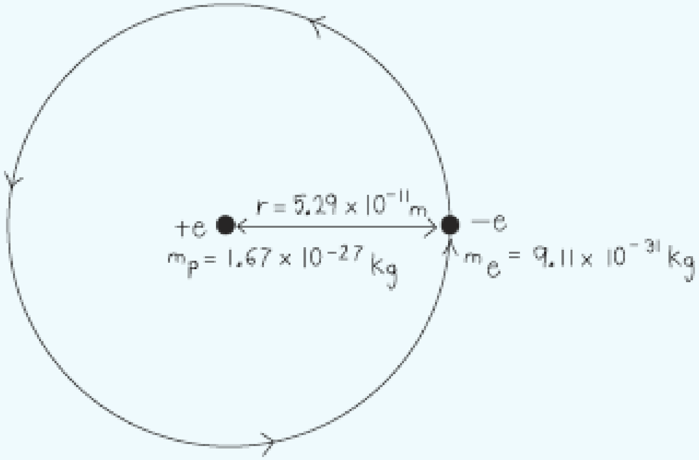
A hydrogen atom consists of one electron and one proton. In an early, simple model of the hydrogen atom called the *Bohr model*, the electron is pictured as moving around the proton in a circular orbit with radius $r = 5.29 \times 10^{-11}$ m. (In Chapter 29, we'll study the Bohr model and also more sophisticated models of atomic structure.) What is the ratio of the magnitude of the electric force between the electron and proton to the magnitude of the gravitational attraction between them? The electron has mass $m_e = 9.11 \times 10^{-31}$ kg, and the proton has mass $m_p = 1.67 \times 10^{-27}$ kg.

SOLUTION

SET UP Figure 17.10 shows our sketch. The distance between the proton and electron is the radius r. Each particle has charge of magnitude e. The electric force is given by Coulomb's law and the gravitational force by Newton's law of gravitation.

SOLVE Coulomb's law gives the magnitude F_e of the electric force between the electron and proton as

$$F_e = k\frac{|q_1 q_2|}{r^2} = k\frac{e^2}{r^2},$$



▲ **FIGURE 17.10** Our sketch for this problem.

where $k = 8.99 \times 10^9\ \mathrm{N \cdot m^2/C^2}$. The gravitational force $\vec{F}_g$ has magnitude F_g:

$$F_g = G\frac{m_1 m_2}{r^2} = G\frac{m_e m_p}{r^2},$$

where $G = 6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}$. The ratio of the two forces is

$$\frac{F_e}{F_g} = \left(\frac{ke^2}{r^2}\right)\left(\frac{r^2}{Gm_e m_p}\right) = \frac{ke^2}{Gm_e m_p}$$

$$= \left(\frac{8.99 \times 10^9\ \mathrm{N \cdot m^2/C^2}}{6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}}\right)$$

$$\times \frac{(1.60 \times 10^{-19}\ \mathrm{C})^2}{(9.11 \times 10^{-31}\ \mathrm{kg})(1.67 \times 10^{-27}\ \mathrm{kg})},$$

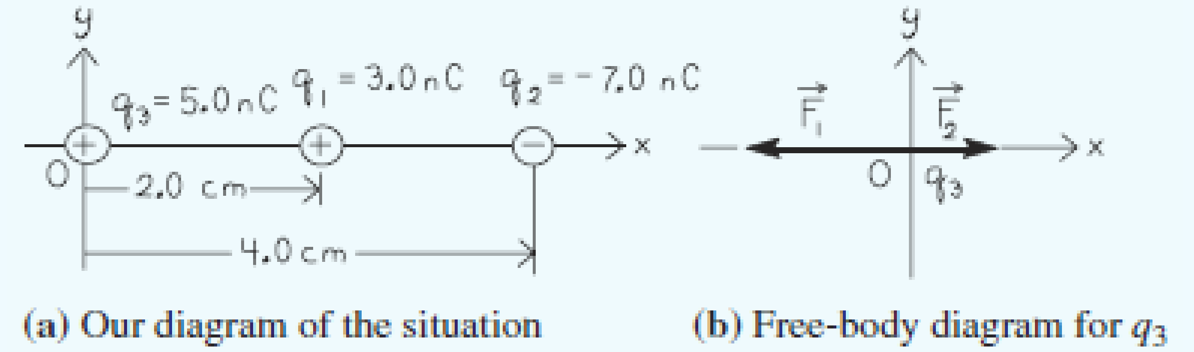
$$\frac{F_e}{F_g} = 2.27 \times 10^{39}.$$

REFLECT In our expression for the ratio, all the units cancel and the ratio is dimensionless. The astonishingly large value of F_e/F_g—about 10^{39}—shows that, in atomic structure, the gravitational force is completely negligible compared with the electrostatic force. The reason gravitational forces dominate in our daily experience

Continued

EXAMPLE 17.3 Adding forces

Two point charges are located on the positive x axis of a coordinate system. Charge $q_1 = 3.0\ \text{nC}$ is 2.0 cm from the origin, and charge $q_2 = -7.0\ \text{nC}$ is 4.0 cm from the origin. What is the total force (magnitude and direction) exerted by these two charges on a third point charge $q_3 = 5.0\ \text{nC}$ located at the origin?



(a) Our diagram of the situation (b) Free-body diagram for q_3

+ve direction

$$F_1 = k\frac{|q_1q_3|}{r_{12}^2}$$
$$= (8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(3.0 \times 10^{-9}\ \text{C})(5.0 \times 10^{-9}\ \text{C})}{(0.020\ \text{m})^2}$$
$$= 3.37 \times 10^{-4}\ \text{N},$$

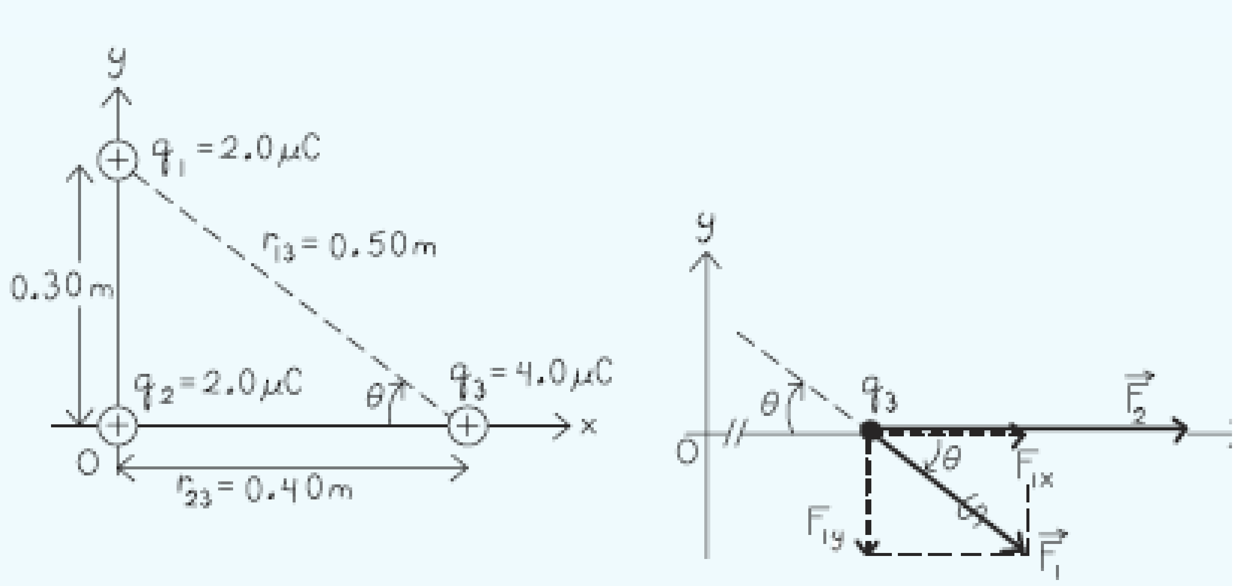
$$F_T = F_2 - F_1$$

$$F_2 = k\frac{|q_2q_3|}{r_{23}^2}$$
$$= (8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(7.0 \times 10^{-9}\ \text{C})(5.0 \times 10^{-9}\ \text{C})}{(0.040\ \text{m})^2}$$
$$= 1.97 \times 10^{-4}\ \text{N}.$$

Vector addition of forces

$$F_1 = k\frac{|q_1 q_3|}{{r_{13}}^2}$$

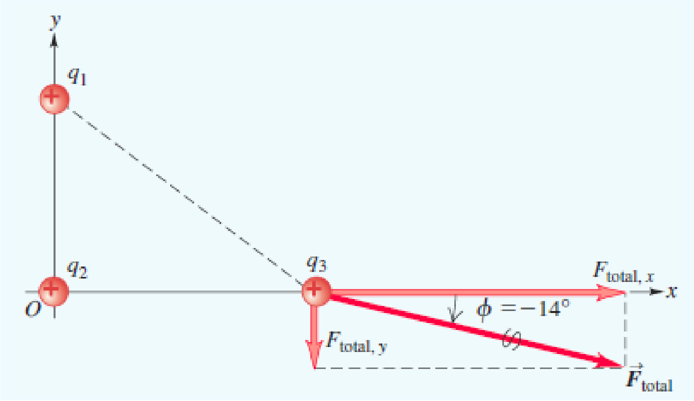
$$F_2 = k\frac{|q_2 q_3|}{{r_{23}}^2}$$



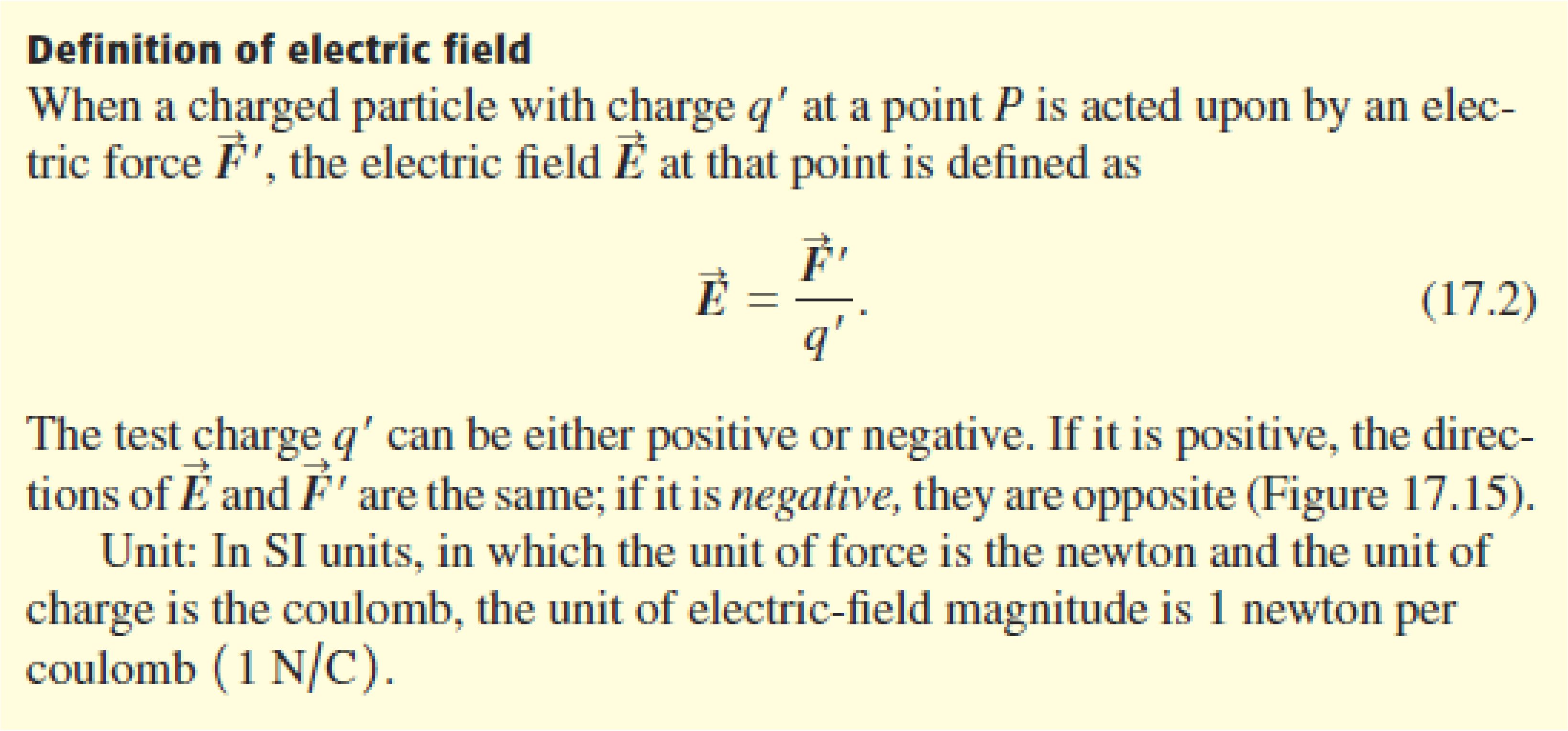
(a) Our sketch of the situation

(b) Free-body diagram for q_3

- Find x & y components of the forces 1 and 2.
- Add all the x forces to get resultant x
- Add all the y forces to get resultant y
- Combine the x and y components using vector concepts.



17.5 Electric Field and Electric Forces



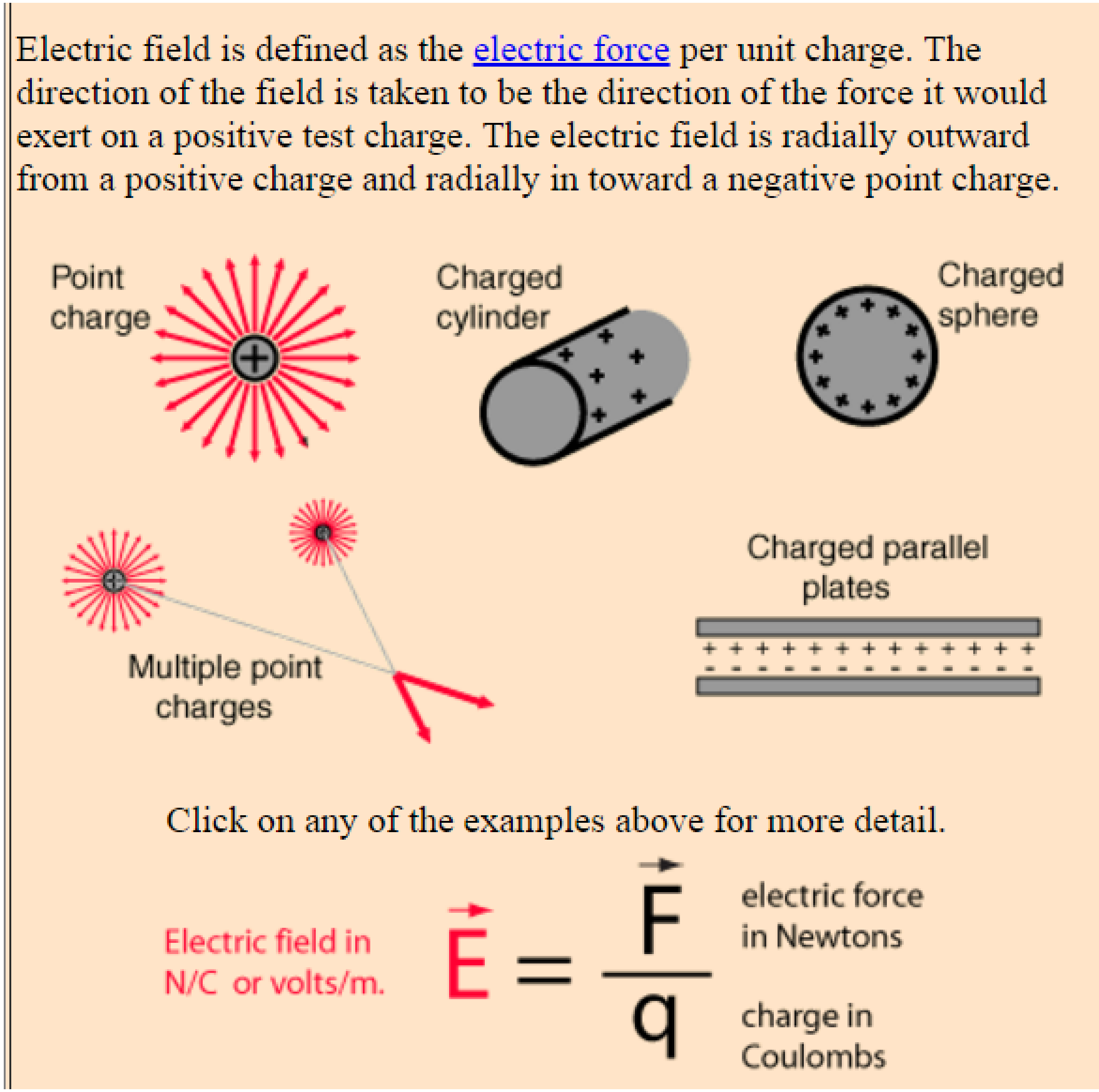
Definition of electric field

When a charged particle with charge q' at a point P is acted upon by an electric force $\vec{F}'$, the electric field $\vec{E}$ at that point is defined as

$$\vec{E} = \frac{\vec{F}'}{q'}. \tag{17.2}$$

The test charge q' can be either positive or negative. If it is positive, the directions of $\vec{E}$ and $\vec{F}'$ are the same; if it is *negative*, they are opposite (Figure 17.15).

Unit: In SI units, in which the unit of force is the newton and the unit of charge is the coulomb, the unit of electric-field magnitude is 1 newton per coulomb (1 N/C).



Electric field is defined as the electric force per unit charge. The direction of the field is taken to be the direction of the force it would exert on a positive test charge. The electric field is radially outward from a positive charge and radially in toward a negative point charge.

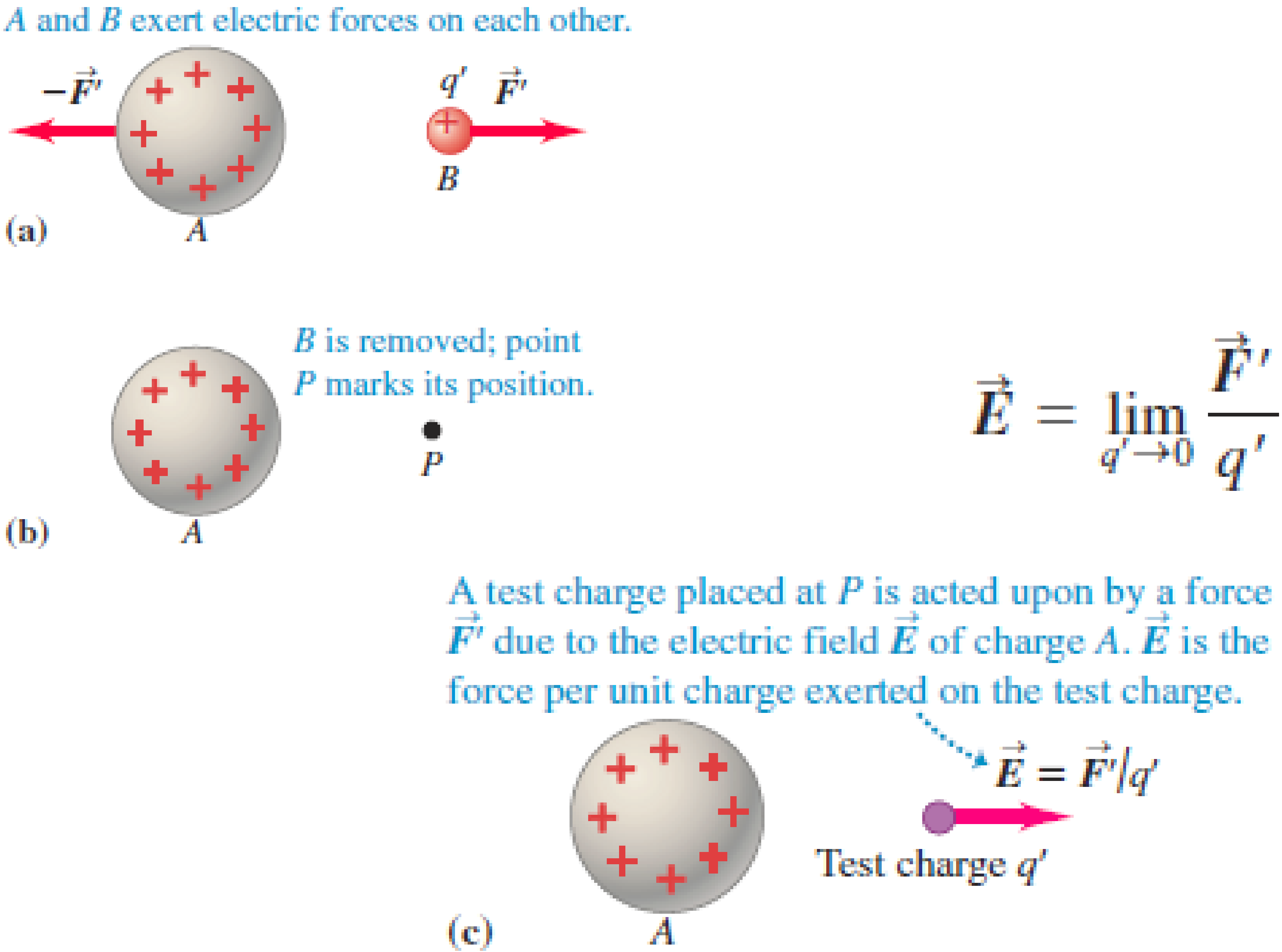
Click on any of the examples above for more detail.

Electric field in N/C or volts/m.

$$\vec{E} = \frac{\vec{F}}{q}$$

electric force in Newtons

charge in Coulombs



A and B exert electric forces on each other.

B is removed; point P marks its position.

A test charge placed at P is acted upon by a force $\vec{F}'$ due to the electric field $\vec{E}$ of charge A. $\vec{E}$ is the force per unit charge exerted on the test charge.

$\vec{E} = \vec{F}'/q'$

Test charge q'

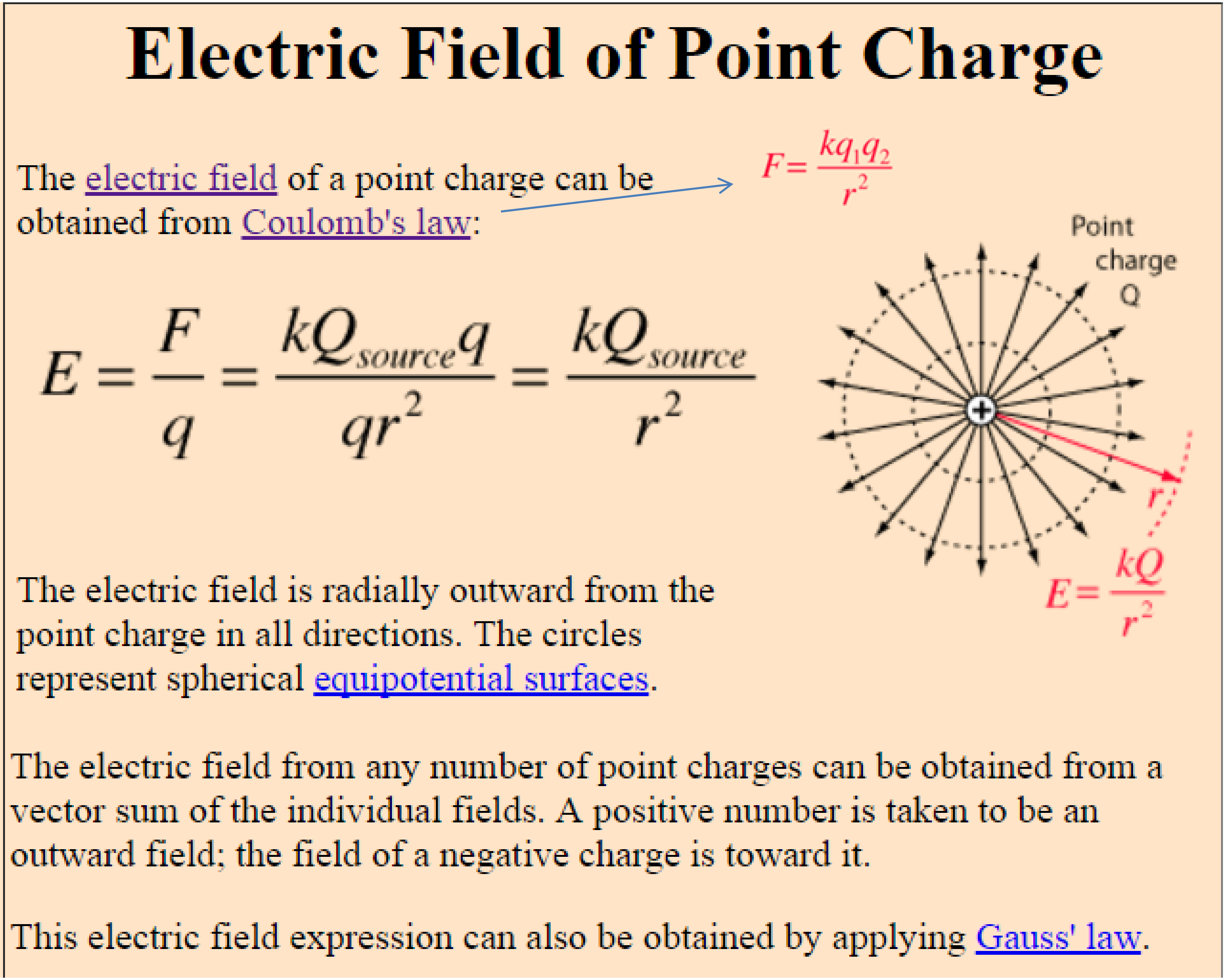
$$\vec{E} = \lim_{q' \to 0} \frac{\vec{F}'}{q'}.$$

The force on a positive test charge points in the direction of the electric field.

The force on a negative test charge points opposite to the electric field.

Principle of superposition
The total electric field at any point due to two or more charges is the vector sum of the fields that would be produced at that point by the individual charges.

$$\vec{E}_{\text{total}} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \cdots.$$



Electric Field of Point Charge

The electric field of a point charge can be obtained from Coulomb's law:

$F=\frac{kq_1q_2}{r^2}$

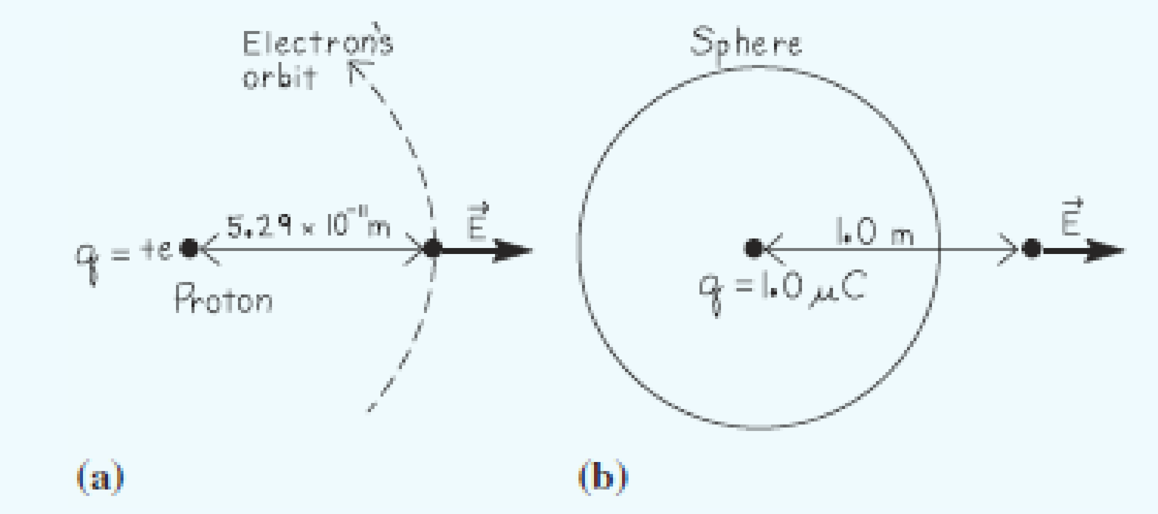
$$E=\frac{F}{q}=\frac{kQ_{source}q}{qr^2}=\frac{kQ_{source}}{r^2}$$

The electric field is radially outward from the point charge in all directions. The circles represent spherical equipotential surfaces.

The electric field from any number of point charges can be obtained from a vector sum of the individual fields. A positive number is taken to be an outward field; the field of a negative charge is toward it.

This electric field expression can also be obtained by applying Gauss' law.

Electric field in a hydrogen atom and Van der Graff (1m from centre)



Acts as single charge at centre

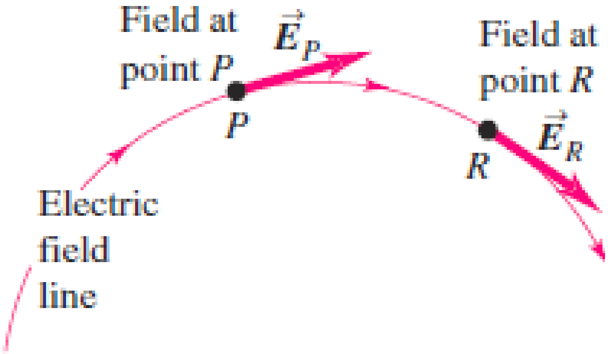
$$\begin{aligned} E &= k\frac{|q|}{r^2} \\ &= (8.99 \times 10^9\ \mathrm{N \cdot m^2/C^2})\frac{(1.60 \times 10^{-19}\ \mathrm{C})}{(5.29 \times 10^{-11}\ \mathrm{m})^2} \\ &= 5.14 \times 10^{11}\ \mathrm{N/C}. \end{aligned}$$

$$\begin{aligned} E = k\frac{|q|}{r^2} &= (8.99 \times 10^9\ \mathrm{N \cdot m^2/C^2})\frac{(1.0 \times 10^{-6}\ \mathrm{C})}{(1.0\ \mathrm{m})^2} \\ &= 9.0 \times 10^3\ \mathrm{N/C}. \end{aligned}$$

Hydrogen atom is MUCH more

EXAMPLE 17.7 Electric field of an electric dipole

17.7 Electric Field Lines

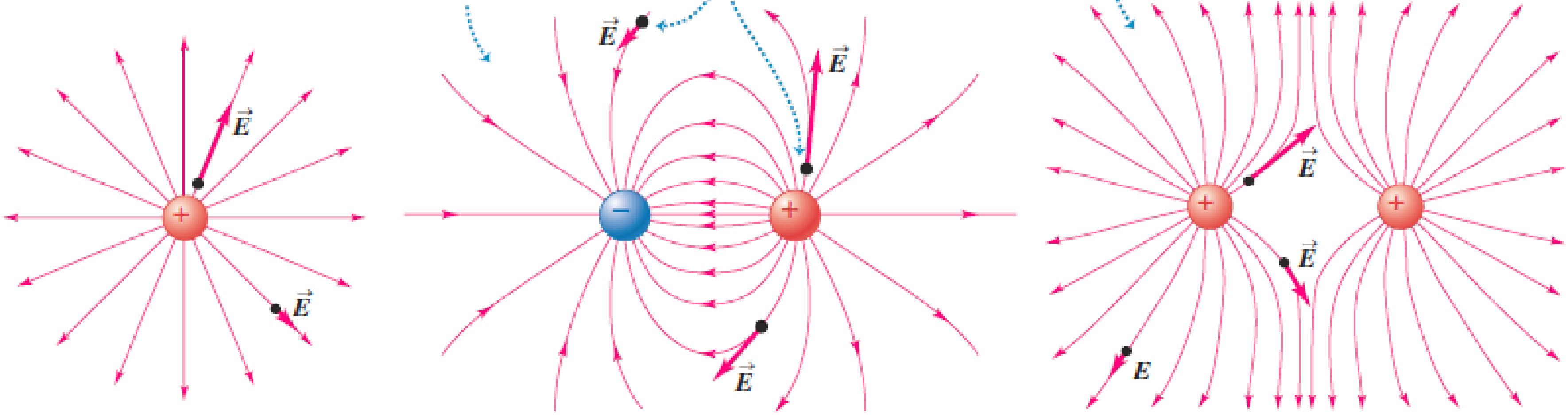


▲ **FIGURE 17.21** The direction of the electric field at any point is tangent to the field line through that point.

Field lines always point *away from* (+) charges and *toward* (−) charges.

At each point in space, the electric field vector is *tangent* to the field line passing through that point.

Field lines are close together where the field is strong, farther apart where it is weaker.

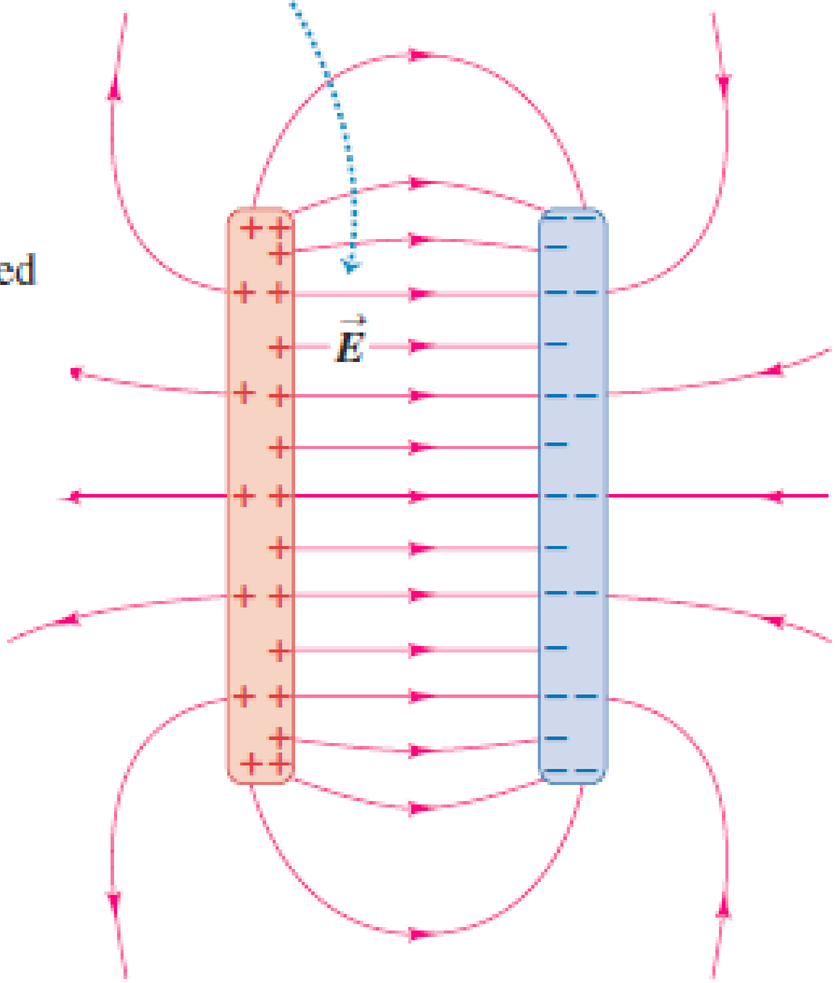


(a) A single positive charge

(b) Two equal and opposite charges (a dipole)

(c) Two equal positive charges

NOTE ► There may be a temptation to think that when a charged particle moves in an electric field, its path always follows a field line. Resist that temptation; the thought is erroneous. The direction of a field line at a given point determines the direction of the particle's *acceleration*, not its velocity. We've seen several examples of motion in which the velocity and acceleration vectors have different directions. ◄



Between the plates of the capacitor, the electric field is nearly uniform, pointing from the positive plate toward the negative one.

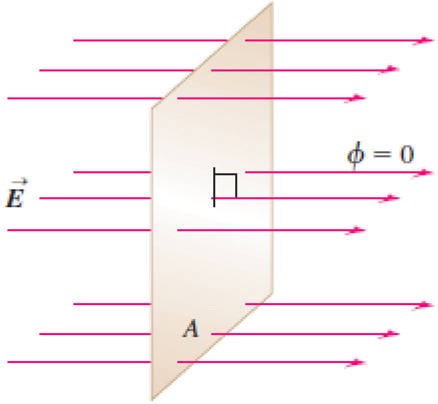
▲ **FIGURE 17.23** The electric field produced by a parallel-plate capacitor (seen in cross section). Between the plates, the field is nearly uniform.

17.8 Gauss’s Law and Field Calculations

Rather than adding many point charges as in coulombs law we can image a Gaussian surface where the electric field goes through.
Gauss’s law is a relation between the field at *all* the points on the surface and the total charge enclosed within the surface.

First we need to define electric flux.

Electric flux



Electric field $\vec{E}$ is perpendicular to area A; the angle between $\vec{E}$ and a line perpendicular to the surface is zero.
The flux is $\Phi_E = EA$.

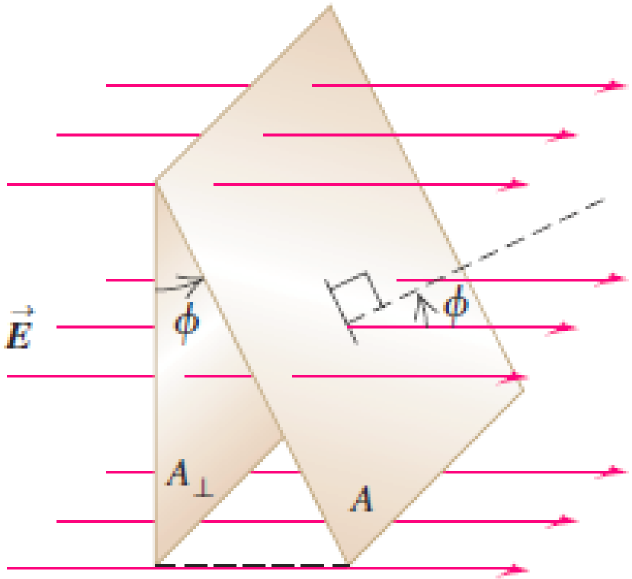
The definition of electric flux involves an area A and the electric field at various points in the area. The area needn't be the surface of a real object; in fact, it will usually be an imaginary area in space. Consider first a small, flat area A perpendicular to a uniform electric field $\vec{E}$ (Figure 17.24a). We denote electric flux by Φ_E; we define the electric flux Φ_E through the area A to be the product of the magnitude E of the electric field and the area A:

$$\Phi_E = EA.$$

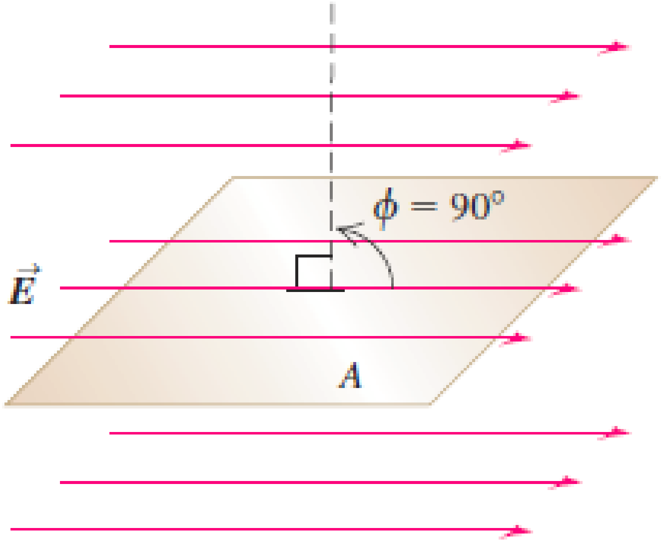
Roughly speaking, we can picture Φ_E in terms of the number of field lines that pass through A. More area means more lines through the area, and a stronger field means more closely spaced lines and therefore more lines per unit area.

$$\Phi_E = E_{\perp} A.$$

$$\Phi_E = EA \cos\phi.$$

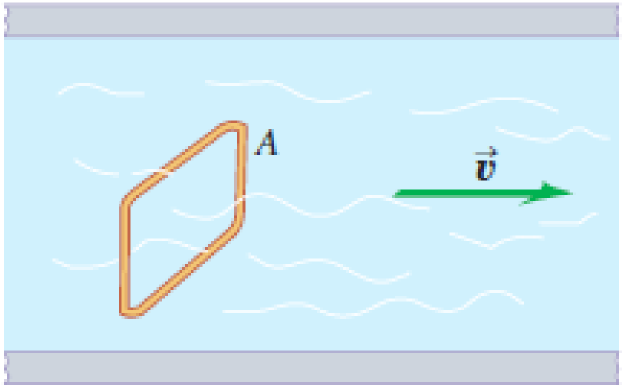


Area A is tilted at an angle ϕ from the perpendicular to $\vec{E}$.
The flux is $\Phi_E = EA \cos\phi$.

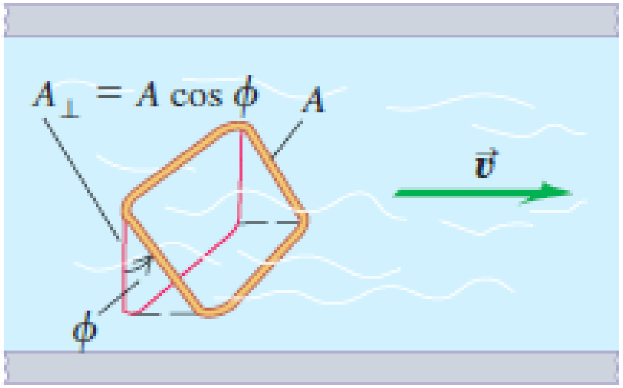


Area A is parallel to $\vec{E}$ (tilted at 90° from the perpendicular to $\vec{E}$).
The flux is $\Phi_E = EA \cos 90° = 0$.

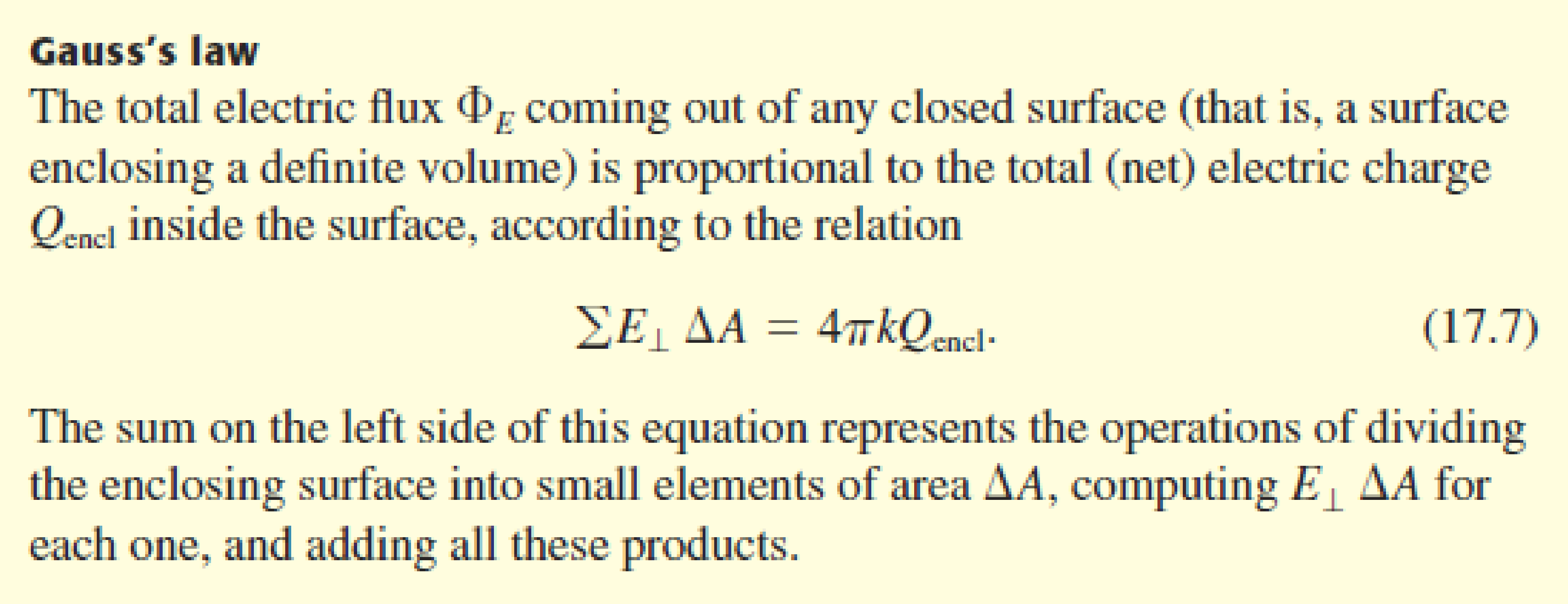
This can be compared to water flow.



(a)



(b)



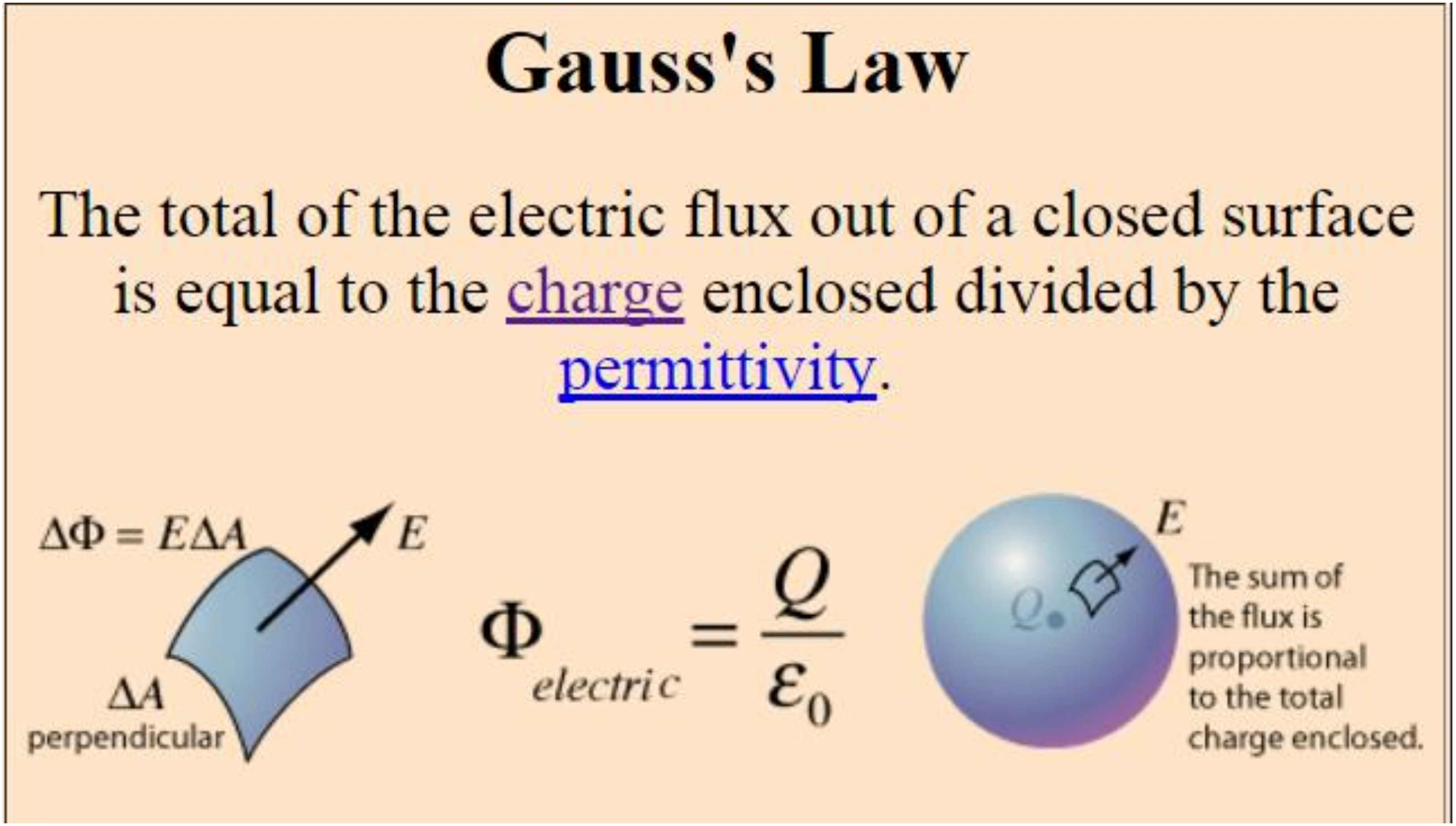
Gauss's law

The total electric flux Φ_E coming out of any closed surface (that is, a surface enclosing a definite volume) is proportional to the total (net) electric charge Q_{encl} inside the surface, according to the relation

$$\sum E_{\perp}\,\Delta A = 4\pi k Q_{\text{encl}}. \tag{17.7}$$

The sum on the left side of this equation represents the operations of dividing the enclosing surface into small elements of area ΔA, computing $E_{\perp}\,\Delta A$ for each one, and adding all these products.

$$\Phi_E = \sum E_{\perp}\,\Delta A = 4\pi k q. = \frac{Q_{\text{encl}}}{\epsilon_0}.$$



Gauss's Law

The total of the electric flux out of a closed surface is equal to the charge enclosed divided by the permittivity.

$$\Delta\Phi = E\Delta A$$

$$\Phi_{electric} = \frac{Q}{\varepsilon_0}$$

The sum of the flux is proportional to the total charge enclosed.

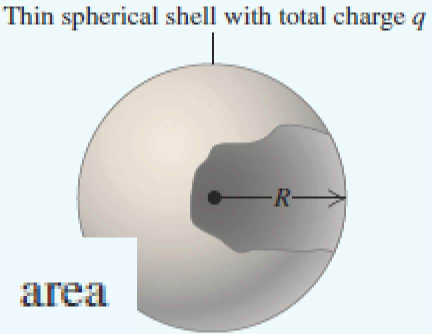
In terms of calculus (just for interest)

Flux $\phi = \int E.\delta A$

Gauss $\phi = \oint E.\delta A = \frac{Q_{enc}}{\varepsilon_o}$

EXAMPLE 17.10 Field due to a spherical shell of charge

A positive charge q is spread uniformly over a thin spherical shell of radius R (Figure 17.29). Find the electric field at points inside and outside the shell.

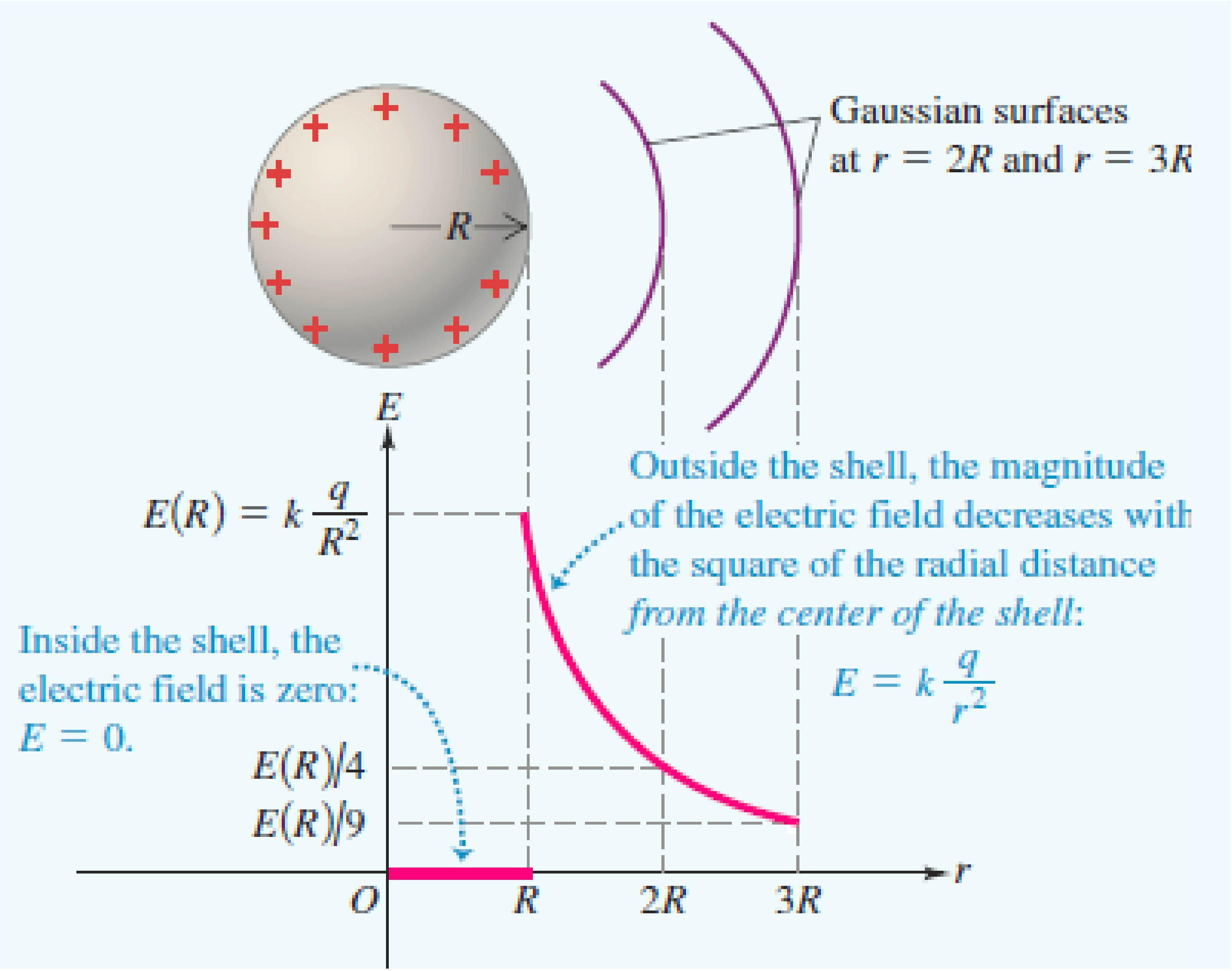


Inside the shell ($r < R$): The Gaussian surface has area $4\pi r^2$. Since, by symmetry, the electric field is uniform over the Gaussian sphere and perpendicular to it at each point, the electric flux is $\Phi_E = EA = E(4\pi r^2)$. The Gaussian surface is inside the shell and encloses none of the charge on the shell, so $Q_{\text{encl}} = 0$.

Gauss's law $\Phi_E = Q_{\text{encl}}/\epsilon_0$ then says that $\Phi_E = E(4\pi r^2) = 0$, so $E = 0$. The electric field is zero at all points inside the shell.

Outside the shell ($r > R$): Again, $\Phi_E = E(4\pi r^2)$. But now all of the shell is inside the Gaussian surface, so $Q_{\text{encl}} = q$. Gauss's law $\Phi_E = Q_{\text{encl}}/\epsilon_0$ then gives $E(4\pi r^2) = q/\epsilon_0$, and it follows that

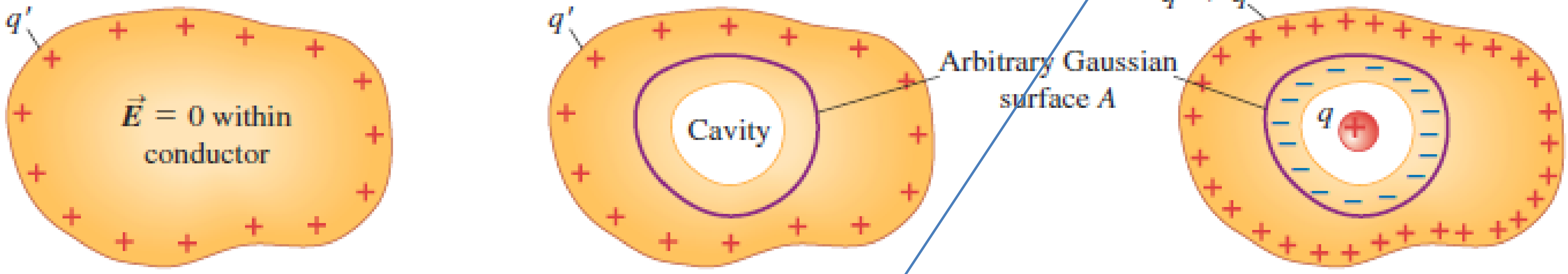
$$E = \frac{q}{4\pi\epsilon_0 r^2} = k\frac{q}{r^2}.$$


Gaussian surfaces
at $r = 2R$ and $r = 3R$
R
E
$E(R) = k\frac{q}{R^2}$
Outside the shell, the magnitude of the electric field decreases with the square of the radial distance *from the center of the shell:*
$E = k\frac{q}{r^2}$
Inside the shell, the electric field is zero:
$E = 0.$
$E(R)/4$
$E(R)/9$
O
R
$2R$
$3R$
r

The charge q' is distributed over the surface of the conductor. The situation is electrostatic, so $\vec{E} = 0$ within the conductor.

Because $\vec{E} = 0$ at all points within the conductor, the electric field at all points on the Gaussian surface must be zero.

For $\vec{E}$ to be zero at all points on the Gaussian surface, the surface of the cavity must have a total charge $-q$.



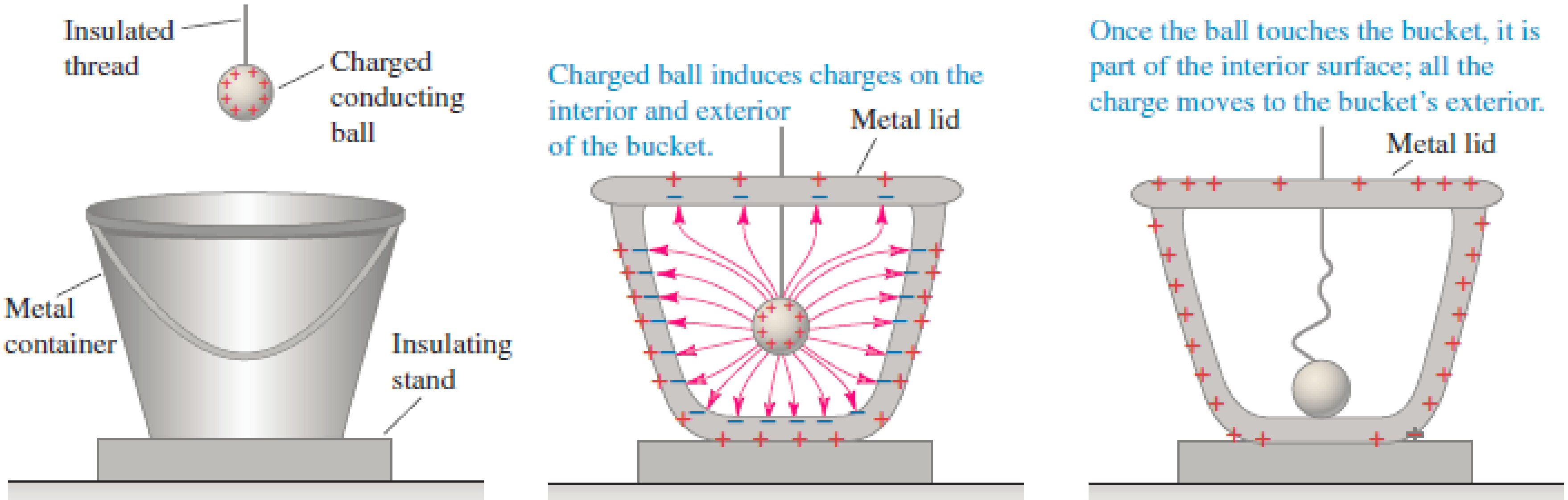
(a) Solid conductor with charge q'

(b) The same conductor with an internal cavity

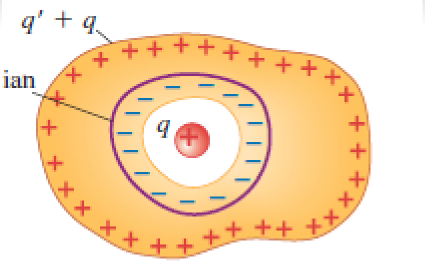
(c) An isolated charge q is placed in the cavity

▲ **FIGURE 17.31** The charge on a solid conductor, on a conductor with a cavity, and on a conductor with a cavity that contains a charge.

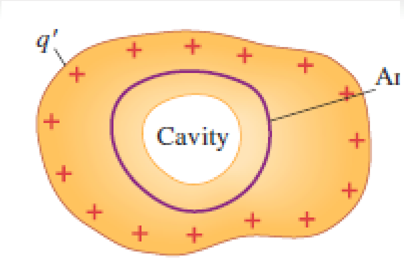
(because the situation is still electrostatic – no moving charges so no electric field in the conductor must be zero)



Faraday Ice Pail



(c) An isolated charge q is placed in the cavity

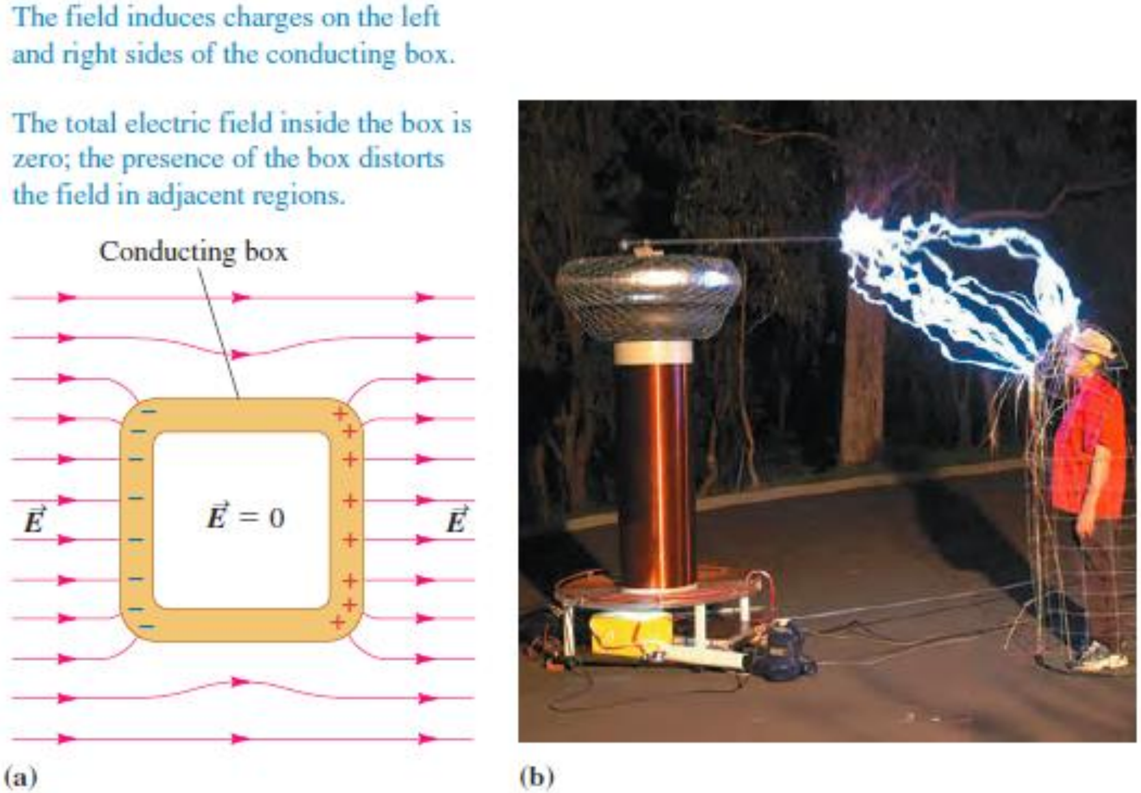


(b) The same conductor with an internal cavity

The surface of the ball becomes, in effect, part of the cavity surface. The situation is now the same as Figure 17.31b; if Gauss's law is correct, the net charge on this surface must be zero. Thus, the ball must lose all its charge. Finally, we pull the ball out, to find that it has indeed lost all its charge.

Faraday cage

▲ Application **A Faraday cage when you need one.** If you find yourself in a thunderstorm while driving, *stay in your car.* If it gets hit by lightning, it will act as a Faraday cage and keep you safe.



The field induces charges on the left and right sides of the conducting box.

The total electric field inside the box is zero; the presence of the box distorts the field in adjacent regions.

▲ **FIGURE 17.34** (a) The effect of putting a conducting box (an electrostatic shield) in a uniform electric field. (b) The conducting cage keeps the operator of this exhibit perfectly safe.

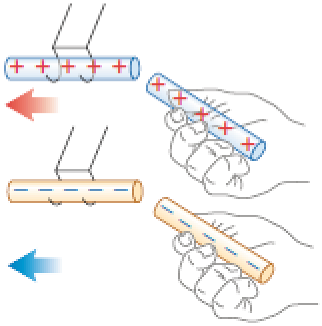
SUMMARY

Electric Charge; Conductors and Insulators

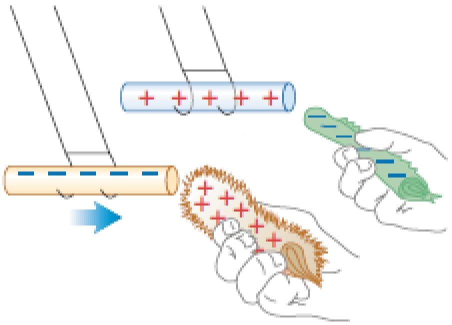
(Sections 17.1–17.3) The fundamental entity in electrostatics is electric charge. There are two kinds of charge: positive and negative. Like charges repel each other; unlike charges attract. **Conductors** are materials that permit electric charge to move within them. **Insulators** permit charge to move much less readily. Most metals are good conductors; most nonmetals are insulators.

All ordinary matter is made of atoms consisting of protons, neutrons, and electrons. The protons and neutrons form the nucleus of the atom; the electrons surround the nucleus at distances much greater than its size. Electrical interactions are chiefly responsible for the structure of atoms, molecules, and solids.

Electric charge is conserved: It can be transferred between objects, but isolated charges cannot be created or destroyed. Electric charge is quantized: Every amount of observable charge is an integer multiple of the charge of an electron or proton.



Like charges repel.

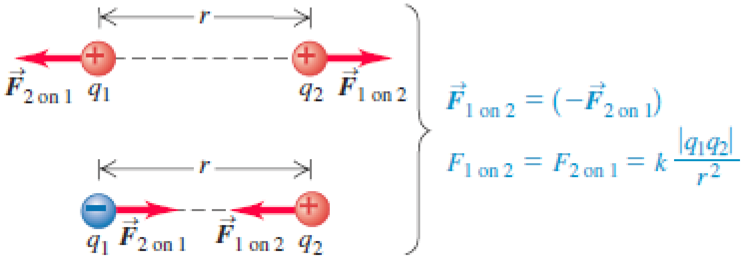


Unlike charges attract.

Coulomb's Law

(Section 17.4) Coulomb's law is the basic law of interaction for point electric charges. For point charges q_1 and q_2 separated by a distance r, the magnitude F of the force each charge exerts on the other is

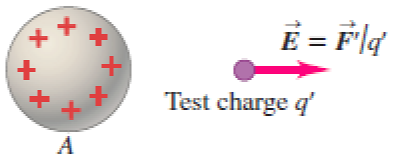
$$F = k\frac{|q_1 q_2|}{r^2}. \qquad (17.1)$$



The force on each charge acts along the line joining the two charges. It is repulsive if q_1 and q_2 have the same sign, attractive if they have opposite signs. The forces form an action–reaction pair and obey Newton's third law.

Electric Field and Electric Forces

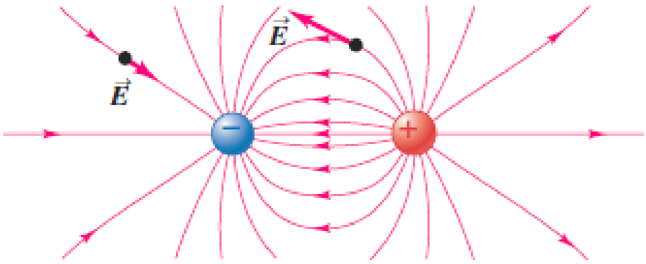
(Sections 17.5 and 17.6) Electric field, a vector quantity, is the force per unit charge exerted on a test charge at any point, provided that the test charge is small enough that it does not disturb the charges that cause the field. The principle of superposition states that the electric field due to any combination of charges is the vector sum of the fields caused by the individual charges. From Coulomb's law, the magnitude of the electric field produced by a point charge is



$$E = k\frac{|q|}{r^2}. \qquad (17.4)$$

Electric Field Lines

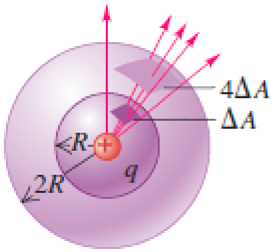
(Section 17.7) **Field lines** provide a graphical representation of electric fields. A field line at any point in space is tangent to the direction of $\vec{E}$ at that point, and the number of lines per unit area (perpendicular to their direction) is proportional to the magnitude of $\vec{E}$ at the point. Field lines point away from positive charges and toward negative charges.



Gauss's Law

(Section 17.8) For a uniform electric field with component $E_\perp$ perpendicular to area A, the **electric flux** through the area is $\Phi_E = E_\perp A$ (Equation 17.6). **Gauss's law** states that the total electric flux Φ_E out of any closed surface (that is, a surface enclosing a definite volume) is proportional to the total electric charge Q_{encl} inside the surface, according to the relation

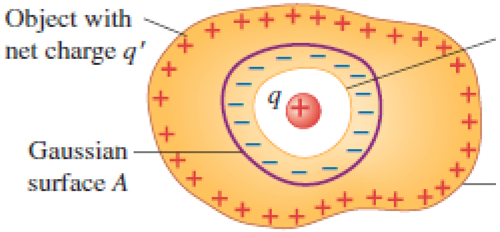
$$\sum E_\perp \, \Delta A = 4\pi k Q_{\text{encl}}. \qquad (17.7)$$



The electric flux through the two concentric spheres is the same.

Charges on Conductors

(Section 17.9) In a static configuration with no net motion of charge, the electric field is always zero within a conductor. The charge on a solid conductor is located entirely on its outer surface. If there is a cavity containing a charge $+q$ within the conductor, the surface of the cavity has a total induced charge $-q$.



For $\vec{E}$ to remain zero across the Gaussian surface, the surface of the cavity must have a charge $-q$.

Surface charge $= q' + q$.

18 Electric Potential and Capacitance

18.1 Electric Potential Energy

Remember the equations from mechanics and the fact that the work done is change in energy

$$W_{a\to b} = F\,s\cos(\emptyset) = U_a - U_b = W_{net} = \Delta KE$$

$$W_{a\to b} = F\,s\cos(\emptyset) = \ U_a - U_b = W_{net} = \Delta KE$$

Apply this to electric fields

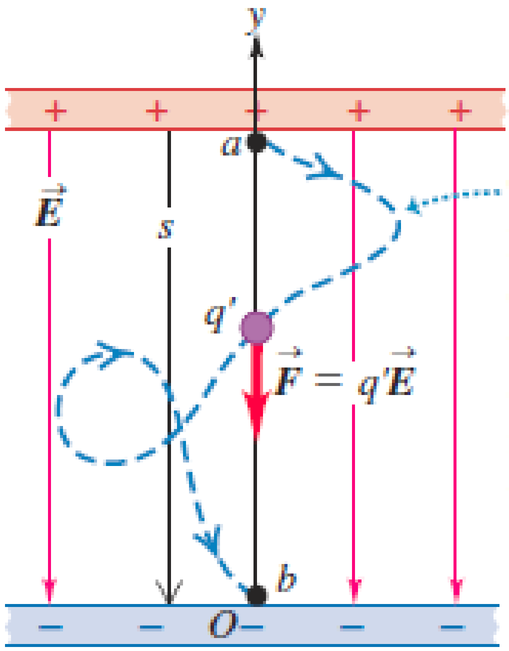
Test charge

$$W_{a\to b} = F\,s = q'\,E\,s$$

Electric potential energy

$$W_{a\to b} = U_a - U_b$$
$$= q'\,E\,(y_a - y_b)$$

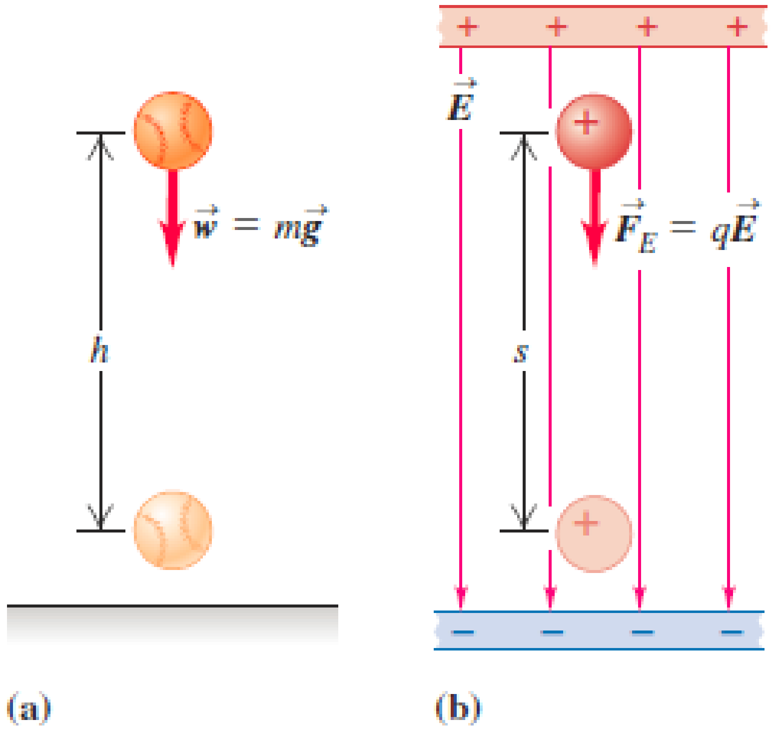
Work done on charge q' by the *constant* electric force between the plates: $W_{a\to b} = q'Es$



The electric force has only a y component, so the work it does on the charge is independent of the charge's path.

▲ FIGURE 18.2 A test charge q' moves from point *a* to point *b* in a uniform electric field.

Comparing gravitational and electrical conservative forces



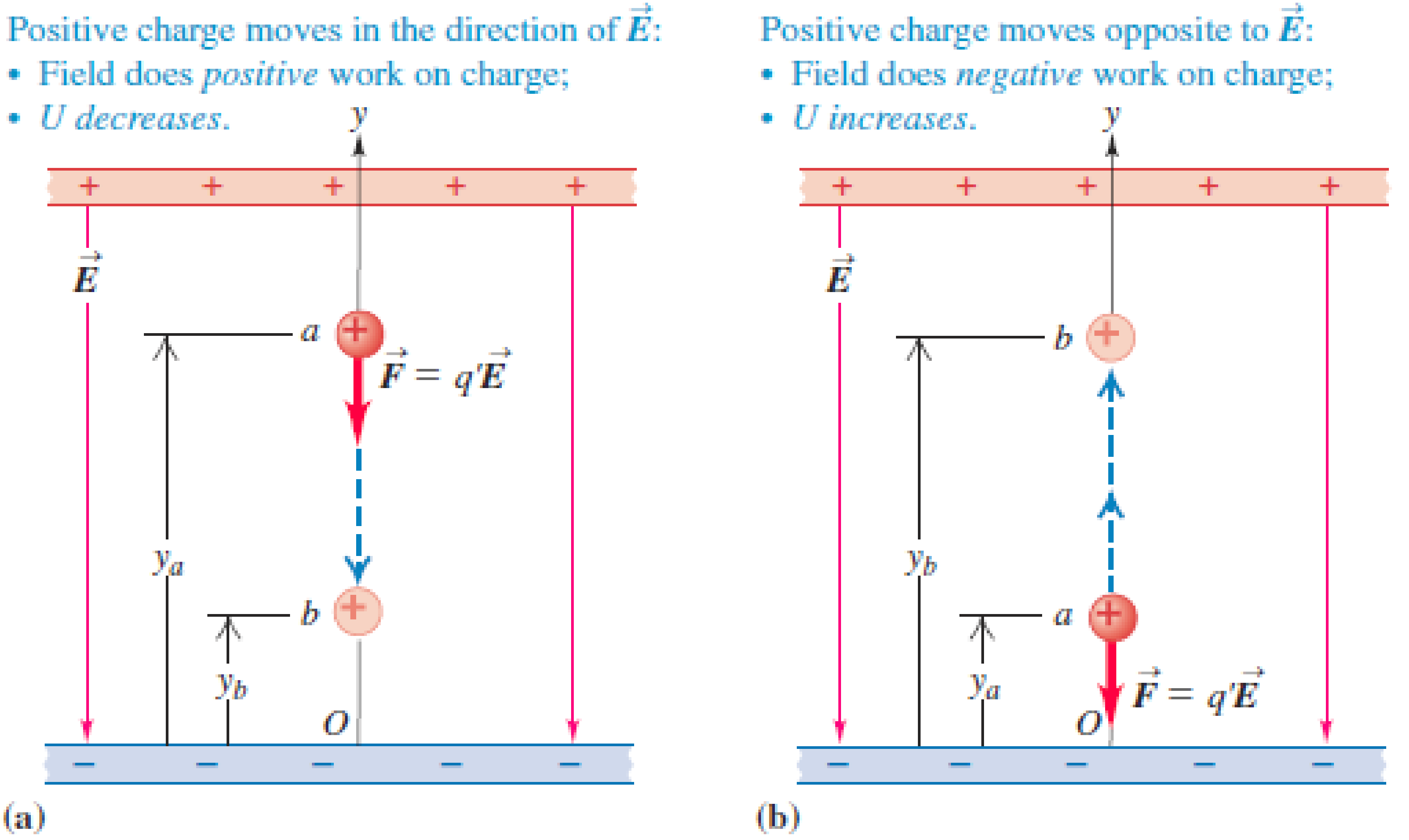
(a) Object moving in a uniform gravitational field:

$$W = -\Delta U_{\text{grav}} = mgh$$

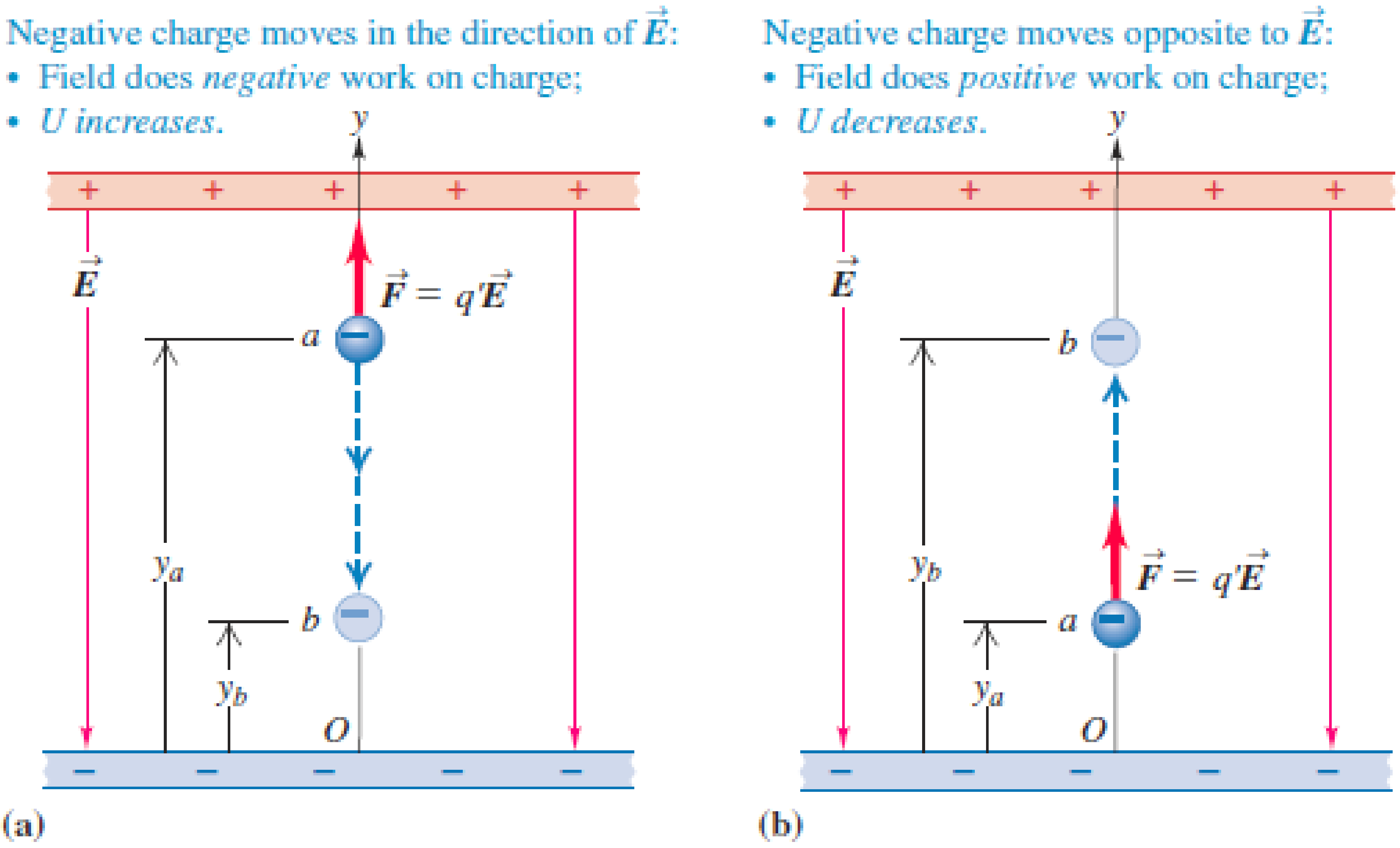
(b) Charge moving in a uniform electric field:

$$W = -\Delta U_E = qEs$$

▲ **FIGURE 18.1** Because electric and gravitational forces are conservative, work done by either can be expressed in terms of a potential energy.



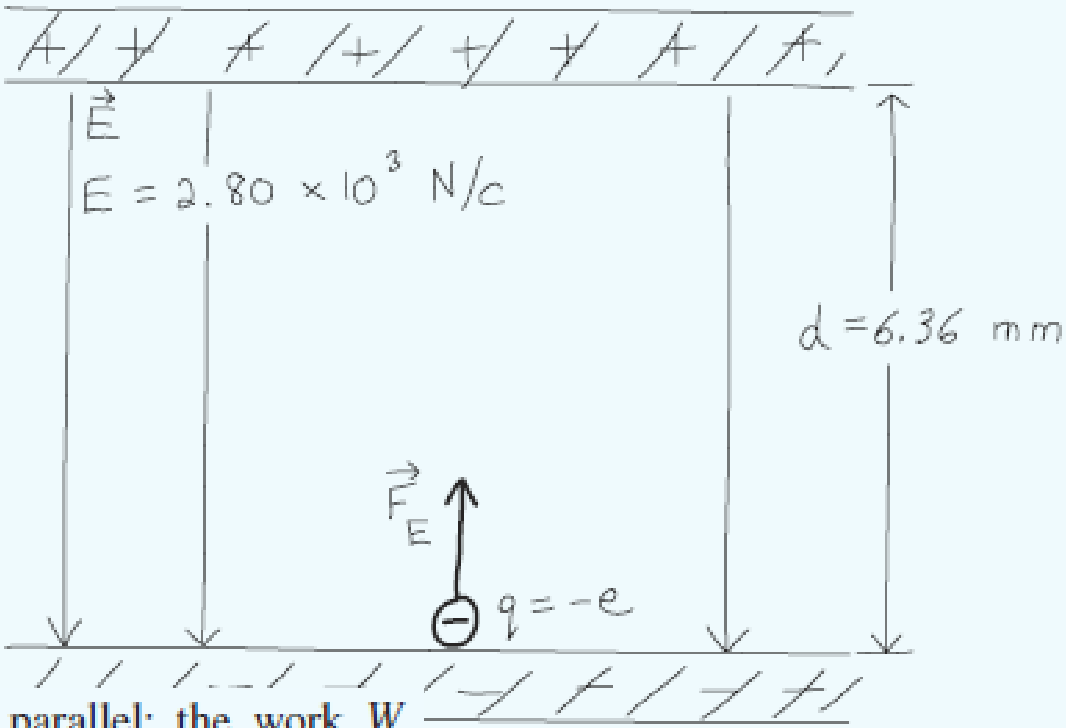
▲ **FIGURE 18.3** The work done by an electric field on a positive charge moving (a) in the direction of and (b) opposite to the electric field.



▲ **FIGURE 18.4** The work done by an electric field on a negative charge moving (a) in the direction of and (b) opposite to the electric field.

EXAMPLE 18.1 Work in a uniform electric field

Two large conducting plates separated by 6.36 mm carry charges of equal magnitude and opposite sign, creating a uniform electric field with magnitude 2.80×10^3 N/C between the plates. An electron moves from the negatively charged plate to the positively charged plate. How much work does the electric field do on the electron?

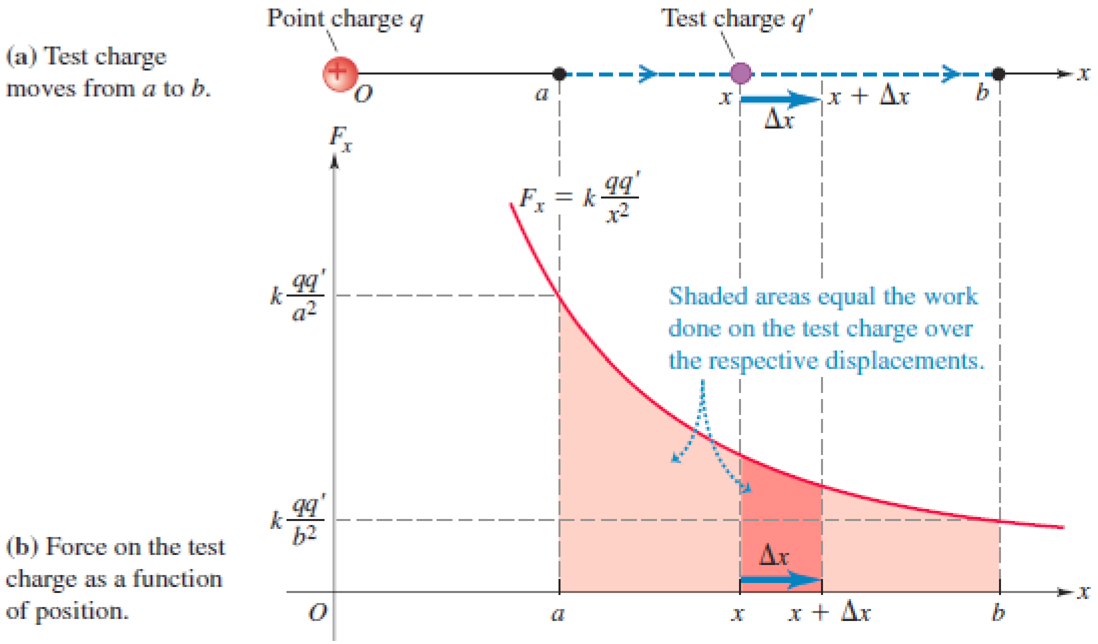


SOLVE The force and displacement are parallel; the work W done by the electric-field force during a displacement of magnitude d is $W = F_e d \cos\phi$ with $\phi = 0$, so

$$
\begin{aligned}
W &= F_e d = eEd \\
&= (1.60 \times 10^{-19}\ \text{C})(2.80 \times 10^3\ \text{N/C})(6.36 \times 10^{-3}\ \text{m}) \\
&= 2.85 \times 10^{-18}\ \text{J}.
\end{aligned}
$$

Potential Energy of Point Charges

It's useful to calculate the work done on a test charge q' when it moves in the electric field caused by a single stationary point charge q.



▲ **FIGURE 18.6** A test charge q' moves radially along a straight line extending from charge q. As it does so, the electric force on it decreases in magnitude.

$$W_{a\rightarrow b} = \frac{k\,q\,q'}{x} = k\,q\,q'\left(\frac{1}{a} - \frac{1}{b}\right)$$

Potential energy of point charges

The potential energy U of a system consisting of a point charge q' located in the field produced by a stationary point charge q, at a distance r from the charge, is

$$U = k\frac{qq'}{r}. \tag{18.8}$$

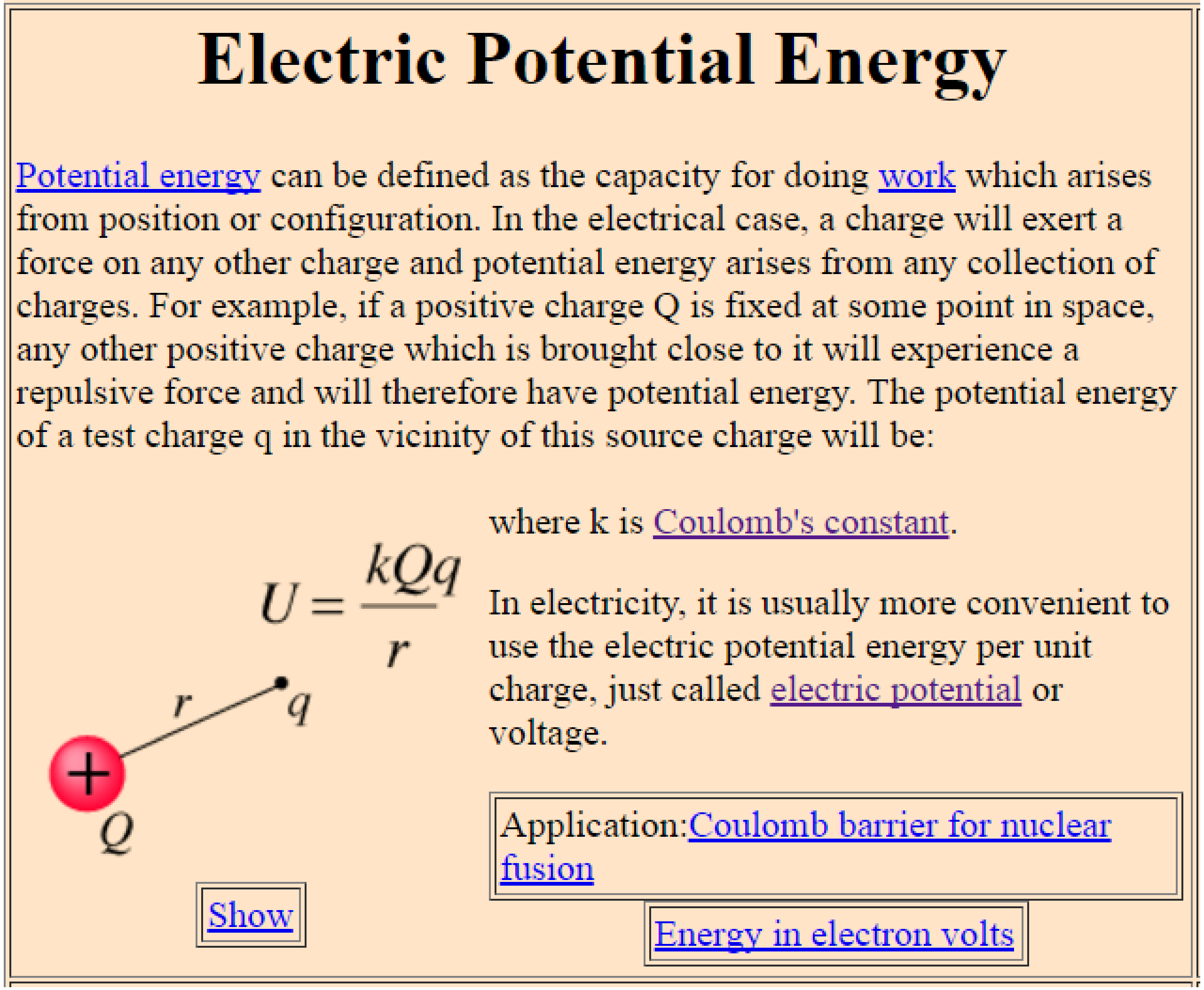
Quantitative Analysis 18.1

Change in potential energy

Consider two positive point charges q_1 and q_2. Their potential energy is defined as zero when they are infinitely far apart, and it increases as they move closer. If q_2 starts at an initial distance r_i from q_1 and moves toward q_1 to a final distance $r_i - \Delta r$ (where Δr is positive), by how much does the system's potential energy change?

SOLUTION The electric potential energy of the two charges depends on the distance r between them: $U = k(q_1 q_2)/r$. Initially, the distance between them is r_i. After q_2 moves a distance Δr toward q_1, the distance is $r_i - \Delta r$. The change in potential energy depends on the reciprocal of these distances, so C must be the answer. More formally, the change in potential energy is

$$\Delta U = U_f - U_i = \frac{kq_1q_2}{r_i - \Delta r} - \frac{kq_1q_2}{r_i}.$$



Electric Potential Energy

Potential energy can be defined as the capacity for doing work which arises from position or configuration. In the electrical case, a charge will exert a force on any other charge and potential energy arises from any collection of charges. For example, if a positive charge Q is fixed at some point in space, any other positive charge which is brought close to it will experience a repulsive force and will therefore have potential energy. The potential energy of a test charge q in the vicinity of this source charge will be:

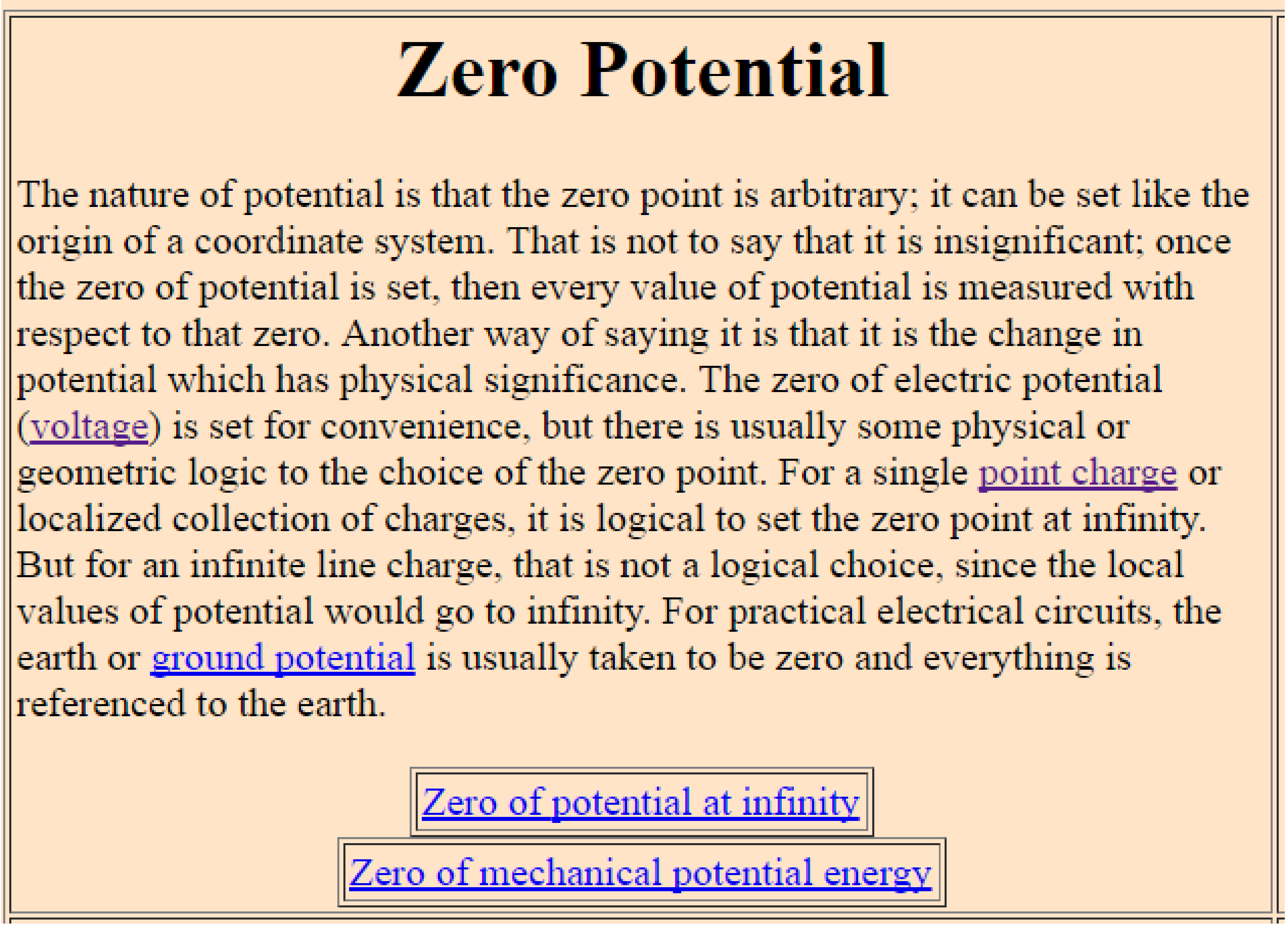
$$U = \frac{kQq}{r}$$

where k is Coulomb's constant.

In electricity, it is usually more convenient to use the electric potential energy per unit charge, just called electric potential or voltage.

Application: Coulomb barrier for nuclear fusion

Energy in electron volts

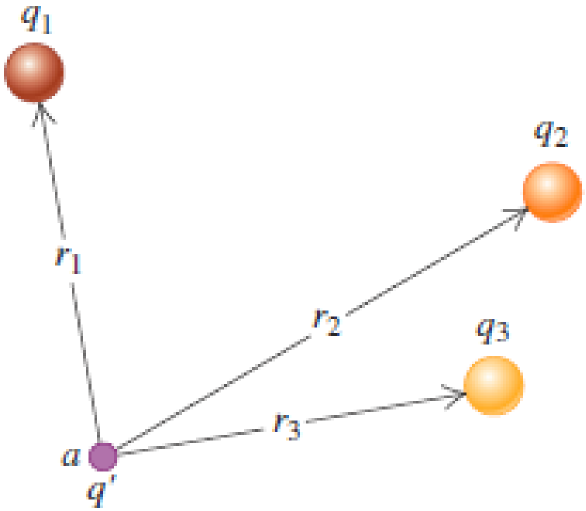


Zero Potential

The nature of potential is that the zero point is arbitrary; it can be set like the origin of a coordinate system. That is not to say that it is insignificant; once the zero of potential is set, then every value of potential is measured with respect to that zero. Another way of saying it is that it is the change in potential which has physical significance. The zero of electric potential (voltage) is set for convenience, but there is usually some physical or geometric logic to the choice of the zero point. For a single point charge or localized collection of charges, it is logical to set the zero point at infinity. But for an infinite line charge, that is not a logical choice, since the local values of potential would go to infinity. For practical electrical circuits, the earth or ground potential is usually taken to be zero and everything is referenced to the earth.

Zero of potential at infinity

Zero of mechanical potential energy

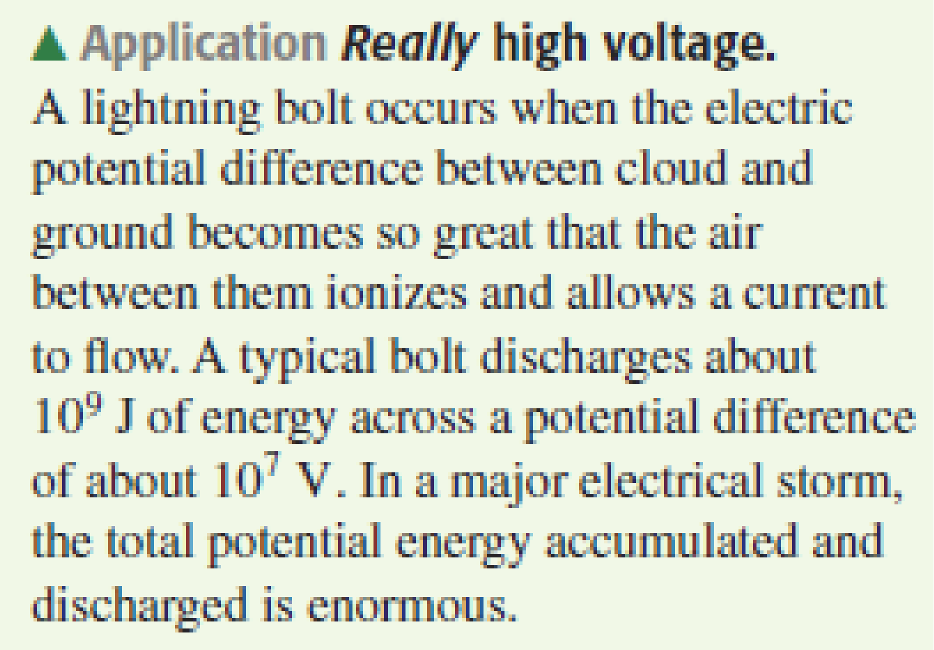


▲ **FIGURE 18.7** Potential energy associated with a charge q' at point a depends on charges q_1, q_2, and q_3 and on their respective distances r_1, r_2, and r_3 from point a.

$$U = kq'\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} + \cdots\right).$$

Making U = 0 at infinity is a convenient reference level for electrostatic problems, but in circuit analysis other reference levels are often more convenient.

18.2 Potential



▲ Application ***Really* high voltage.**
A lightning bolt occurs when the electric potential difference between cloud and ground becomes so great that the air between them ionizes and allows a current to flow. A typical bolt discharges about 10^9 J of energy across a potential difference of about 10^7 V. In a major electrical storm, the total potential energy accumulated and discharged is enormous.

Definition of potential

The electric potential V at any point in an electric field is the electric potential energy U per unit charge associated with a test charge q' at that point:

$$V = \frac{U}{q'}, \qquad \text{or} \qquad U = q'V. \tag{18.10}$$

Potential energy and charge are both scalars, so potential is a scalar quantity.

Unit: From Equation 18.10, the units of potential are energy divided by charge. The SI unit of potential, 1 J/C, is called one **volt** (1 V), in honor of the Italian scientist Alessandro Volta (1745–1827):

$$1\ \text{V} = 1\ \text{volt} = 1\ \text{J/C} = 1\ \text{joule/coulomb}.$$

In the context of electric circuits, potential is often called voltage.
For instance, a 9 V battery has a difference in electric potential (potential difference) of 9 V between its two terminals. A 20,000 V power line has a potential difference of 20,000 V between itself and the ground.

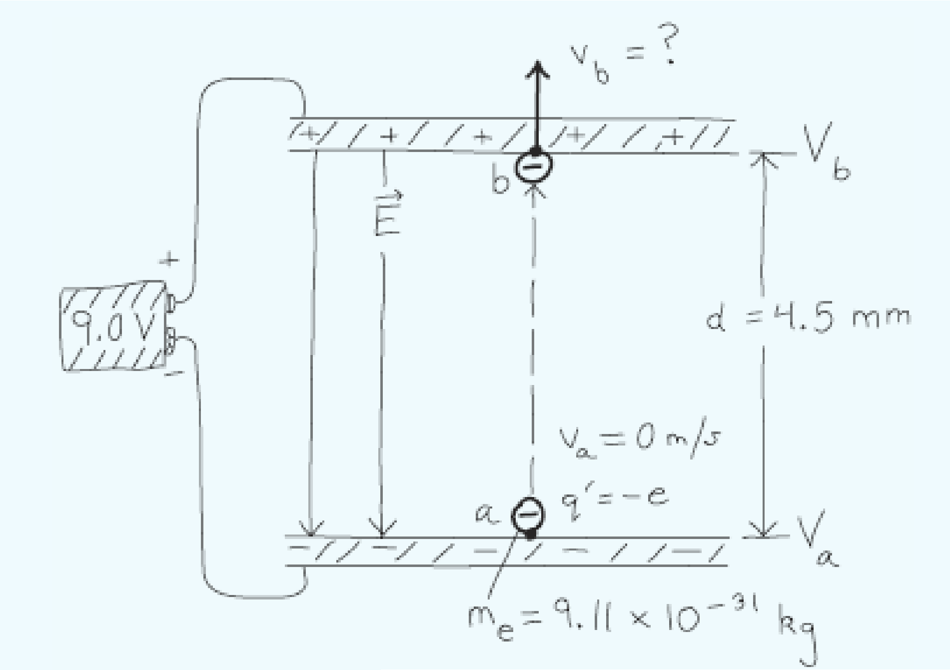
To put Equation 18.2 on a "work per unit charge" basis, we divide both sides by q', obtaining

$$\frac{W_{a \to b}}{q'} = \frac{U_a}{q'} - \frac{U_b}{q'} = V_a - V_b, \tag{18.11}$$

where $V_a = U_a/q'$ is the potential energy per unit charge at point a and V_b is that at b. We call V_a and V_b the *potential at point a* and *potential at point b*, respectively. The potential difference $V_a - V_b$ is called *the potential of a with respect to b*.

EXAMPLE 18.2 Parallel plates and conservation of energy

A 9.0 V battery is connected across two large parallel plates that are separated by 4.5 mm of air, creating a potential difference of 9.0 V between the plates. **(a)** What is the electric field in the region between the plates? **(b)** An electron is released from rest at the negative plate. If the only force on the electron is the electric force exerted by the electric field of the plates, what is the speed of the electron as it reaches the positive plate? The mass of an electron is $m_e = 9.11 \times 10^{-31}$ kg.



$$E = \frac{V_b - V_a}{d} = \frac{9.0\ \text{V}}{4.5 \times 10^{-3}\ \text{m}} = 2.0 \times 10^3\ \text{V/m}.$$

Part (b): Conservation of energy applied to points a and b at the corresponding plates gives

$$K_a + U_a = K_b + U_b.$$

Also, $U = q'V$, where $q' = -e$, the charge of an electron. Using this expression to replace U in the conservation-of-energy equation gives

$$K_a + q'V_a = K_b + q'V_b.$$

The electron is released from rest from point a, so $K_a = 0$. We next solve for K_b:

$$\begin{aligned} K_b &= q'(V_a - V_b) = -e(V_a - V_b) = +e(V_b - V_a) \\ &= (1.60 \times 10^{-19}\,\mathrm{C})(9.0\,\mathrm{V}) \\ &= 1.44 \times 10^{-18}\,\mathrm{J}. \end{aligned}$$

Then $K_b = \frac{1}{2}m_e v_b^2$ gives

$$v_b = \sqrt{\frac{2K_b}{m_e}} = \sqrt{\frac{2(1.44 \times 10^{-18}\,\mathrm{J})}{9.11 \times 10^{-31}\,\mathrm{kg}}} = 1.8 \times 10^6\,\mathrm{m/s}.$$

Potential of a point charge

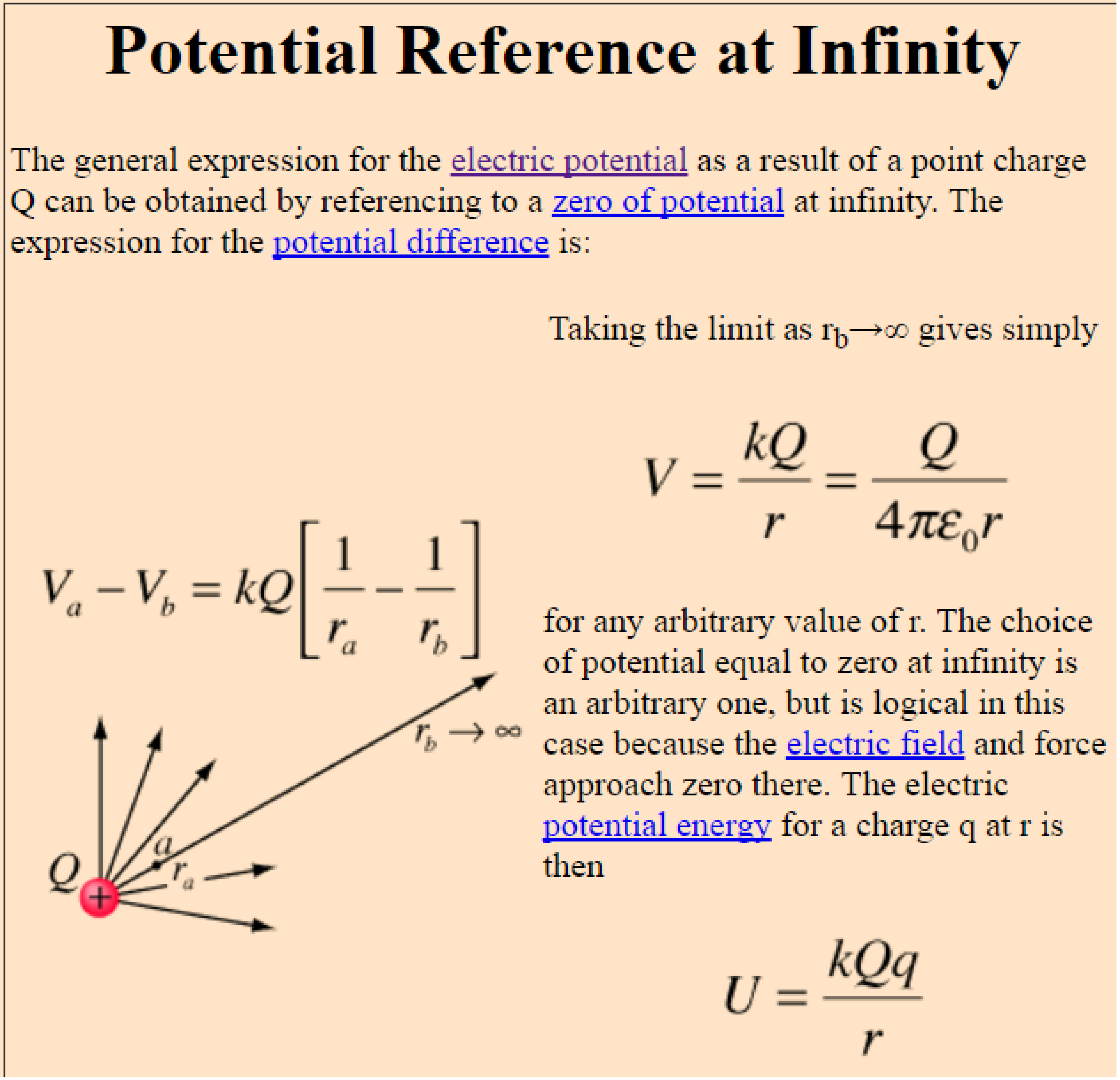
When a test charge q' is a distance r from a point charge q, the potential V is

$$V = \frac{U}{q'} = k\frac{q}{r}, \tag{18.12}$$

where k is the same constant as in Coulomb's law (Equation 17.1).

Similarly, to find the potential V at a point due to any collection of point charges $q_1, q_2, q_3, \ldots$ at distances $r_1, r_2, r_3, \ldots$, respectively, from q', we divide Equation 18.9 by q':

$$V = \frac{U}{q'} = k\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} + \cdots\right). \tag{18.13}$$



Potential Reference at Infinity

The general expression for the electric potential as a result of a point charge Q can be obtained by referencing to a zero of potential at infinity. The expression for the potential difference is:

$$V_a - V_b = kQ\left[\frac{1}{r_a} - \frac{1}{r_b}\right]$$

Taking the limit as $r_b \to \infty$ gives simply

$$V = \frac{kQ}{r} = \frac{Q}{4\pi\varepsilon_0 r}$$

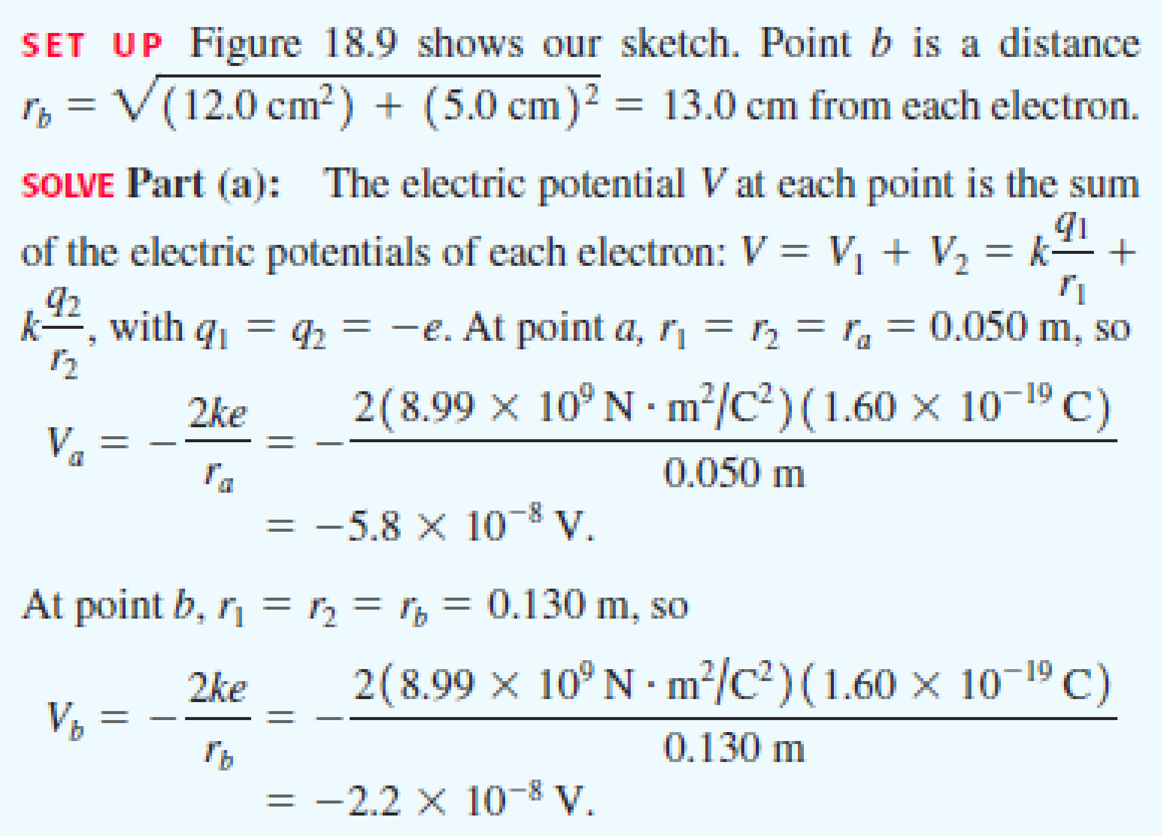
for any arbitrary value of r. The choice of potential equal to zero at infinity is an arbitrary one, but is logical in this case because the electric field and force approach zero there. The electric potential energy for a charge q at r is then

$$U = \frac{kQq}{r}$$

EXAMPLE 18.3 Potential of two point charges

Two electrons are held in place 10.0 cm apart. Point *a* is midway between the two electrons, and point *b* is 12.0 cm directly above point *a*.

Calculate the electric potential at point *a* and at point *b*.



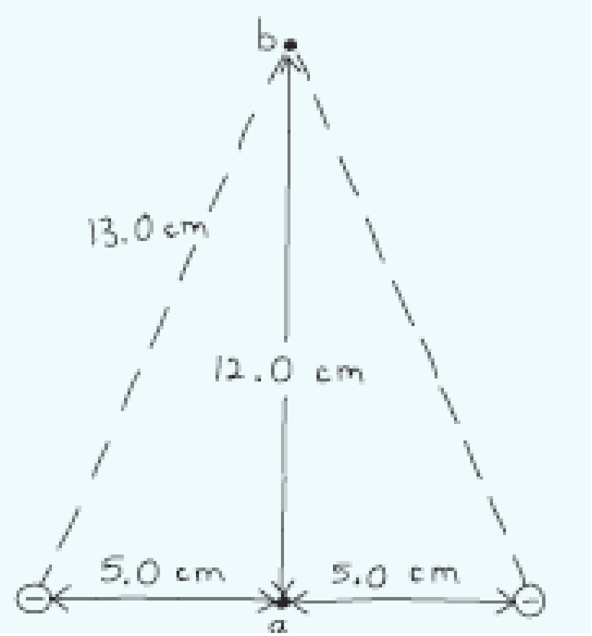
SET UP Figure 18.9 shows our sketch. Point b is a distance $r_b = \sqrt{(12.0\ \text{cm}^2) + (5.0\ \text{cm})^2} = 13.0\ \text{cm}$ from each electron.

SOLVE Part (a): The electric potential V at each point is the sum of the electric potentials of each electron: $V = V_1 + V_2 = k\frac{q_1}{r_1} + k\frac{q_2}{r_2}$, with $q_1 = q_2 = -e$. At point a, $r_1 = r_2 = r_a = 0.050\ \text{m}$, so

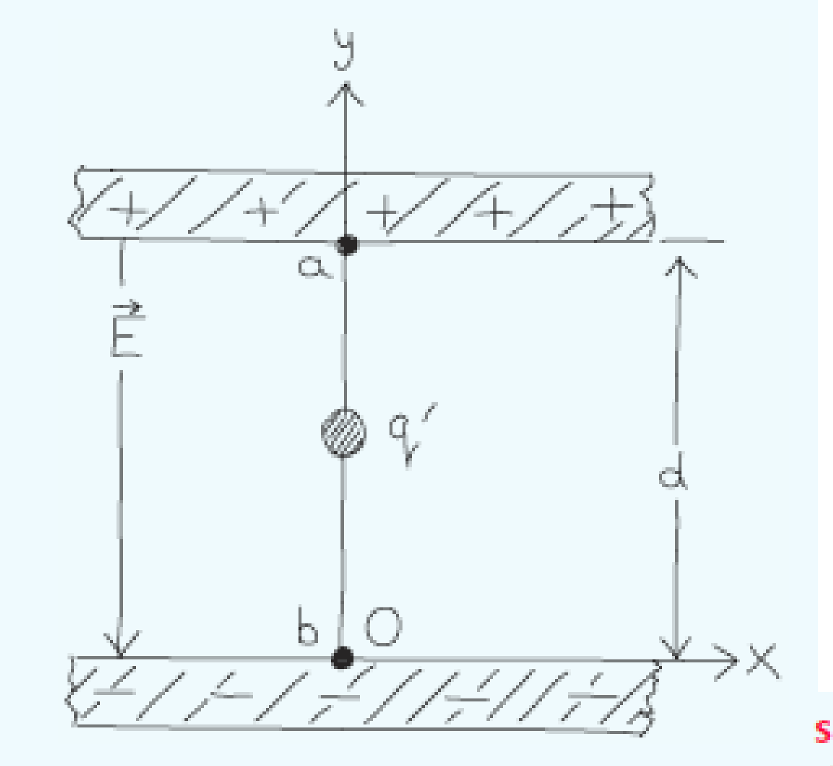
$$V_a = -\frac{2ke}{r_a} = -\frac{2(8.99 \times 10^9\ \text{N} \cdot \text{m}^2/\text{C}^2)(1.60 \times 10^{-19}\ \text{C})}{0.050\ \text{m}}$$
$$= -5.8 \times 10^{-8}\ \text{V}.$$

At point b, $r_1 = r_2 = r_b = 0.130\ \text{m}$, so

$$V_b = -\frac{2ke}{r_b} = -\frac{2(8.99 \times 10^9\ \text{N} \cdot \text{m}^2/\text{C}^2)(1.60 \times 10^{-19}\ \text{C})}{0.130\ \text{m}}$$
$$= -2.2 \times 10^{-8}\ \text{V}.$$

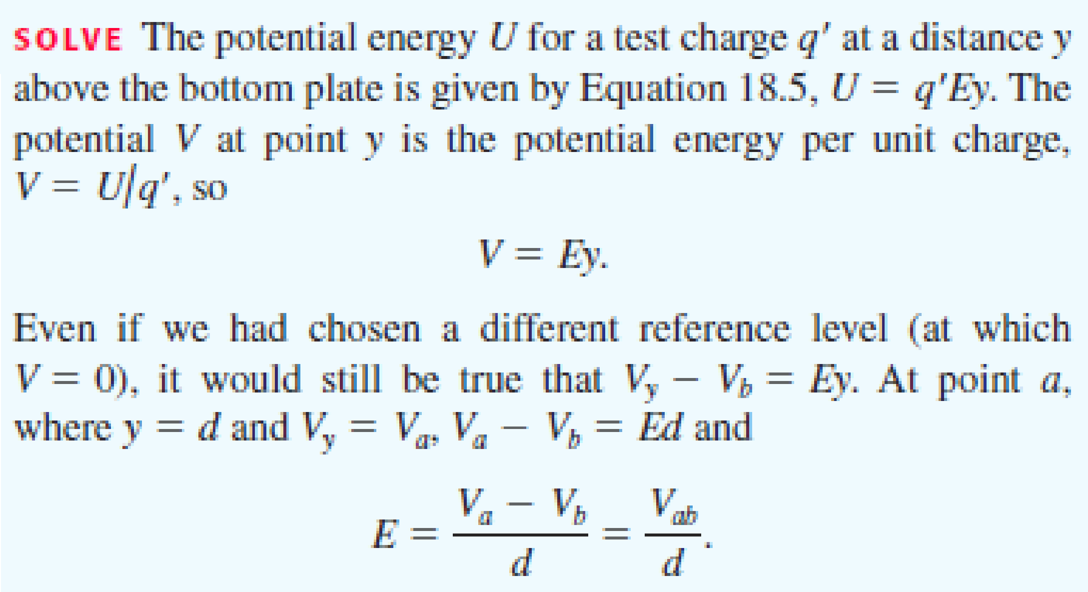


Parallel plates



Remember that potential is simply *potential energy per unit charge*.

We **choose** the potential *V* to be zero at y = 0 (point *b* in our sketch).



SOLVE The potential energy U for a test charge q' at a distance y above the bottom plate is given by Equation 18.5, $U = q'Ey$. The potential V at point y is the potential energy per unit charge, $V = U/q'$, so

$$V = Ey.$$

Even if we had chosen a different reference level (at which $V = 0$), it would still be true that $V_y - V_b = Ey$. At point a, where $y = d$ and $V_y = V_a$, $V_a - V_b = Ed$ and

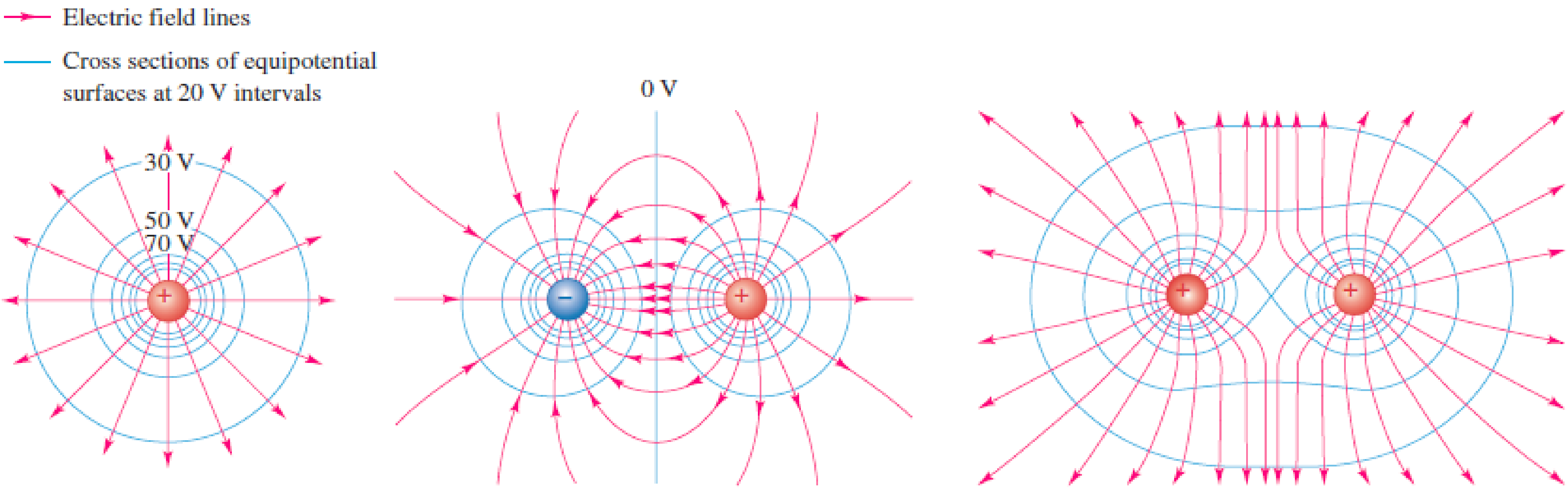
$$E = \frac{V_a - V_b}{d} = \frac{V_{ab}}{d}.$$

18.3 Equipotential Surfaces

An equipotential surface is defined as a surface on which the potential is the same at every point.

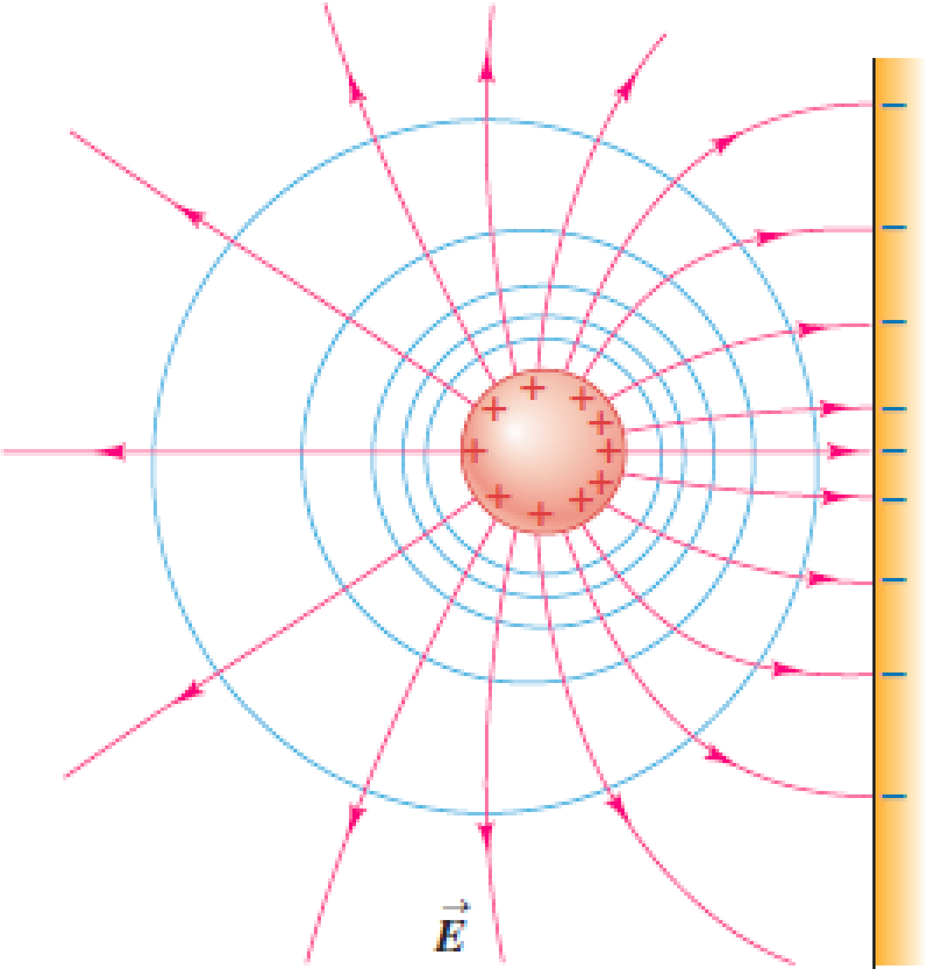
No point can be at two different potentials, so equipotential surfaces for different potentials can never touch or intersect.

The potential energy for a test charge is the same at every point on a given equipotential surface, so the field does no work on a test charge when it moves from point to point on such a surface.



(a) A single positive charge (b) An electric dipole (c) Two equal positive charges

▲ **FIGURE 18.11** Equipotential surfaces and electric field lines for assemblies of point charges. How would the diagrams change if the charges were reversed?



▲ **FIGURE 18.13** When charges are at rest, a conducting surface is always an equipotential surface. Field lines are perpendicular to a conducting surface.

E must be perpendicular to the surface at every point. Field lines and equipotential surfaces are always mutually perpendicular.

We can prove that when all charges are at rest, the electric field just outside a conductor must be perpendicular to the surface at every point.

It follows that, in an electrostatic situation, a conducting surface is always an equipotential surface.

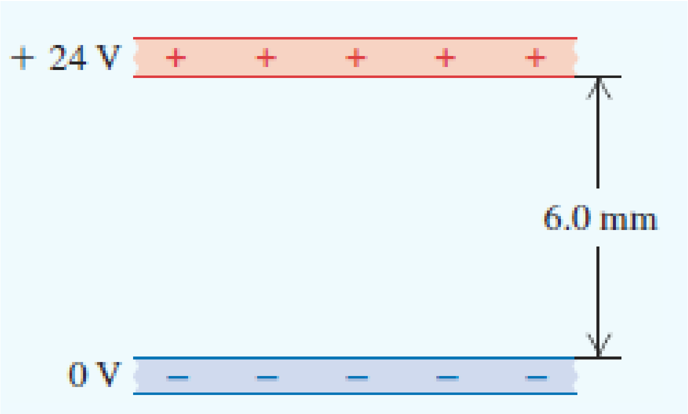
Electric field represented as potential gradient

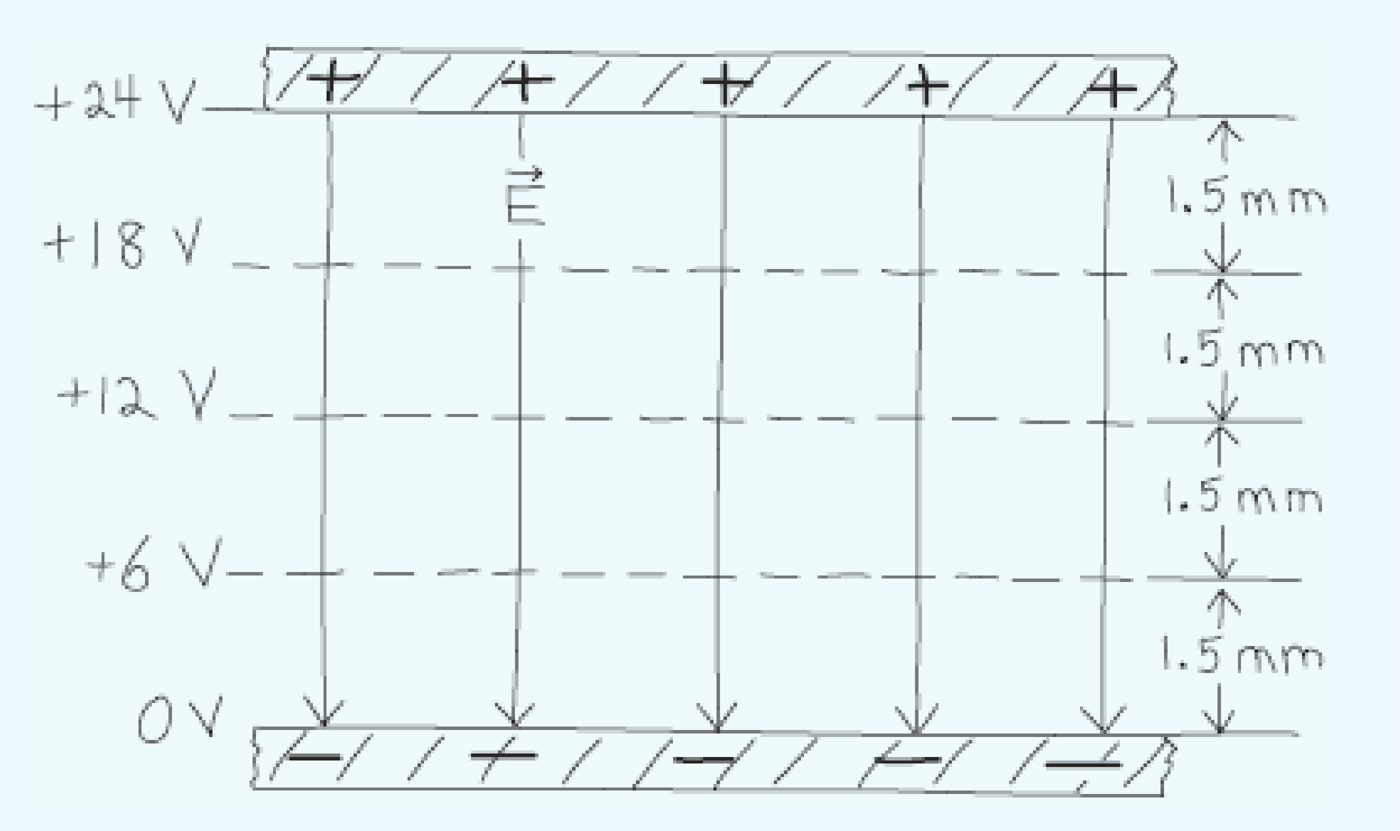
The magnitude of the electric field at any point on an equipotential surface equals the rate of change of potential, ΔV, with distance Δs as the point moves perpendicularly from the surface to an adjacent one a distance Δs away:

$$E = -\frac{\Delta V}{\Delta s}. \tag{18.14}$$

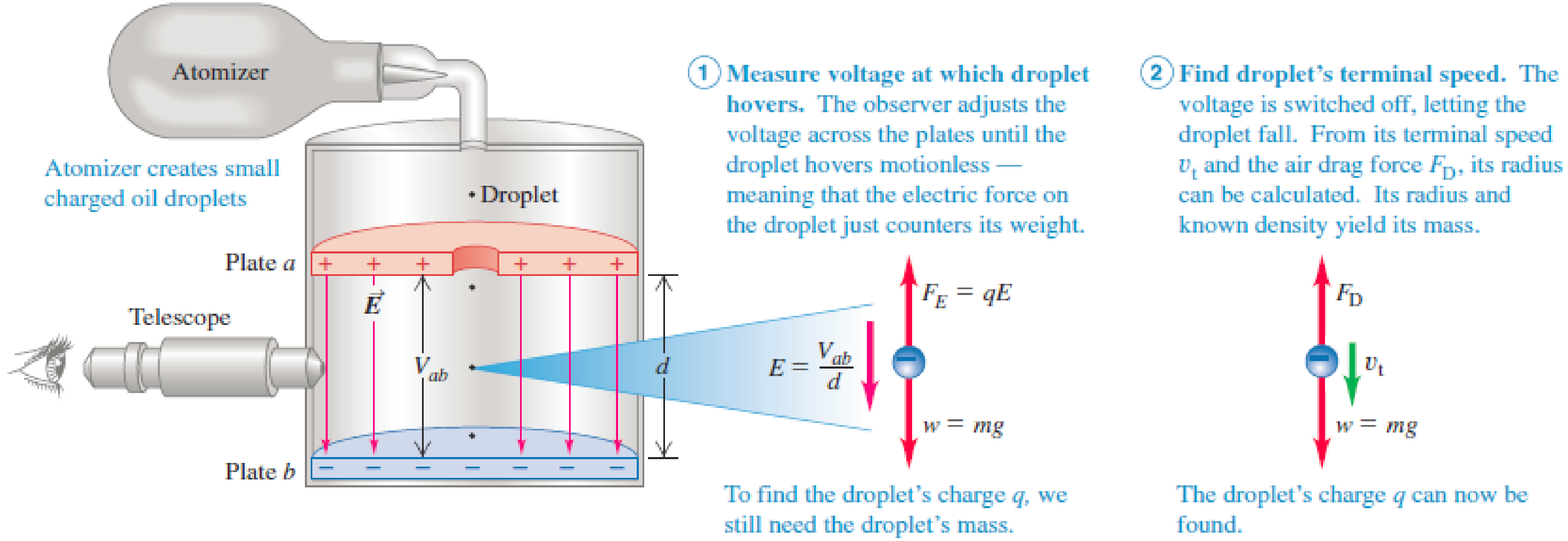
The negative sign shows that when a point moves in the direction of the electric field, the potential decreases. The quantity $\Delta V/\Delta s$, representing a rate of change of V with distance, is called the **potential gradient.** We see that this is an alternative name for electric field.

Equipotential surfaces within a capacitor





18.4 The Millikan Oil-Drop Experiment



Atomizer creates small charged oil droplets

① **Measure voltage at which droplet hovers.** The observer adjusts the voltage across the plates until the droplet hovers motionless — meaning that the electric force on the droplet just counters its weight.

② **Find droplet's terminal speed.** The voltage is switched off, letting the droplet fall. From its terminal speed v_t and the air drag force F_D, its radius can be calculated. Its radius and known density yield its mass.

To find the droplet's charge q, we still need the droplet's mass.

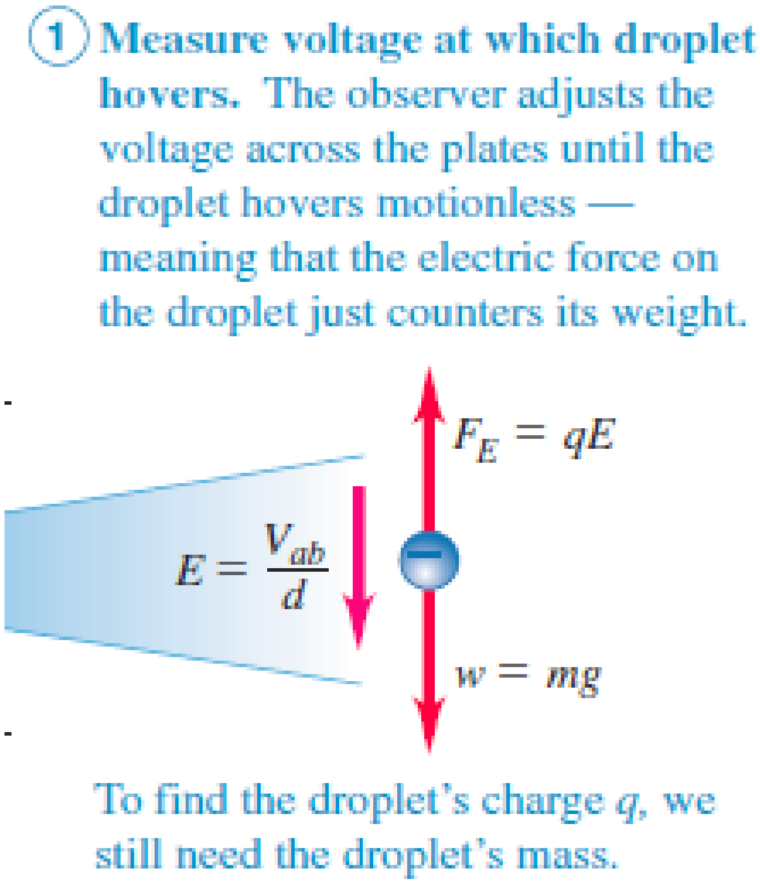
The droplet's charge q can now be found.

(a) Schematic diagram of apparatus

(b)

(c)

▲ **FIGURE 18.16** The Millikan oil-drop experiment, which demonstrated that charge is quantized and provided the first determination of e.



① **Measure voltage at which droplet hovers.** The observer adjusts the voltage across the plates until the droplet hovers motionless — meaning that the electric force on the droplet just counters its weight.

To find the droplet's charge q, we still need the droplet's mass.

Droplet stationary

$$qE = mg$$

So

$$q = \frac{mg}{E}$$

we can find E from

$$E = \frac{V}{d}$$

and find m from

$$m = \rho V = \rho \frac{4\pi r^2}{3}$$

and r from terminal velocity

Now we can measure the charge on a droplet.
Each droplet will have a different charge (+ or -).

So with MANY measurements and knowing that

$$q = \pm n\ e$$

We can determine e.

(Where n is an integer and e is the charge on an electron)

An electron has a charge of

$$1.602\ x\ 10^{-19}\ C$$

Electrovolt

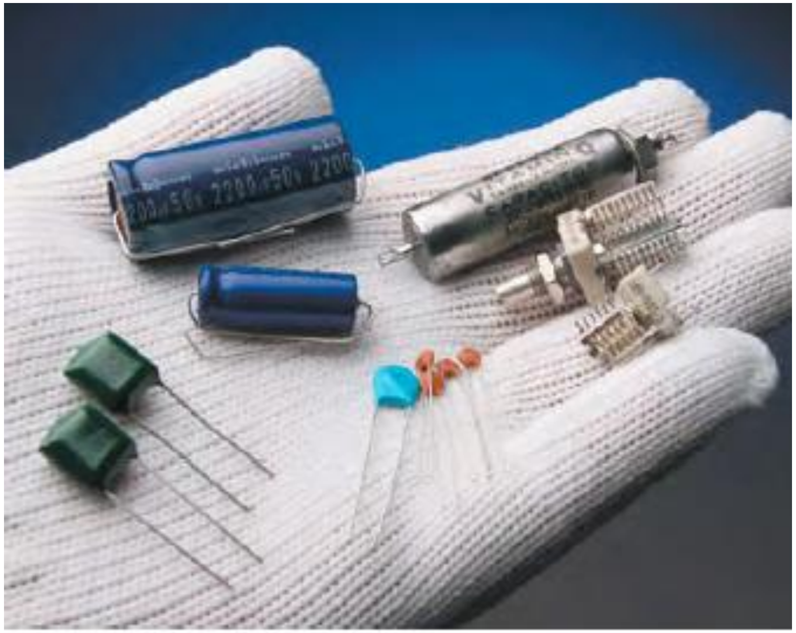
An electrovolt is a unit of energy

If we move an electron through a potential difference of 1V

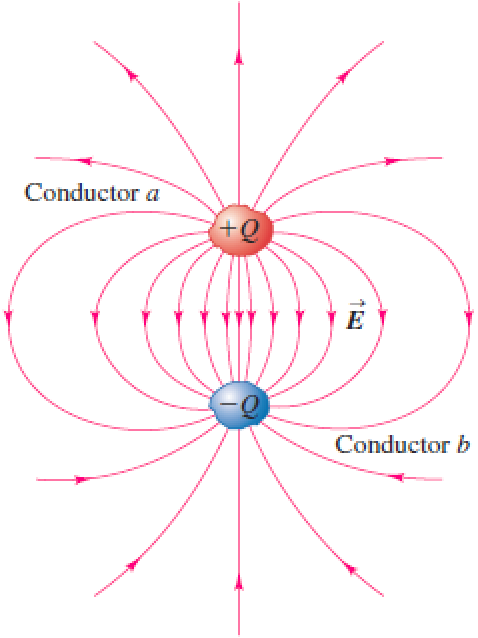
$\Delta U = qV = 1.602\ x\ 10^{-19}$ x 1

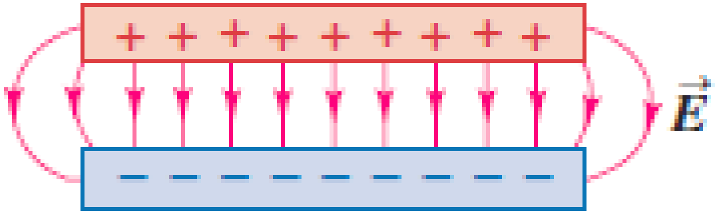
$$1\ eV = 1.602\ x\ 10^{-19}\ \mathrm{J}$$

18.5 Capacitors



▲ **FIGURE 18.17** An assortment of practical capacitors.





Definition of capacitance

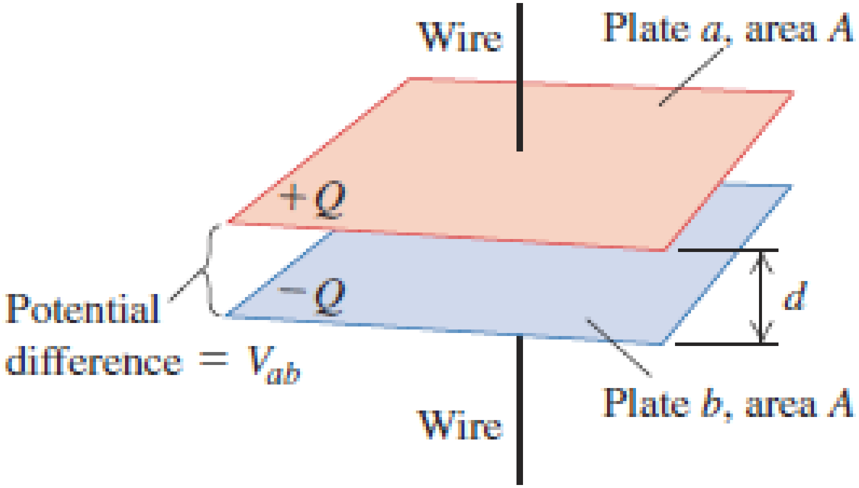
The capacitance C of a capacitor is the ratio of the magnitude of the charge Q on *either* conductor to the magnitude of the potential difference V_{ab} between the conductors:

$$C = \frac{Q}{V_{ab}}. \tag{18.15}$$

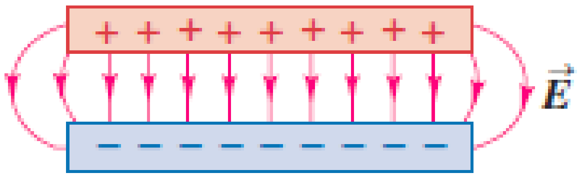
Unit: The SI unit of capacitance is called 1 **farad** (1 F), in honor of Michael Faraday. From Equation 18.15, 1 farad is equal to 1 *coulomb per volt* $(1\ \mathrm{C/V})$: $1\ \mathrm{F} = 1\ \mathrm{C/V}$.

In circuit diagrams, a capacitor is represented by either of these symbols:

Parallel plate capacitors



(a) A basic parallel-plate capacitor



(b) Electric field due to a parallel-plate capacitor

▲ **FIGURE 18.19** The elements of a parallel-plate capacitor.

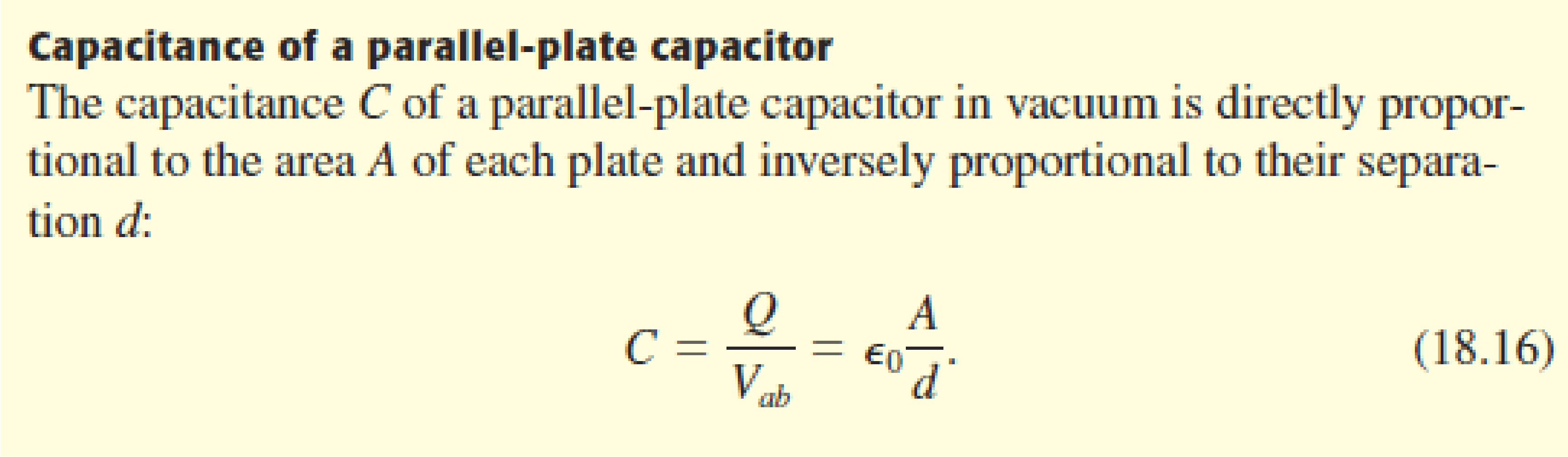
We can define the surface charge density as

$$\sigma = \frac{Q}{A}$$

Where Q is the charge on the plates and A is the area of the plates

$$E = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 A} = \frac{V}{d}$$

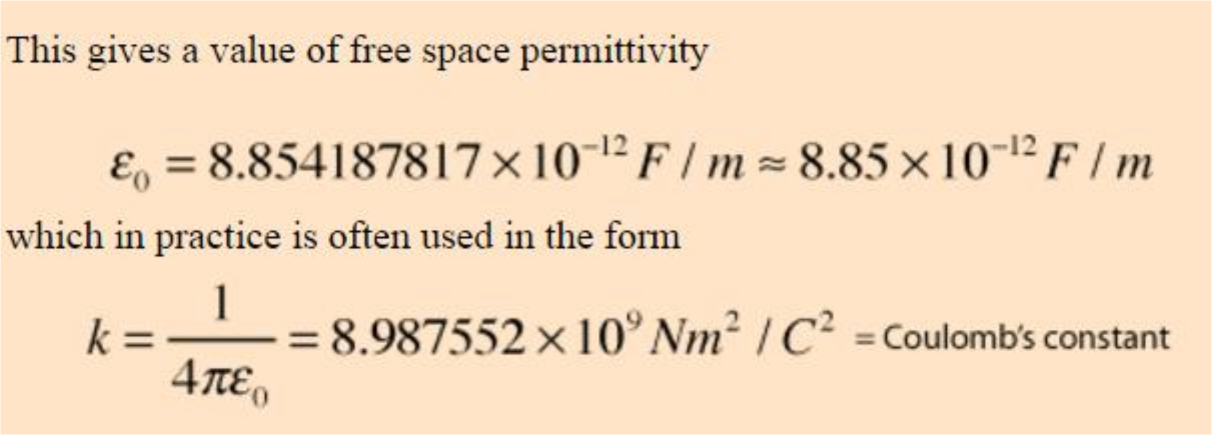
$k = 1/4\pi\epsilon_0$, where $\epsilon_0 = 8.854 \times 10^{-12}\ \mathrm{C^2/N \cdot m^2}$.



Capacitance of a parallel-plate capacitor

The capacitance C of a parallel-plate capacitor in vacuum is directly proportional to the area A of each plate and inversely proportional to their separation d:

$$C = \frac{Q}{V_{ab}} = \epsilon_0 \frac{A}{d}. \qquad (18.16)$$

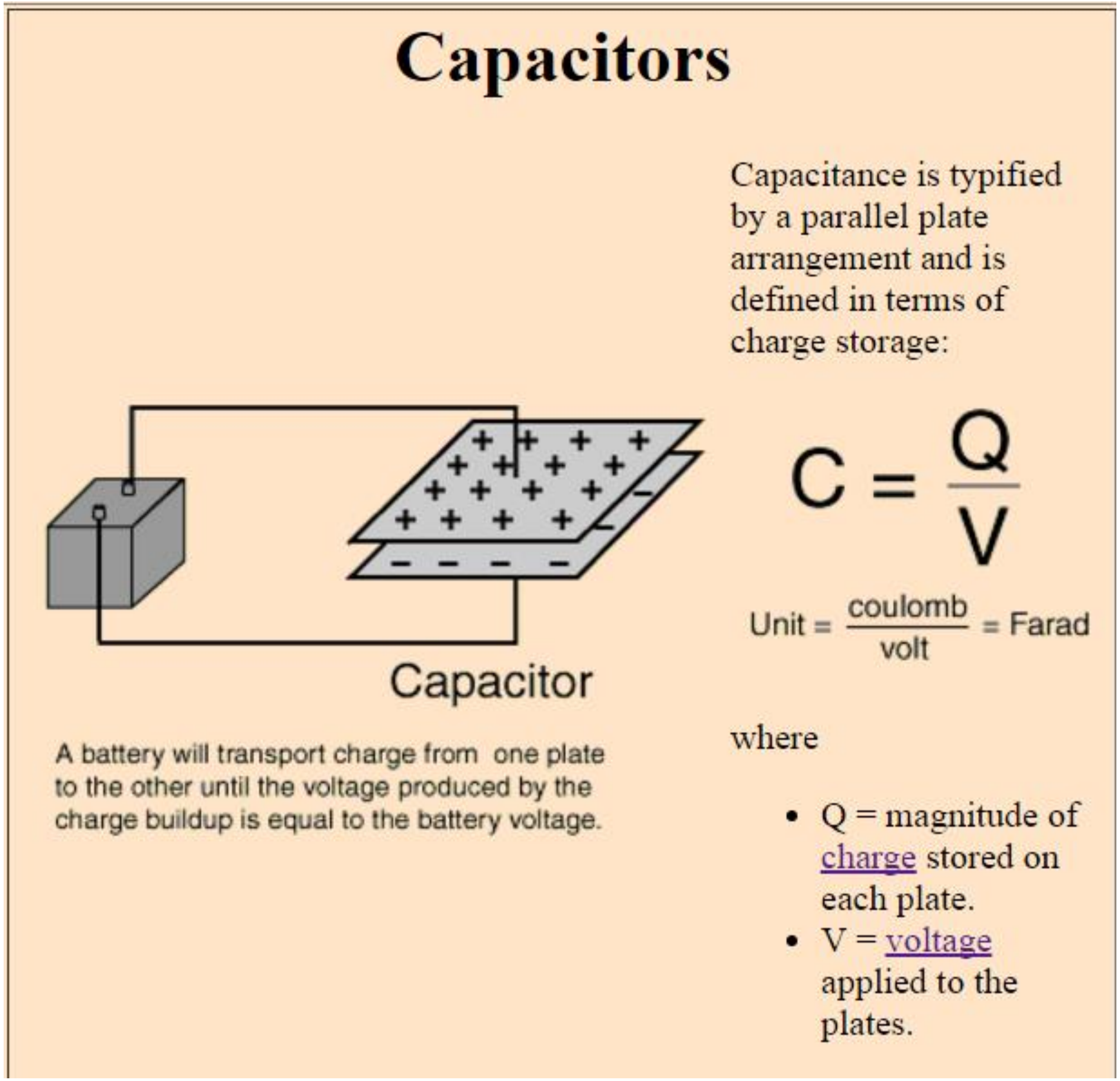


This gives a value of free space permittivity

$$\varepsilon_0 = 8.854187817 \times 10^{-12} F/m \approx 8.85 \times 10^{-12} F/m$$

which in practice is often used in the form

$$k = \frac{1}{4\pi\varepsilon_0} = 8.987552 \times 10^{9} Nm^2/C^2$$ = Coulomb's constant



Capacitors

Capacitance is typified by a parallel plate arrangement and is defined in terms of charge storage:

$$C = \frac{Q}{V}$$

$$\text{Unit} = \frac{\text{coulomb}}{\text{volt}} = \text{Farad}$$

Capacitor

A battery will transport charge from one plate to the other until the voltage produced by the charge buildup is equal to the battery voltage.

where

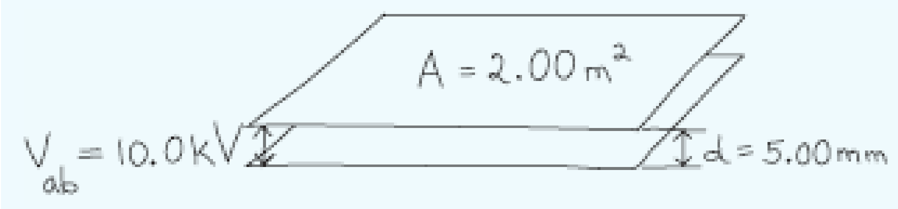
- Q = magnitude of charge stored on each plate.
- V = voltage applied to the plates.

EXAMPLE 18.7 Properties of a parallel-plate capacitor

The plates of a parallel-plate capacitor are 5.00 mm apart and 2.00 m^2 in area. A potential difference of 10.0 kV is applied across the capacitor. Compute (a) the capacitance, (b) the charge on each plate, and (c) the magnitude of the electric field in the region between the plates.

a

$$C = \frac{\epsilon_0 A}{d} = \frac{(8.85 \times 10^{-12}\ \mathrm{F/m})(2.00\ \mathrm{m}^2)}{5.00 \times 10^{-3}\ \mathrm{m}}$$
$$= 3.54 \times 10^{-9}\ \mathrm{F} = 0.00354\ \mu\mathrm{F}.$$



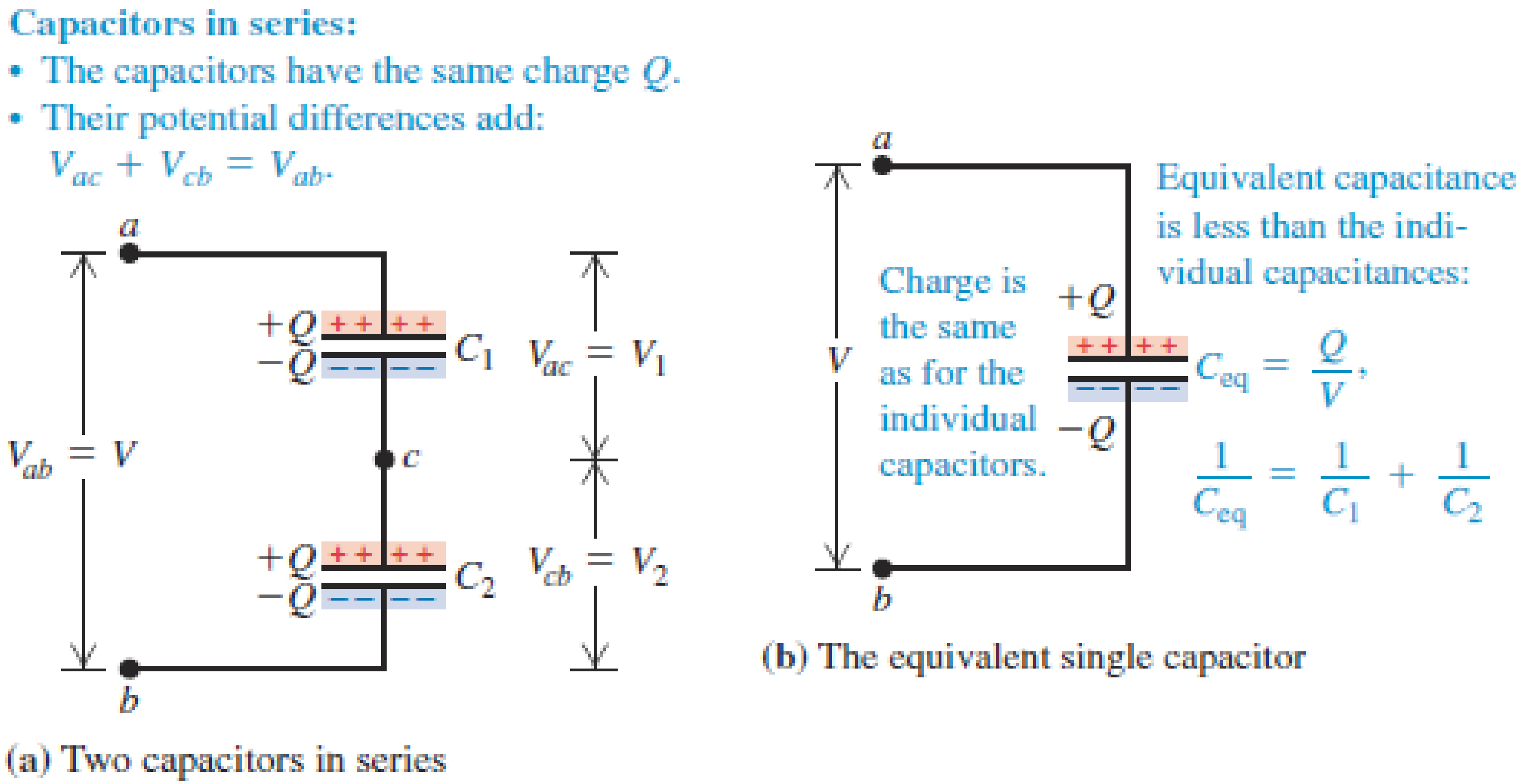
b

$$Q = CV_{ab} = (3.54 \times 10^{-9}\ \mathrm{F})(1.00 \times 10^{4}\ \mathrm{V})$$
$$= 3.54 \times 10^{-5}\ \mathrm{C} = 35.4\ \mu\mathrm{C}.$$

c

$$E = \frac{V_{ab}}{d} = \frac{1.00 \times 10^{4}\ \mathrm{V}}{5.00 \times 10^{-3}\ \mathrm{m}} = 2.00 \times 10^{6}\ \mathrm{V/m}.$$

18.6 Capacitors in Series and in Parallel



Capacitors in series:

- The capacitors have the same charge Q.
- Their potential differences add: $V_{ac} + V_{cb} = V_{ab}$.

Charge is the same as for the individual capacitors.

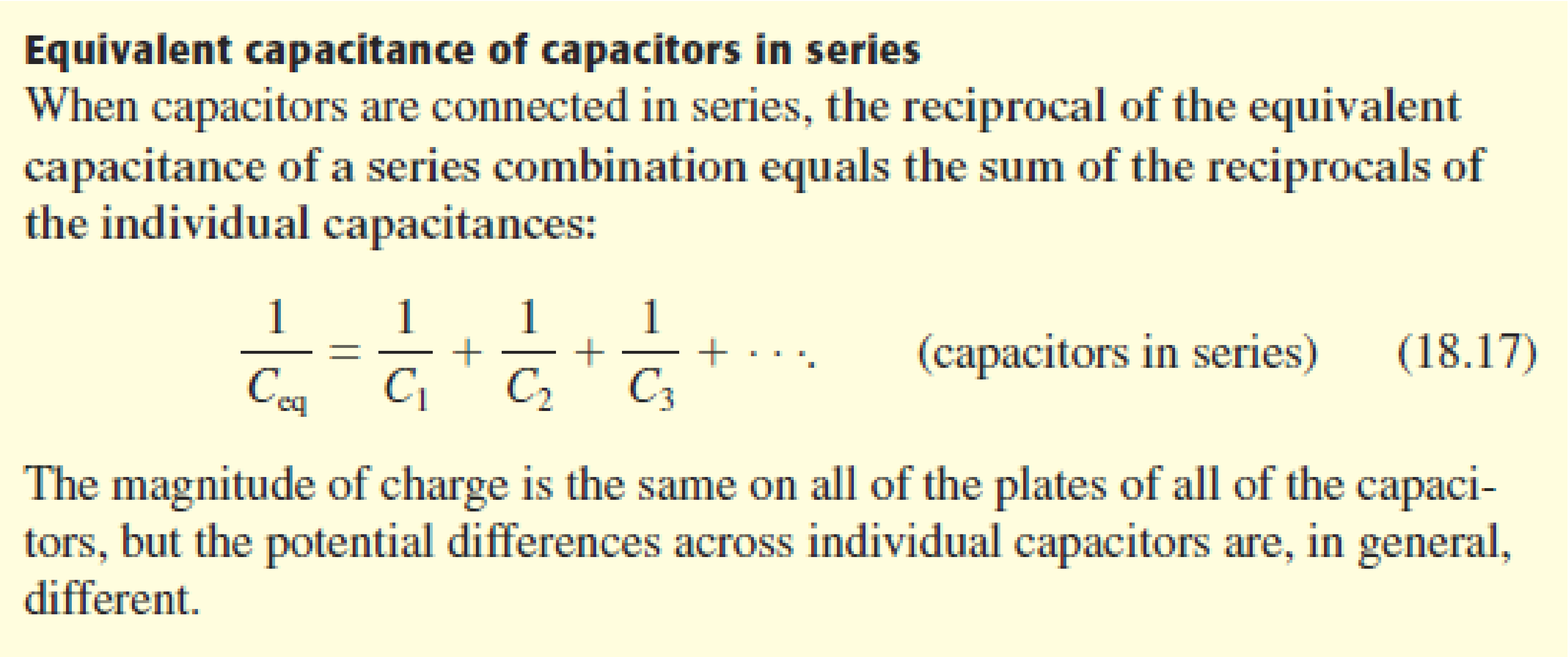
Equivalent capacitance is less than the individual capacitances:

$$C_{eq} = \frac{Q}{V},$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

(a) Two capacitors in series

(b) The equivalent single capacitor



Equivalent capacitance of capacitors in series

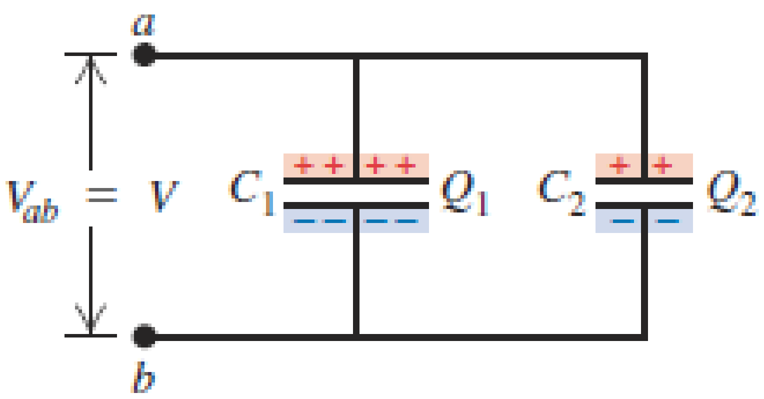
When capacitors are connected in series, the reciprocal of the equivalent capacitance of a series combination equals the sum of the reciprocals of the individual capacitances:

$$\frac{1}{C_{\mathrm{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots. \qquad \text{(capacitors in series)} \qquad (18.17)$$

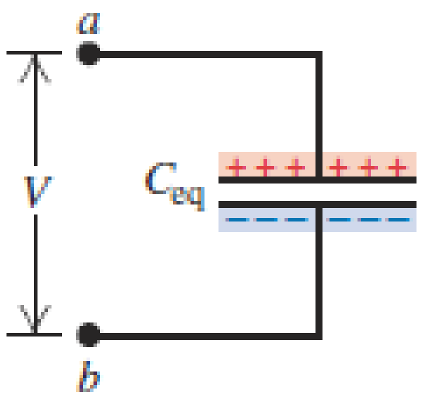
The magnitude of charge is the same on all of the plates of all of the capacitors, but the potential differences across individual capacitors are, in general, different.

Capacitors in parallel:

- The capacitors have the same potential V.
- The charge on each capacitor depends on its capacitance: $Q_1 = C_1V$, $Q_2 = C_2V$.



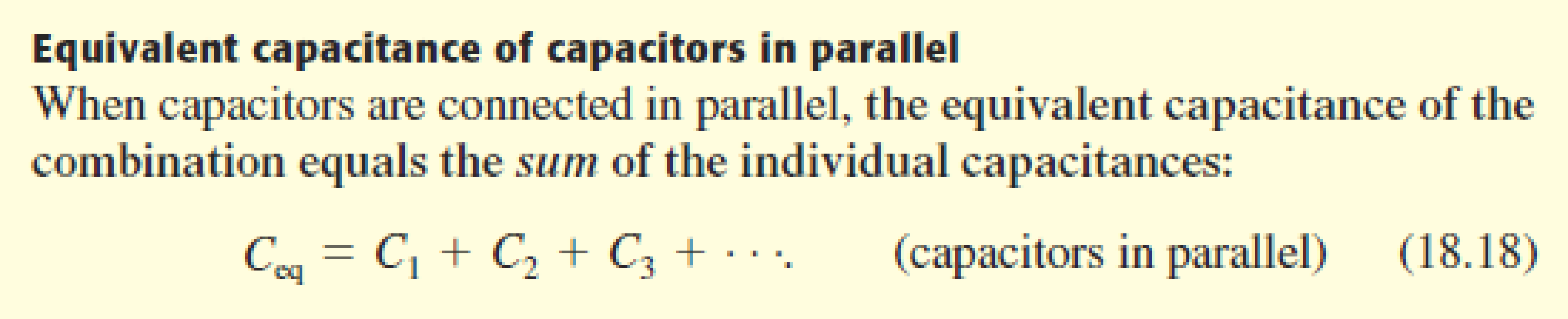
(a) Capacitors connected in parallel



Charge is the sum of the individual charges:
$Q = Q_1 + Q_2$

Equivalent capacitance:
$C_{eq} = C_1 + C_2$.

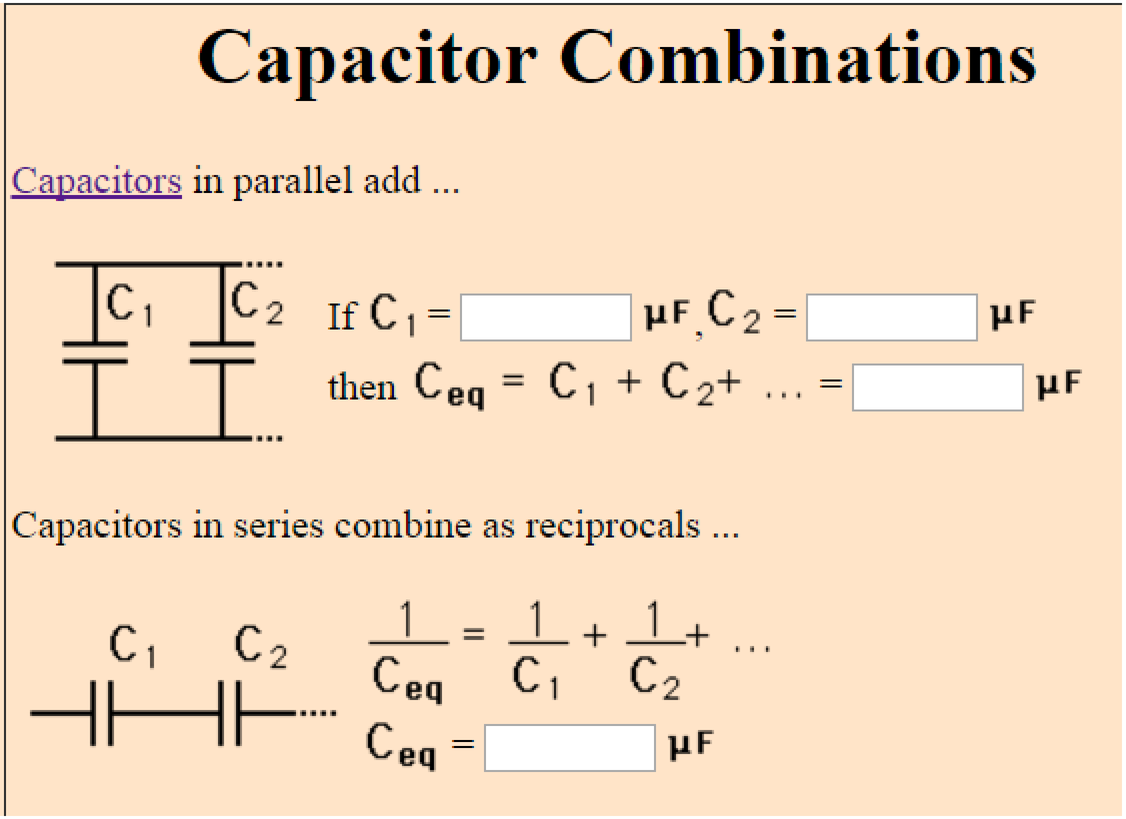
(b) The equivalent single capacitor



Equivalent capacitance of capacitors in parallel

When capacitors are connected in parallel, the equivalent capacitance of the combination equals the *sum* of the individual capacitances:

$$C_{eq} = C_1 + C_2 + C_3 + \cdots. \qquad \text{(capacitors in parallel)} \qquad (18.18)$$



Capacitor Combinations

Capacitors in parallel add ...

If $C_1 =$ ___ μF, $C_2 =$ ___ μF

then $C_{eq} = C_1 + C_2 + \dots =$ ___ μF

Capacitors in series combine as reciprocals ...

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \dots$$

$C_{eq} =$ ___ μF

EXAMPLE 18.8 Capacitors in series and in parallel

Two capacitors, one with $C_1 = 6.0\ \mu\text{F}$ and the other with $C_2 = 3.0\ \mu\text{F}$, are connected to a potential difference of $V_{ab} = 18\ \text{V}$. Find the equivalent capacitance, and find the charge and potential difference for each capacitor when the two capacitors are connected (a) in series and (b) in parallel.

SOLUTION

SET UP Figure 18.24 shows our sketches of the two situations. We remember that when capacitors are connected in series, the charges are the same on the two capacitors and the potential differences add. When they are connected in parallel, the potential differences are the same and the charges add.

SOLVE Part (a): The equivalent capacitance for the capacitors in series is given by Equation 18.17:

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{C_1 + C_2}{C_1 C_2}.$$

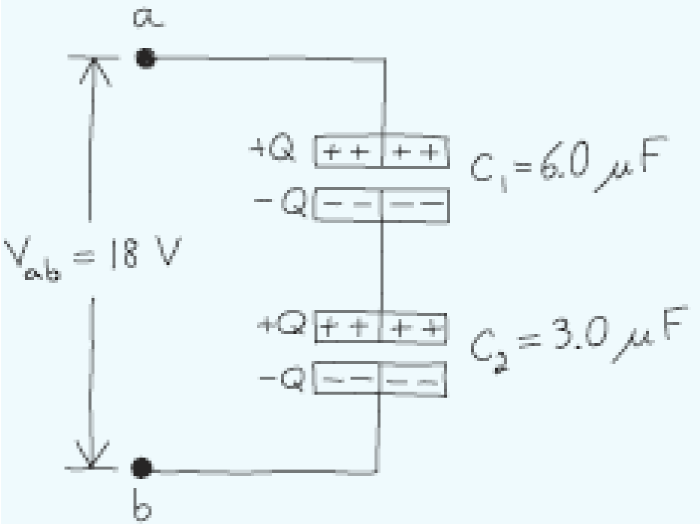
Thus,

$$C_{\text{eq}} = \frac{C_1 C_2}{C_1 + C_2} = \frac{(6.0\ \mu\text{F})(3.0\ \mu\text{F})}{6.0\ \mu\text{F} + 3.0\ \mu\text{F}} = 2.0\ \mu\text{F}.$$

The charge is $Q = C_{\text{eq}}V = (2.0\ \mu\text{F})(18\ \text{V}) = 36\ \mu\text{C}$, the same for both capacitors. The voltages are

$$V_1 = \frac{Q}{C_1} = \frac{36\ \mu\text{C}}{6.0\ \mu\text{F}} = 6.0\ \text{V} \quad \text{and} \quad V_2 = \frac{Q}{C_2} = \frac{36\ \mu\text{C}}{3.0\ \mu\text{F}} = 12.0\ \text{V}.$$

Note that $V_1 + V_2 = V_{ab}$ (i.e., 6.0 V + 12 V = 18 V).

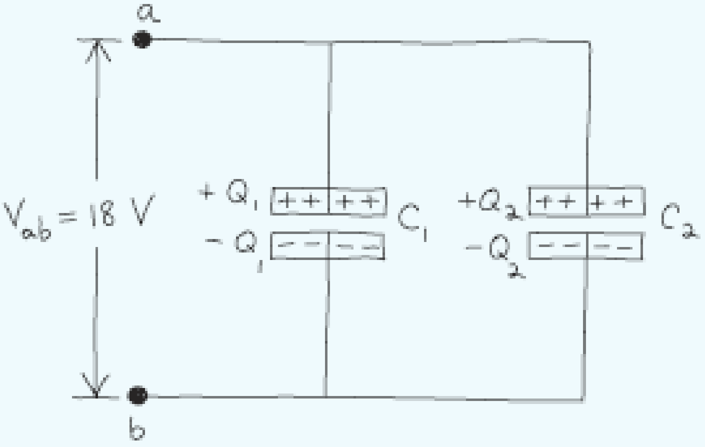


Part (b): When capacitors are connected in parallel, the potential differences are the same and the charges add. The equivalent capacitance is given by Equation 18.18:

$$C_{eq} = C_1 + C_2 = 6.0\ \mu F + 3.0\ \mu F = 9.0\ \mu F.$$

The potential difference for the equivalent capacitor is equal to the potential difference for each capacitor:

$$V_1 = V_2 = V_{ab} = 18\ V.$$



The charges of the capacitors are

$$Q_1 = C_1V = (6.0\ \mu F)(18\ V) = 108\ \mu C,$$
$$Q_2 = C_2V = (3.0\ \mu F)(18\ V) = 54\ \mu C.$$

The total charge is $Q_1 + Q_2 = Q$, so the charge on the equivalent capacitor is $Q = C_{eq}V = (9.0\ \mu F)(18\ V) = 162\ \mu C$.

18.7 Electric Field Energy

Many of the most important applications of capacitors depend on their ability to store energy.

The capacitor plates, with opposite charges, separated and attracted toward each other, are analogous to a stretched spring or an object lifted in the earth’s gravitational field.

The potential energy corresponds to the energy input required to charge the capacitor and to the work done by the electrical forces when it discharges. This work is analogous to the work done by a spring or the earth’s gravity when the system returns from its displaced position to the reference position.

Energy in a capacitor

$$V = \frac{\Delta W}{\Delta q}$$

$$\Delta W = V\,\Delta q = \frac{q}{C}\,\Delta q$$

$$U = W_{total} = \left(\frac{V}{2}\right)Q = \left(\frac{V}{2}\right)CV = \frac{1}{2}CV^2$$

Where V/2 is the average potential difference during the charging process.

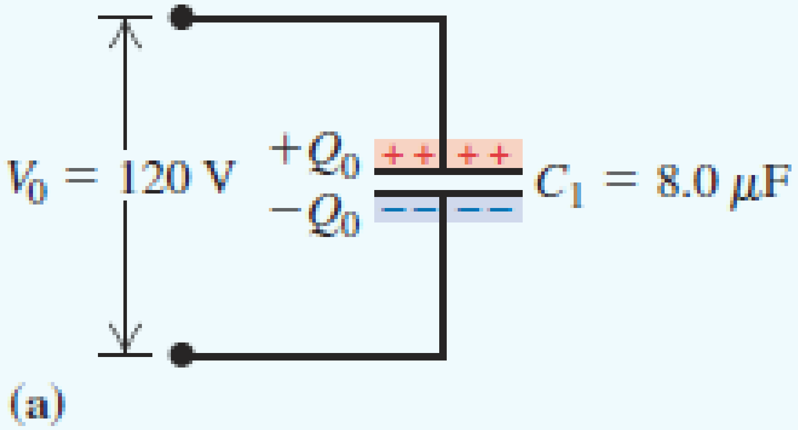
Energy density in an Electric field

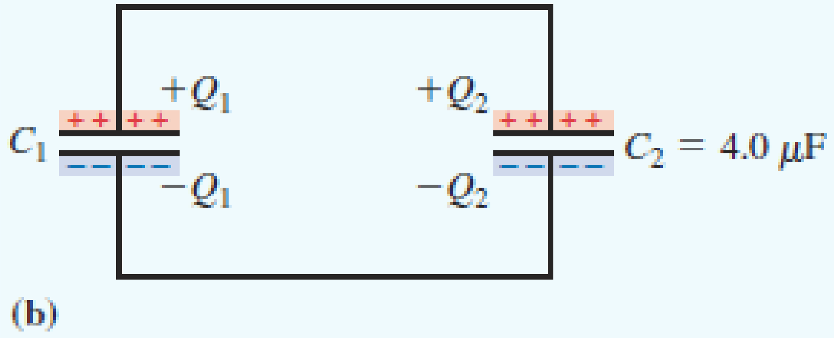
Since $C = \frac{\epsilon_o A}{d} \quad and \quad V = Ed$

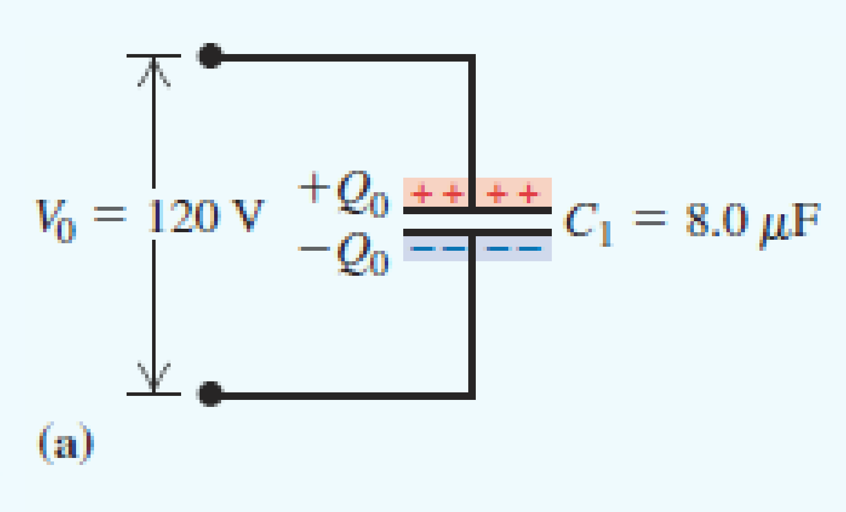
$$u = energy\,density = \frac{Energy}{Volume} = \frac{\frac{1}{2}CV^2}{Ad} = \frac{1}{2}\epsilon_o E^2$$

EXAMPLE 18.9 Stored energy

A capacitor with $C_1 = 8.0\ \mu F$ is connected to a potential difference $V_0 = 120$ V, as shown in Figure 18.25a. (a) Find the magnitude of charge Q_0 and the total energy stored after the capacitor has become fully charged. (b) Without any charge being lost from the plates, the capacitor is disconnected from the source of potential difference and connected to a second capacitor $C_2 = 4.0\ \mu F$ that is initially uncharged (Figure 18.25b). After the charge has finished redistributing between the two capacitors, find the charge and potential difference for each capacitor, and find the total stored energy.

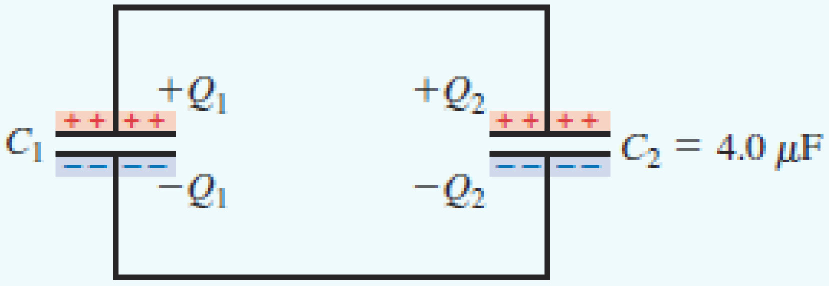






SOLVE **Part (a):** For the original capacitor, we use the potential difference and the capacitance to find the charge: $Q_0 = C_1V_0 = (8.0\ \mu\text{F})(120\ \text{V}) = 960\ \mu\text{C}$. To find the stored energy, we use Equation 18.19:

$$U = \frac{1}{2}Q_0V_0 = \frac{1}{2}(960 \times 10^{-6}\ \text{C})(120\ \text{V}) = 0.058\ \text{J}.$$



(b)

Part (b): From conservation of charge, $Q_1 + Q_2 = Q_0$. Since V is the same for both capacitors, $Q_1 = C_1V$ and $Q_2 = C_2V$. When we substitute these equations into the conservation-of-charge equation, we find that $C_1V + C_2V = Q_0$ and

$$V = \frac{Q_0}{C_1 + C_2} = \frac{960\ \mu\text{C}}{12\ \mu\text{F}} = 80\ \text{V}.$$

Then $Q_1 = C_1V = 640\ \mu\text{C}$ and $Q_2 = C_2V = 320\ \mu\text{C}$.

The final total stored energy is the sum of the energies stored by each capacitor:

$$\frac{1}{2}Q_1V + \frac{1}{2}Q_2V = \frac{1}{2}(Q_1 + Q_2)V = \frac{1}{2}Q_0V$$

$$= \frac{1}{2}(960 \times 10^{-6}\ \text{C})(80\ \text{V}) = 0.038\ \text{J}.$$

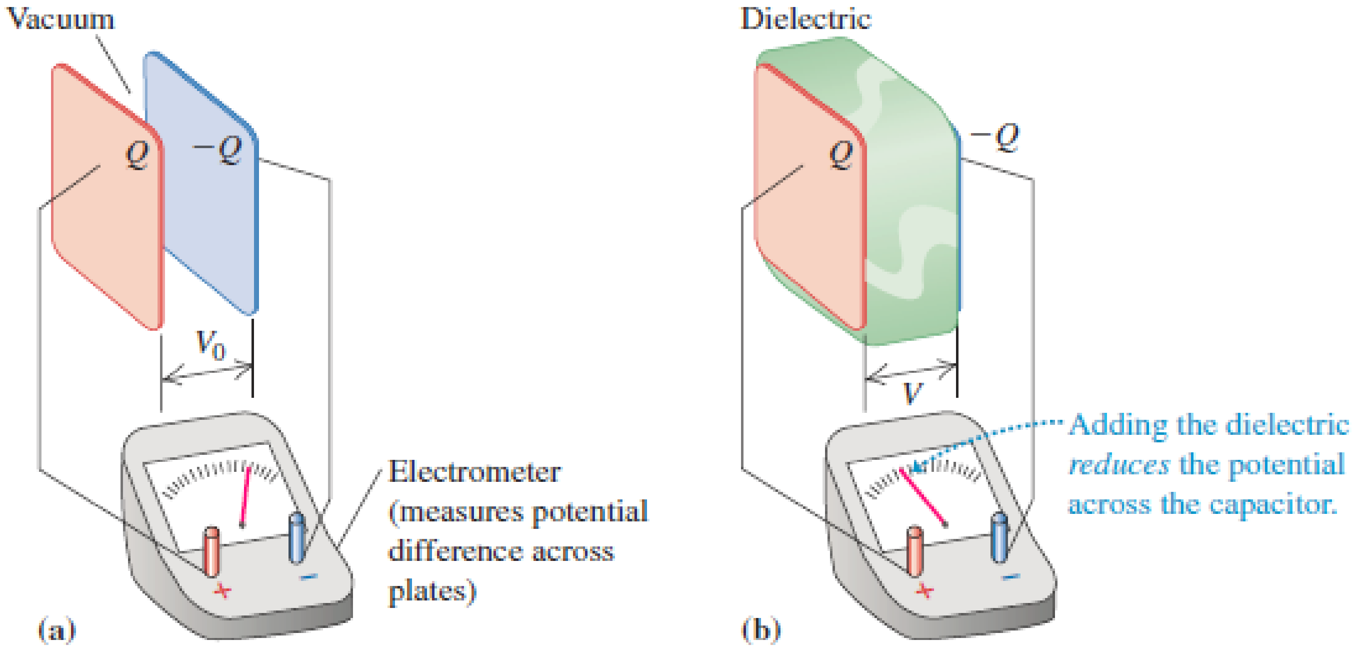
18.8 Dielectrics

Placing a solid dielectric between the plates of a capacitor serves three functions.

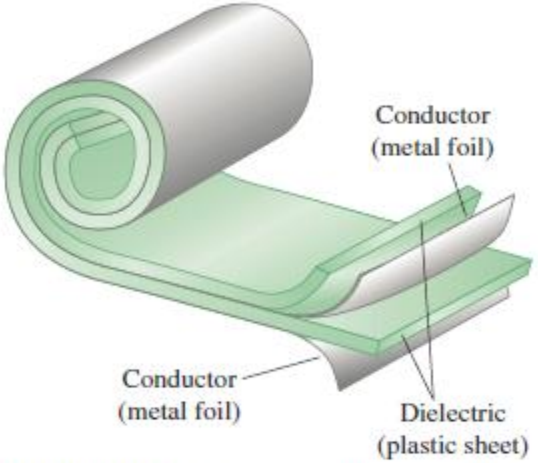
First, it solves the mechanical problem of maintaining two large metal sheets at a very small separation without actual contact.

Second, many insulating materials can tolerate stronger electric fields without breakdown than can air.

Third, the capacitance of a capacitor of given dimensions is *greater* when there is a dielectric material between the plates than when there is air or vacuum.



Adding the dielectric *reduces* the potential across the capacitor.



▲ **FIGURE 18.26** A common type of parallel-plate capacitor is made from a rolled-up sandwich of metal foil and plastic film.

Dielectric constant of the material, *K*

$$K = \frac{C}{C_o}$$

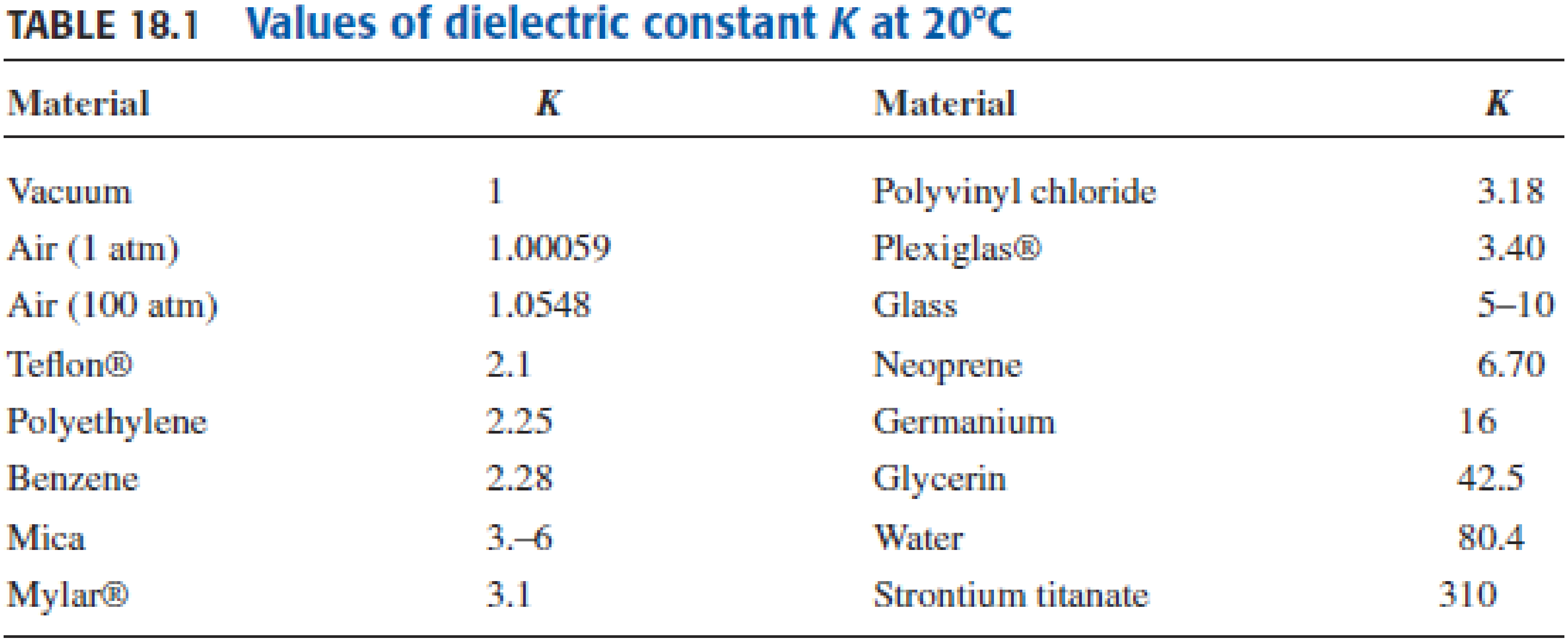


TABLE 18.1 Values of dielectric constant *K* at 20°C

Material	*K*	Material	*K*
Vacuum	1	Polyvinyl chloride	3.18
Air (1 atm)	1.00059	Plexiglas®	3.40
Air (100 atm)	1.0548	Glass	5–10
Teflon®	2.1	Neoprene	6.70
Polyethylene	2.25	Germanium	16
Benzene	2.28	Glycerin	42.5
Mica	3.–6	Water	80.4
Mylar®	3.1	Strontium titanate	310

Where C_0 is the capacitance in a vacuum

For a given charge density σ_i, the induced charges on the dielectric's surfaces reduce the electric field between the plates.

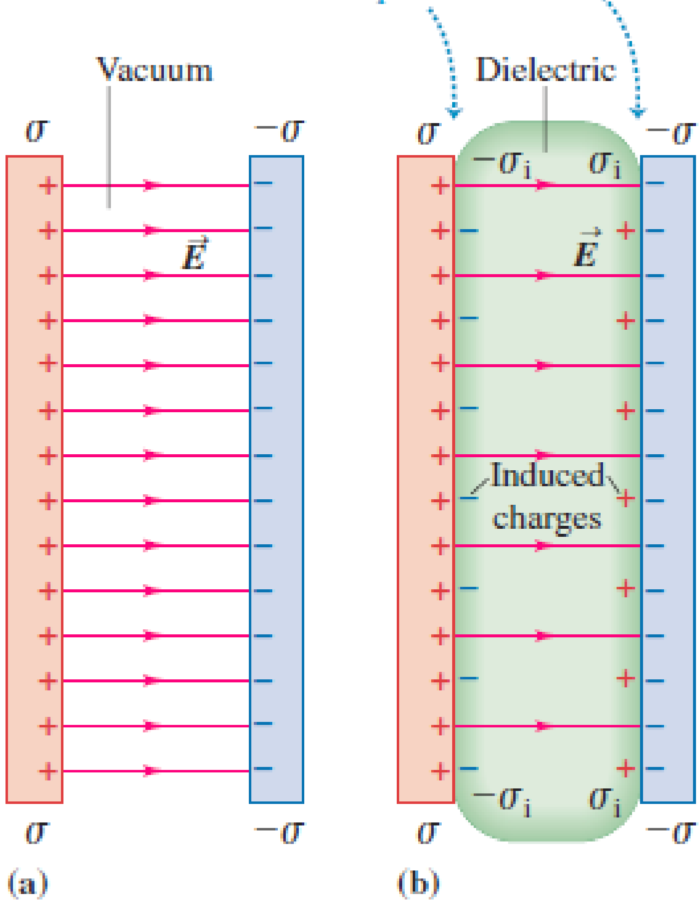


FIGURE 18.28 The effect of a dielectric on the electric field between the plates of a capacitor.

$$K = \frac{C}{C_o}$$

$$C = K\, C_o$$

$$V = \frac{V_o}{K}$$

$$E = \frac{E_o}{K}$$

SOLVE **Part (a):** The presence of the dielectric increases the capacitance. Without the dielectric, the capacitance is

$$C_0 = \frac{\epsilon_0 A}{d} = \frac{(8.85 \times 10^{-12}\ \mathrm{F/m})(0.200\ \mathrm{m}^2)}{0.010\ \mathrm{m}}$$
$$= 1.77 \times 10^{-10}\ \mathrm{F} = 177\ \mathrm{pF}.$$

The original charge on the capacitor is

$$Q = C_0 V_0 = (1.77 \times 10^{-10}\ \mathrm{F})(3.00 \times 10^3\ \mathrm{V})$$
$$= 5.31 \times 10^{-7}\ \mathrm{C} = 0.531\ \mu\mathrm{C}.$$

After the dielectric is inserted, the charge is still $Q = 0.531\ \mu\mathrm{C}$, but now $V = 1.00$ kV, so

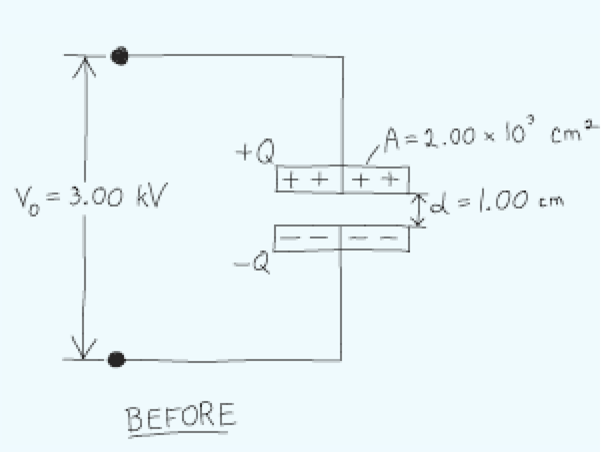
$$C = \frac{Q}{V} = \frac{0.531 \times 10^{-6}\ \mathrm{C}}{1.00 \times 10^3\ \mathrm{V}} = 5.31 \times 10^{-10}\ \mathrm{F} = 531\ \mathrm{pF}.$$

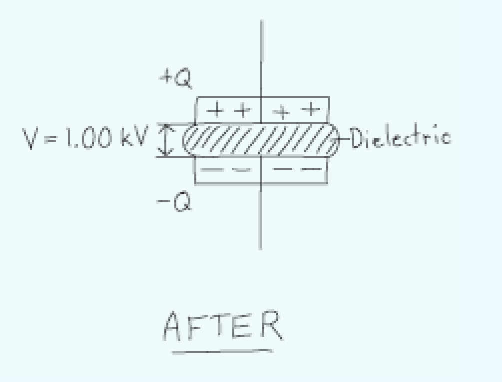
Part (b): By definition, the dielectric constant is

$$K = C/C_0 = (531\ \mathrm{pF})/(177\ \mathrm{pF}) = 3.00.$$

Note that this is also

$$K = V_0/V = (3.00\ \mathrm{kV})/(1.00\ \mathrm{kV}) = 3.00.$$



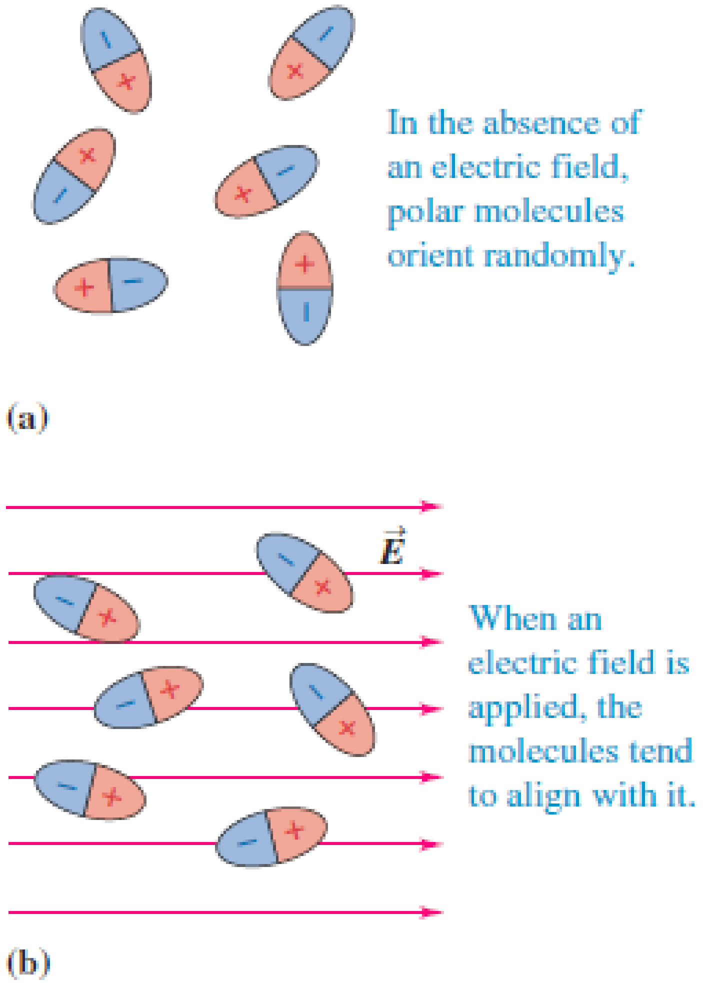


Dielectric Breakdown

The maximum electric field a material can withstand without the occurrence of breakdown is called its dielectric strength.

▲ **FIGURE 18.30** Dielectric breakdown in the laboratory and in nature. The left-hand photo shows a block of Plexiglas® subjected to a very strong electric field; the pattern was etched by flowing charge.

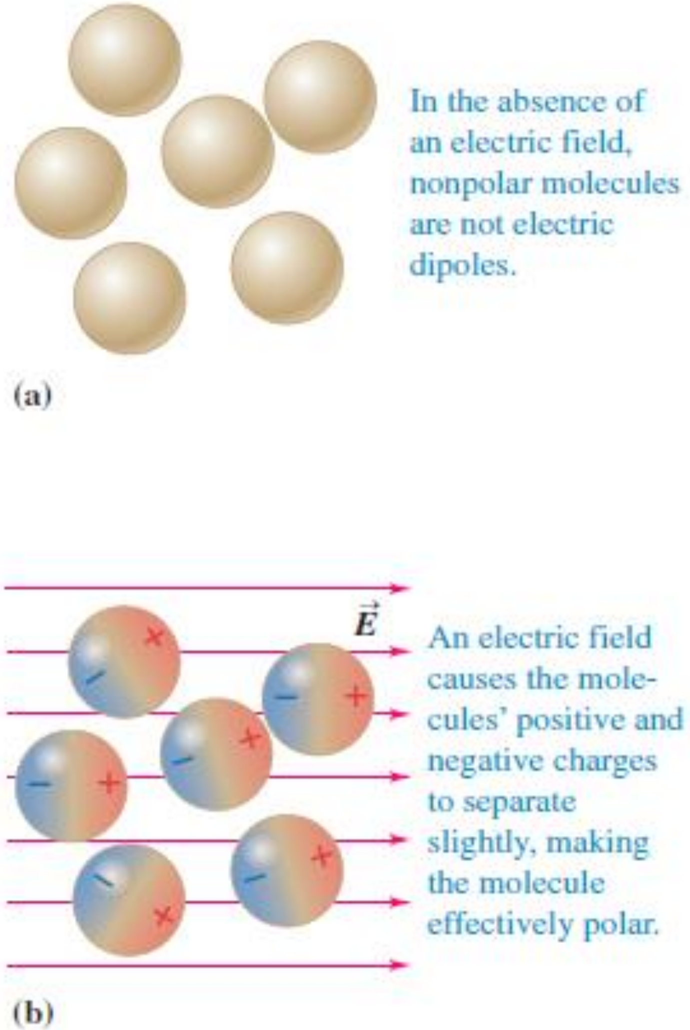
18.9 Molecular Model of Induced Charge



In the absence of an electric field, polar molecules orient randomly.

When an electric field is applied, the molecules tend to align with it.

▲ **FIGURE 18.31** The effect of an electric field on a group of polar molecules.

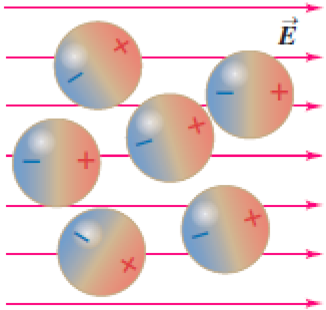


In the absence of an electric field, nonpolar molecules are not electric dipoles.

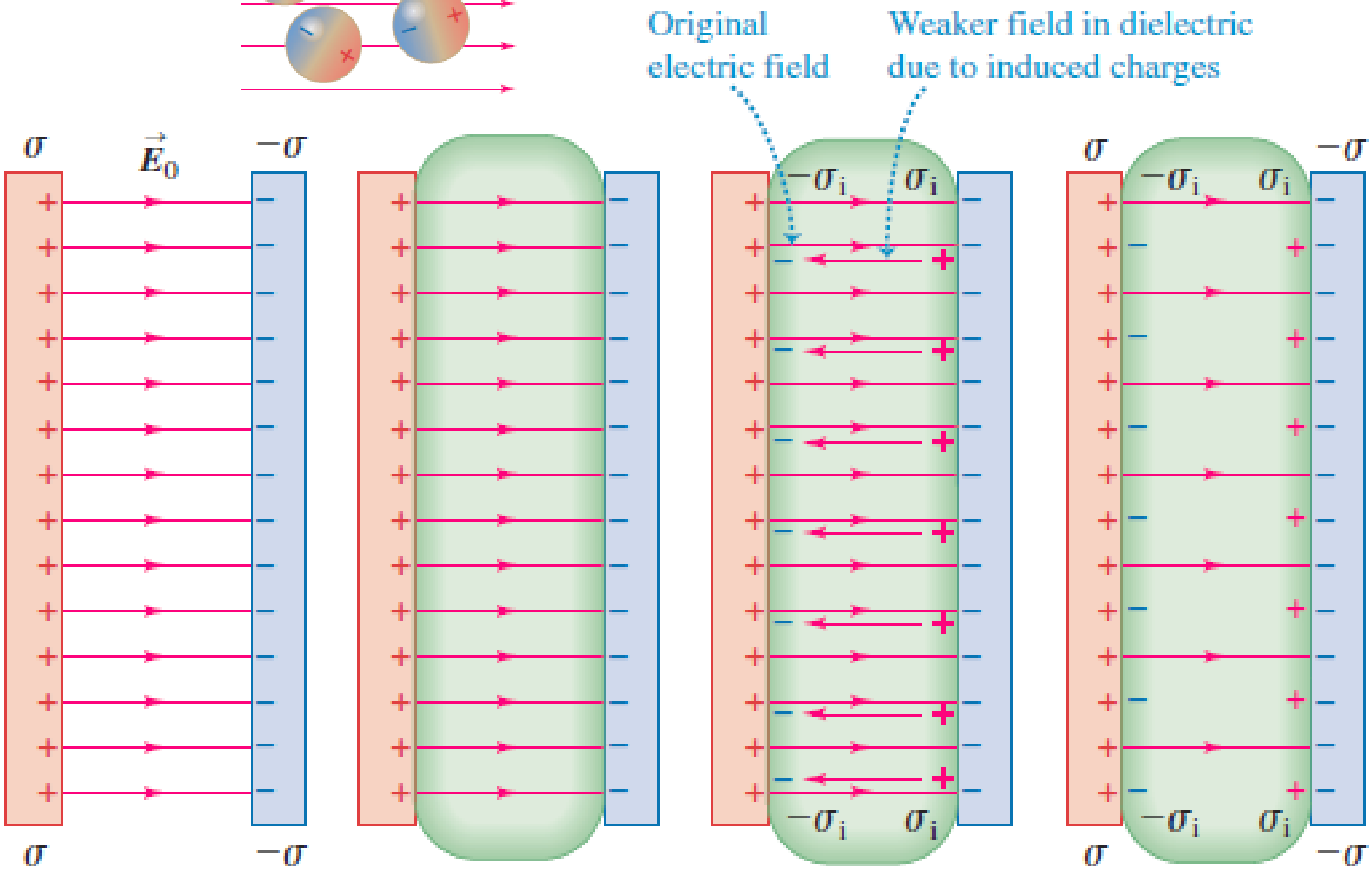
An electric field causes the molecules' positive and negative charges to separate slightly, making the molecule effectively polar.

▲ **FIGURE 18.32** The effect of an electric field on a group of nonpolar molecules.

Induced negative



Induced positive



(a) No dielectric

(b) Dielectric just inserted

(c) Induced charges create electric field

(d) Resultant field

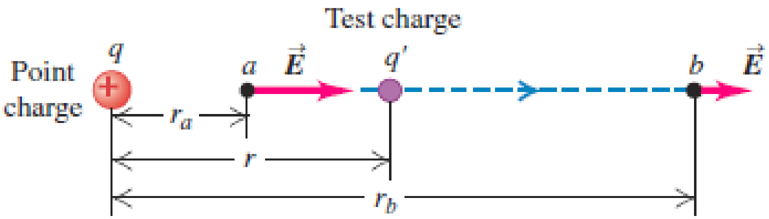
▲ **FIGURE 18.33** How a dielectric reduces the electric field between capacitor plates.

SUMMARY

Electric Potential Energy

(Section 18.1) The work W done by the electric-field force on a charged particle moving in a field can be represented in terms of potential energy U: $W_{a \to b} = U_a - U_b$ (Equation 18.2). For a charge q' that undergoes a displacement $\vec{s}$ parallel to a uniform electric field, the change in potential energy is $U_a - U_b = q'Es$ (Equation 18.5). The potential energy for a point charge q' moving in the field produced by a point charge q at a distance r from q' is

$$U = k\frac{qq'}{r}. \qquad (18.8)$$



Potential

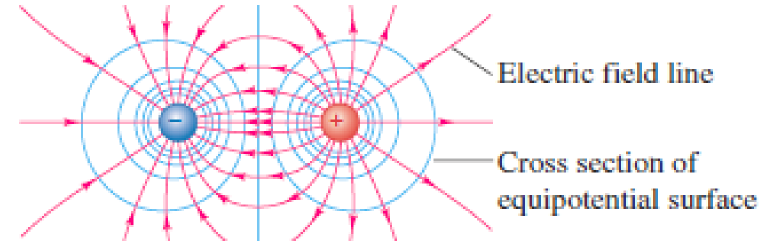
(Section 18.2) **Potential,** a scalar quantity denoted by V, is potential energy per unit charge. The potential at any point due to a point charge is

$$V = \frac{U}{q'} = k\frac{q}{r}. \qquad (18.12)$$

A positive test charge tends to "fall" from a high-potential region to a low-potential region.

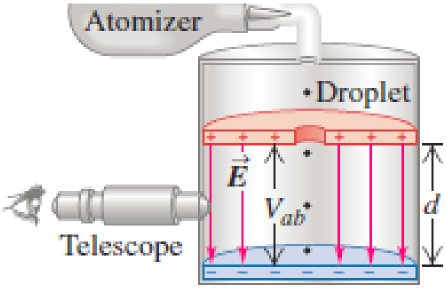
Equipotential Surfaces

(Section 18.3) An **equipotential surface** is a surface on which the potential has the same value at every point. At a point where a field line crosses an equipotential surface, the two are perpendicular. When all charges are at rest, the surface of a conductor is always an equipotential surface, and all points in the interior of a conductor are at the same potential.



The Millikan Oil-Drop Experiment

(Section 18.4) The Millikan oil-drop experiment determined the electric charge of individual electrons by measuring the motion of electrically charged oil drops in an electric field. The size of a drop is determined by measuring its terminal speed of fall under gravity and the drag force of air.



Capacitors

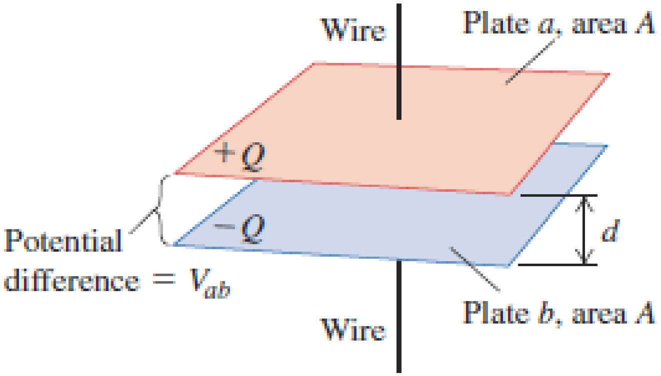
(Sections 18.5 and 18.6) A **capacitor** consists of any pair of conductors separated by vacuum or an insulating material. The **capacitance** C is defined as $C = Q/V_{ab}$ (Equation 18.14). A **parallel-plate capacitor** is made with two parallel plates, each with area A, separated by a distance d. If they are separated by vacuum, the capacitance is $C = \epsilon_0(A/d)$ (Equation 18.16).

When capacitors with capacitances C_1, C_2, C_3, . . . are connected in series, the equivalent capacitance C_{eq} is given by

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots. \qquad (18.17)$$

When they are connected in parallel, the equivalent capacitance is

$$C_{eq} = C_1 + C_2 + C_3 + \cdots. \qquad (18.18)$$

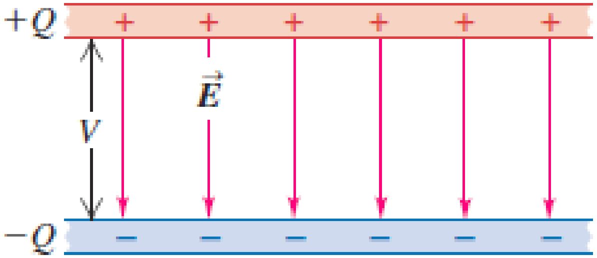


Electric Field Energy

(Section 18.7) The energy U required to charge a capacitor C to a potential difference V and a charge Q is equal to the energy stored in the capacitor and is given by

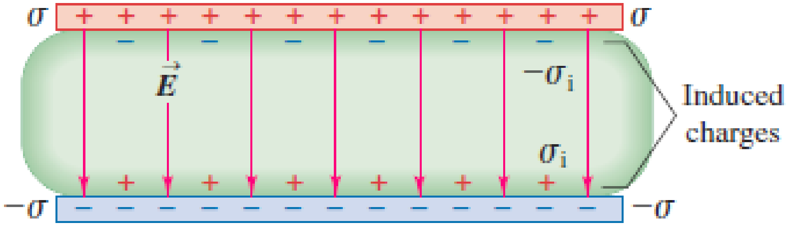
$$U = W_{total} = \left(\frac{V}{2}\right)Q = \frac{Q^2}{2C} = \frac{1}{2}CV^2. \qquad (18.19)$$

This energy can be thought of as residing in the electric field between the conductors; the energy density u (energy per unit volume) is $u = \frac{1}{2}\epsilon_0 E^2$ (Equation 18.20).



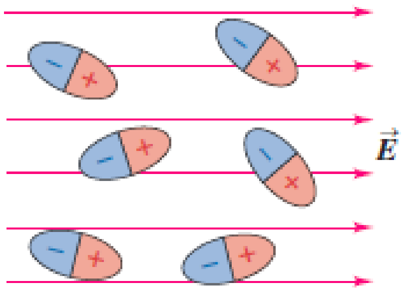
Dielectrics

(Section 18.8) When the space between the conductors is filled with a dielectric material, the capacitance *increases* by a factor K called the dielectric constant of the material. When the charges $\pm Q$ on the plates remain constant, charges induced on the surface of the dielectric *decrease* the electric field and potential difference between conductors by the same factor K. Under sufficiently strong fields, dielectrics become conductors, a phenomenon called dielectric breakdown. The maximum field that a material can withstand without breakdown is called its dielectric strength.



Molecular Model of Induced Charge

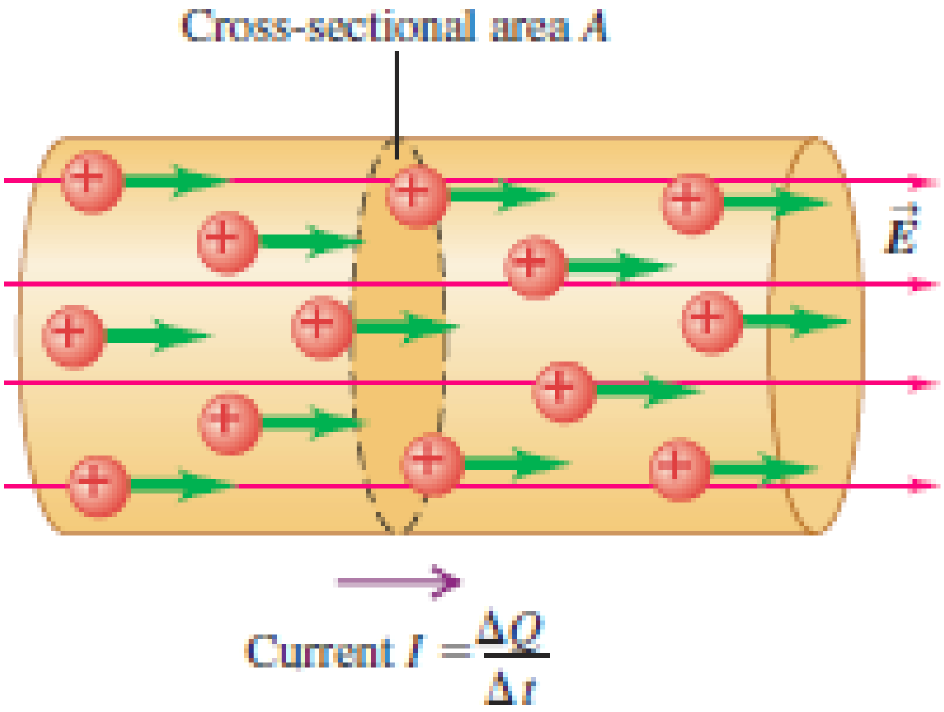
(Section 18.9) A *polar molecule* has equal amounts of positive and negative charge, but a lopsided distribution, with excess positive charge concentrated on one side of the molecule and negative charge on the other. When placed in an electric field, polar molecules tend to partially align with the field. For a material containing polar molecules, this microscopic alignment appears as an induced surface charge density. Even a molecule that is not ordinarily polar attains a lopsided charge distribution when it is placed in an electric field: The field pushes the positive charges in the molecule in the direction of the field and pushes the negative charges in the opposite direction.



19 Current, Resistance, and Direct-Current Circuits

19.1 Current
Charges in motion

In this chapter, we shift our emphasis to situations in which non-zero electric fields exist inside conductors, causing motion of the mobile charges within the conductors. A current (also called *electric current*) is any motion of charge from one region of a conductor to another.



Definition of current

When a net charge ΔQ passes through a cross section of conductor during time Δt, the current is

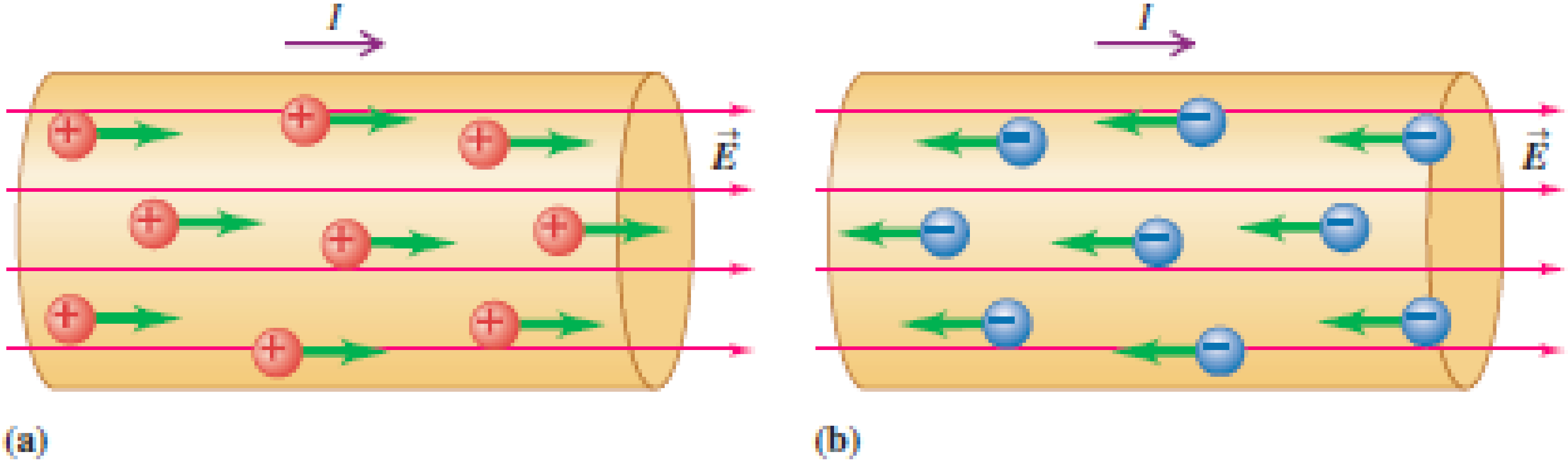
$$I = \frac{\Delta Q}{\Delta t}. \qquad (19.1)$$

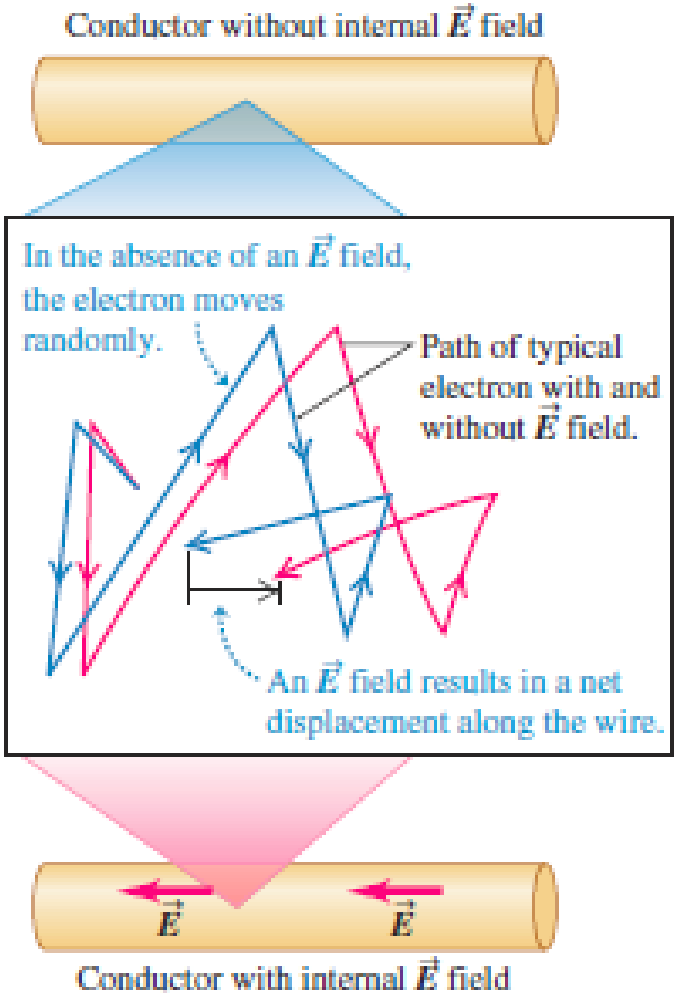
Unit: 1 coulomb/second = 1 C/s = 1 ampere = 1 A.

Current is a *scalar* quantity. The SI unit of current is the **ampere**

A **conventional current** is treated as a flow of positive charges, regardless of whether the free charges in the conductor are positive, negative, or both.

In a metallic conductor, the moving charges are electrons — but the *current* still points in the direction positive charges would flow.





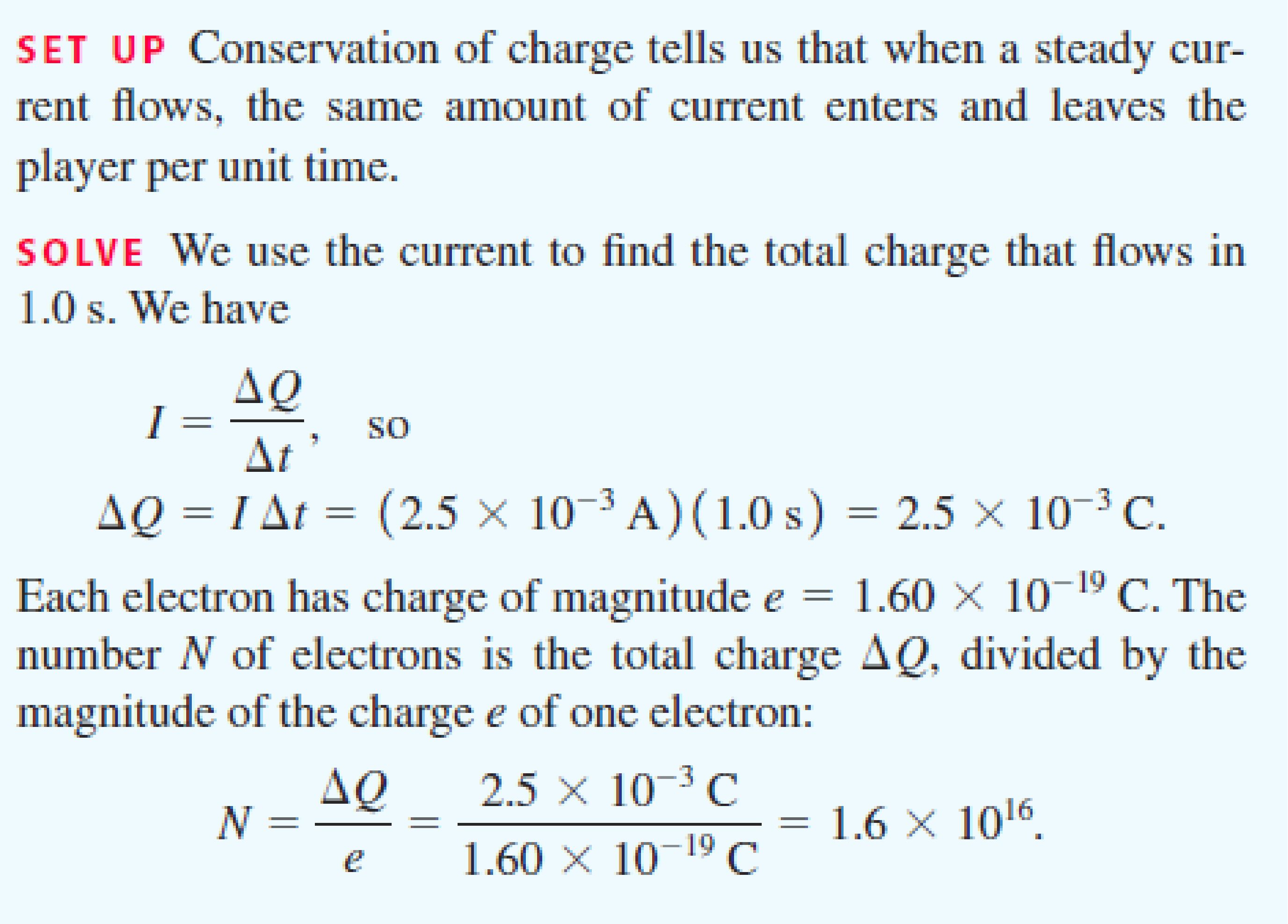
▲ **FIGURE 19.3** The presence of an electric field imposes a small drift (greatly exaggerated here) on an electron's random motion.

The current at any instant is the same at all cross sections.

When you turn on a light switch, the light comes on almost instantaneously because the electric fields in the conductors travel with a speed approaching the speed of light. You don’t have to wait for individual electrons to travel from the switch to the bulb!

EXAMPLE 19.1 How many electrons?

One of the circuits in a small portable CD player operates on a current of 2.5 mA. How many electrons enter and leave this part of the player in 1.0 s?



SET UP Conservation of charge tells us that when a steady current flows, the same amount of current enters and leaves the player per unit time.

SOLVE We use the current to find the total charge that flows in 1.0 s. We have

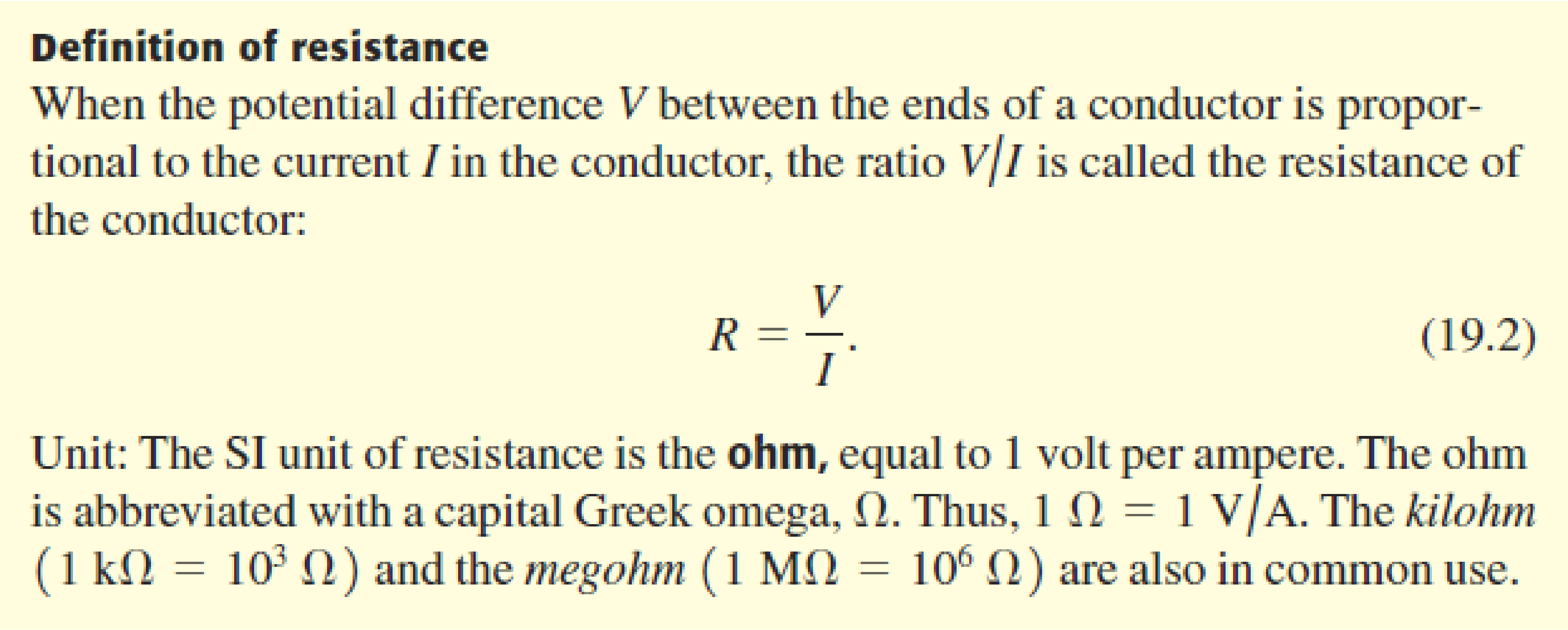
$$I = \frac{\Delta Q}{\Delta t}, \quad \text{so}$$

$$\Delta Q = I\,\Delta t = (2.5 \times 10^{-3}\,\text{A})(1.0\,\text{s}) = 2.5 \times 10^{-3}\,\text{C}.$$

Each electron has charge of magnitude $e = 1.60 \times 10^{-19}$ C. The number N of electrons is the total charge ΔQ, divided by the magnitude of the charge e of one electron:

$$N = \frac{\Delta Q}{e} = \frac{2.5 \times 10^{-3}\,\text{C}}{1.60 \times 10^{-19}\,\text{C}} = 1.6 \times 10^{16}.$$

19.2 Resistance and Ohm’s Law

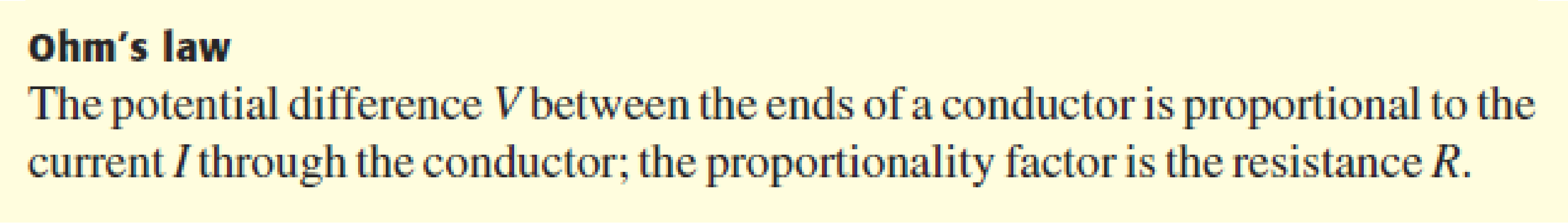


Definition of resistance

When the potential difference V between the ends of a conductor is proportional to the current I in the conductor, the ratio V/I is called the resistance of the conductor:

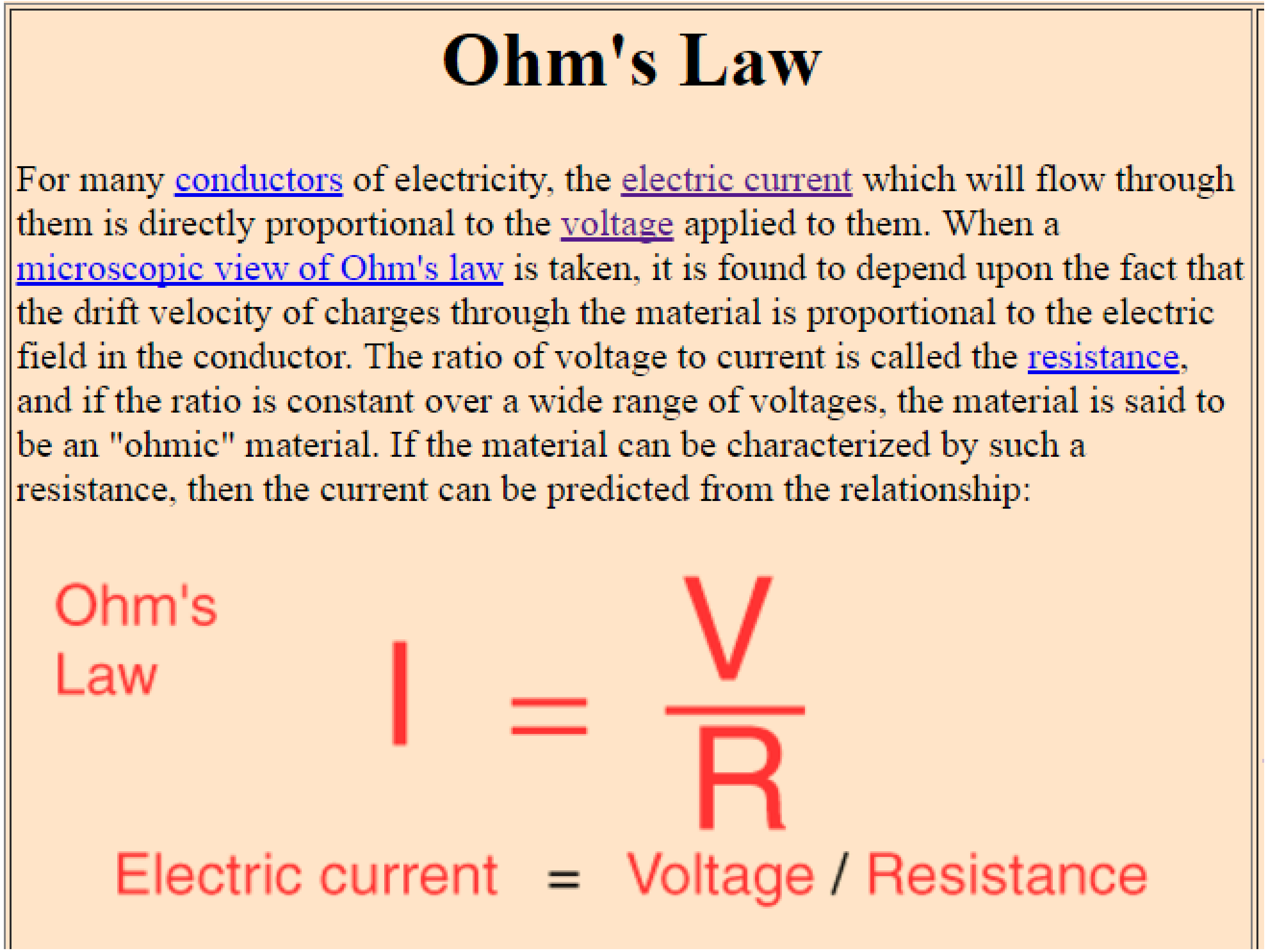
$$R = \frac{V}{I}. \tag{19.2}$$

Unit: The SI unit of resistance is the **ohm,** equal to 1 volt per ampere. The ohm is abbreviated with a capital Greek omega, Ω. Thus, $1\ \Omega = 1\ \mathrm{V/A}$. The *kilohm* $(1\ \mathrm{k}\Omega = 10^3\ \Omega)$ and the *megohm* $(1\ \mathrm{M}\Omega = 10^6\ \Omega)$ are also in common use.



Ohm’s law

The potential difference V between the ends of a conductor is proportional to the current I through the conductor; the proportionality factor is the resistance R.



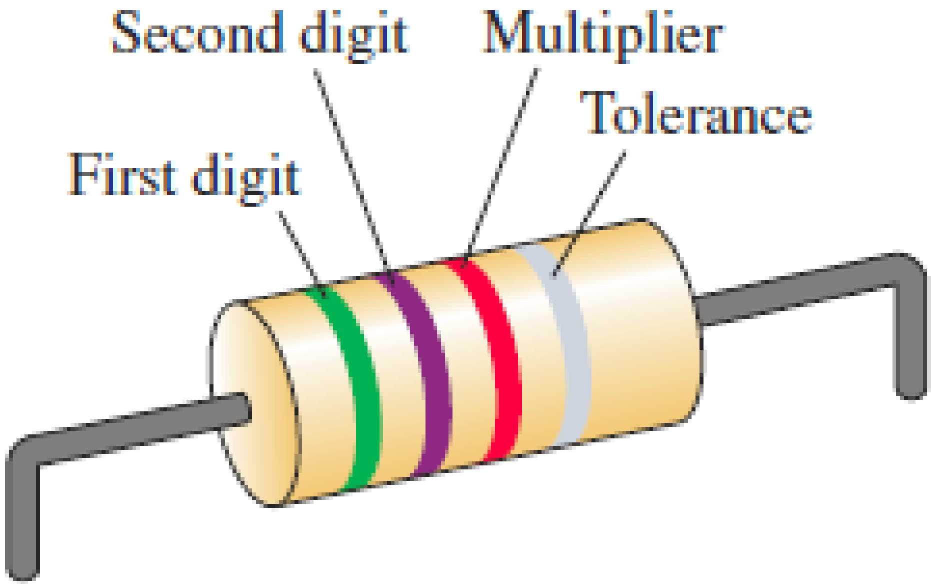
Ohm's Law

For many conductors of electricity, the electric current which will flow through them is directly proportional to the voltage applied to them. When a microscopic view of Ohm's law is taken, it is found to depend upon the fact that the drift velocity of charges through the material is proportional to the electric field in the conductor. The ratio of voltage to current is called the resistance, and if the ratio is constant over a wide range of voltages, the material is said to be an "ohmic" material. If the material can be characterized by such a resistance, then the current can be predicted from the relationship:

Ohm's Law

$$I = \frac{V}{R}$$

Electric current = Voltage / Resistance



▲ **FIGURE 19.4** Commercial resistors use a code consisting of colored bands to indicate their resistance.

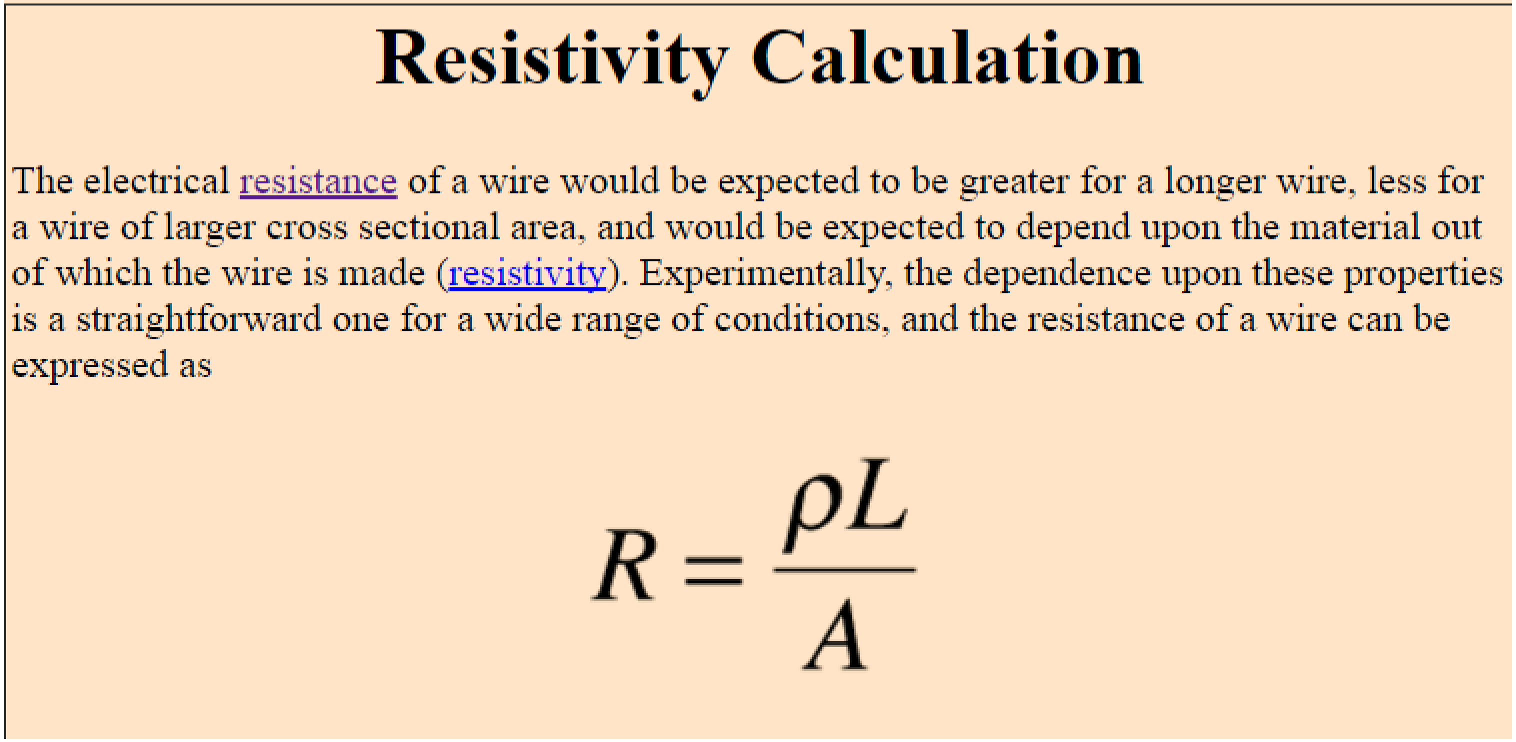
Definition of resistivity

The resistance R is proportional to the length L and inversely proportional to the cross-sectional area A, with a proportionality factor ρ called the **resistivity** of the material. That is,

$$R = \rho\frac{L}{A}, \tag{19.3}$$

where ρ, in general different for different materials, characterizes the conduction properties of a material.

Unit: The SI unit of resistivity is 1 ohm · meter = 1 Ω · m.



Resistivity Calculation

The electrical resistance of a wire would be expected to be greater for a longer wire, less for a wire of larger cross sectional area, and would be expected to depend upon the material out of which the wire is made (resistivity). Experimentally, the dependence upon these properties is a straightforward one for a wide range of conditions, and the resistance of a wire can be expressed as

$$R = \frac{\rho L}{A}$$

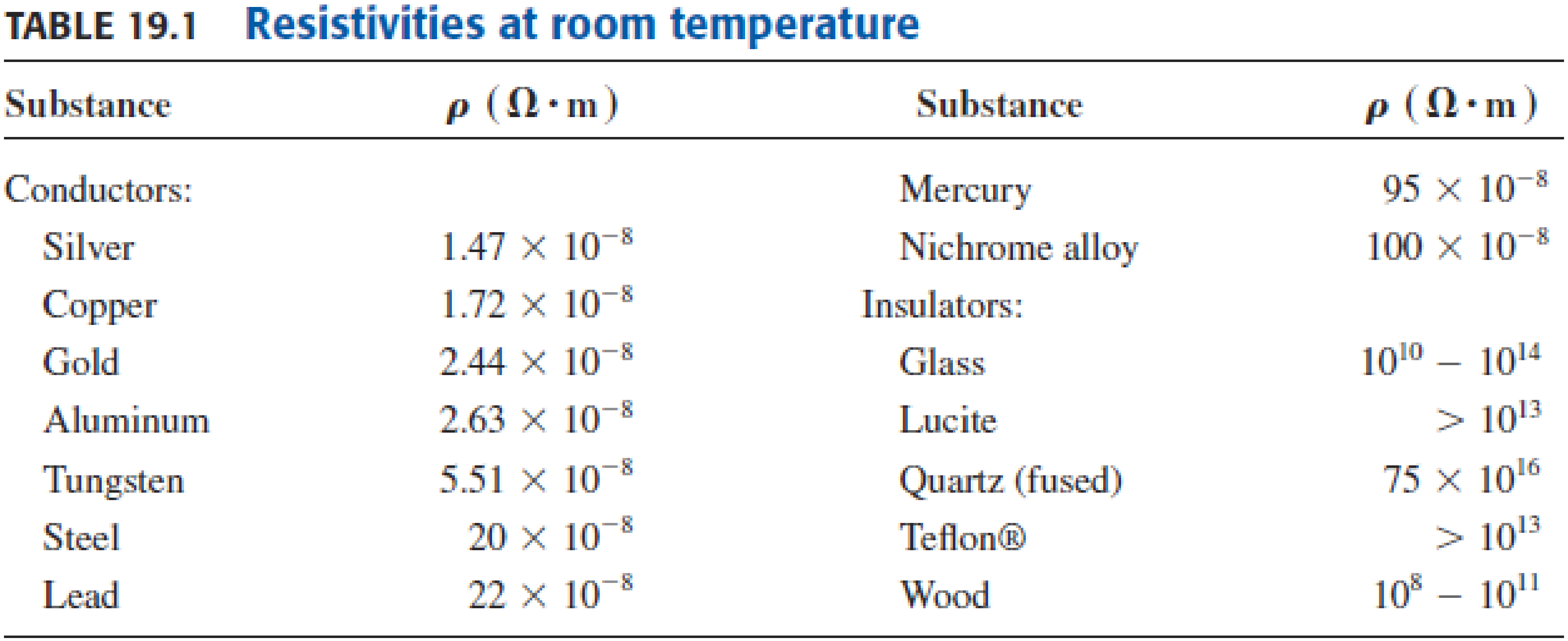


TABLE 19.1 Resistivities at room temperature

Substance	$\rho\ (\Omega \cdot m)$	Substance	$\rho\ (\Omega \cdot m)$
Conductors:		Mercury	95×10^{-8}
Silver	1.47×10^{-8}	Nichrome alloy	100×10^{-8}
Copper	1.72×10^{-8}	Insulators:	
Gold	2.44×10^{-8}	Glass	$10^{10} - 10^{14}$
Aluminum	2.63×10^{-8}	Lucite	$> 10^{13}$
Tungsten	5.51×10^{-8}	Quartz (fused)	75×10^{16}
Steel	20×10^{-8}	Teflon®	$> 10^{13}$
Lead	22×10^{-8}	Wood	$10^{8} - 10^{11}$

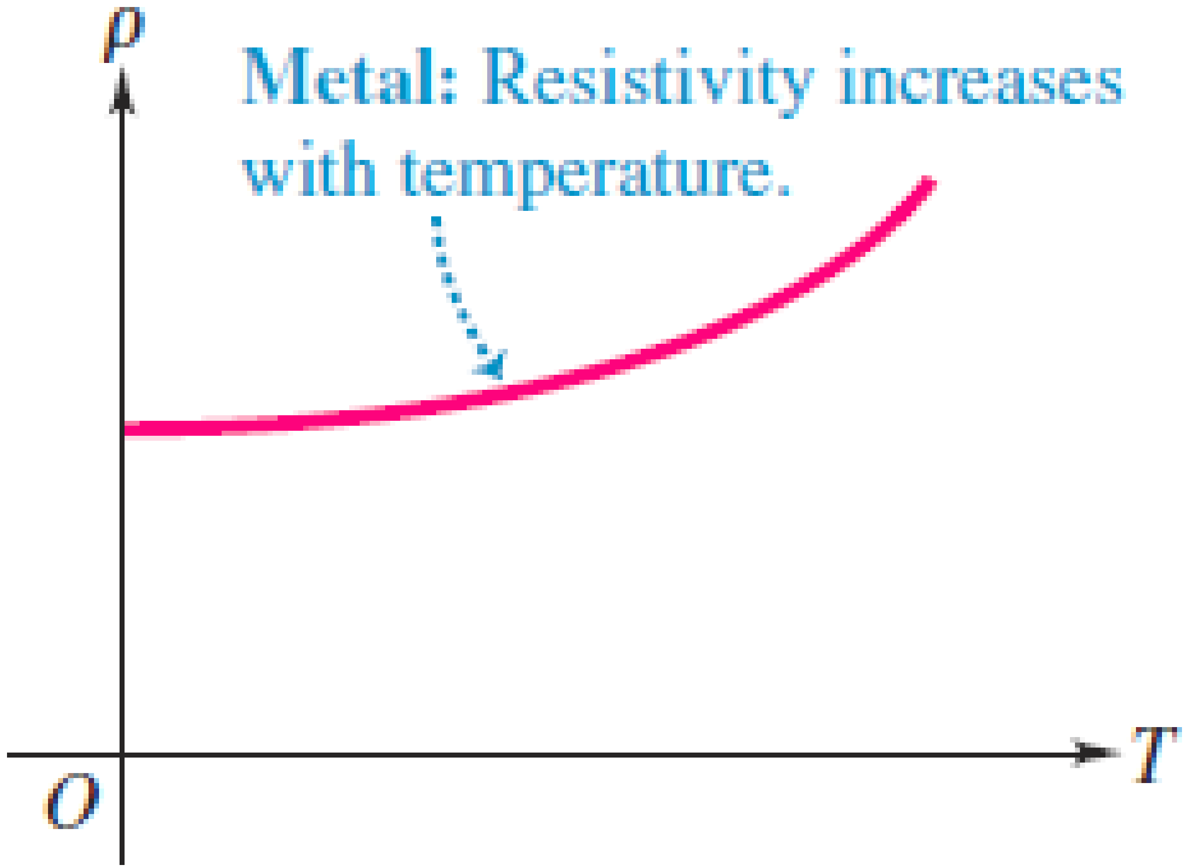
$$R = \rho \frac{L}{A}$$

Temperature Dependence of Resistance

The resistance of every conductor varies somewhat with temperature. The resistivity of a *metallic* conductor nearly always increases with increasing temperature (Figure 19.5a). Over a small temperature range (up to 100 C° or so), the change in resistivity of a metal is approximately proportional to the temperature change. If R_0 is the resistance at a reference temperature T_0 (often taken as 0°C or 20°C) and R_T is the resistance at temperature T, then the variation of R with temperature is described approximately by the equation

$$R_T = R_0[1 + \alpha(T - T_0)]. \tag{19.4}$$

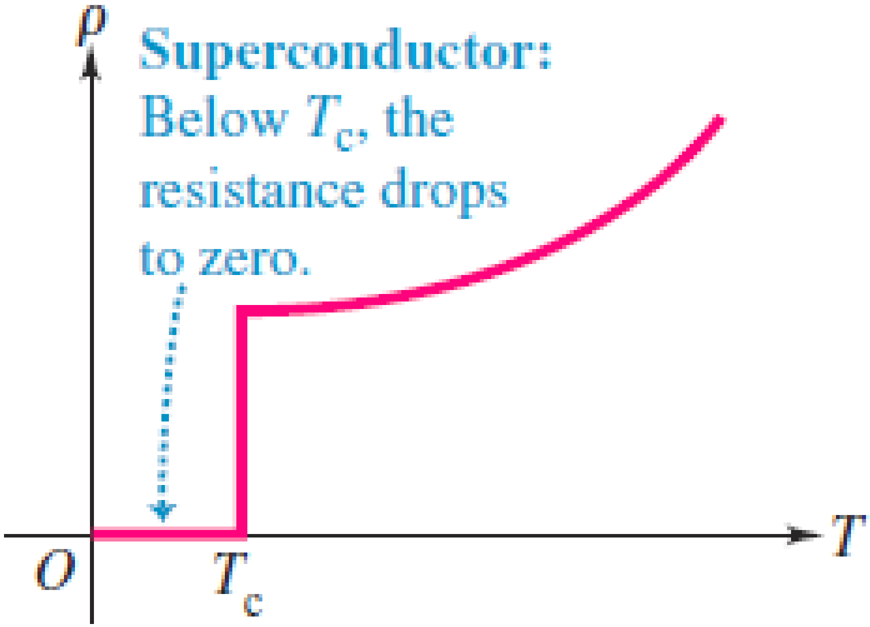
The factor α is called the **temperature coefficient of resistivity.** For common metals, α typically has a value of 0.003 to 0.005 $(\mathrm{C}^\circ)^{-1}$. That is, an increase in temperature of 1 C° increases the resistance by 0.3% to 0.5%.



$$R_T = R_0[1 + \alpha(T - T_0)].$$

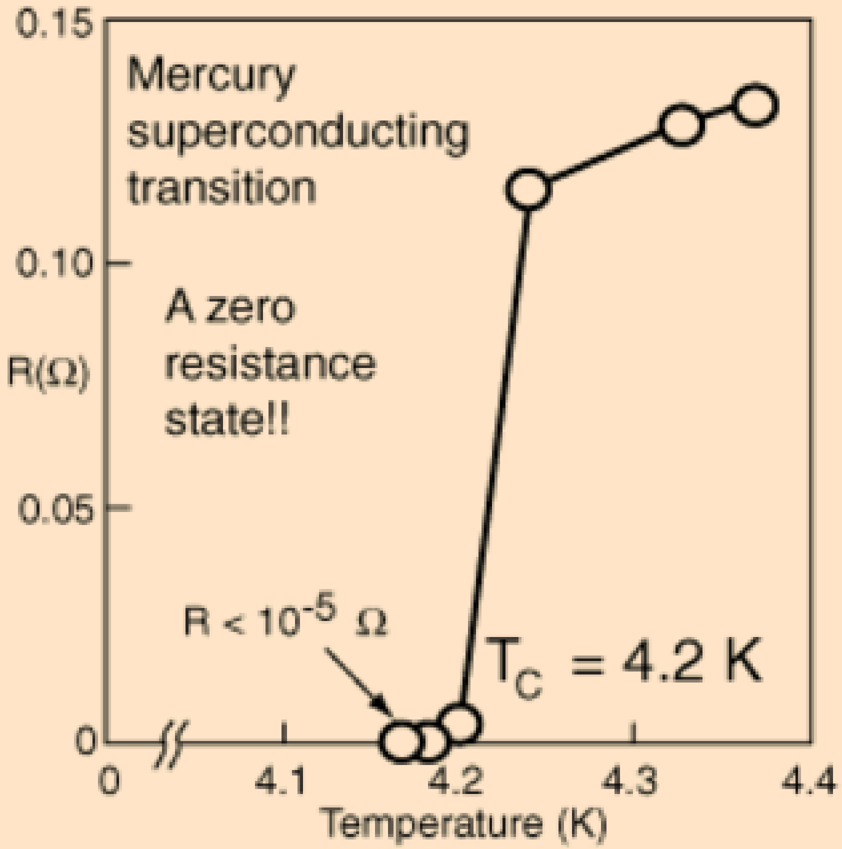
Superconductivity

▲ **FIGURE 19.6** A maglev train in Shanghai. Maglev (“magnetic-levitation”) trains use superconducting electromagnets to create magnetic fields strong enough to levitate a train off the tracks.



(b)

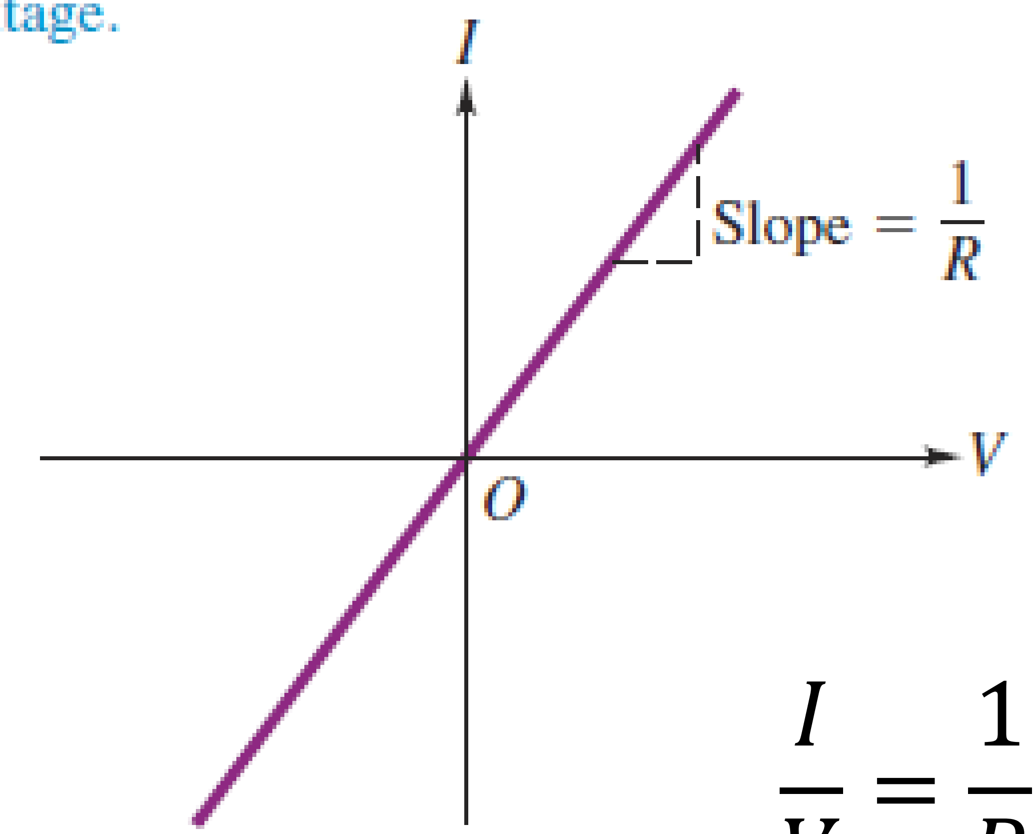
The Discovery of Superconductivity



H. K. Onnes, Commun. Phys. Lab.12,120, (1911)

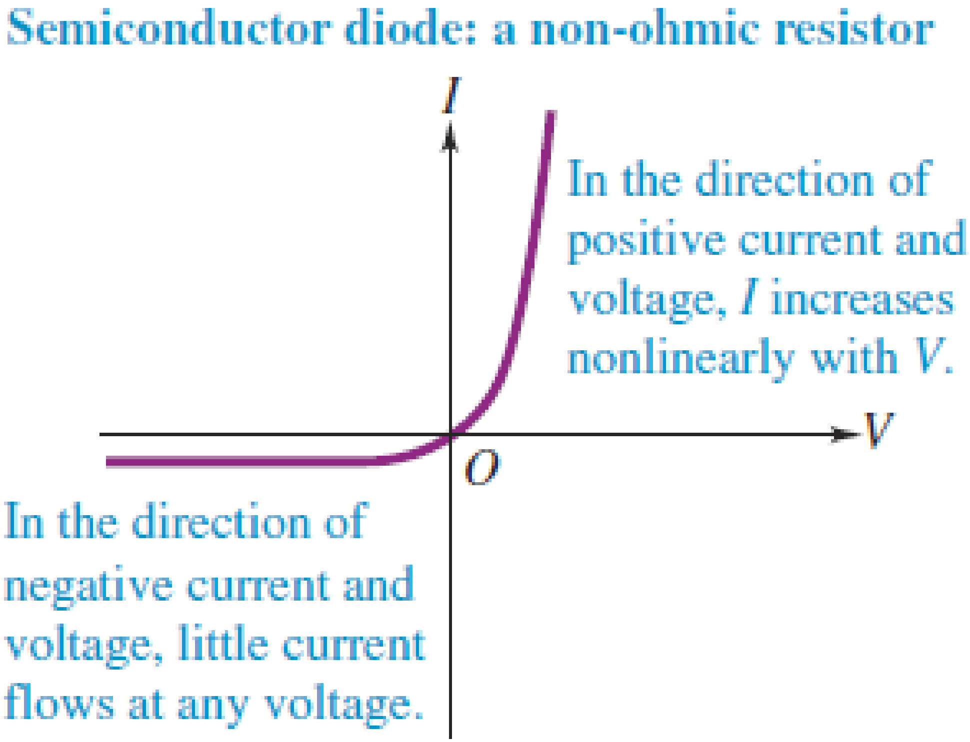
H. Kamerlingh Onnes found that the resistivity of mercury suddenly dropped to zero at 4.2K, a phase transition to a zero resistance state. This phenomenon was called superconductivity, and the temperature at which it occurred is called its critical temperature.

Ohmic resistor (e.g., typical metal wire): At a given temperature, current is proportional to voltage.



$$\frac{I}{V} = \frac{1}{R}$$

Non-ohmic conductors



Semiconductor diode: a non-ohmic resistor

In the direction of positive current and voltage, *I* increases nonlinearly with *V*.

In the direction of negative current and voltage, little current flows at any voltage.

This is a graph for a semiconductor diode, a device that is decidedly *non*-ohmic. Notice that the resistance of a diode depends on the *direction* of the current. Diodes act like one-way valves for current; they are used to perform a wide variety of logic functions in computer circuitry.

EXAMPLE 19.2 Resistance in your stereo system

Suppose you're hooking up a pair of stereo speakers. **(a)** You happen to have on hand some 20-m-long pieces of 16 gauge copper wire (diameter 1.3 mm); you use them to connect the speakers to the amplifier. These wires are longer than needed, but you just coil up the excess length instead of cutting them. What is the resistance of one of these wires? **(b)** To improve the performance of the system, you purchase 3.0-m-long speaker cables that are made with 8 gauge copper wire (diameter 3.3 mm). What is the resistance of one of these cables?

SOLUTION

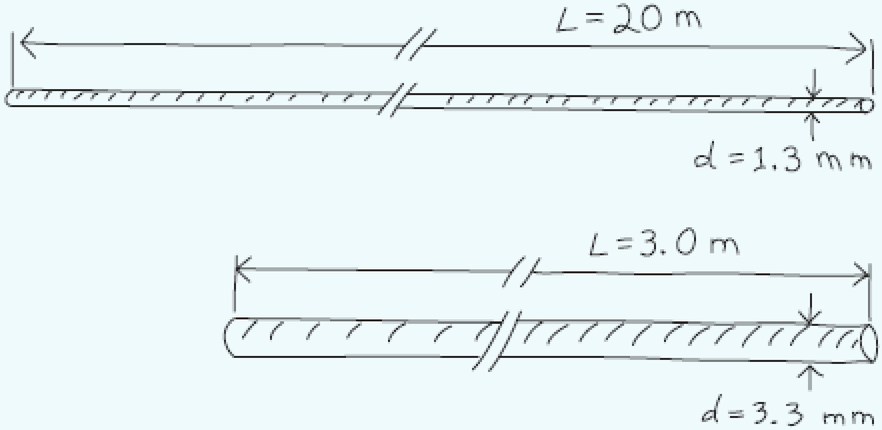
SET UP Figure 19.8 shows our sketch. The resistivity of copper at room temperature is $\rho = 1.72 \times 10^{-8}\ \Omega \cdot \text{m}$ (Table 19.1). The cross-sectional area A of a wire is related to its radius by $A = \pi r^2$.

SOLVE To find the resistances, we use Equation 19.3, $R = \rho L/A$.

Part (a): $$R = \frac{(1.72 \times 10^{-8}\ \Omega \cdot \text{m})(20\ \text{m})}{\pi(6.5 \times 10^{-4}\ \text{m})^2} = 0.26\ \Omega.$$

Part (b): $$R = \frac{(1.72 \times 10^{-8}\ \Omega \cdot \text{m})(3.0\ \text{m})}{\pi(1.65 \times 10^{-3}\ \text{m})^2} = 6.0 \times 10^{-3}\ \Omega.$$

REFLECT The shorter, fatter wires offer over forty times less resistance than the longer, skinnier ones.



▲**FIGURE 19.8** Our sketch for this problem.

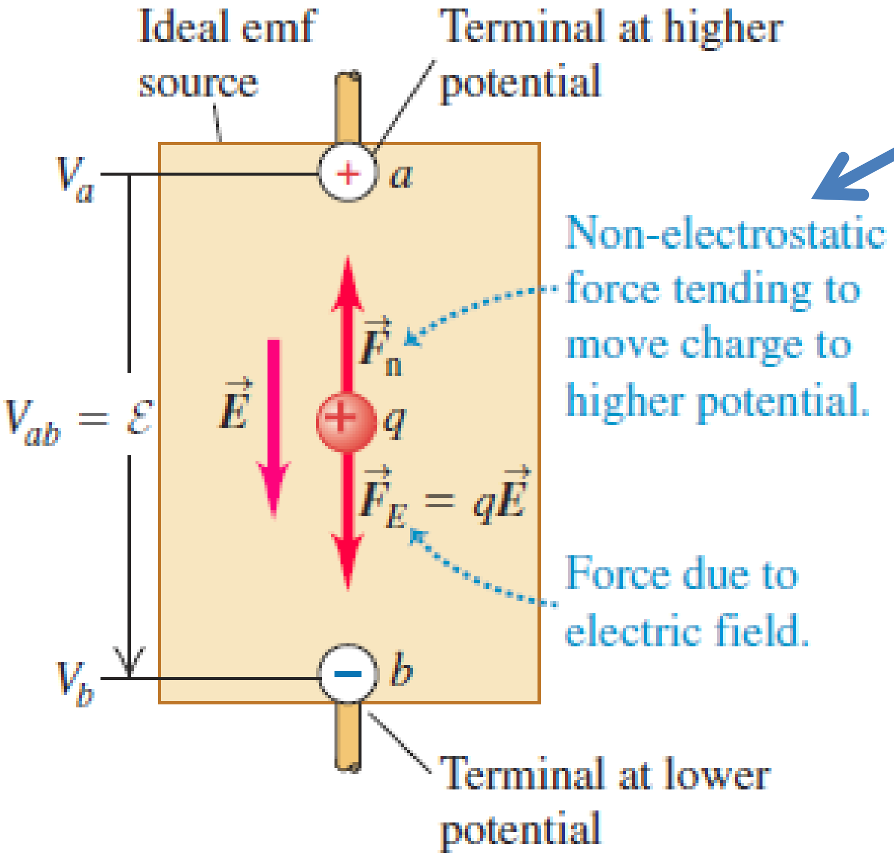
Practice Problem: 14 gauge copper wire has a diameter of 1.6 mm. What length of this wire has a resistance of 1.0 Ω? *Answer:* 120 m.

19.3 Electromotive Force and Circuits

The influence that moves charge from lower to higher potential (despite the electric-field forces in the opposite direction) is called electromotive force (abbreviated emf and pronounced “ee-em-eff”).

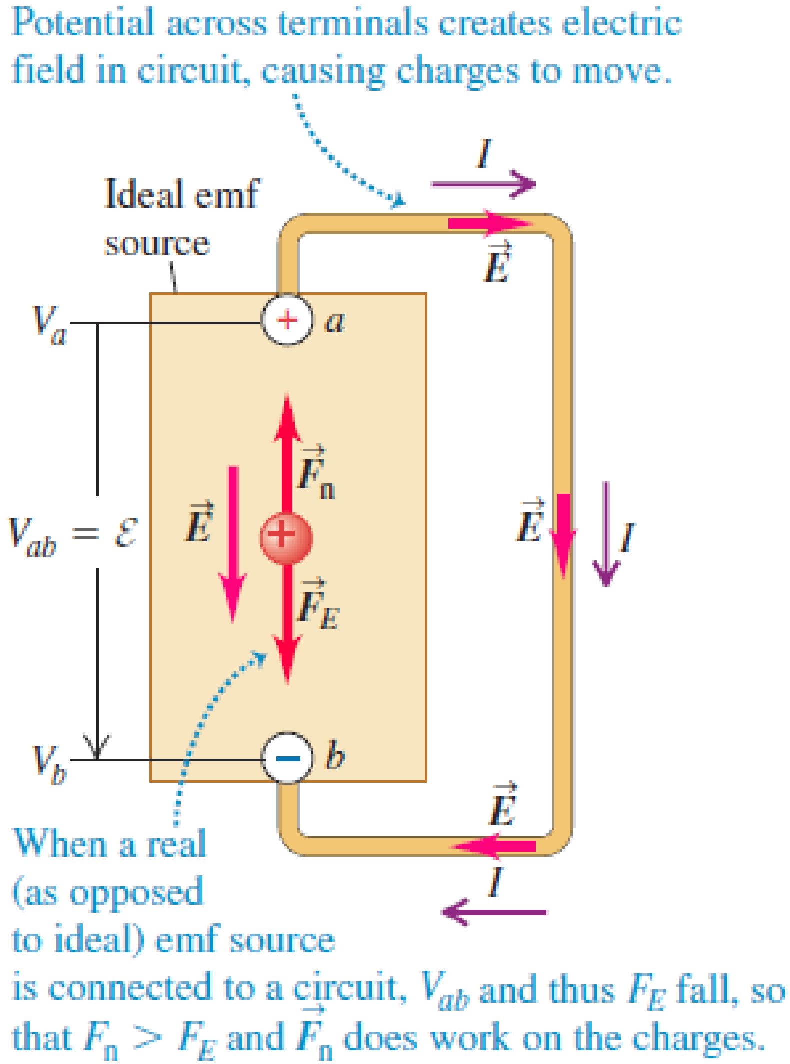
A battery with an emf of 1.5 V does 1.5 J of work on every coulomb of charge that passes through it.

We’ll use the symbol $\mathcal{E}$ for emf.



When the emf source is not part of a closed circuit, $F_n = F_E$ and there is no net motion of charge between the terminals.

The nature of this additional influence depends on the source. In a battery, it is due to chemical processes; in an electric generator, it results from magnetic forces.



Potential across terminals creates electric field in circuit, causing charges to move.

When a real (as opposed to ideal) emf source is connected to a circuit, V_{ab} and thus F_E fall, so that $F_n > F_E$ and $\vec{F}_n$ does work on the charges.

▲ **FIGURE 19.11** Schematic diagram of an ideal emf source in a complete circuit.

No complete circuit

$$V_{ab} = \varepsilon$$

Ideal source

$$V_{ab} = \varepsilon = \mathrm{IR}$$

Real source with internal resistance

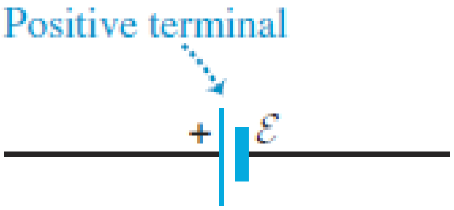
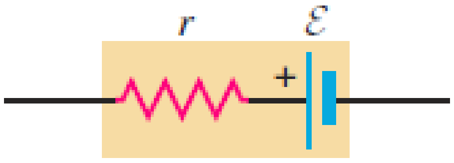
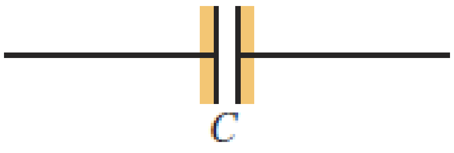
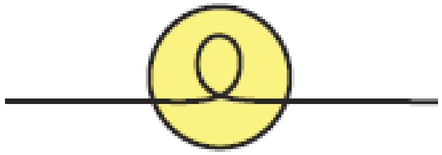
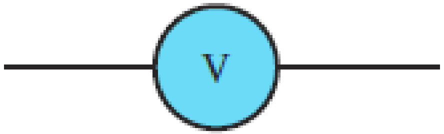
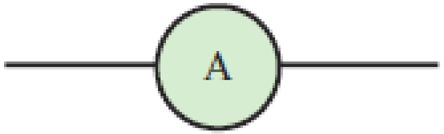
$$V_{ab} = \varepsilon - \mathrm{Ir}$$

Internal Resistance in a Source of emf

Real sources of emf don't behave exactly like the ideal sources we've described because charge that moves through the material of any real source encounters *resistance*. We call this the **internal resistance** of the source, denoted by r. If this resistance behaves according to Ohm's law, r is constant. The current through r has an associated drop in potential equal to Ir. The terminal potential difference V_{ab} is then

$$V_{ab} = \mathcal{E} - Ir. \qquad \text{(source with internal resistance)} \qquad (19.7)$$

TABLE 19.2 Circuit symbols used in this chapter

	Wire with negligible resistance
	Resistor
	emf source
	emf source with internal resistance
	Capacitor
	Switch (open)
	Switch (closed)
	Bulb
	Voltmeter (measures potential difference between its terminals)
	Ammeter (measures current)
	Ground

EXAMPLE 19.5 A dim flashlight

As a flashlight battery ages, its emf stays approximately constant, but its internal resistance increases. A fresh battery has an emf of 1.5 V and negligible internal resistance. When the battery needs replacement, its emf is still 1.5 V, but its internal resistance has increased to 1000 Ω. If this old battery is supplying 1.0 mA to a lightbulb, what is its terminal voltage?

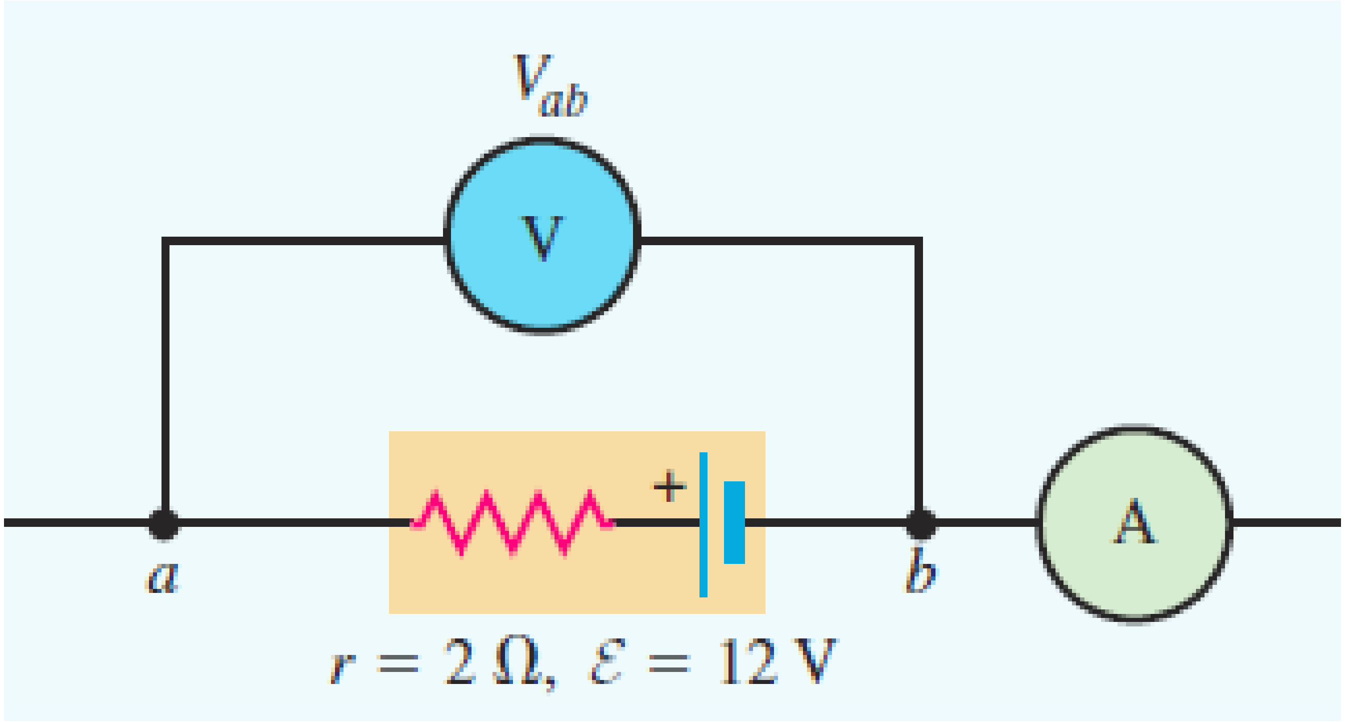
SET UP AND SOLVE The terminal voltage of a new battery is 1.5 V. The terminal voltage of the old, worn-out battery is given by $V_{ab} = \mathcal{E} - Ir$, so

$$V_{ab} = 1.5\ \text{V} - (1.0 \times 10^{-3}\ \text{A})(1000\ \Omega) = 0.5\ \text{V}.$$

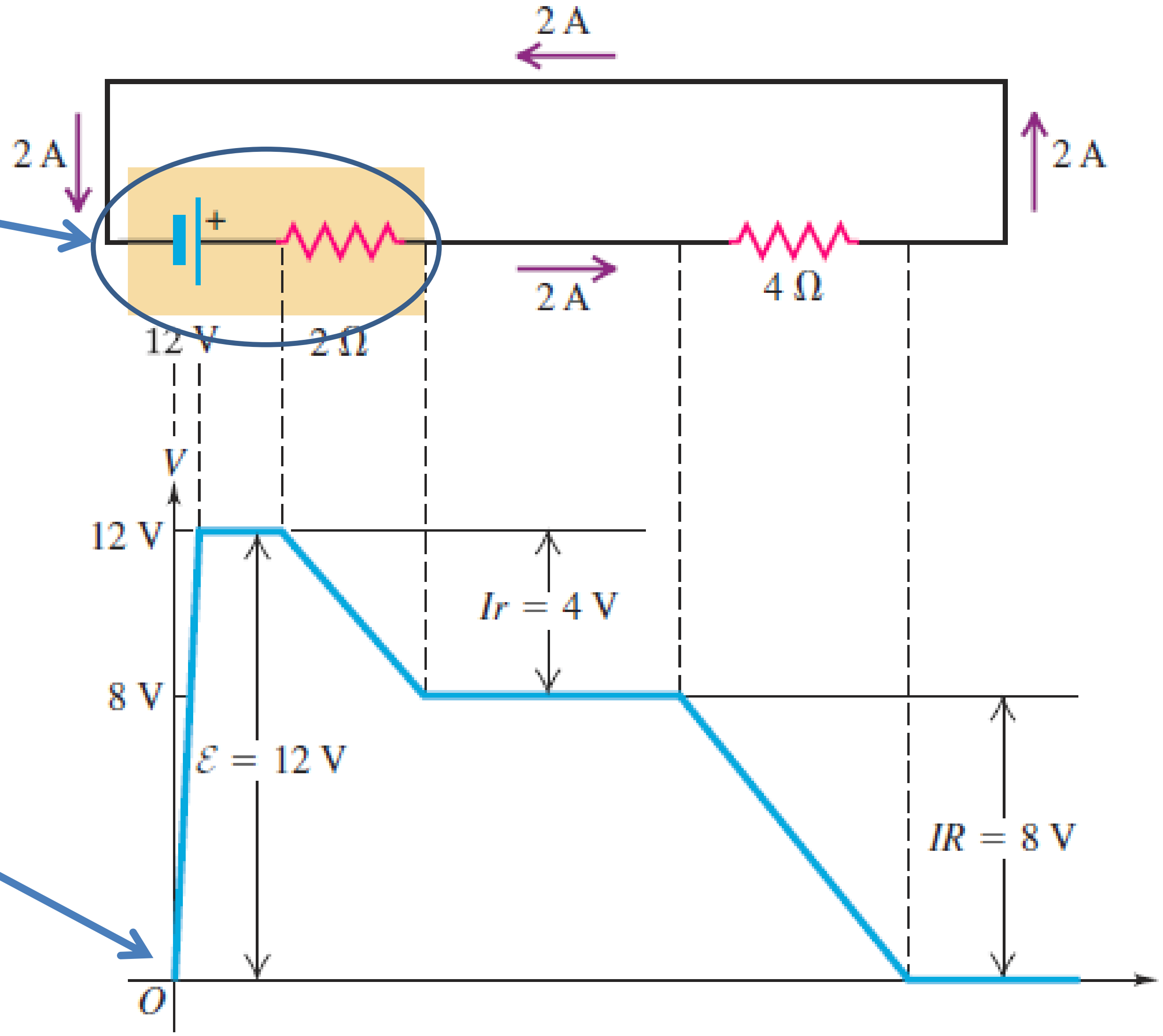
It’s important to understand how the meters in the circuit work.

The symbol V in a circle represents an ideal voltmeter. It measures the potential difference between the two points in the circuit where it is connected, but *no current flows through the volt-meter.*
The symbol A in a circle represents an ideal ammeter. It measures the current that flows through it, but *there is no potential difference between its terminals.*

Thus, the behavior of a circuit doesn’t change when an ideal ammeter or voltmeter is connected to it.


V_{ab}
V
A
a
b
+
$r = 2\ \Omega,\ \mathcal{E} = 12\ \mathrm{V}$

How the potential changes in a circuit.



Battery or cell

Take the potential to be zero at the negative terminal of the battery.

▲ **FIGURE 19.16** Potential rises and drops in the circuit.

19.4 Energy and Power in Electric Circuits

$$Since\ I = \frac{\Delta Q}{\Delta t}\ \ and\ V = \frac{\Delta W}{\Delta Q}$$

Then

$$\Delta W = V\Delta Q = VI\Delta t$$

This work represents electrical energy transferred *into* the circuit element. The time rate of energy transfer is *power,* denoted by P. Dividing the preceding equation by Δt, we obtain the *time rate* at which the rest of the circuit delivers electrical energy to the circuit element:

$$\frac{\Delta W}{\Delta t} = P = V_{ab} I. \tag{19.9}$$

$$P = V_{ab}I = I^2R = \frac{V_{ab}{}^2}{R}. \tag{19.10}$$

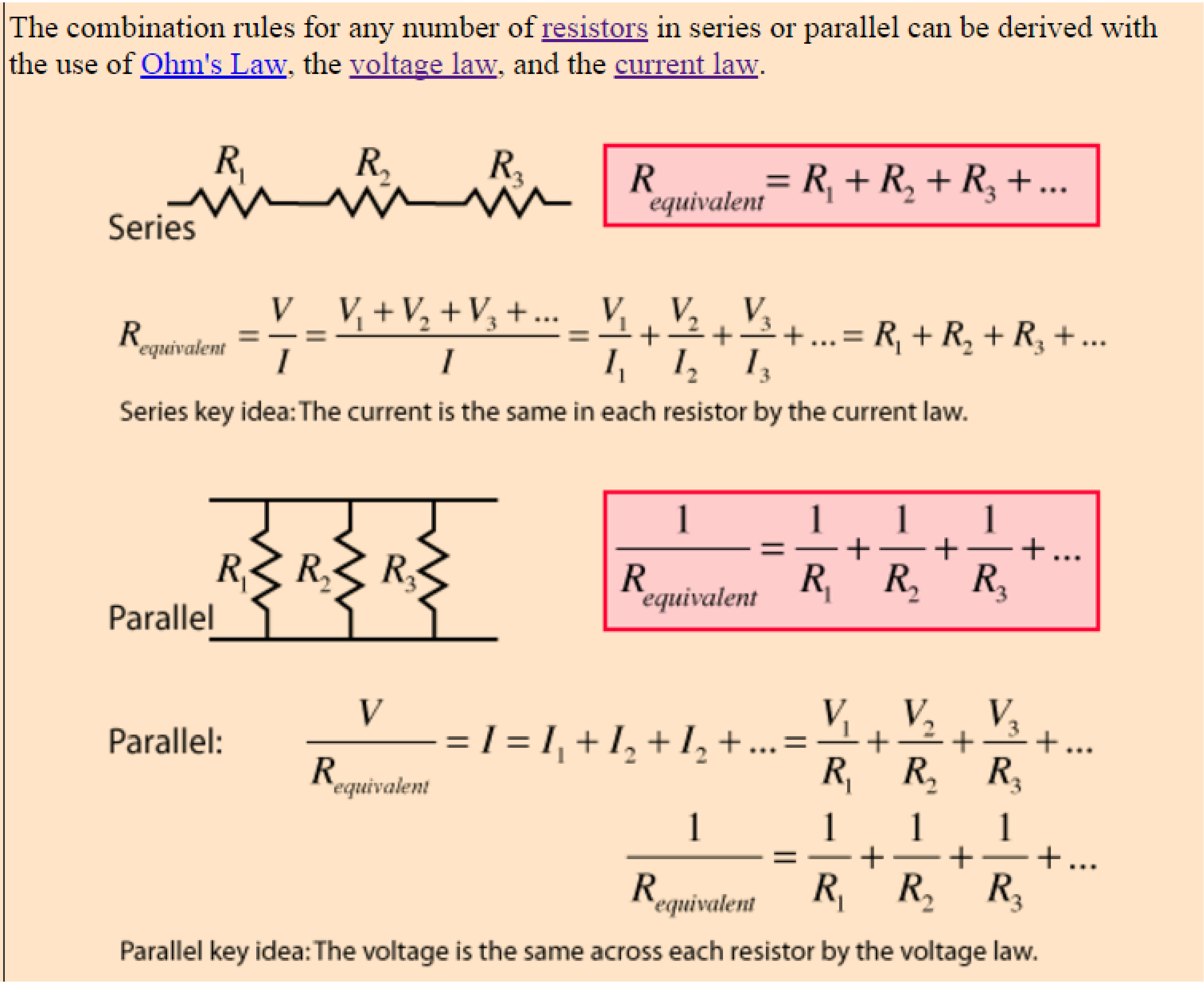
What becomes of this energy? The moving charges collide with atoms in the resistor and transfer some of their energy to these atoms, increasing the internal energy of the material. Either the temperature of the resistor increases, or there is a flow of heat out of it, or both. We say that energy is *dissipated* in the resistor at a rate I^2R. Too high a temperature can change the resistance unpredictably; the resistor may melt or even explode. Of course, some devices, such as electric heaters, are designed to get hot and transfer heat to their surroundings. But every resistor has a *power rating:* the maximum power that the device can dissipate without becoming overheated and damaged. In practical applications, the power rating of a resistor is often just as important a characteristic as its resistance.

▲ Application **Cheap light** If you've had incandescent flashlights or bicycle lights and changed to lights that use light-emitting diodes (LEDs), you know the large difference in energy consumption. A halogen bicycle headlight might go through a set of batteries in 3 hours, but an even brighter LED headlight will last 30 hours. Why the difference? The answer is that any incandescent bulb (including a halogen bulb) works by using the dissipation of electrical energy to heat a filament white hot. Some of the energy is converted to visible light, but most is lost as heat. In an LED, electrical energy is used to move semiconductor electrons to a region where they emit light. Most of the electrical energy, then, emerges as light; little is lost as heat.

Power Output of a Source

Using $V = \varepsilon - Ir$

$$P = VI = \varepsilon I - I^2 r$$



The combination rules for any number of resistors in series or parallel can be derived with the use of Ohm's Law, the voltage law, and the current law.

$$\boxed{R_{equivalent} = R_1 + R_2 + R_3 + \dots}$$

$$R_{equivalent} = \frac{V}{I} = \frac{V_1 + V_2 + V_3 + \dots}{I} = \frac{V_1}{I_1} + \frac{V_2}{I_2} + \frac{V_3}{I_3} + \dots = R_1 + R_2 + R_3 + \dots$$

Series key idea: The current is the same in each resistor by the current law.

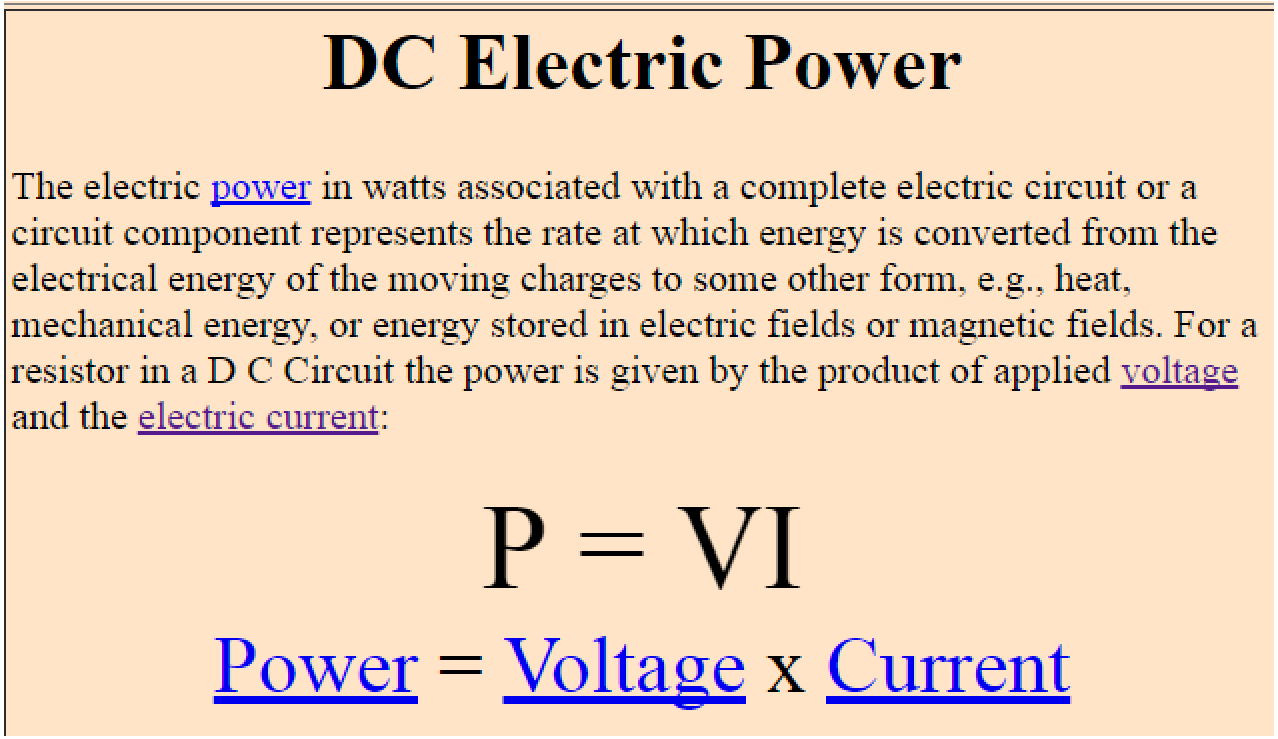
$$\boxed{\frac{1}{R_{equivalent}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots}$$

Parallel:

$$\frac{V}{R_{equivalent}} = I = I_1 + I_2 + I_2 + \dots = \frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} + \dots$$

$$\frac{1}{R_{equivalent}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots$$

Parallel key idea: The voltage is the same across each resistor by the voltage law.

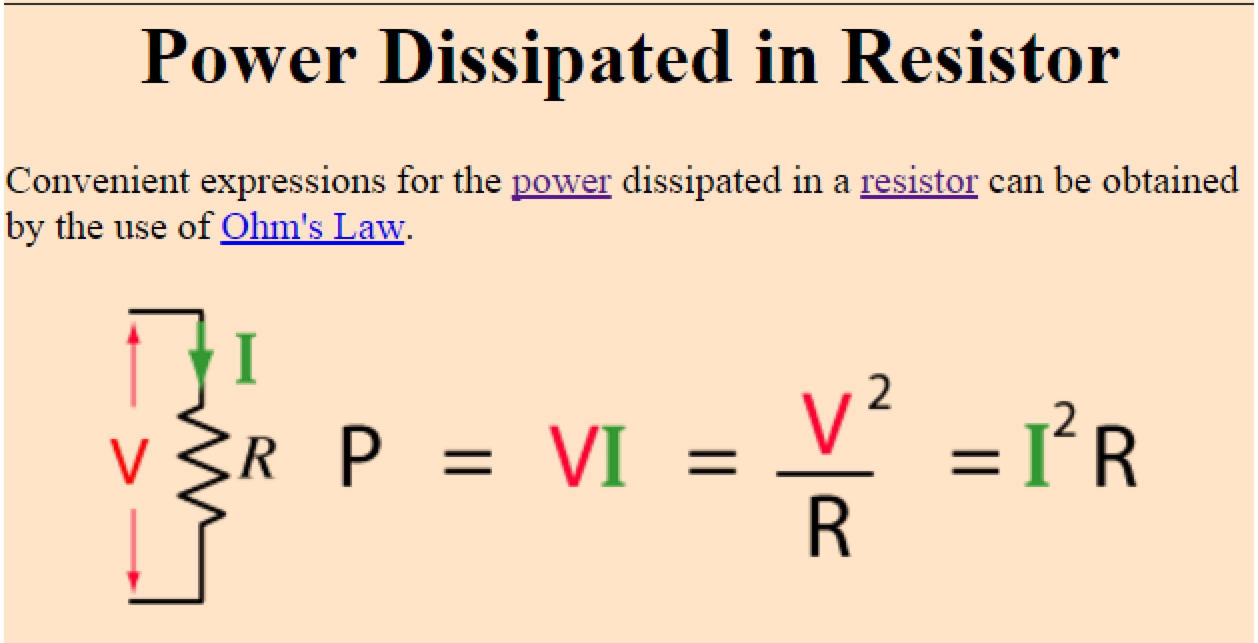


DC Electric Power

The electric power in watts associated with a complete electric circuit or a circuit component represents the rate at which energy is converted from the electrical energy of the moving charges to some other form, e.g., heat, mechanical energy, or energy stored in electric fields or magnetic fields. For a resistor in a D C Circuit the power is given by the product of applied voltage and the electric current:

$$P = VI$$

Power = Voltage x Current

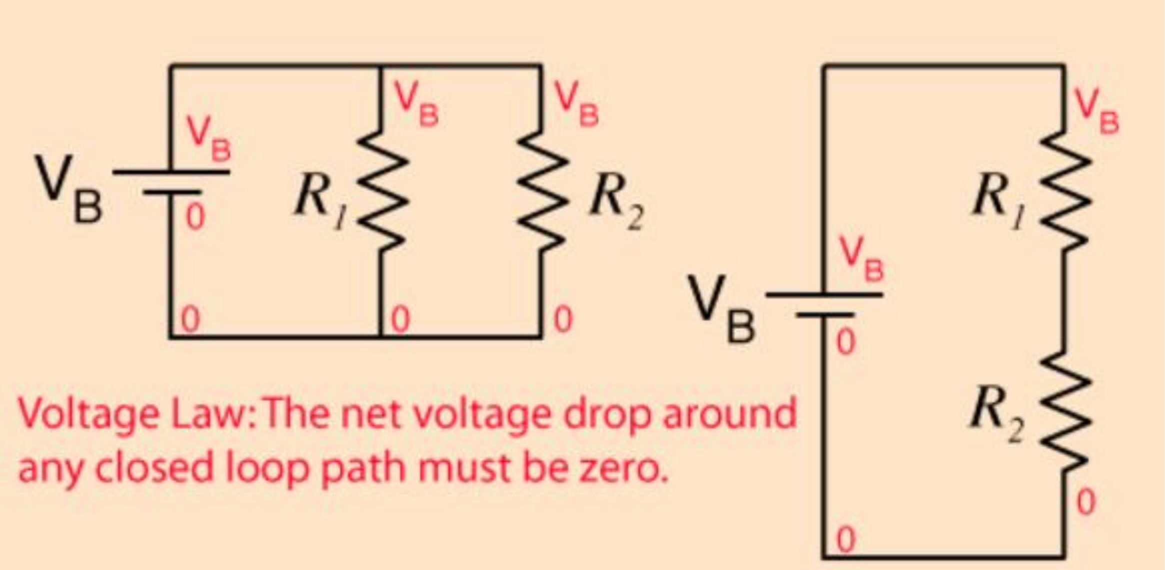


Power Dissipated in Resistor

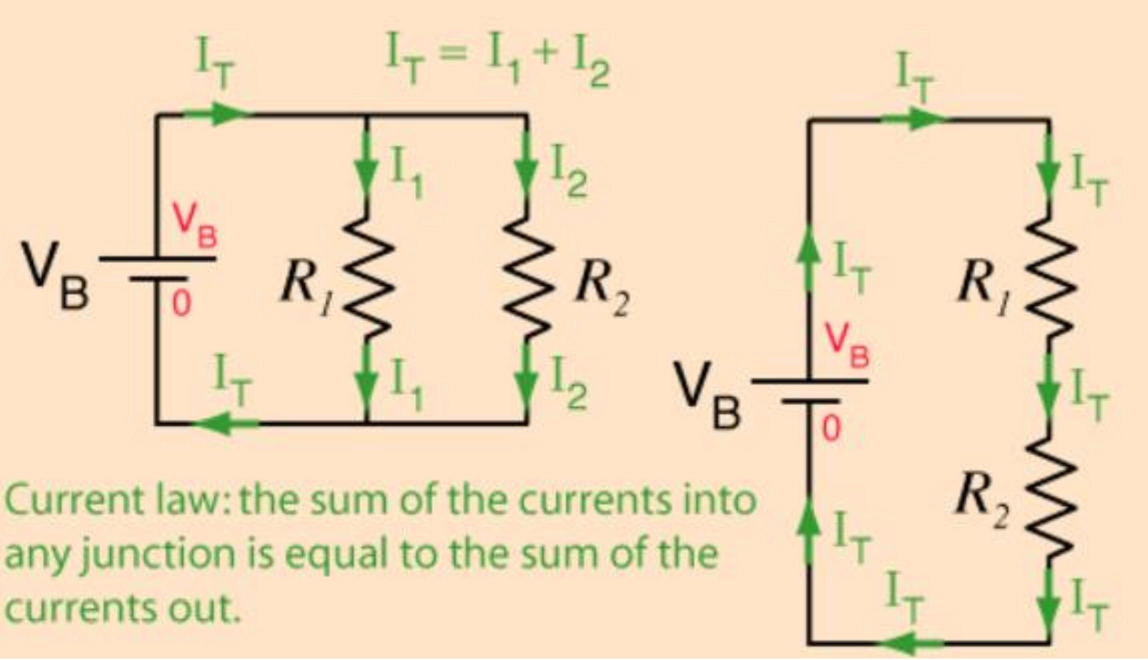
Convenient expressions for the power dissipated in a resistor can be obtained by the use of Ohm's Law.

$$P = VI = \frac{V^2}{R} = I^2 R$$

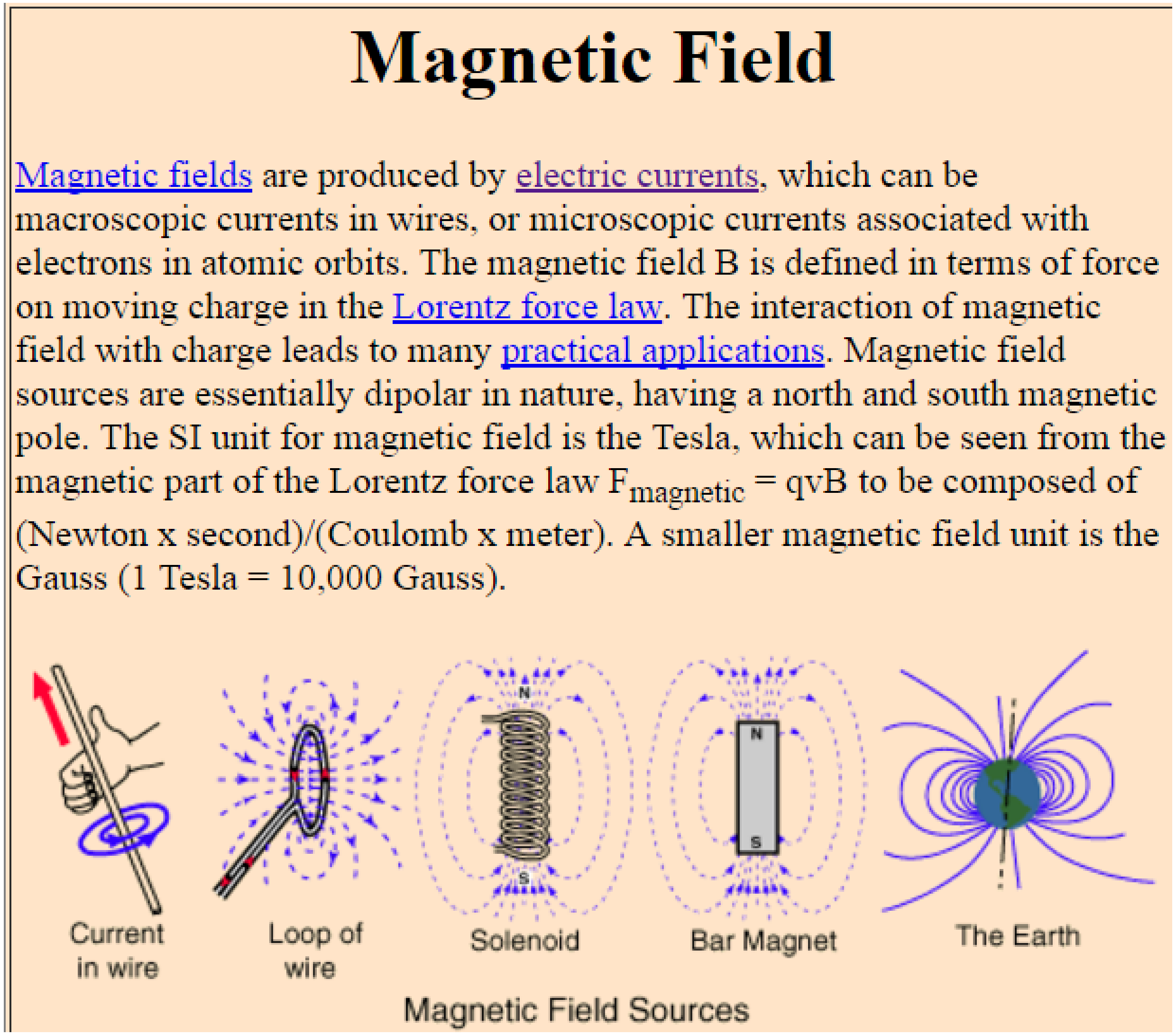
Kirchoff



Voltage Law: The net voltage drop around any closed loop path must be zero.



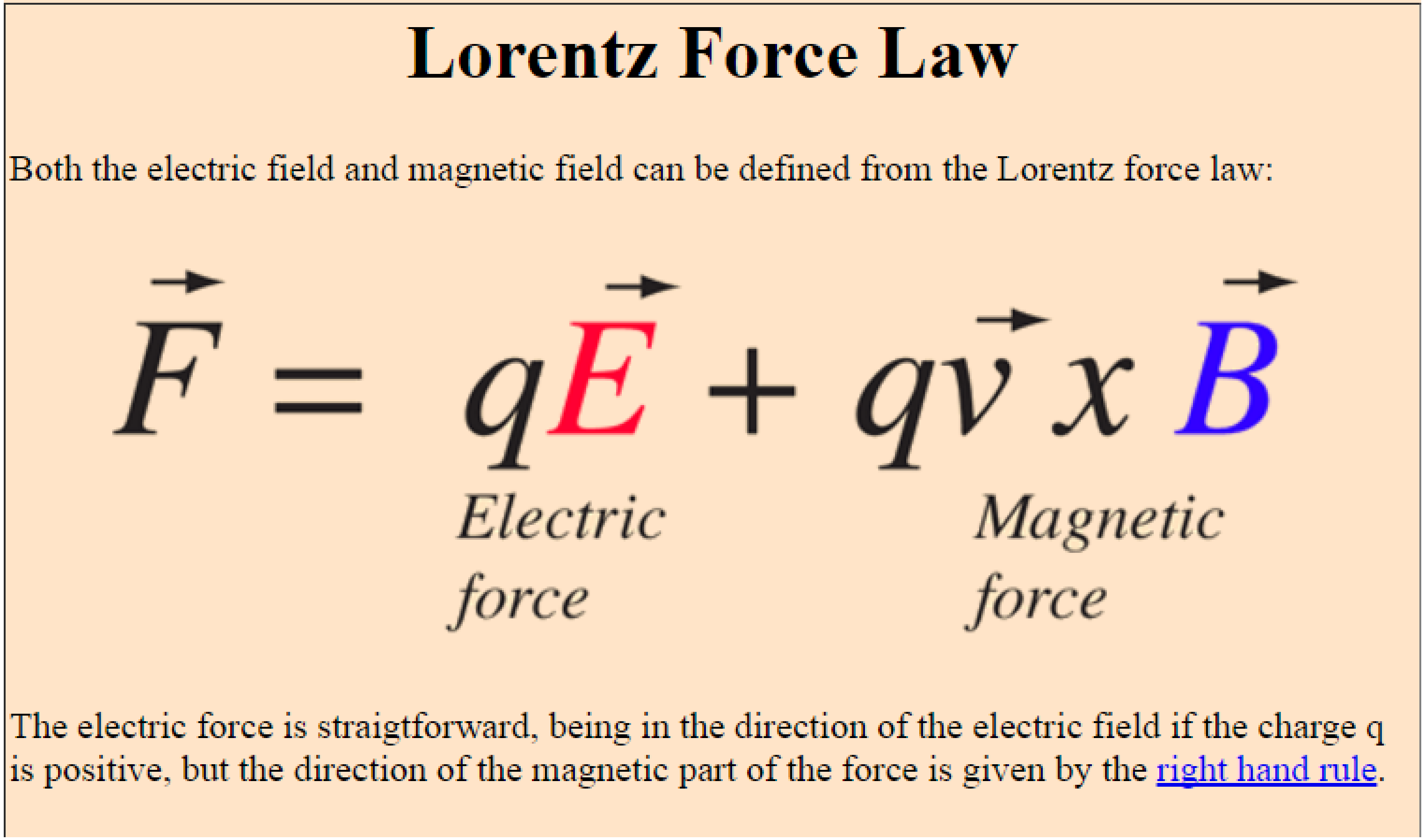
Current law: the sum of the currents into any junction is equal to the sum of the currents out.



Magnetic Field

Magnetic fields are produced by electric currents, which can be macroscopic currents in wires, or microscopic currents associated with electrons in atomic orbits. The magnetic field B is defined in terms of force on moving charge in the Lorentz force law. The interaction of magnetic field with charge leads to many practical applications. Magnetic field sources are essentially dipolar in nature, having a north and south magnetic pole. The SI unit for magnetic field is the Tesla, which can be seen from the magnetic part of the Lorentz force law $F_{magnetic} = qvB$ to be composed of (Newton x second)/(Coulomb x meter). A smaller magnetic field unit is the Gauss (1 Tesla = 10,000 Gauss).

Magnetic Field Sources

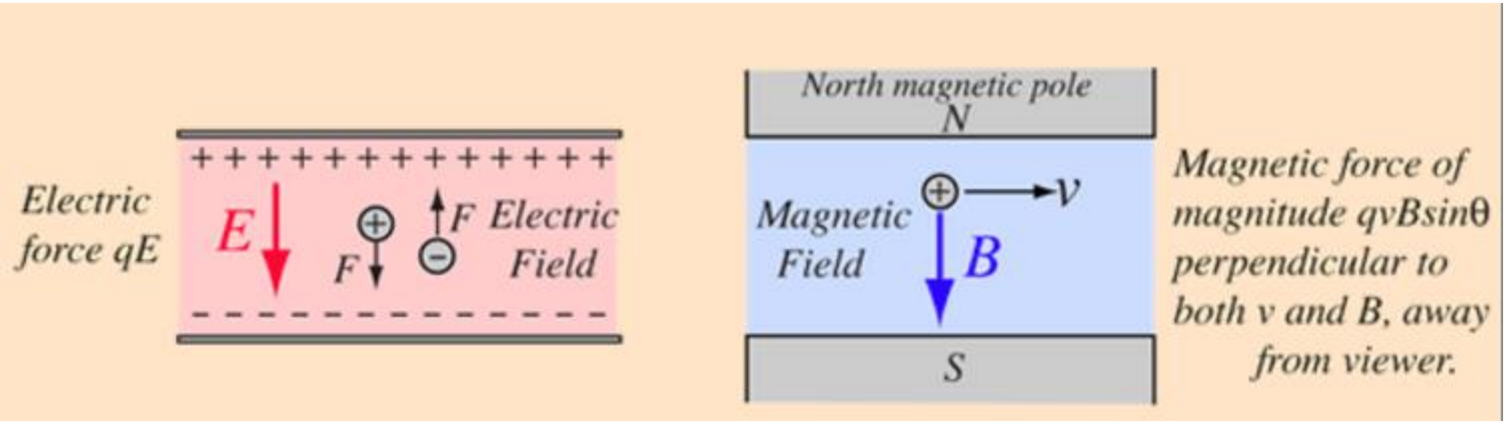


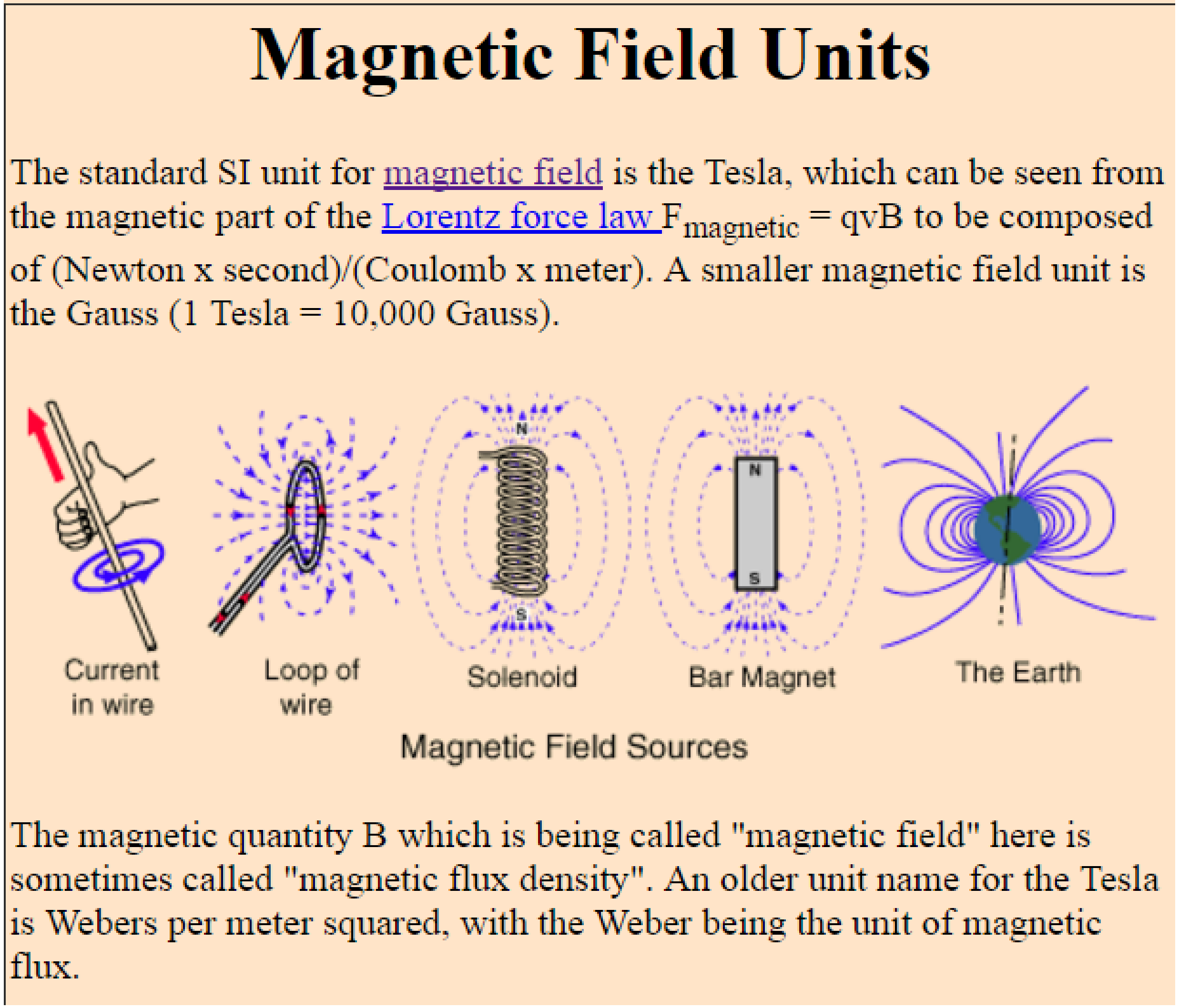
Lorentz Force Law

Both the electric field and magnetic field can be defined from the Lorentz force law:

$$\vec{F} = \underset{\text{Electric force}}{q\vec{E}} + \underset{\text{Magnetic force}}{q\vec{v} \; x \; \vec{B}}$$

The electric force is straigtforward, being in the direction of the electric field if the charge q is positive, but the direction of the magnetic part of the force is given by the right hand rule.


Electric force qE
+ + + + + + + + + + + + + +
E
F
F
Electric Field
- - - - - - - - - - - - - -
North magnetic pole
N
Magnetic Field
v
B
S
Magnetic force of magnitude qvBsinθ perpendicular to both v and B, away from viewer.

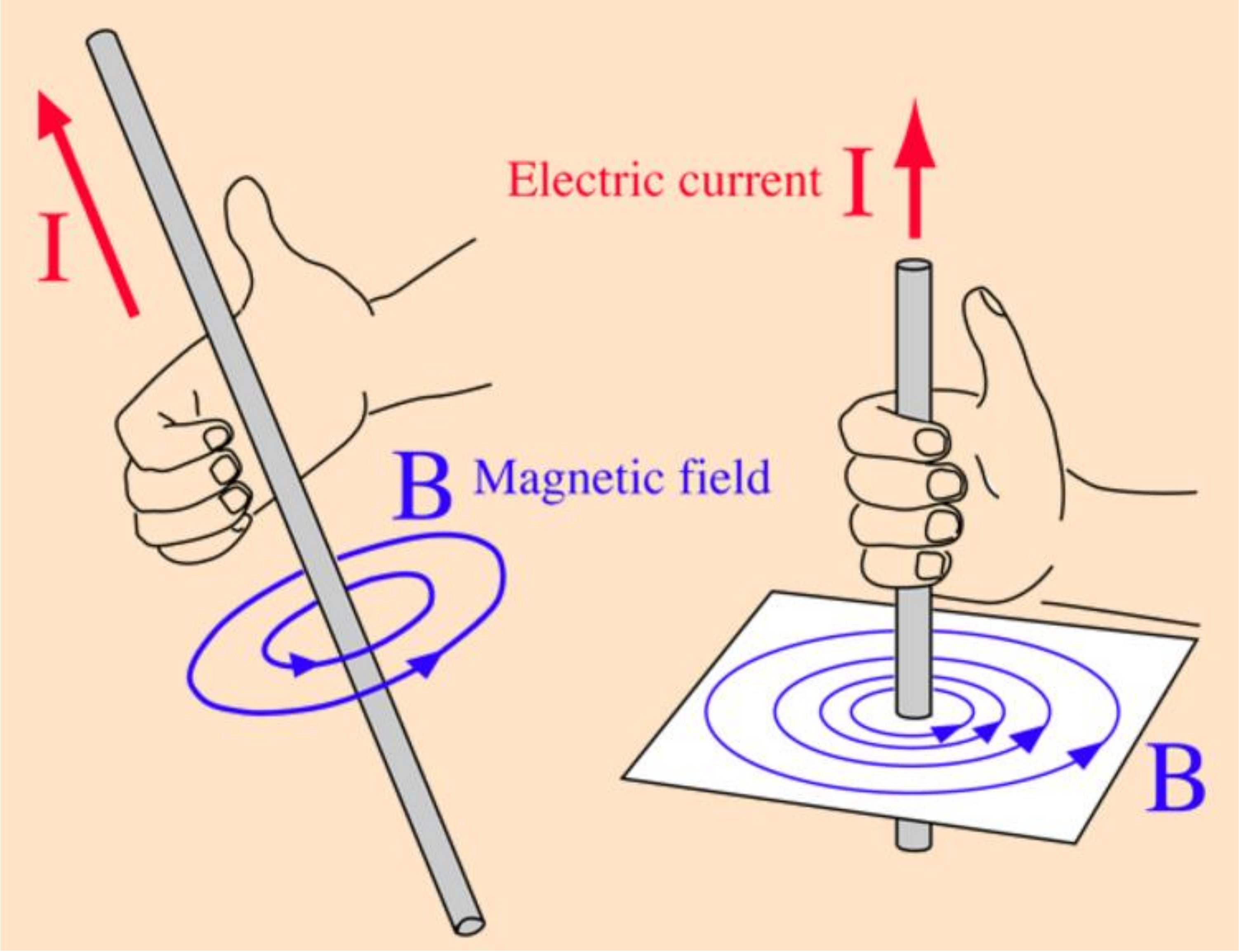


Magnetic Field Units

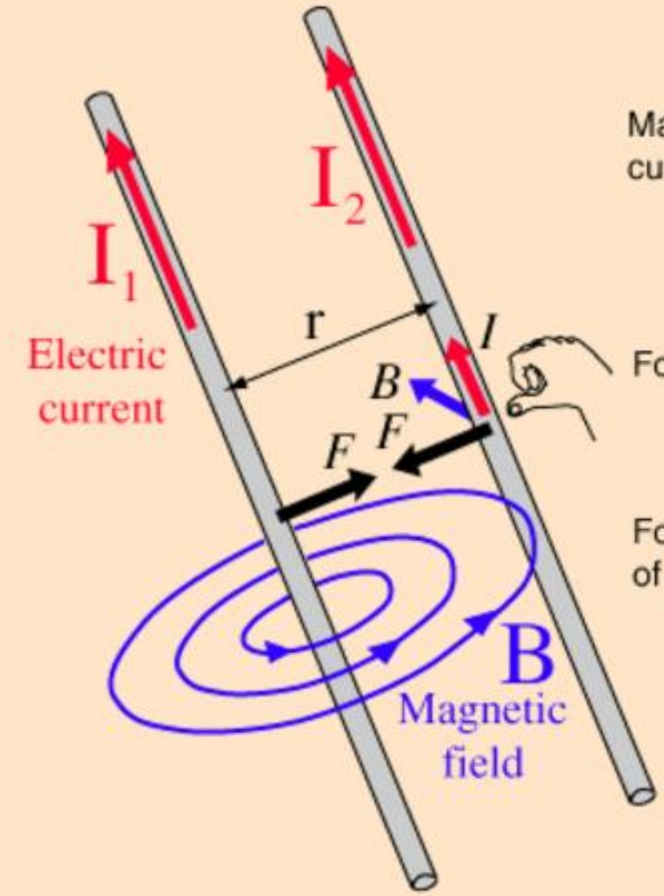
The standard SI unit for magnetic field is the Tesla, which can be seen from the magnetic part of the Lorentz force law $F_{magnetic} = qvB$ to be composed of (Newton x second)/(Coulomb x meter). A smaller magnetic field unit is the Gauss (1 Tesla = 10,000 Gauss).

Magnetic Field Sources

The magnetic quantity B which is being called "magnetic field" here is sometimes called "magnetic flux density". An older unit name for the Tesla is Webers per meter squared, with the Weber being the unit of magnetic flux.


I
Electric current I
B Magnetic field
B

Magnetic Force Between Wires



Magnetic field at wire 2 from current in wire 1:

$$B = \frac{\mu_0 I_1}{2\pi r}$$

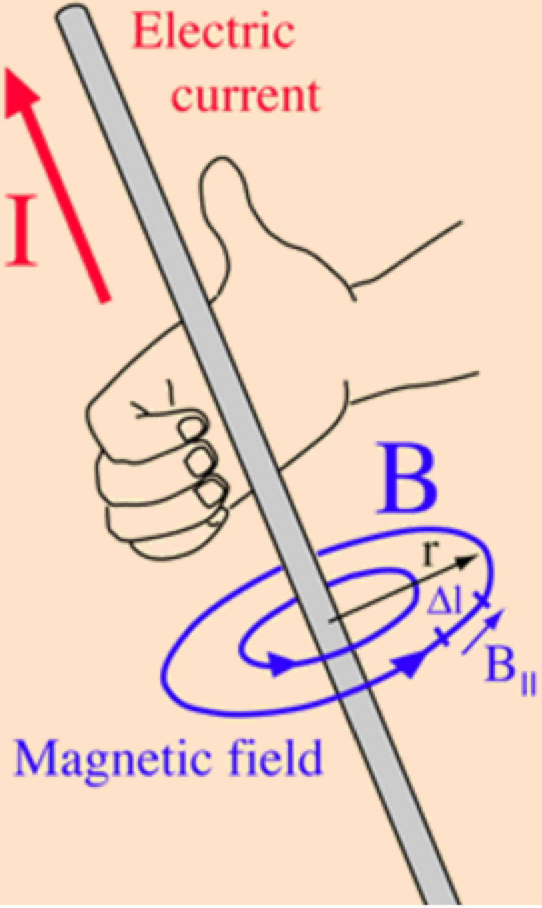
Force on a length ΔL of wire 2:

$$F = I_2 \Delta L B$$

Force per unit length in terms of the currents:

$$\frac{F}{\Delta L} = \frac{\mu_0 I_1 I_2}{2\pi r}$$

Magnetic Field of Current



The magnetic field of an infinitely long straight wire can be obtained by applying Ampere's law. Ampere's law takes the form

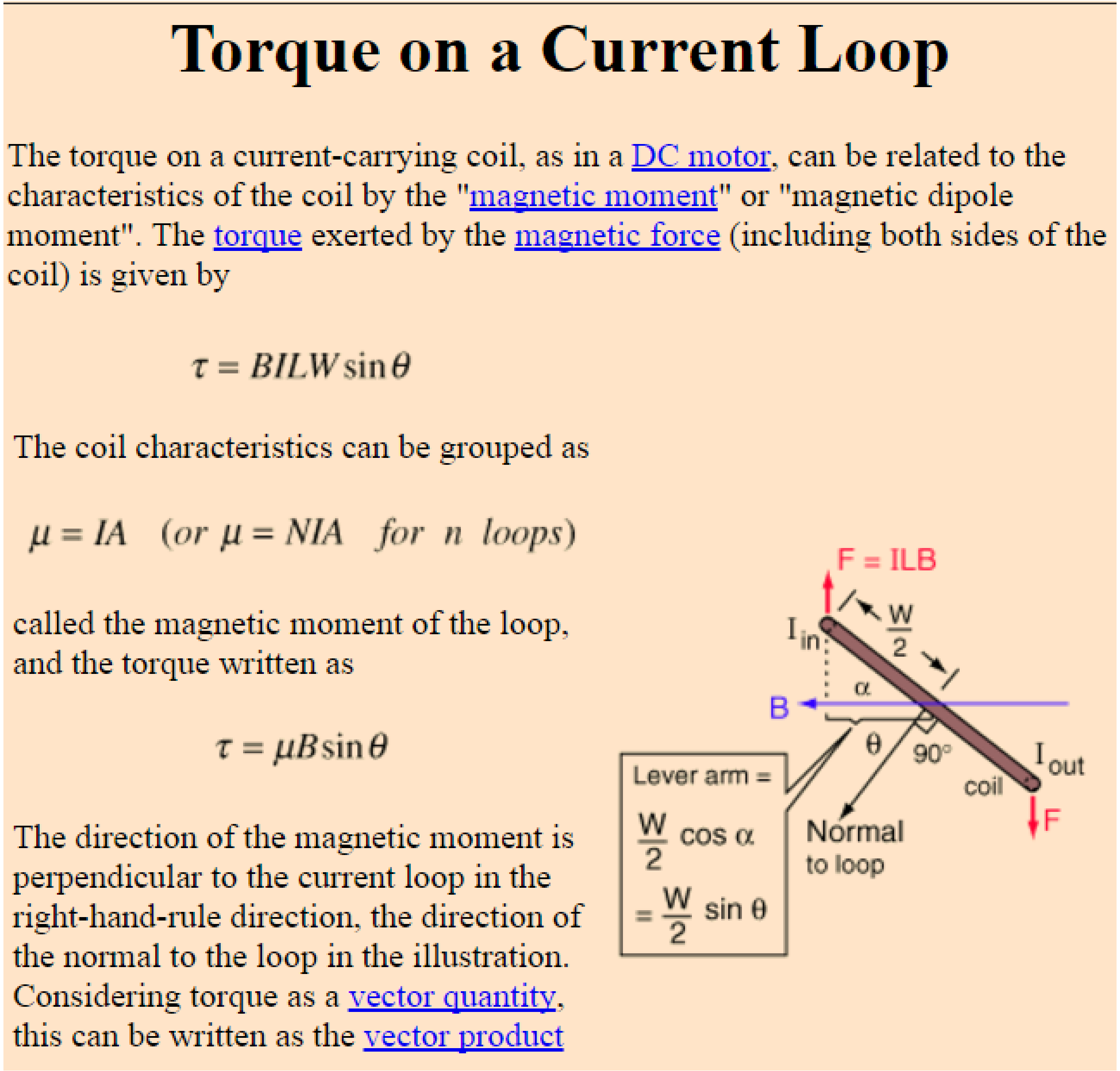
$$\sum B_{||} \Delta l = \mu_0 I$$

and for a circular path centered on the wire, the magnetic field is everywhere parallel to the path. The summation then becomes just

$$\sum B_{||} \Delta l = B 2\pi r$$

$$B = \frac{\mu_0 I}{2\pi r}$$

The constant μ_0 is the permeability of free space.



Torque on a Current Loop

The torque on a current-carrying coil, as in a DC motor, can be related to the characteristics of the coil by the "magnetic moment" or "magnetic dipole moment". The torque exerted by the magnetic force (including both sides of the coil) is given by

$$\tau = BILW\sin\theta$$

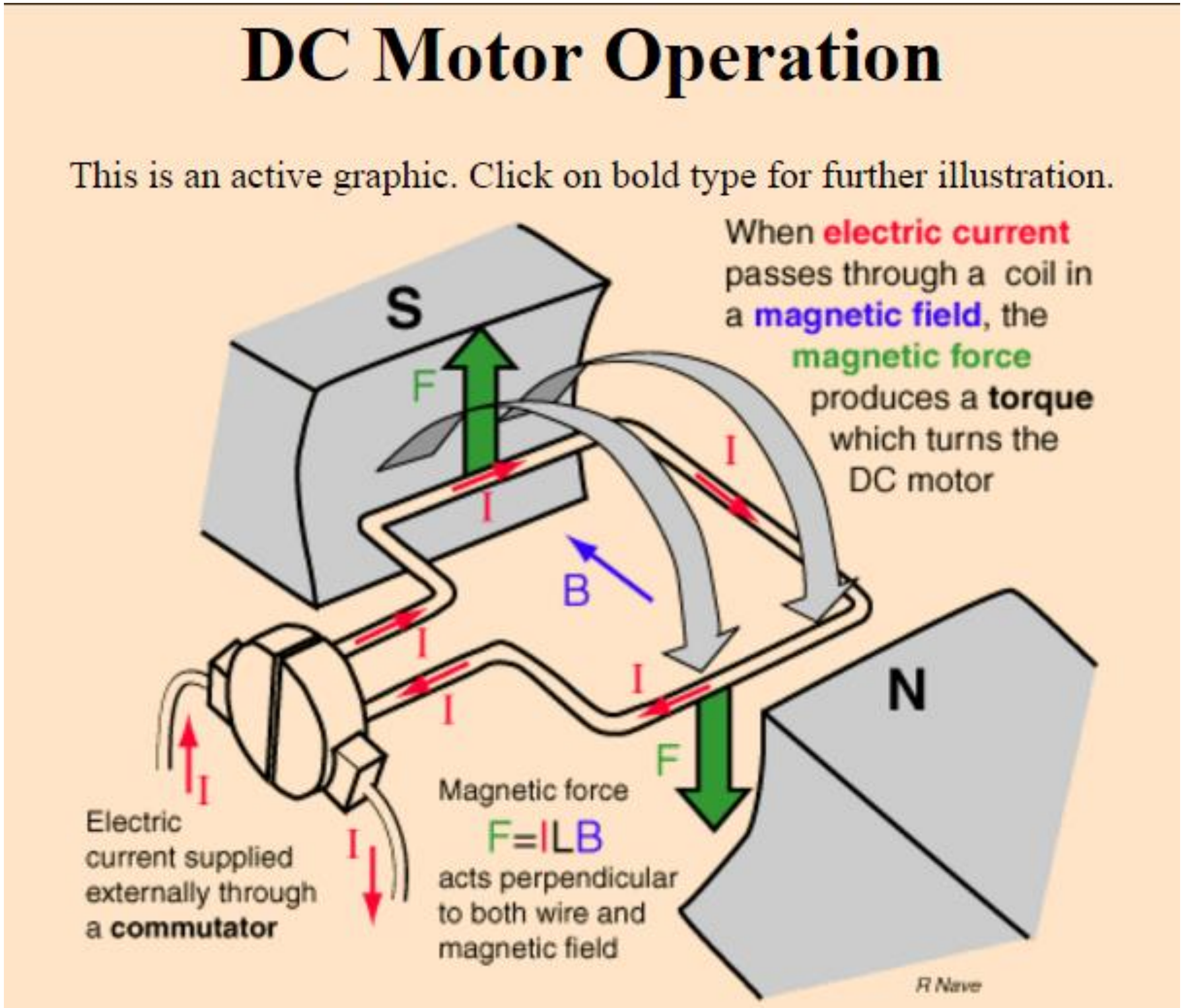
The coil characteristics can be grouped as

$$\mu = IA \quad (or\ \mu = NIA \quad for\ \ n\ \ loops)$$

called the magnetic moment of the loop, and the torque written as

$$\tau = \mu B\sin\theta$$

The direction of the magnetic moment is perpendicular to the current loop in the right-hand-rule direction, the direction of the normal to the loop in the illustration. Considering torque as a vector quantity, this can be written as the vector product



DC Motor Operation

This is an active graphic. Click on bold type for further illustration.

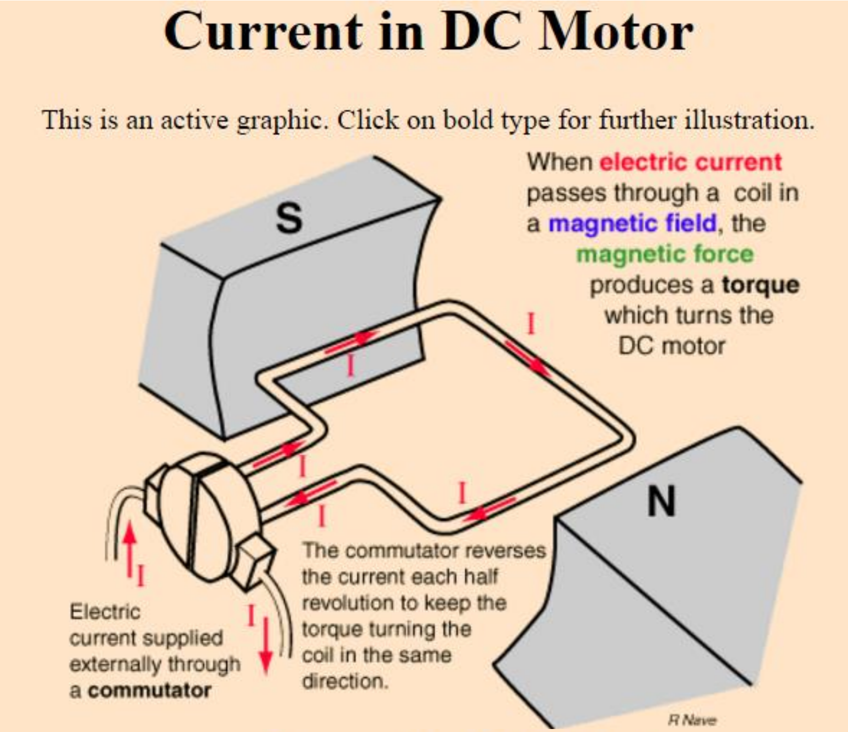
When **electric current** passes through a coil in a **magnetic field**, the **magnetic force** produces a **torque** which turns the DC motor

Electric current supplied externally through a **commutator**

Magnetic force

$F = ILB$

acts perpendicular to both wire and magnetic field



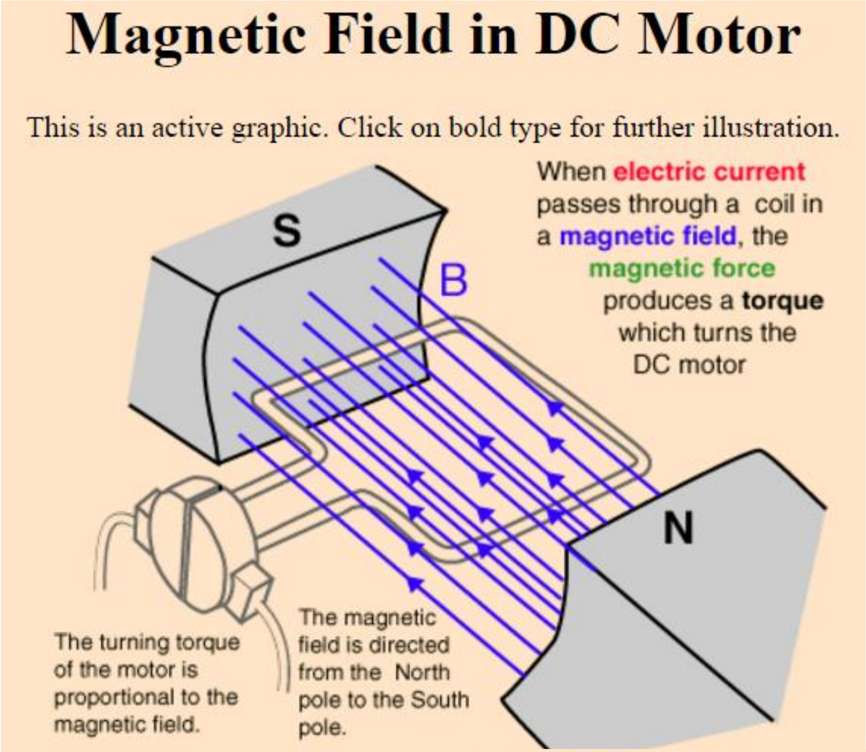
Current in DC Motor

This is an active graphic. Click on bold type for further illustration.

When **electric current** passes through a coil in a **magnetic field**, the **magnetic force** produces a **torque** which turns the DC motor

The commutator reverses the current each half revolution to keep the torque turning the coil in the same direction.

Electric current supplied externally through a **commutator**



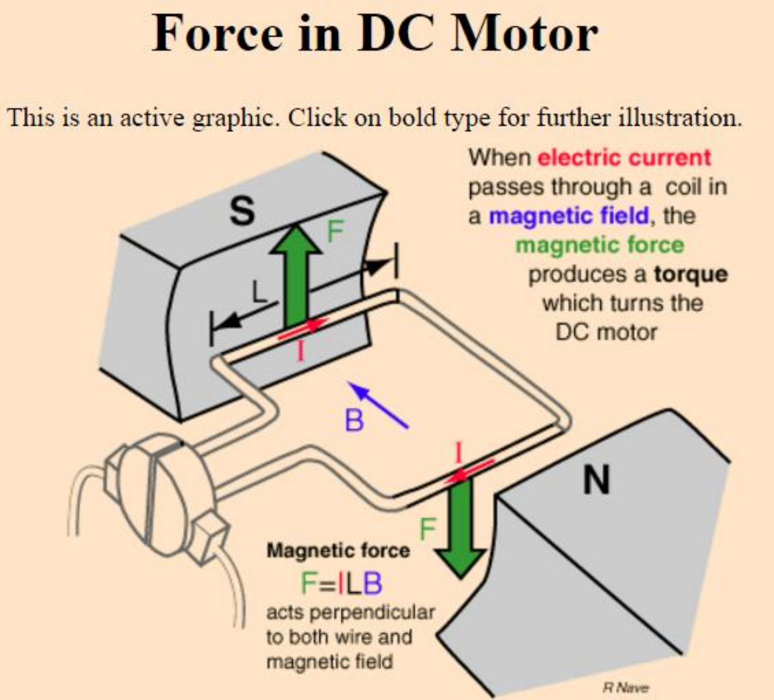
Magnetic Field in DC Motor

This is an active graphic. Click on bold type for further illustration.

When **electric current** passes through a coil in a **magnetic field**, the **magnetic force** produces a **torque** which turns the DC motor

The turning torque of the motor is proportional to the magnetic field.

The magnetic field is directed from the North pole to the South pole.



Force in DC Motor

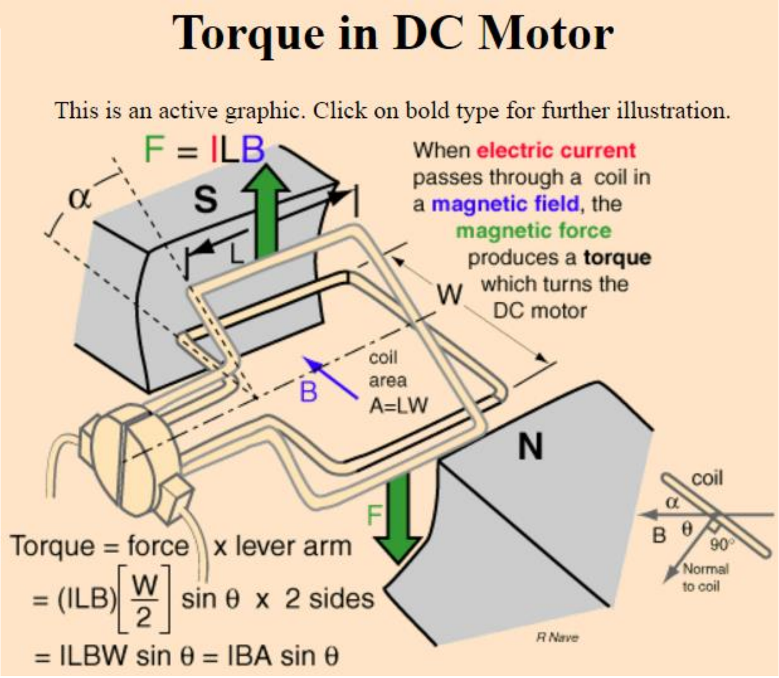
This is an active graphic. Click on bold type for further illustration.

When **electric current** passes through a coil in a **magnetic field**, the **magnetic force** produces a **torque** which turns the DC motor

Magnetic force

$F=ILB$

acts perpendicular to both wire and magnetic field



Torque in DC Motor

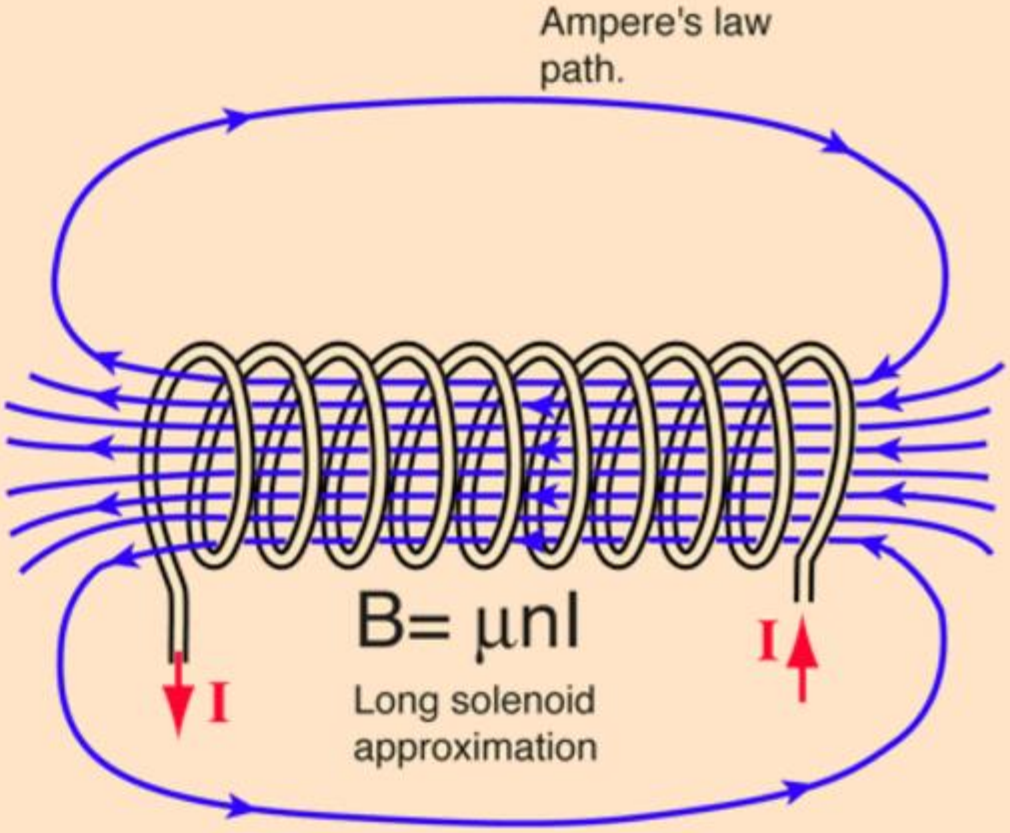
This is an active graphic. Click on bold type for further illustration.

When **electric current** passes through a coil in a **magnetic field**, the **magnetic force** produces a **torque** which turns the DC motor

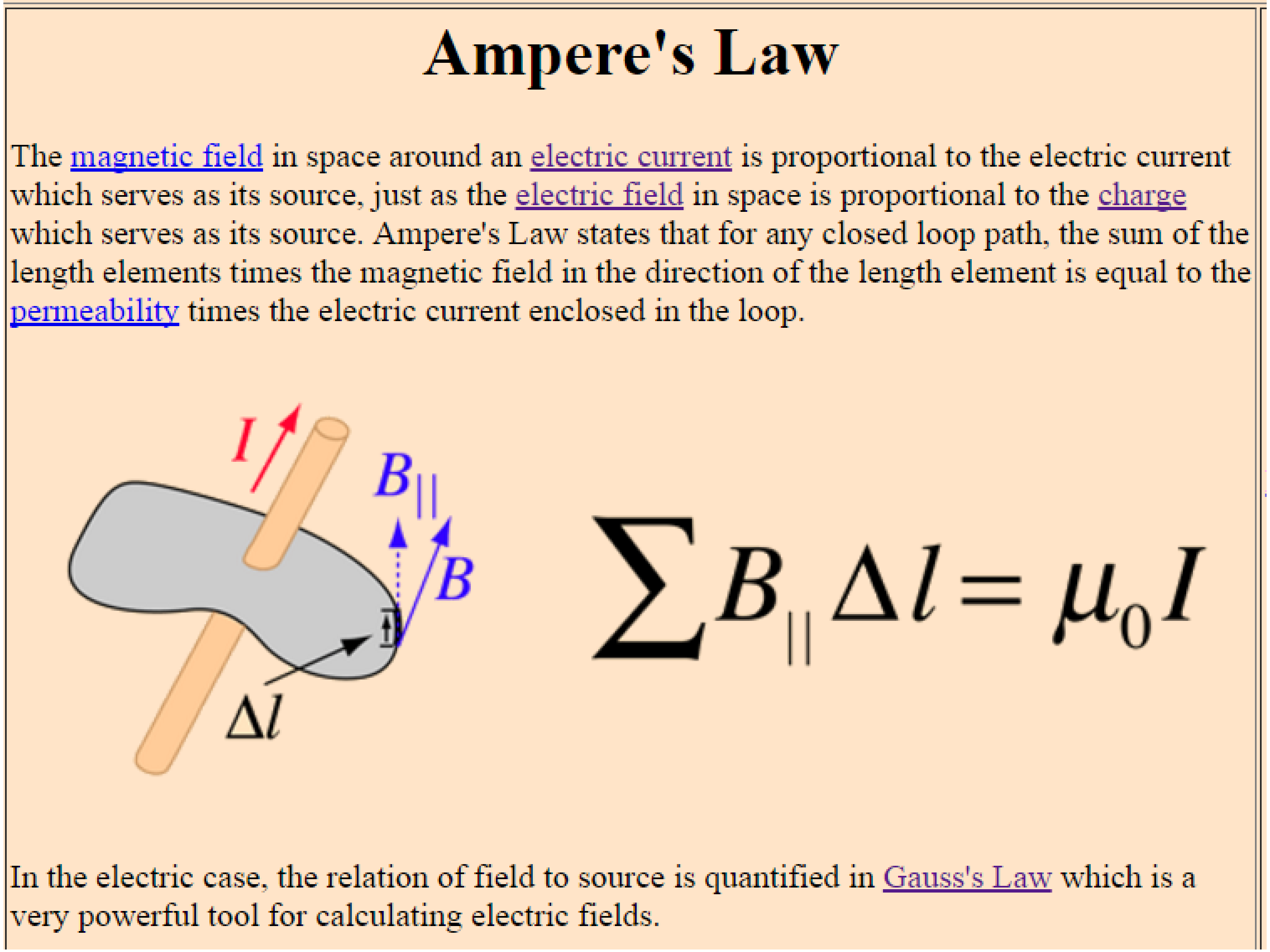
$$\text{Torque} = \text{force} \times \text{lever arm}$$
$$= (ILB)\left[\frac{W}{2}\right]\sin\theta \times 2 \text{ sides}$$
$$= ILBW\sin\theta = IBA\sin\theta$$

Solenoid

A long straight coil of wire can be used to generate a nearly uniform magnetic field similar to that of a bar magnet. Such coils, called solenoids, have an enormous number of practical applications. The field can be greatly strengthened by the addition of an iron core. Such cores are typical in electromagnets.



The magnetic field is concentrated into a nearly uniform field in the center of a long solenoid. The field outside is weak and divergent.

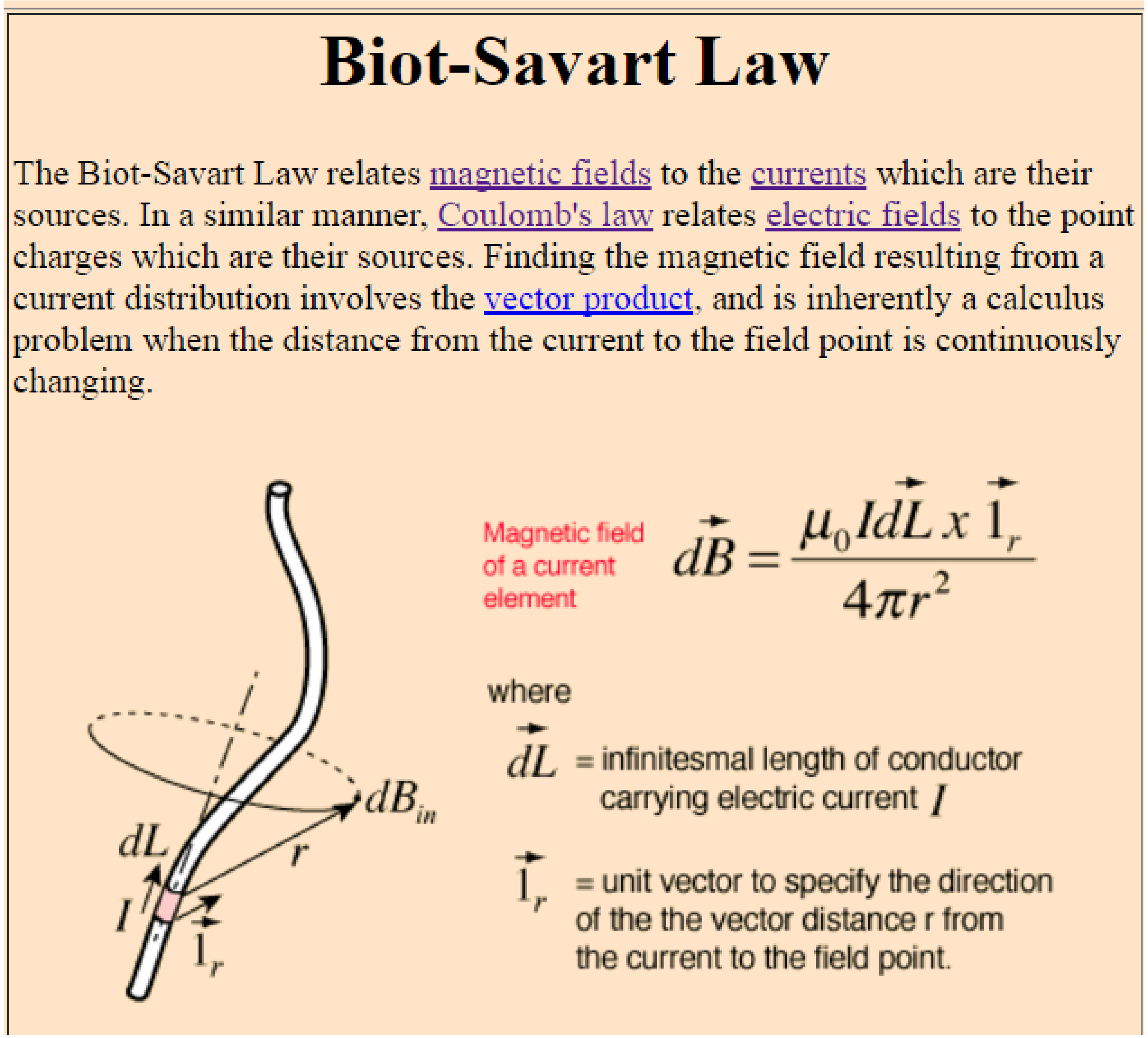


Ampere's Law

The magnetic field in space around an electric current is proportional to the electric current which serves as its source, just as the electric field in space is proportional to the charge which serves as its source. Ampere's Law states that for any closed loop path, the sum of the length elements times the magnetic field in the direction of the length element is equal to the permeability times the electric current enclosed in the loop.

$$\sum B_{||} \Delta l = \mu_0 I$$

In the electric case, the relation of field to source is quantified in Gauss's Law which is a very powerful tool for calculating electric fields.



Biot-Savart Law

The Biot-Savart Law relates magnetic fields to the currents which are their sources. In a similar manner, Coulomb's law relates electric fields to the point charges which are their sources. Finding the magnetic field resulting from a current distribution involves the vector product, and is inherently a calculus problem when the distance from the current to the field point is continuously changing.

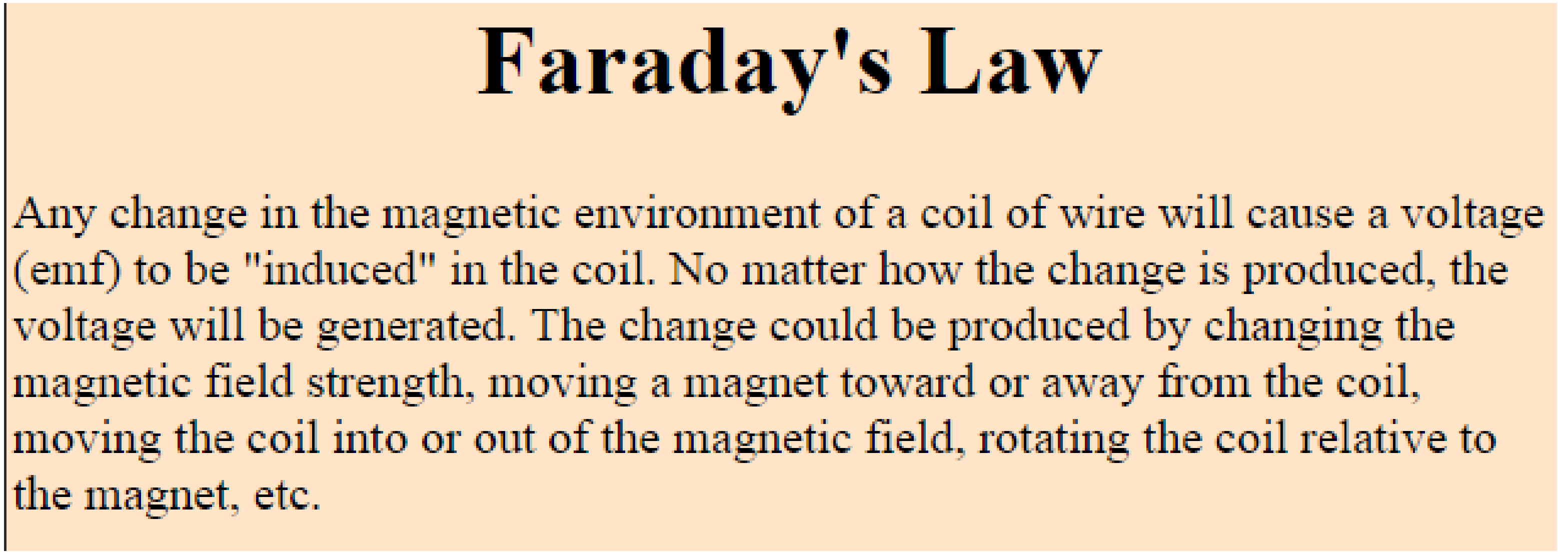
Magnetic field of a current element

$$d\vec{B} = \frac{\mu_0 I d\vec{L} \, x \, \vec{1}_r}{4\pi r^2}$$

where

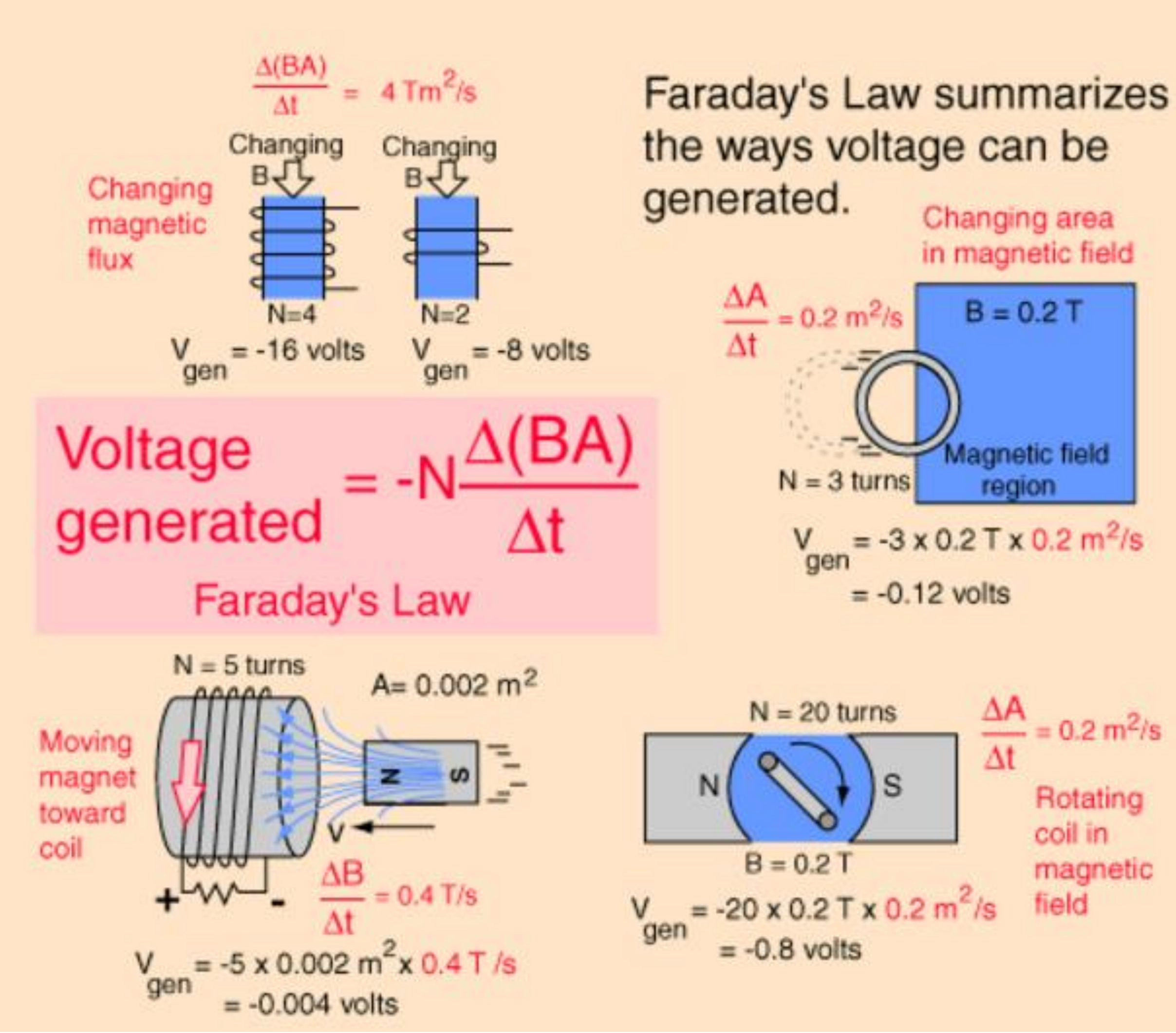
$d\vec{L}$ = infinitesmal length of conductor carrying electric current I

$\vec{1}_r$ = unit vector to specify the direction of the the vector distance r from the current to the field point.



Faraday's Law

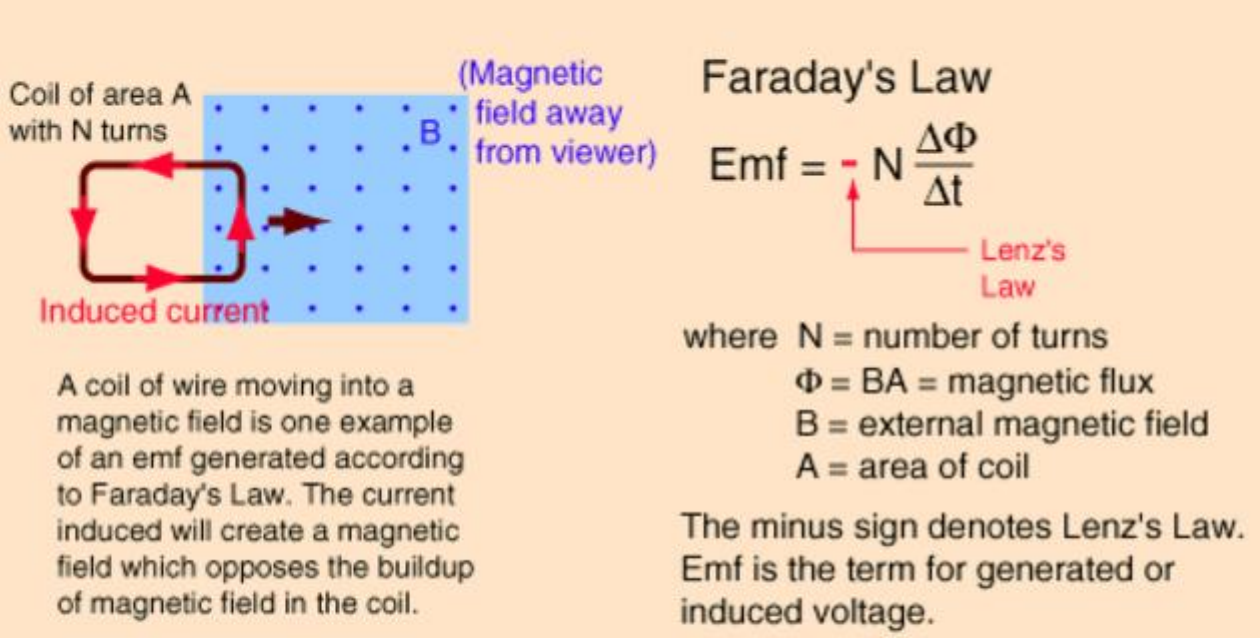
Any change in the magnetic environment of a coil of wire will cause a voltage (emf) to be "induced" in the coil. No matter how the change is produced, the voltage will be generated. The change could be produced by changing the magnetic field strength, moving a magnet toward or away from the coil, moving the coil into or out of the magnetic field, rotating the coil relative to the magnet, etc.



Faraday's Law summarizes the ways voltage can be generated.

$$\text{Voltage generated} = -N\frac{\Delta(BA)}{\Delta t}$$

Faraday's Law



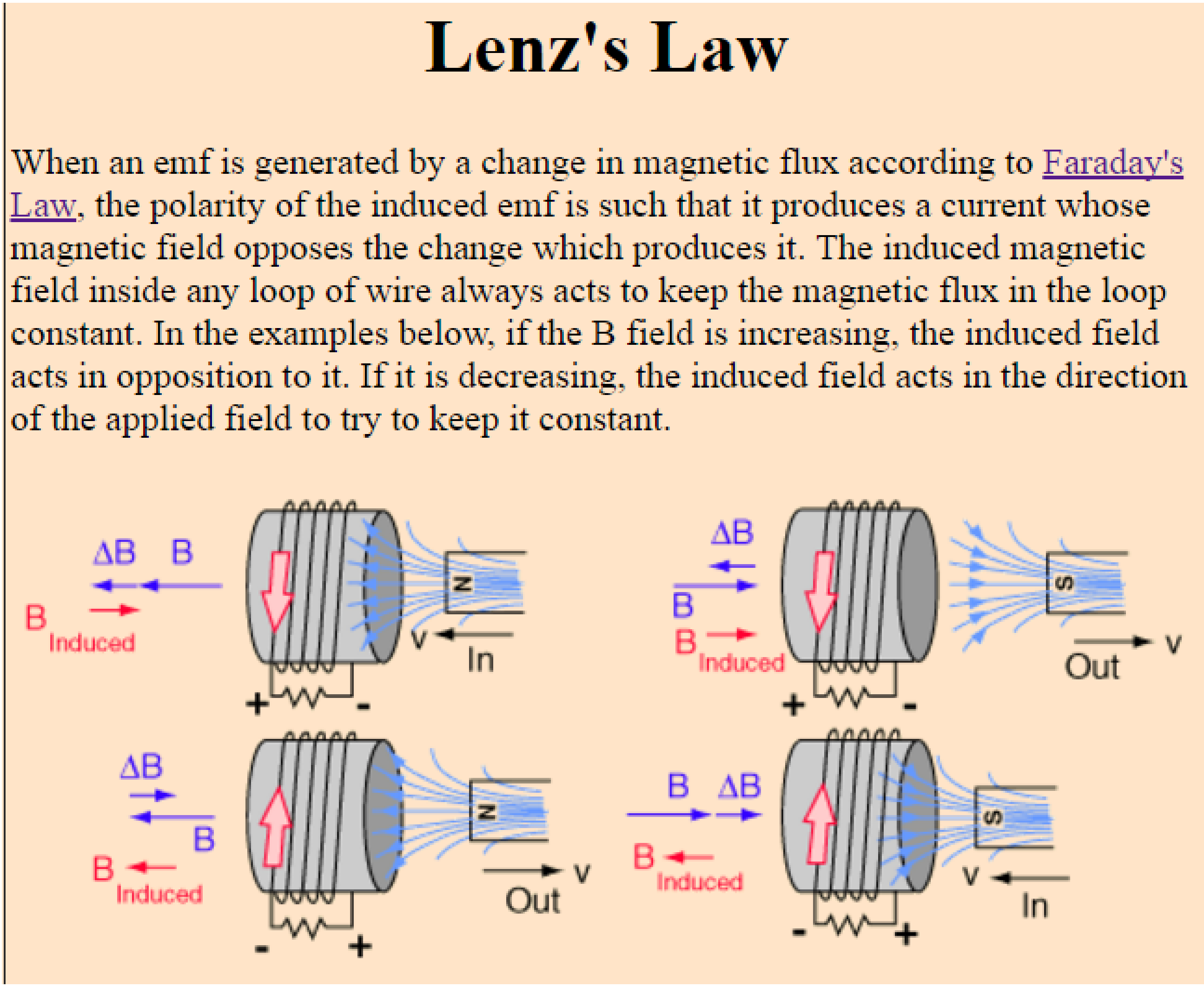
A coil of wire moving into a magnetic field is one example of an emf generated according to Faraday's Law. The current induced will create a magnetic field which opposes the buildup of magnetic field in the coil.

Faraday's Law

$$\text{Emf} = \underset{\text{Lenz's Law}}{-} N \frac{\Delta\Phi}{\Delta t}$$

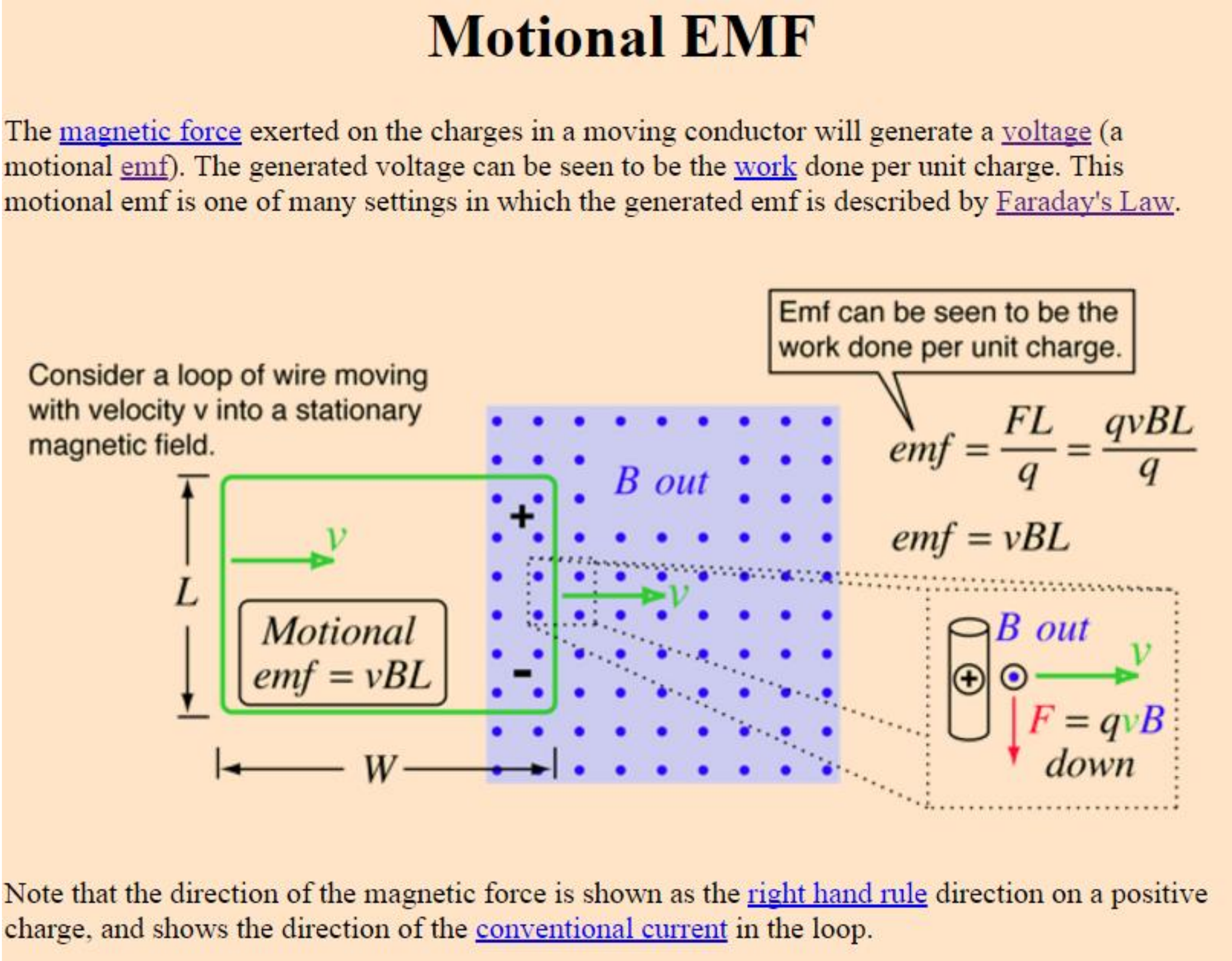
where N = number of turns
Φ = BA = magnetic flux
B = external magnetic field
A = area of coil

The minus sign denotes Lenz's Law. Emf is the term for generated or induced voltage.



Lenz's Law

When an emf is generated by a change in magnetic flux according to Faraday's Law, the polarity of the induced emf is such that it produces a current whose magnetic field opposes the change which produces it. The induced magnetic field inside any loop of wire always acts to keep the magnetic flux in the loop constant. In the examples below, if the B field is increasing, the induced field acts in opposition to it. If it is decreasing, the induced field acts in the direction of the applied field to try to keep it constant.



Motional EMF

The magnetic force exerted on the charges in a moving conductor will generate a voltage (a motional emf). The generated voltage can be seen to be the work done per unit charge. This motional emf is one of many settings in which the generated emf is described by Faraday's Law.

Consider a loop of wire moving with velocity v into a stationary magnetic field.

Emf can be seen to be the work done per unit charge.

$$emf = \frac{FL}{q} = \frac{qvBL}{q}$$

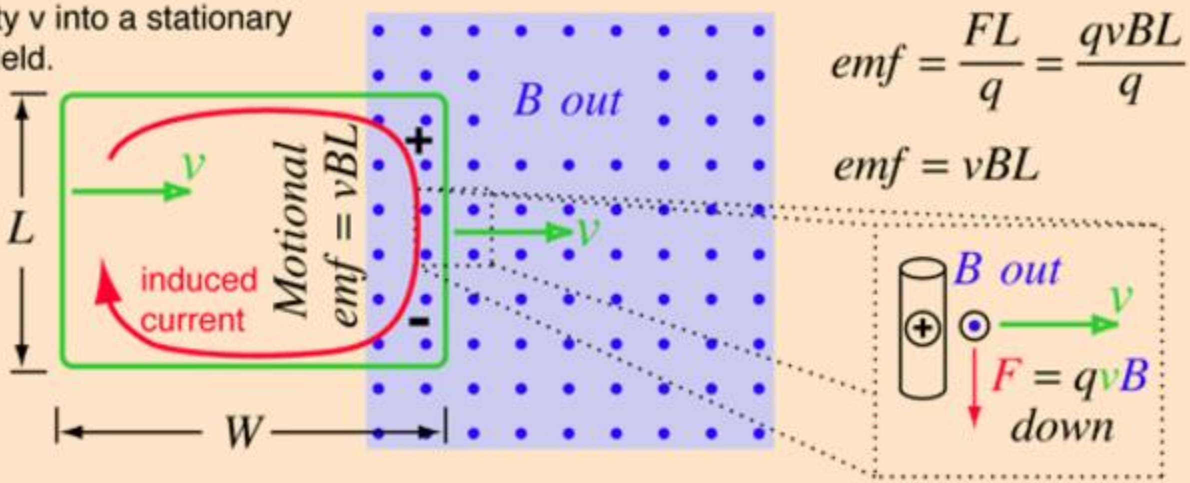
$$emf = vBL$$

Motional $emf = vBL$

Note that the direction of the magnetic force is shown as the right hand rule direction on a positive charge, and shows the direction of the conventional current in the loop.

Motional EMF and Faraday's Law

Consider a loop of wire moving with velocity v into a stationary magnetic field.

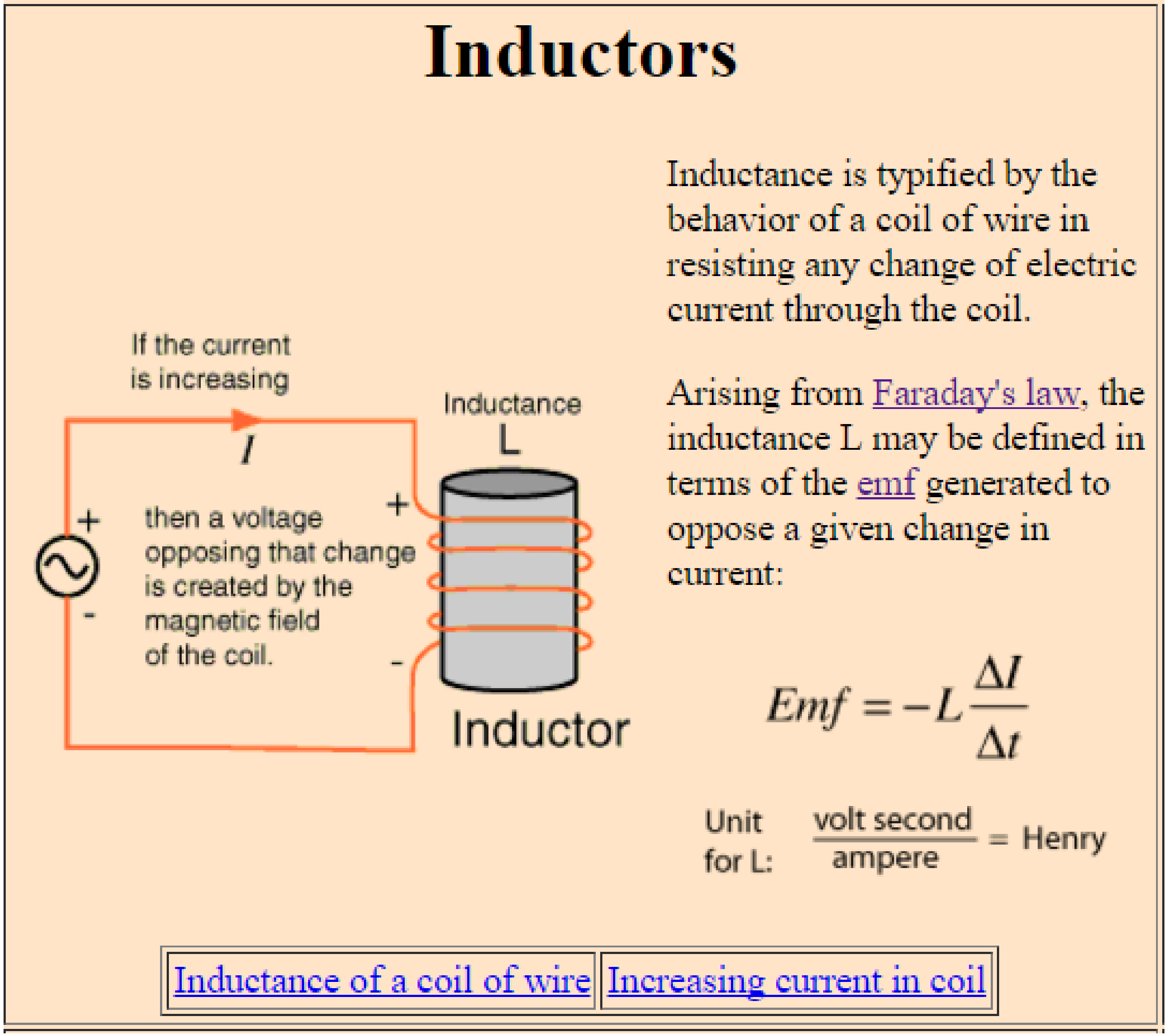


$$emf = \frac{FL}{q} = \frac{qvBL}{q}$$

$$emf = vBL$$

The motional emf expression is an application of Faraday's Law, as can be seen from:

$$emf = \underset{\text{special case}}{BLv} = BL\frac{\Delta W}{\Delta t} = \frac{\Delta(BA)}{\Delta t} = \underset{\text{more general case}}{\frac{\Delta\Phi}{\Delta t}}$$



Inductors

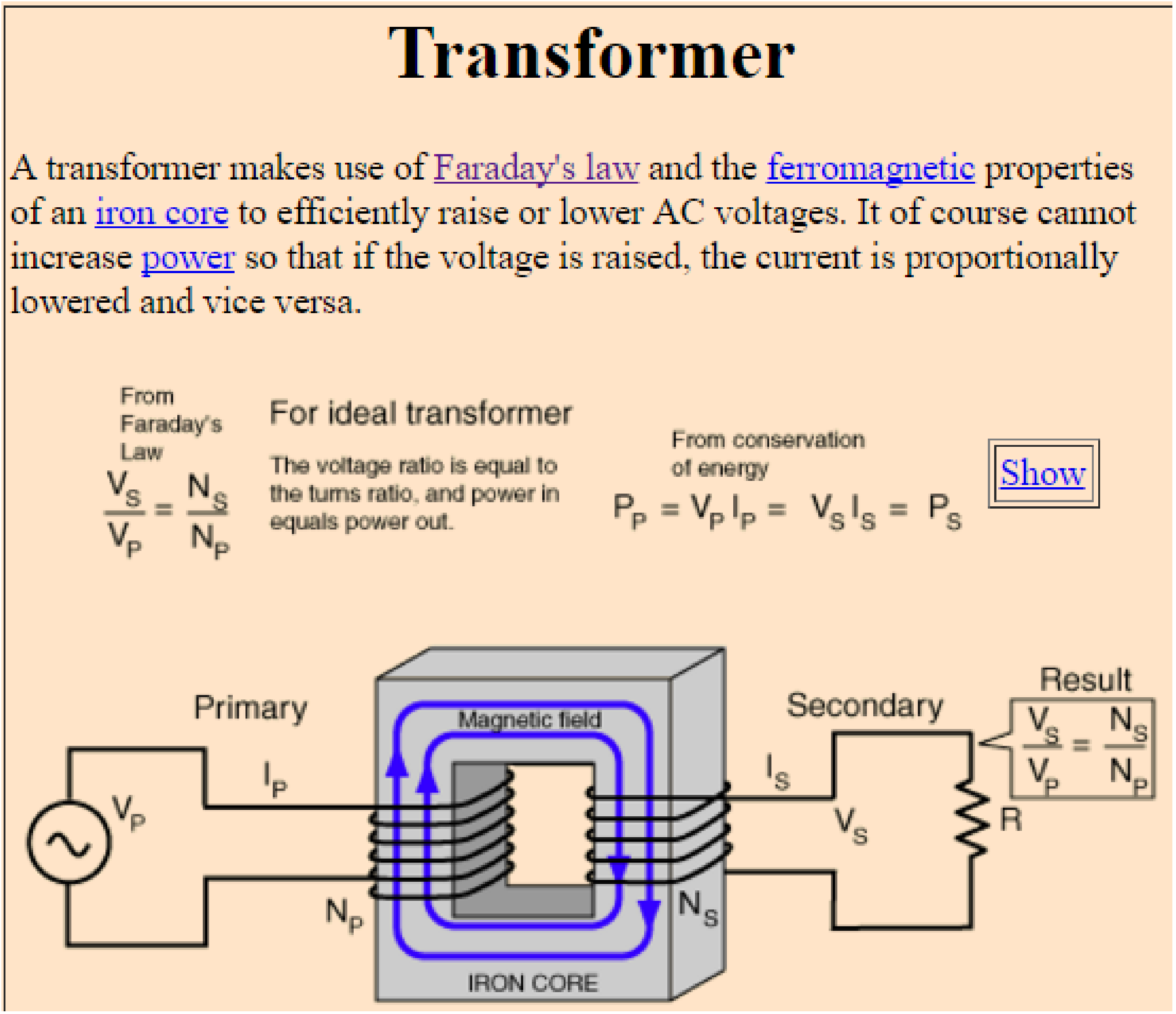
Inductance is typified by the behavior of a coil of wire in resisting any change of electric current through the coil.

Arising from Faraday's law, the inductance L may be defined in terms of the emf generated to oppose a given change in current:

$$Emf = -L\frac{\Delta I}{\Delta t}$$

Unit for L: $\frac{\text{volt second}}{\text{ampere}} = \text{Henry}$

Inductance of a coil of wire | Increasing current in coil



Transformer

A transformer makes use of Faraday's law and the ferromagnetic properties of an iron core to efficiently raise or lower AC voltages. It of course cannot increase power so that if the voltage is raised, the current is proportionally lowered and vice versa.

From Faraday's Law

$$\frac{V_S}{V_P} = \frac{N_S}{N_P}$$

For ideal transformer

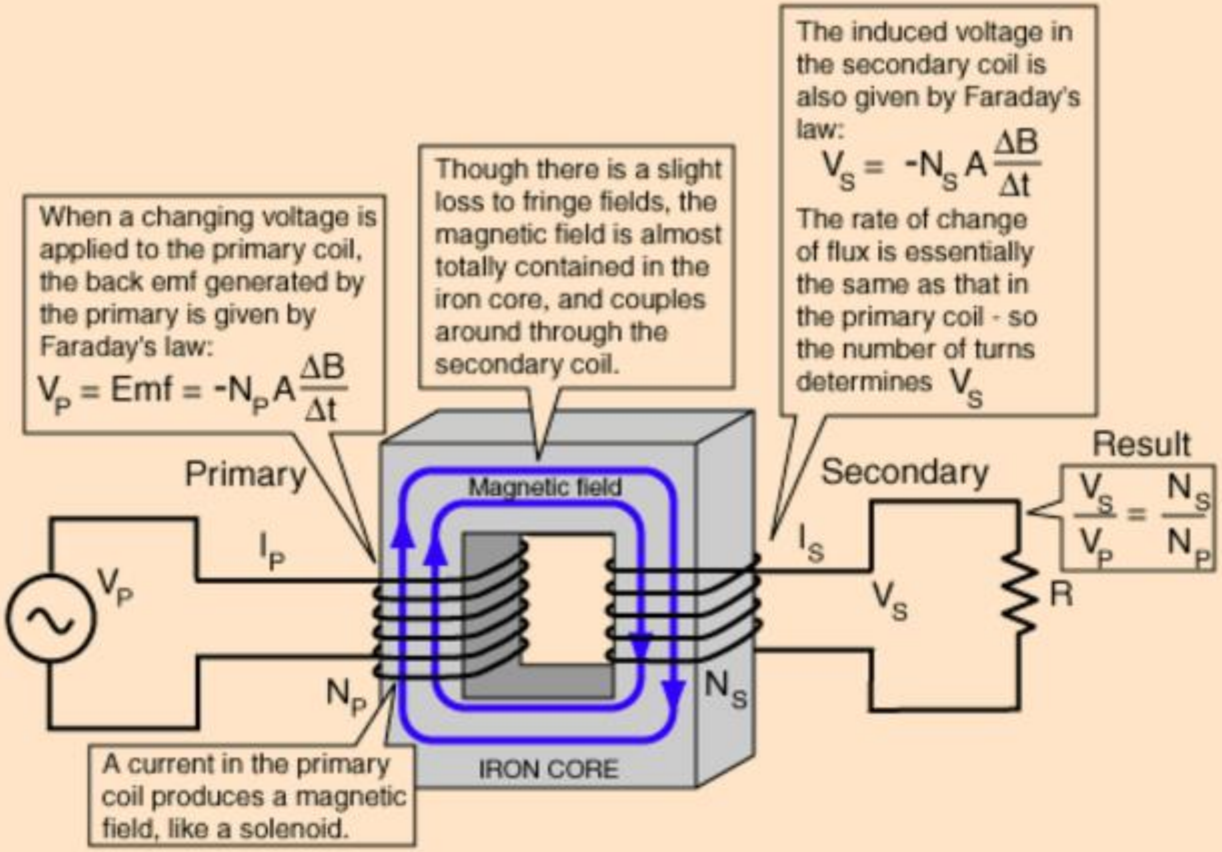
The voltage ratio is equal to the turns ratio, and power in equals power out.

From conservation of energy

$$P_P = V_P I_P = V_S I_S = P_S$$

Show

Transformer and Faraday's Law



When a changing voltage is applied to the primary coil, the back emf generated by the primary is given by Faraday's law:

$$V_P = \text{Emf} = -N_P A \frac{\Delta B}{\Delta t}$$

Though there is a slight loss to fringe fields, the magnetic field is almost totally contained in the iron core, and couples around through the secondary coil.

The induced voltage in the secondary coil is also given by Faraday's law:

$$V_S = -N_S A \frac{\Delta B}{\Delta t}$$

The rate of change of flux is essentially the same as that in the primary coil - so the number of turns determines V_S

Result

$$\frac{V_S}{V_P} = \frac{N_S}{N_P}$$

A current in the primary coil produces a magnetic field, like a solenoid.