Fluid Mechanics

CHAPTER 13

Fluid Mechanics

- Density
- Pressure in a static fluid
- Pascal's law
- Absolute and gauge pressure
- Buoyancy
- Fluid flow
- Real fluids

Density

Density is mass per unit volume

$$\rho = \frac{m}{V}$$

Density

TABLE 13.1 Densities of Some Common Substances

Material	Density (kg/m^3)*	Material	Density (kg/m^3)*
Gases		Concrete	2×10^3
Air (1 atm, 20° C)	1.20	Aluminum	2.7×10^3
		Iron, steel	7.8×10^3
Liquids		Brass	8.6×10^3
Benzene	0.90×10^3	Copper	8.9×10^3
Ethanol	0.81×10^3	Silver	10.5×10^3
Water	1.00×10^3	Lead	11.3×10^3
Seawater	1.03×10^3	Gold	19.3×10^3
Blood	1.06×10^3	Platinum	21.4×10^3
Solids		Mercury	13.6×10^3
Glycerin	1.26×10^3	White-dwarf star	10^{10}
Ice	0.92×10^3	Neutron star	10^{18}

Example 1

Find

a)the mass of air, and

b)the weight of air

within a fairly spacious room with dimensions of length 5.0 m, width 4.0m and a ceiling height of 3.0 m ?

What would be the mass and weight of an equal volume of water?

Solution…

First we need the volume of the room:

Volume Room = Length * Width * Height

Volume Room = 5m * 4m * 3m = 60 m^3

From data on density

Thus we have: Density = Mass / Volume

Mass = Density_{Air} * Volume = 1.2 kg/m^3 * 60m^3 = 72kg

Weight = Mass x Accel. Gravity = 72 * 10 = 720N

For an equal Volume of water:

Mass = Density_{Water}*Volume = 1000kgm3 * 60 = 60000kg

Weight_{water} = 60000 * 10 = 6 x 10^5N

Pressure

Pressure is **normal** force (or perpendicular force) per unit area.

A Force parallel to a surface doesn't apply pressure to it !!

$$\boldsymbol{P = \frac{F_{\perp}}{A}}$$

- Units of pressure:

$$1 \text{ atm} = 1.013\times10^{5} \text{ Pa} = 1.013 \text{ bar} = 14.7 \text{ psi}$$

The Standard Unit of Pressure is **Pascals or Newtons per metre^2,** this is what we use in equations !!

Example

For the same room in the previous example, find the total force on the floor due to the air above the surface, if the air pressure is at 1.00 atm.

Floor Area from earlier = Length * Width

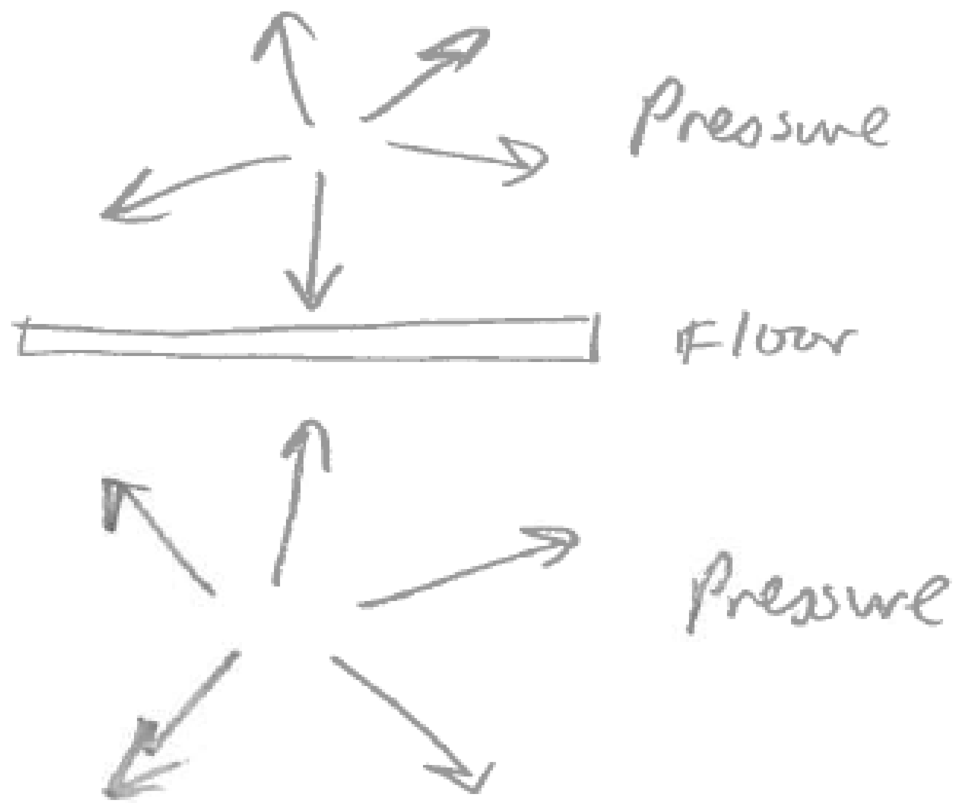
Floor Area = 5m * 4m = $20m^2$

Force = Pressure * Area

Force = 1.103×10^5 Pa * $20m^2$ = 2.21×10^6 N

This is about 225,000 kg of mass !!

Why does the floor not collapse ?



Pressure below $\approx$ above

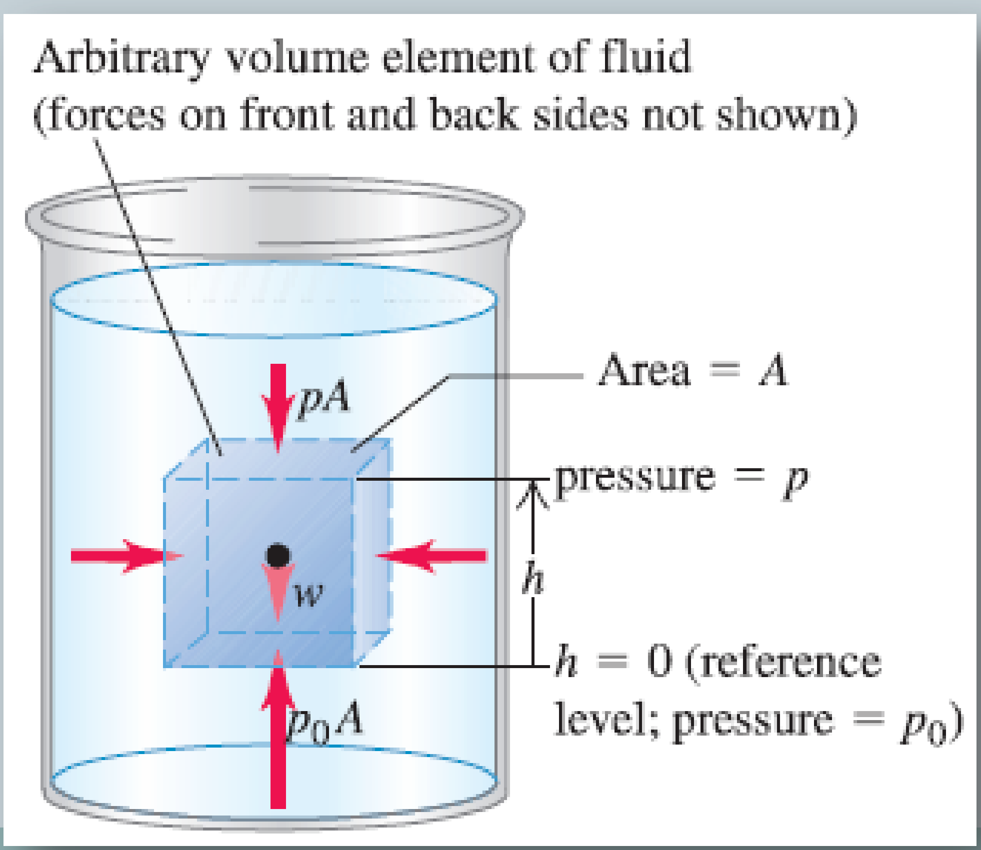
Pressure in a Fluid

Fluid pressure at a point at a distance *h* below the surface of a **static** fluid is given by

$$p = \rho g h$$

Characteristics of Pressure in Fluids

- Pressure is perpendicular to the surface of the container and to the surface of any object immersed in it.
- Pressure acts in all directions



Characteristics of Pressure in Fluids

- Pressure is independent of shape or area of container
- Pressure depends on depth below surface

Total Pressure below a free surface

$$p_{abs} = p_{atm} + \rho g h$$

$\rho g h$ is sometimes known as the *gauge* pressure

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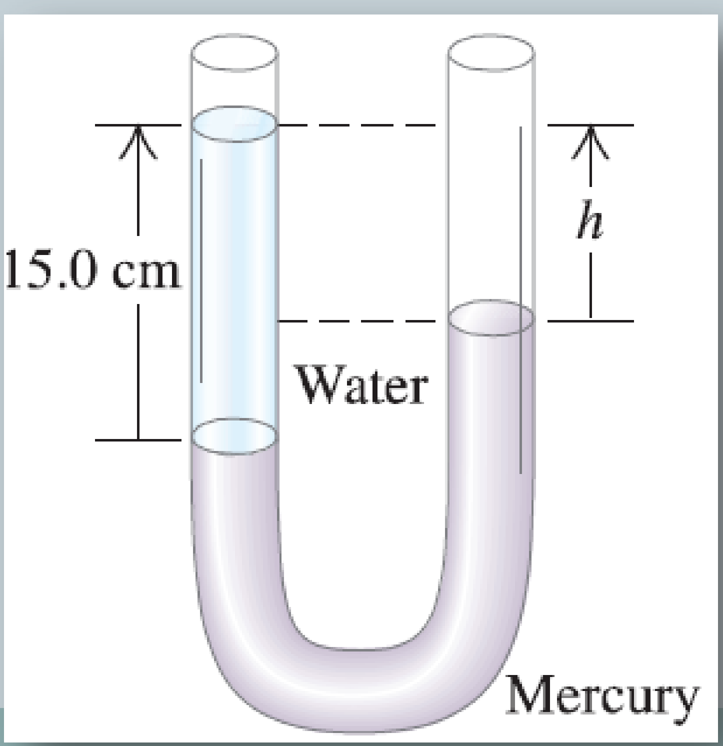
$\rho g h$ is sometimes known as the *gauge* pressure

Example (U shaped Pressure)

A U-shaped tube open to the air at both ends contains some mercury. A quantity of water is carefully poured into the left arm of the U-shaped tube until the vertical height of the water column is 15.0 cm.

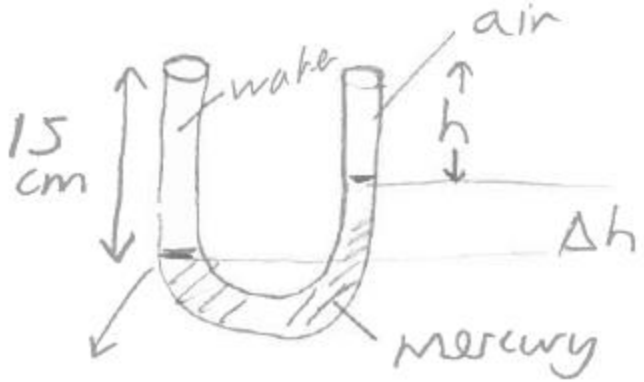
(a) What is the gauge pressure at the water-mercury interface?

(b) Calculate the vertical distance h from the top of the mercury in the right-hand arm of the tube to the top of the water in the left-hand arm.



Gauge pressure

$$P_{abs} = P_{atm} + \underbrace{\rho g h}_{\text{Gauge pressure}}$$



$$\text{Gauge pressure} = \rho g h$$
$$= 1000(9.8)\,0.15$$
$$= 1470 \text{ Pa}$$

Since forces are balanced (Area constant)
This must equal Pressure of mercury

$$1470 = 13600 \times (9.8)\,\Delta h$$

$$h = 15 - \Delta h$$

Convert to meters

Solution…

A) We know: $P_{abs} = p_{atm} + \rho g h$

$$P_{abs} = 1.013 \times 10^5 + (1000 * 9.8 * 0.15)$$

$$P_{abs} = 1.013 \times 10^5 + 1470$$

$$p_{abs} = 1.0277 \times 10^5 \text{ Pa (at the H2O/Hg interface)}$$

$$P_{gauge} = 1470\ Pa$$

B) The additional pressure from the water must equate to the additional height for the Mercury:

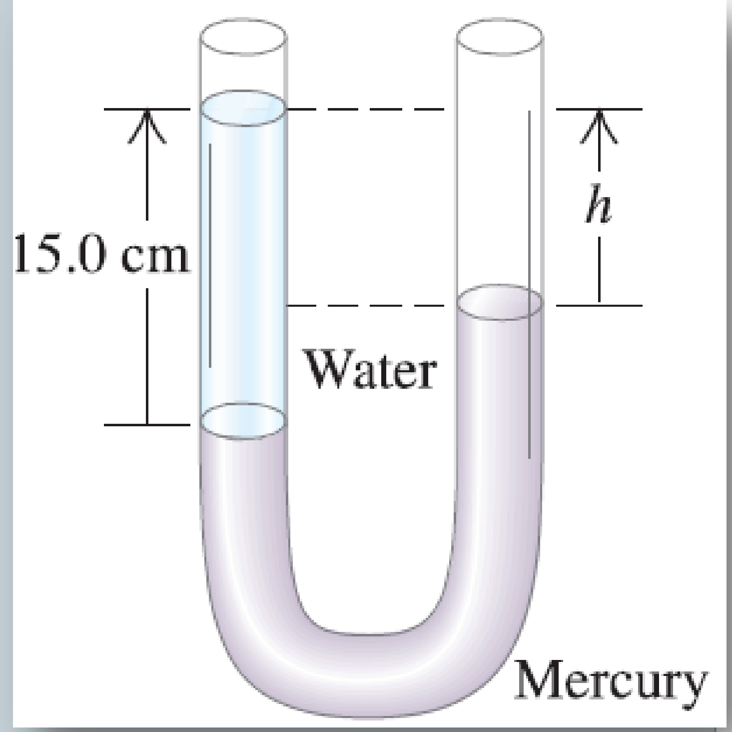
Additional pressure from Water = 1470 Pa, this must equate to pgh for the mercury: **1470 = 13600 * 9.8 * Δh**

$$\Delta h = 1.103 \times 10^{-2} \text{ metres or } 1.103 \text{ cm}$$

Is this the final answer ?

Solution Continued…

The **Δh** term indicates the **Change in Height** of the Mercury and not in fact **h** itself…



Thus, h = 15cm – Δh

h = 15cm – 1.103cm

h = 13.897 cm or 0.139m

Why don't you try for a Glycerin solution of height 40cm instead of Water… (Glycerin Density = 1.26 x 10^3 kg/m^3

Glycerin Solution…

$$p_{total} = p_{atm} + \rho g h$$

p_{total} =1.013 x 10^5 + (1260 * 9.8 * 0.40)

p_{total} = 1.0145 x 10^5 Pa + 4939.2 Pa

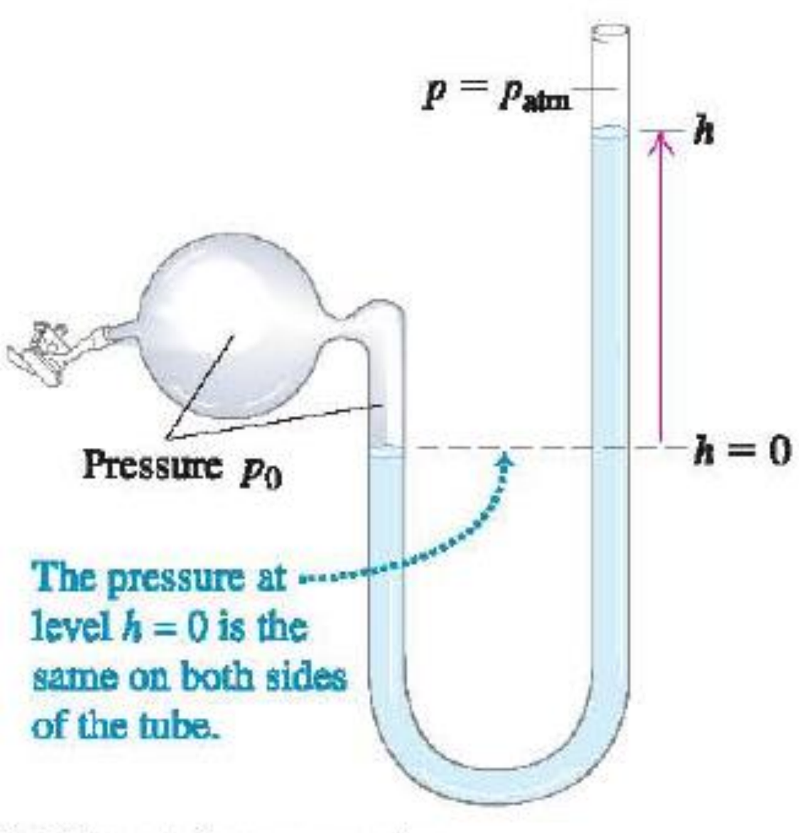
p_{total} = 1.0639 x 10^5 Pa

So, 4939.2 = 13600 * 9.8 * Δh

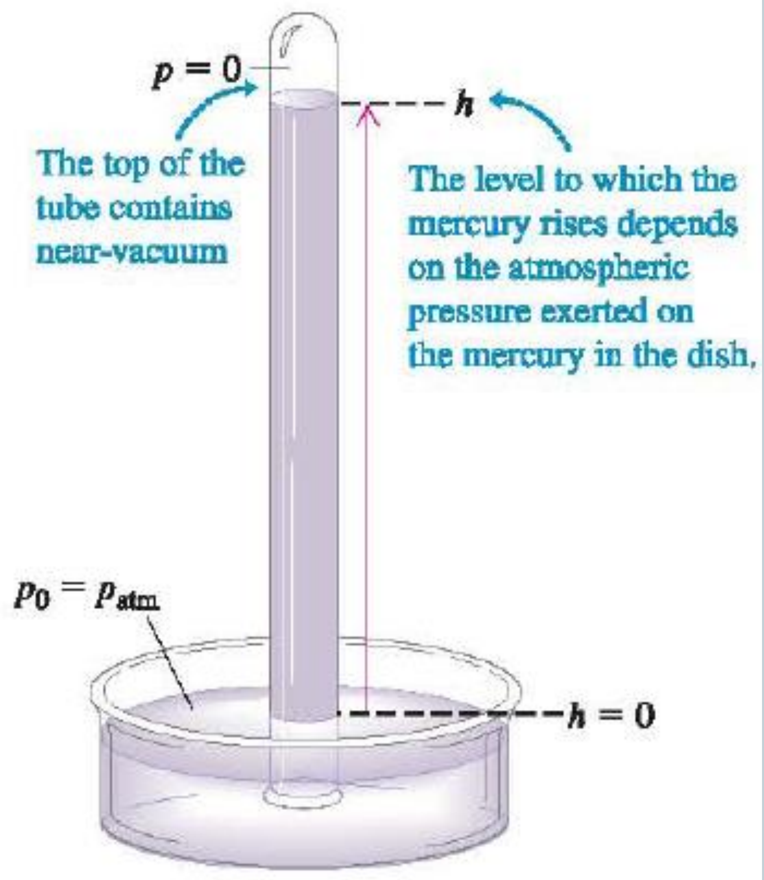
Δh = 0.0371 m or 3.71 cm

Therefore: h = 40 – 3.71 = 36.29 cm or 0.363 m

Pressure Gauges



(a) Open-tube manometer



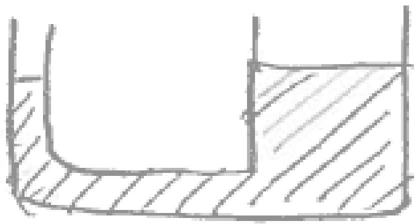
(b) Mercury barometer

Pascal’s Law

Pressure applied to an enclosed *incompressible* fluid is transmitted *undiminished* to every portion of the fluid and the walls of the containing vessel.

The pressure depends only on **depth**.

The shape of the container does not make any difference!!



Hydraulics

$$P = \frac{F}{A} \Rightarrow \frac{F_1}{A_1} = \frac{F_2}{A_2}$$ Pascals law

$$WD = Fd$$

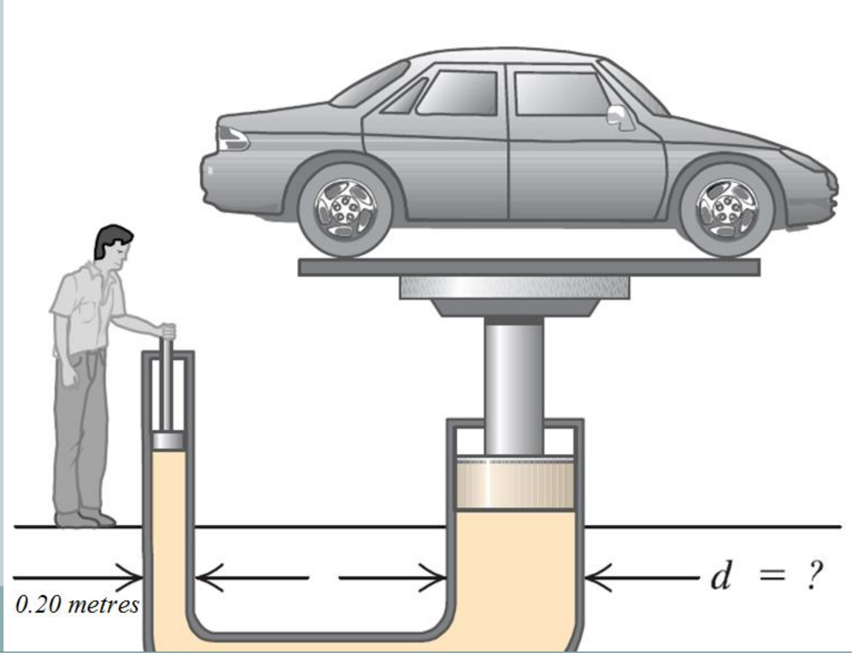
$$Vol = d_1 A_1 = d_2 A_2$$

An Application of Pascal's Law: Hydraulic Lift

You are designing a hydraulic lift for an automobile garage. It will consist of two oil-filled cylindrical pipes of different diameters. A worker pushes down on a piston at one end, raising the car on a platform at the other end. To handle a full range of jobs, you must be able to lift cars up to 2000 kg, plus the 500 kg platform on which they are parked. To avoid injury to your workers, the maximum amount of force a worker should need to exert is 250 N.

(a) What should be the diameter of the pipe under the platform?

(b) If the worker pushes down with a stroke 250 cm long, by how much will he raise the car at the other end?



Solution…

a) We need to apply Pascal's Law:

$$P = \frac{F_1}{A_1} = \frac{F_2}{A_2}$$

Here, we need the **Area** of a pipe, which is **πr^2.**

So we have:

$$\frac{F_1}{\pi {r_1}^2} = \frac{F_2}{\pi {r_2}^2}$$

Re-arrange this for r_2: as we are looking for the second radius:

$$r_2 = \sqrt{\frac{F_2 \pi {r_1}^2}{\pi F_1}}$$

Solution cont…

Now we can put numbers in, but remember, we are dealing with ***Forces*** and ***not Masses***…

F_1 = 250 N and F_2 = (2500 x 10) = 25000 N

$$r_2 = \sqrt{\frac{25000 * \pi * 0.1^2}{\pi * 250}} = 1\ metre$$

Now we have to calculate b) How far does the car move for a piston stroke of 2.50 metres….

There are in fact 2 ways to do this…….

Solution continued…

1st Method: Volume of fluid must be constant, thus:

$$Volume\ of\ fluid\ moved = d_1 A_1 = d_2 A_2$$

$$d_2 = \frac{d_1 A_1}{A_2} = \frac{d_1 \pi {r_1}^2}{\pi {r_2}^2} = \frac{2.5 * \pi * 0.1^2}{\pi * 1^2} = 2.5 * 10^{-2} m$$

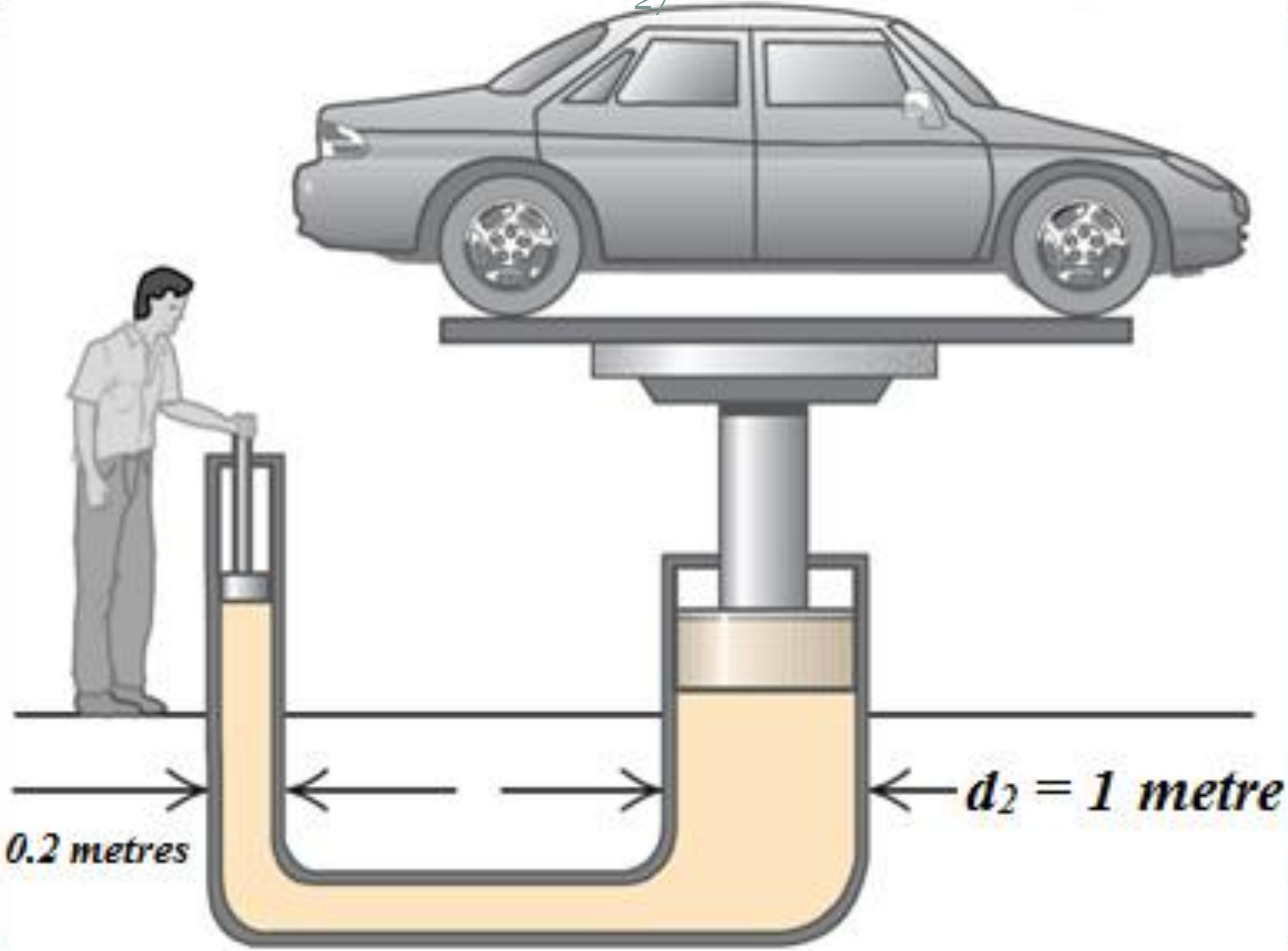
2nd Method: Word Done on both sides must be equal:

$$Work\ Done = F_1 d_1 = F_2 d_2$$

$$d_2 = \frac{F_1 d_1}{F_2} = \frac{250 * 2.5}{25000} = \frac{500}{25000} = 2.5 * 10^{-2} m$$

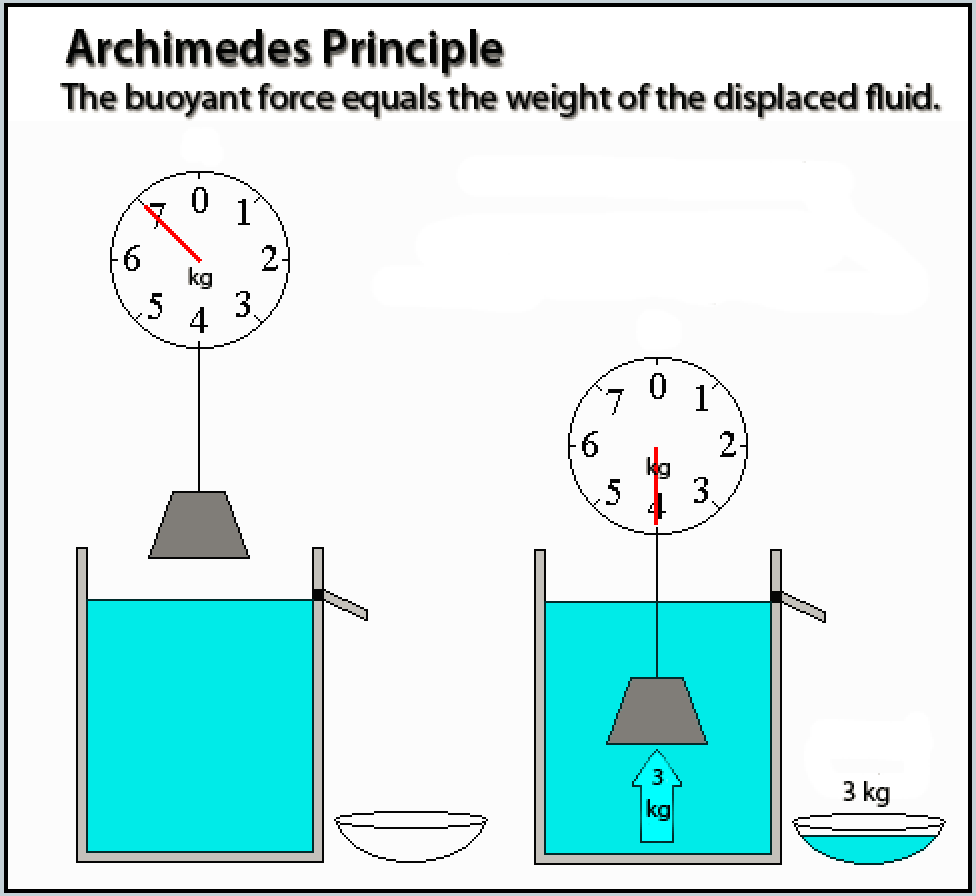
So Both Results are the **Same** – Which is Expected !!

Pressure applied to an enclosed *incompressible* fluid is transmitted *undiminished* to every portion of the fluid and the walls of the containing vessel.

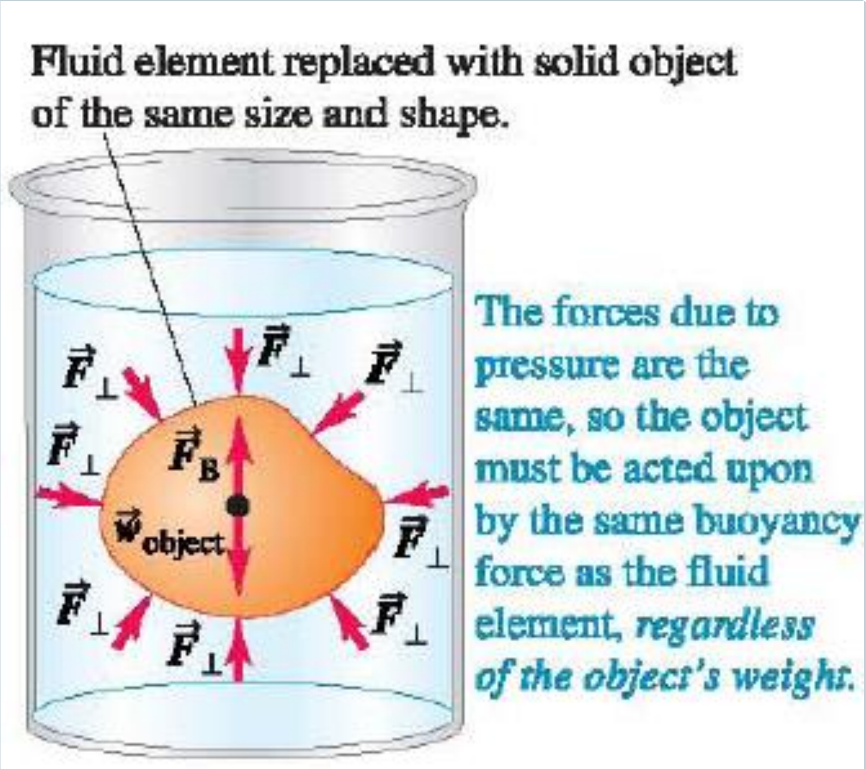


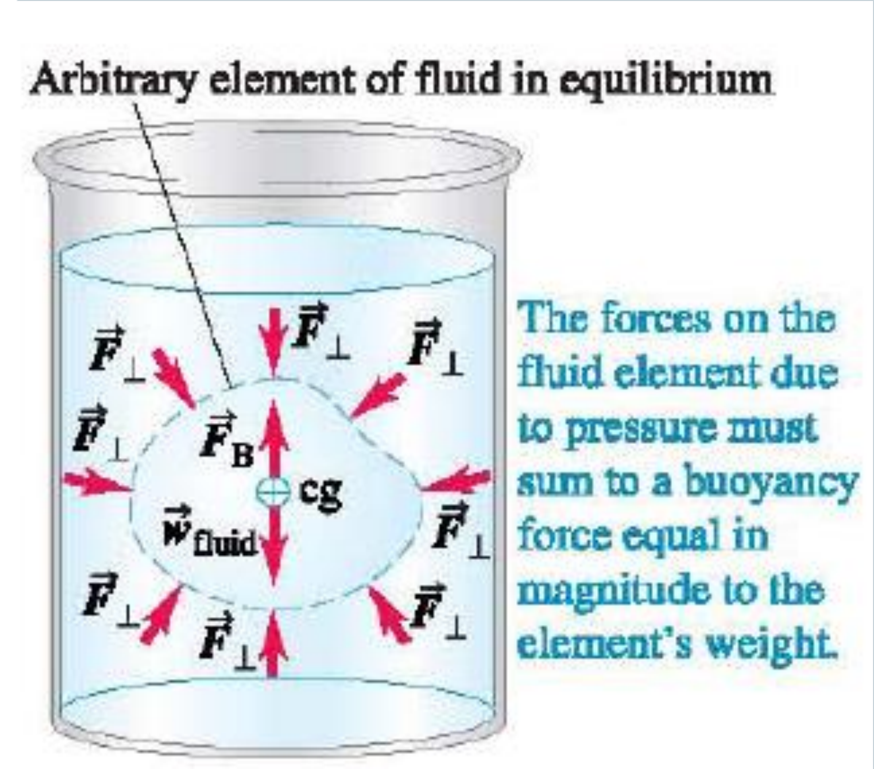
Buoyancy: Archimedes's Principle

- When an object is completely or partially immersed in a fluid, the fluid exerts an upward force on the object equal to the weight of the fluid that is displaced by the object.

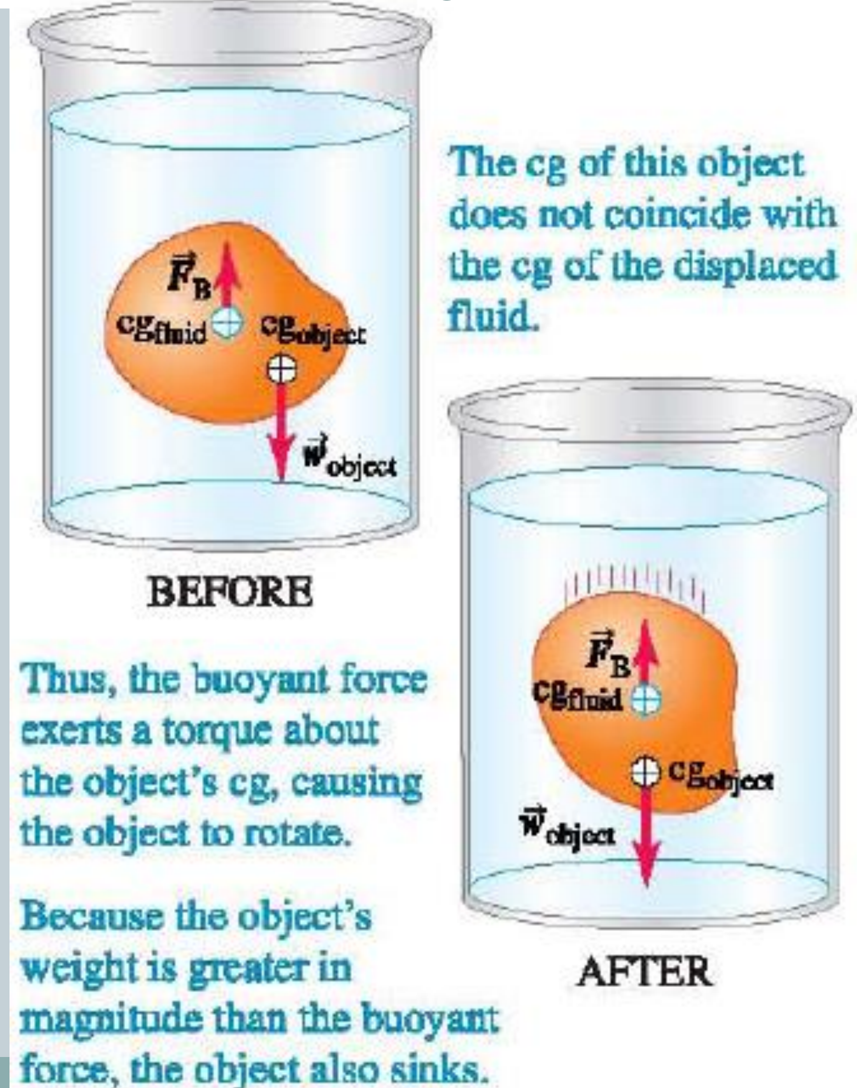


Archimedes's Principle: Buoyancy





Archimedes's Principle: Buoyancy



Example

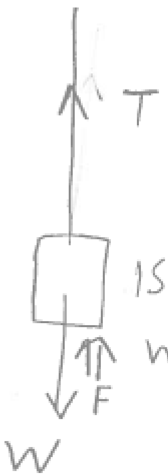
- A 15.0 kg solid-gold statue is being raised from a sunken treasure ship.

(a) Find the tension in the hoisting cable when the statue is completely immersed.

(b) Find the tension when the statue is completely out of the water.

The density of gold is $19.3 \times 10^3\ kg/m^3$.

When an object is completely or partially immersed in a fluid, the fluid exerts an upward force on the object equal to the weight of the fluid that is displaced by the object.



15.0 kg

$W = mg$

$= 15.0\ 9.8$

$= 147\ N$

$\rho = \frac{M}{V}$ of Gold

$V = \frac{m}{\rho}$

$= \frac{15}{19.3 \times 10^3}$

$= 7.8 \times 10^{-4}\ m^3$

mass of water displaced

$m = \rho V$

$= 1.00 \times 10^3 (7.8 \times 10^{-4})$

$= 0.78\ kg$

$W = mg$

$= 0.78\ (9.8) = 7.6\ N$

(a) moving at constant velocity

$T + W_{\text{water displaced}} = W_{\text{gold}}$

$T = W_{\text{gold}} - W_{\text{water displaced}}$

$= 147 - 7.6$

$= 139.4\ N$

(b) $T = W_{\text{gold}}$

$= 147\ N$

Fluid Flow

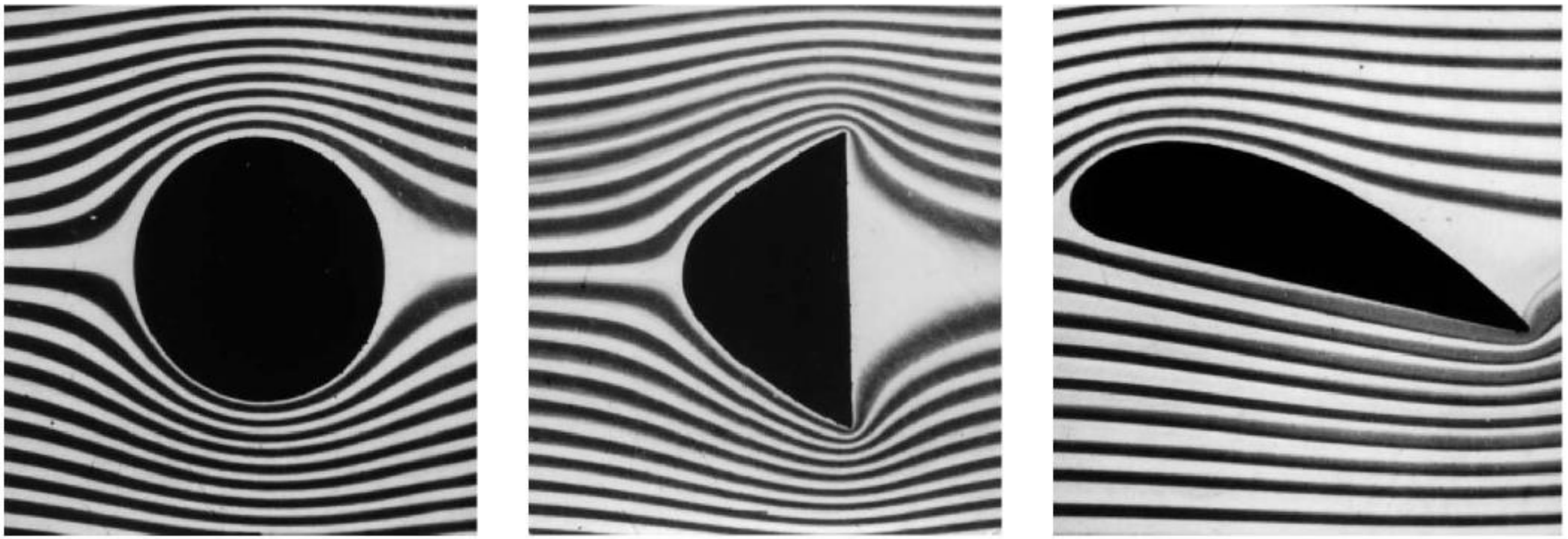
- An *ideal fluid* is incompressible and has no internal friction or viscosity.
- Most liquids are almost incompressible.

- Laminar

Turbulent

Laminar Flow

- Adjacent layers of fluid slide past each other smoothly
- There is a clear flow pattern
- **Steady flow** is when the pattern doesn't change with time



(a) (b) (c)

Turbulent Flow

- Sudden changes in the velocities of the particles
- Irregular flow
- No steady-state pattern

- http://www.youtube.com/watch?v=WG-YCpAGgQQ

Density, Pressure and Fluids

$$p = \frac{m}{V}$$

$$P = \frac{F}{A}$$

$$P = P_{atm} + pgh$$

Extra questions (if time)

A 10.0 kg Aluminum statue is being raised from a sunken treasure ship.

(a) Find the tension in the hoisting cable when the statue is completely immersed.

(b) Find the tension when the statue is completely out of the water.

The density of Aluminum is $2.7 \times 10^3 \ kg/m^3$.

The density of water is 1000 kg/m^3

A hydraulic lift consists of two oil-filled cylindrical pipes of different diameters. A worker pushes down on a piston of diameter 0.1 m at one end, raising the car on a platform at the other end. It must be able to lift cars up to 3000 kg, plus the 500 kg platform on which they are parked. The maximum amount of force a worker should need to exert is 200 N.

(a) What should be the diameter of the pipe under the platform?

(b) If the worker pushes down with a stroke 300 cm long, by how much will he raise the car at the other end?