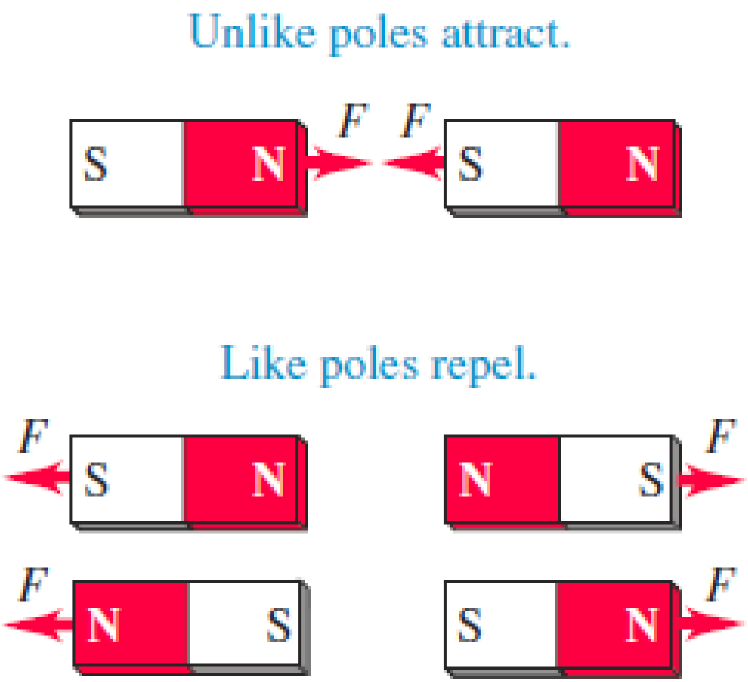
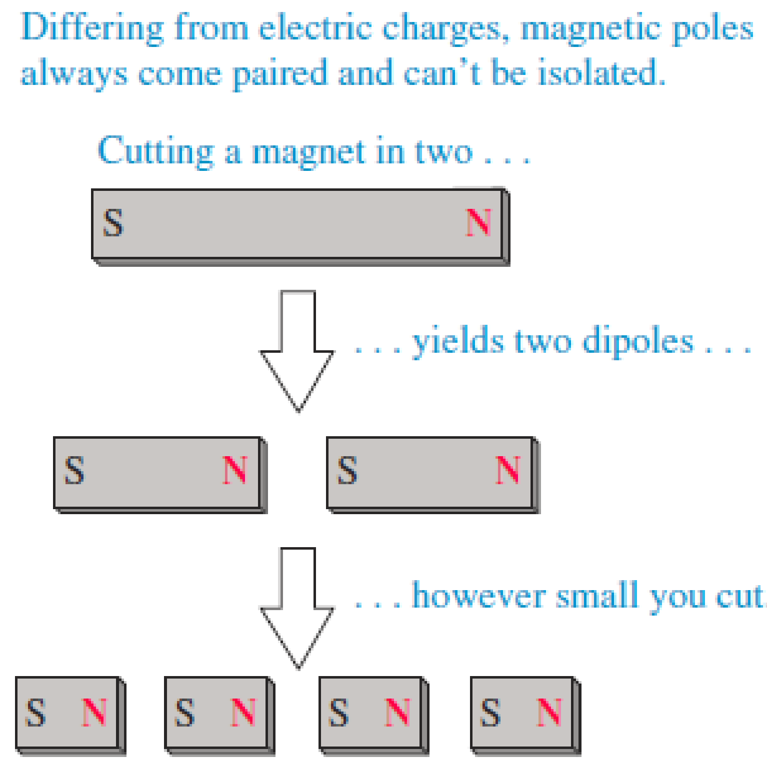
20 Magnetic Field and Magnetic Forces

In industrial settings, electromagnets are often used to pick up and move iron-containing material, such as this shredded scrap. How can electric currents cause magnetic forces? We'll learn in this chapter.

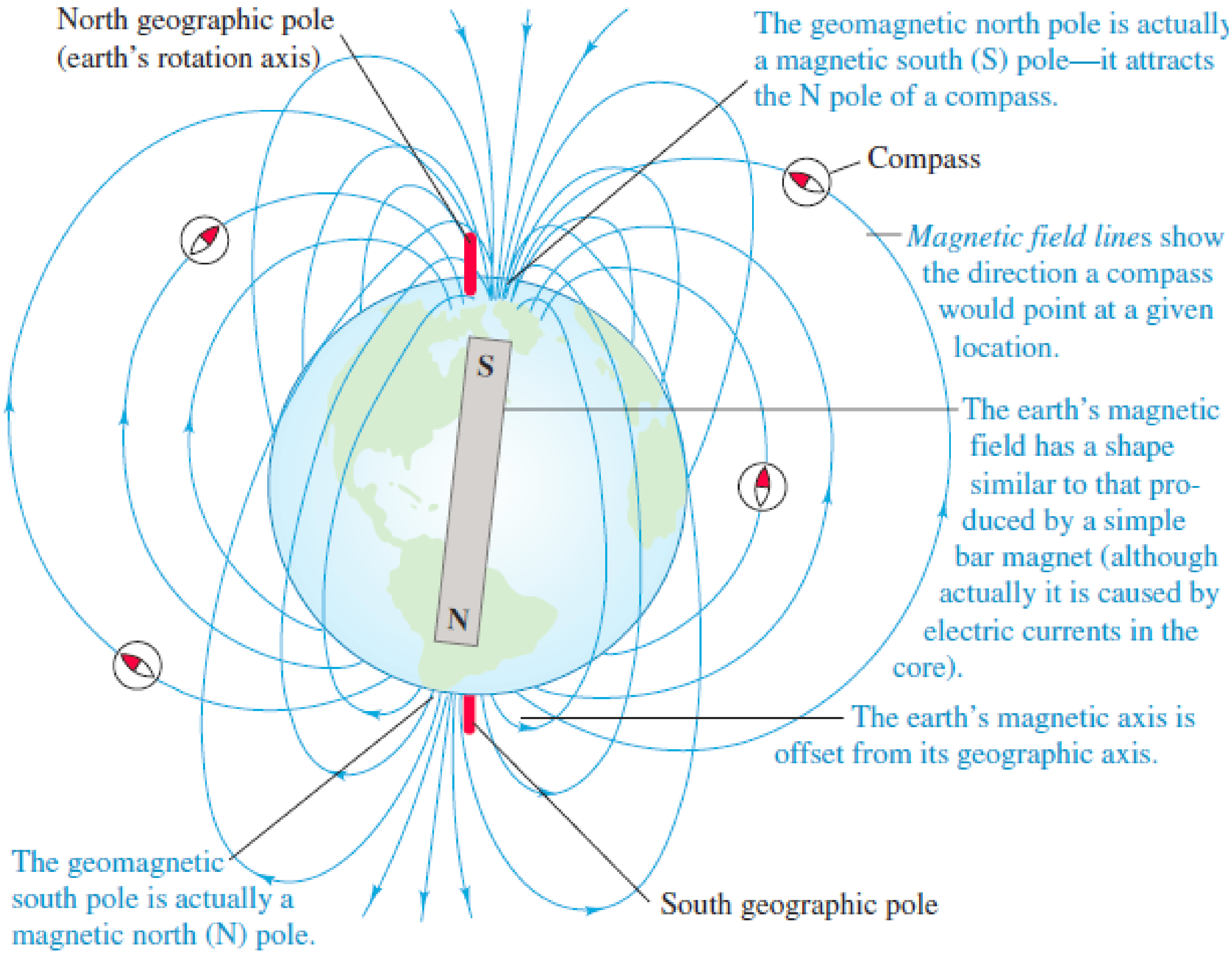
▲ **FIGURE 20.1** This bar magnet picks up steel filings—but not the copper filings in the pile. Later in this chapter, we'll learn why some metals are strongly magnetic and others are not.



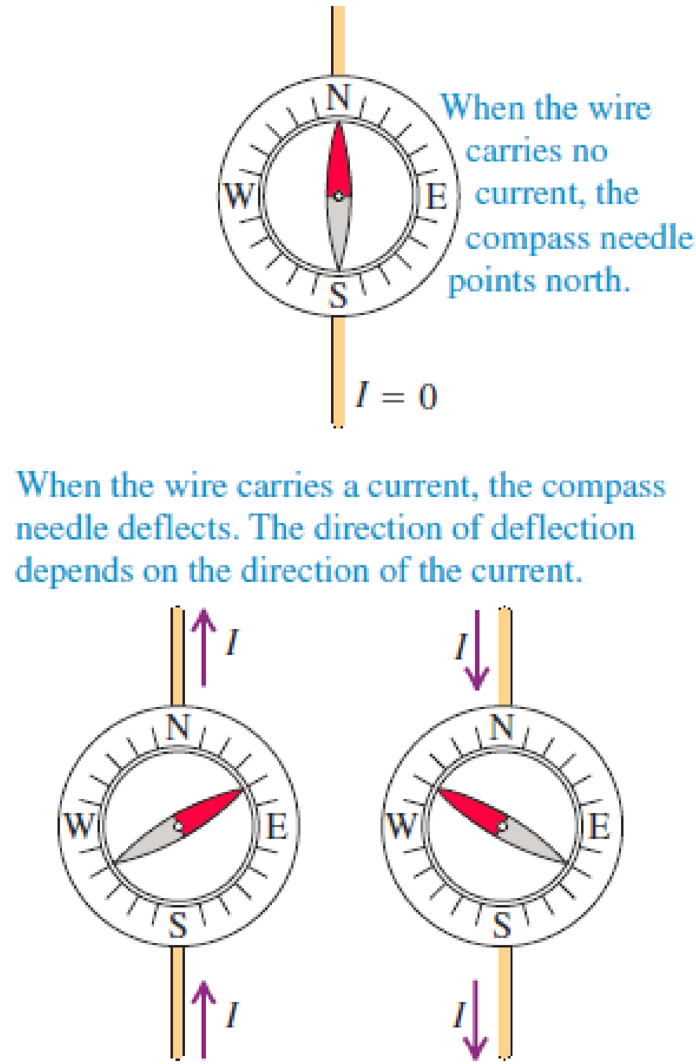
▲ **FIGURE 20.2** Unlike magnetic poles attract each other; like magnetic poles repel each other.



▲ **FIGURE 20.3** Magnets always have paired N and S poles.



▲ **FIGURE 20.4** A compass placed at any point in the earth's magnetic field will point in the direction of the field line at that point. Representing the earth's field as that of a tilted bar magnet is only a crude approximation of its fairly complex configuration.



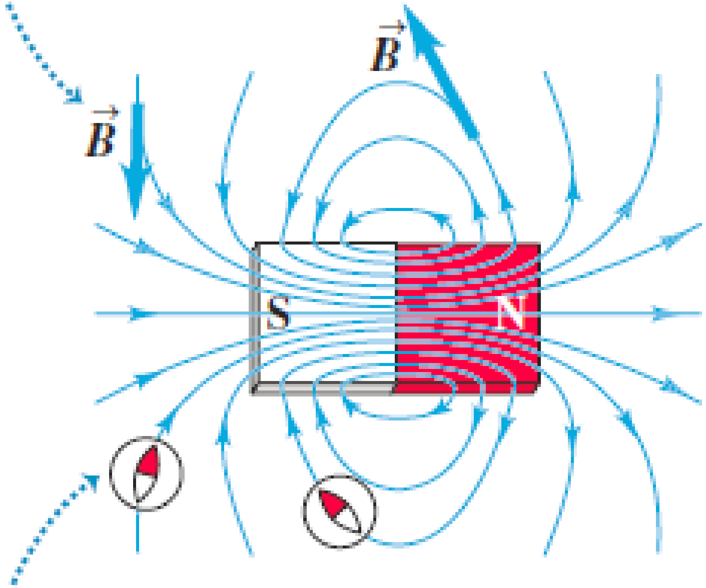
▲ **FIGURE 20.5** The behavior of a compass placed directly over a wire (seen from above). If the compass were placed directly *under* the wire, the deflection of the needle would be reversed.

In 1819, the Danish scientist Hans Christian Oersted, observed that a compass needle was deflected by a current-carrying wire.

A few years later, it was found that moving a magnet near a conducting loop can cause a current in the loop and that a changing current in one conducting loop can cause a current in a separate loop.

At each point, the field line is tangent to the magnetic field vector $\vec{B}$.

The more densely the field lines are packed, the stronger the field is at that point.



At each point, the field lines point in the same direction a compass would . . .

. . . therefore, magnetic field lines point *away from* N poles and *toward* S poles.

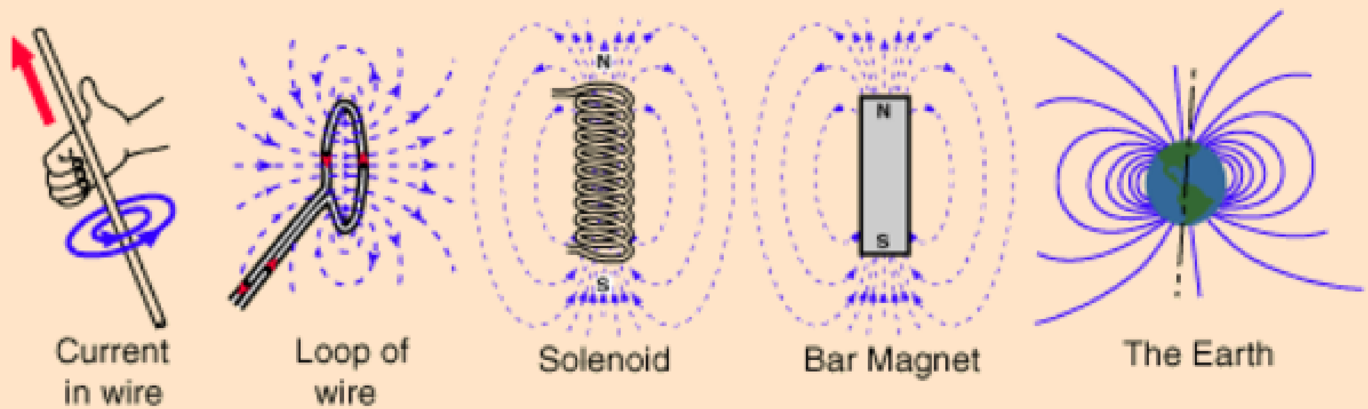
▲ **FIGURE 20.6** Magnetic field lines in a plane through the center of a permanent magnet.

The N pole of the compass needle always tends to point in the direction of the magnetic filed **B**

Because the direction of **B** at each point is unique, field lines never intersect.

Magnetic Field

Magnetic fields are produced by electric currents, which can be macroscopic currents in wires, or microscopic currents associated with electrons in atomic orbits. The magnetic field B is defined in terms of force on moving charge in the Lorentz force law. The interaction of magnetic field with charge leads to many practical applications. Magnetic field sources are essentially dipolar in nature, having a north and south magnetic pole. The SI unit for magnetic field is the Tesla, which can be seen from the magnetic part of the Lorentz force law $F_{magnetic} = qvB$ to be composed of (Newton x second)/(Coulomb x meter). A smaller magnetic field unit is the Gauss (1 Tesla = 10,000 Gauss).



Magnetic Field Sources

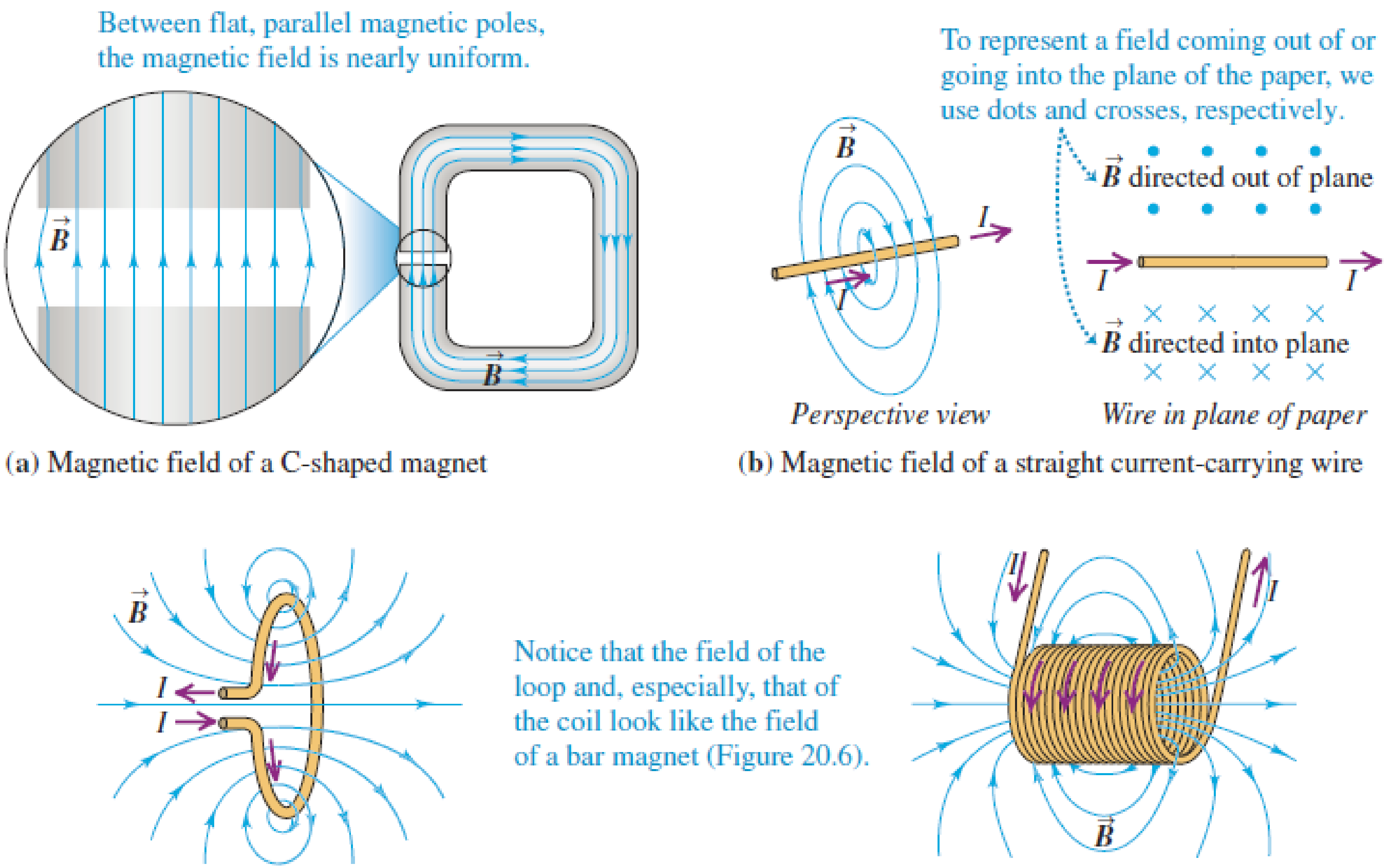
20.2 Magnetic Field and Magnetic Force

To introduce the concept of a magnetic field, let's review our formulation of *electrical* interactions in Chapter 17, where we introduced the concept of an *electric* field. We represented electrical interactions in two steps:

1. A distribution of electric charge at rest creates an electric field $\vec{E}$ at all points in the surrounding space.
2. The electric field exerts a force $\vec{F} = q\vec{E}$ on any other charge q that is present in the field.

We can describe magnetic interactions in the same way:

1. A permanent magnet, a moving charge, or a current creates a **magnetic field** at all points in the surrounding space.
2. The magnetic field exerts a force $\vec{F}$ on any other moving charge or current that is present in the field.

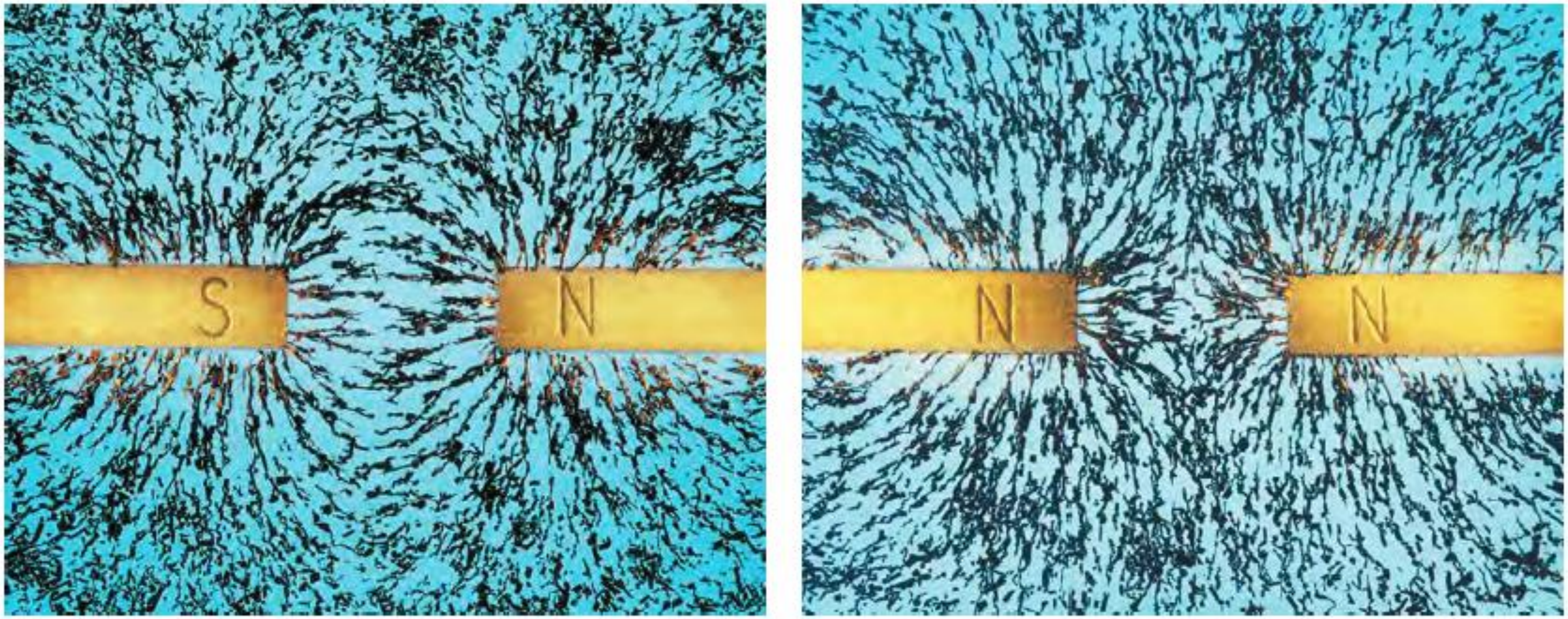


(a) Magnetic field of a C-shaped magnet

(b) Magnetic field of a straight current-carrying wire

(c) Magnetic fields of a current-carrying loop and a current-carrying coil (solenoid)

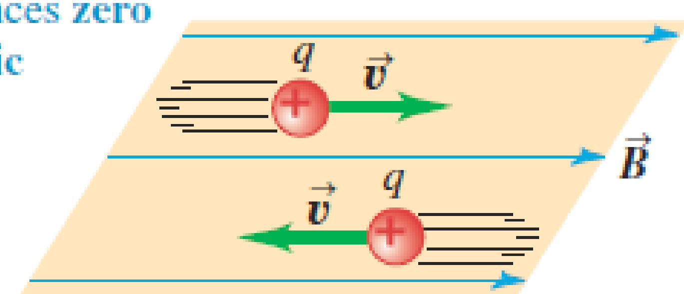
▲ **FIGURE 20.7** Some examples of magnetic fields.



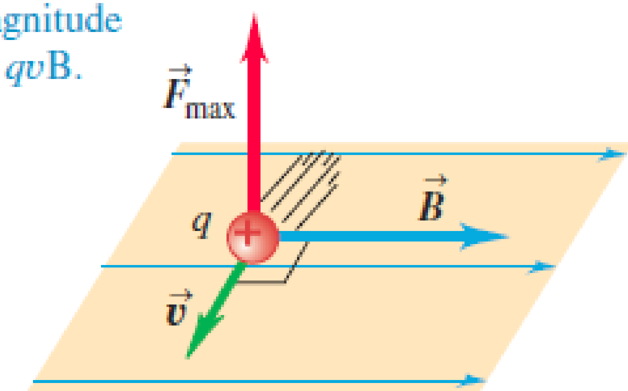
▲ **FIGURE 20.8** Magnetic field lines made visible by iron filings, which line up tangent to the field lines like little compass needles.

Magnetic Force

A charge moving **parallel** to a magnetic field experiences **zero magnetic force.**

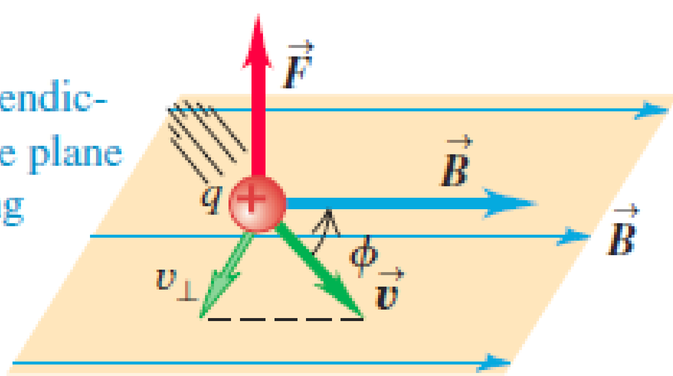


A charge moving **perpendicular** to a magnetic field experiences a maximal magnetic force with magnitude $F_{max} = qv\mathrm{B}$.



A charge moving at an angle ϕ to a magnetic field experiences a magnetic force with magnitude $F = |q|v_{\perp}B = |q|vB \sin \phi$.

$\vec{F}$ is perpendicular to the plane containing $\vec{v}$ and $\vec{B}$.



Magnitude of the magnetic force

When a charged particle moves with velocity $\vec{\boldsymbol{v}}$ in a magnetic field $\vec{\boldsymbol{B}}$, the magnitude F of the force exerted on it is

$$F = |q|v_{\perp}B = |q|vB\sin\phi, \tag{20.1}$$

where $|q|$ is the magnitude of the charge and ϕ is the angle measured from the direction of $\vec{\boldsymbol{v}}$ to the direction of $\vec{\boldsymbol{B}}$,

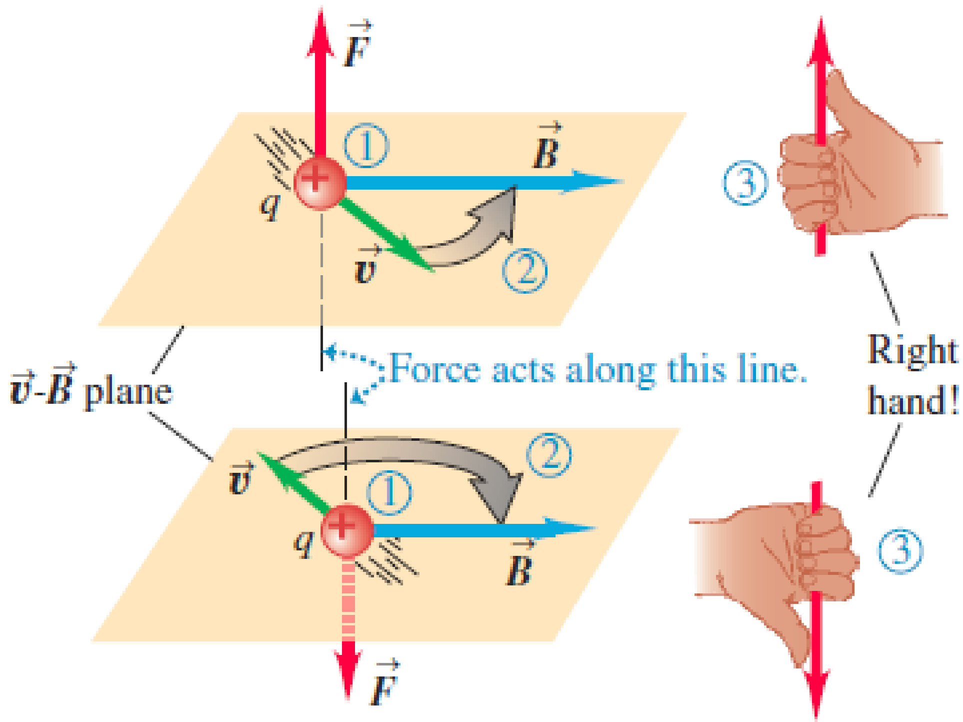
The direction of the magnetic force

Right-hand rule for the direction of magnetic force on a **positive** charge moving in a magnetic field:

① Place the $\vec{v}$ and $\vec{B}$ vectors tail to tail.

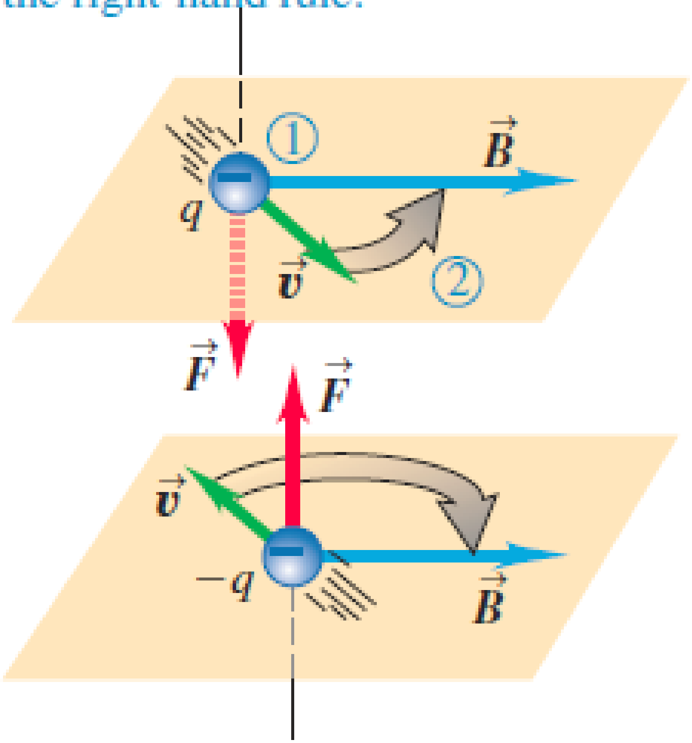
② Imagine turning $\vec{v}$ toward $\vec{B}$ in the $\vec{v}$-$\vec{B}$ plane (through the smaller angle).

③ The force acts along a line perpendicular to the $\vec{v}$-$\vec{B}$ plane. Curl the fingers of your *right hand* around this line in the same direction you rotated $\vec{v}$. Your thumb now points in the direction the force acts.



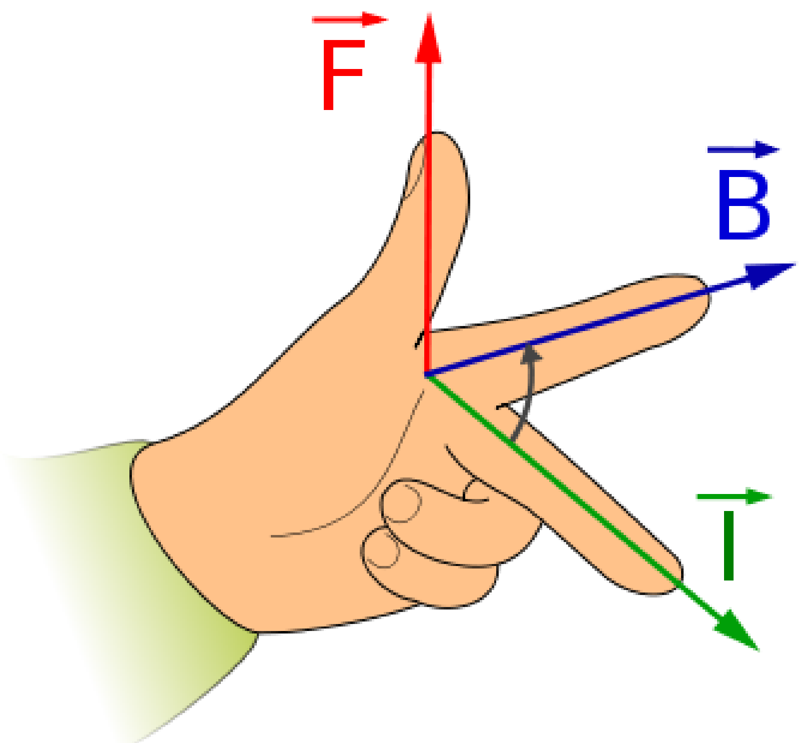
Right hand > curl fingers from **v to B** > Thumb is Force direction

If the charge is negative, the direction of the force is *opposite* to that given by the right-hand rule.



An equivalent rule is Fleming's **left**-hand rule.
This also gives you the direction of the force that acts on a current if you know the magnetic field.
The current, I, is the velocity of a positive particle.

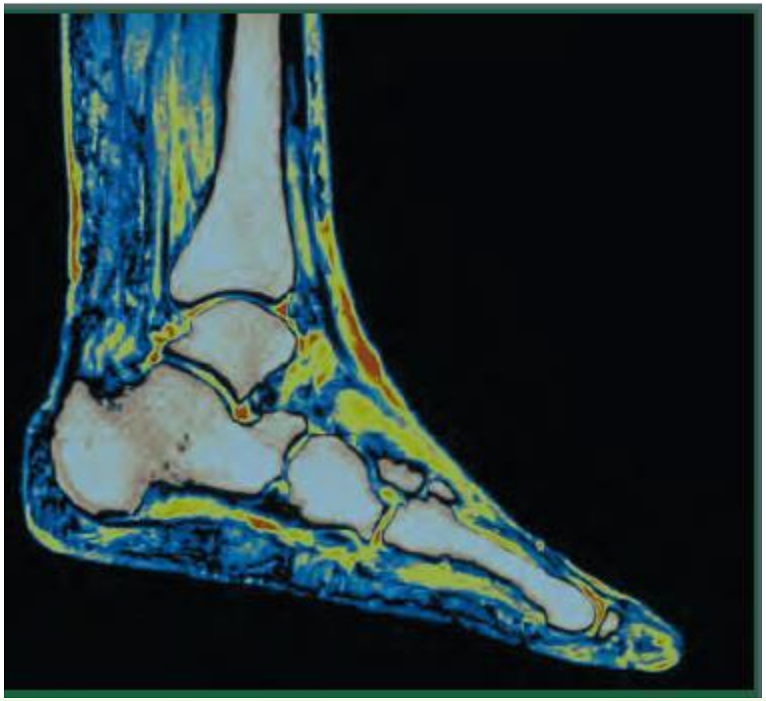
Alternative



Equation 20.1 can be interpreted in a different but equivalent way. Because ϕ is the angle between the directions of vectors $\vec{\boldsymbol{v}}$ and $\vec{\boldsymbol{B}}$, we may interpret $B \sin \phi$ as the component of $\vec{\boldsymbol{B}}$ perpendicular to $\vec{\boldsymbol{v}}$—that is, $B_{\perp}$. With this notation, the force expression (Equation 20.1) becomes

$$F = |q| v B_{\perp}. \tag{20.2}$$

This form is equivalent to Equation 20.1, but it's sometimes more convenient to use, especially in problems involving *currents* rather than individual particles.



◀ **BIO Application Spin doctor?** The incredible detail shown in this false-color magnetic resonance image (MRI) of the foot comes from an analysis of the behavior of spinning hydrogen nuclei in a magnetic field. The patient is placed in a strong magnetic field of about 1.5 T, over 10,000 times stronger than the earth's. Each spinning hydrogen nucleus in the imaged tissue acts like a tiny electromagnet, aligning itself either with or against the magnetic field. A pulse of electromagnetic energy of about 50 MHz causes these tiny spinning magnets to flip their orientation. As the nucleii flip back following the pulse, they produce a signal that is proportional to the amount of hydrogen in any imaged tissue. Therefore, hydrogen-rich fatty tissue looks quite different from hydrogen-deficient bone, making MRI imaging ideal for analyzing soft-tissue details that are invisible in x-ray analysis.

Definition of the tesla

$$1 \text{ tesla} = 1 \text{ T} = 1 \text{ N}/(\text{A} \cdot \text{m}).$$

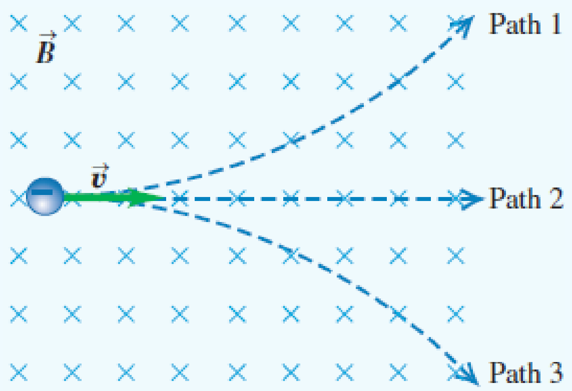
The cgs unit of B, the **gauss** $(1 \text{ G} = 10^{-4} \text{ T})$, is also in common use. Instruments for measuring magnetic field are sometimes called gaussmeters or teslameters.

Conceptual Analysis 20.1

Direction of magnetic force

Which of the three paths, 1, 2, or 3, does the electron in Figure 20.12 follow? (Remember that the blue ×'s represent a magnetic field pointing *into* the page, as explained in Figure 20.7b.)

A. Path 1.
B. Path 2, because the force on it is zero.
C. Path 2, because the force on it is perpendicular to the page. (We see the path projected onto the plane of the paper.)
D. Path 3.

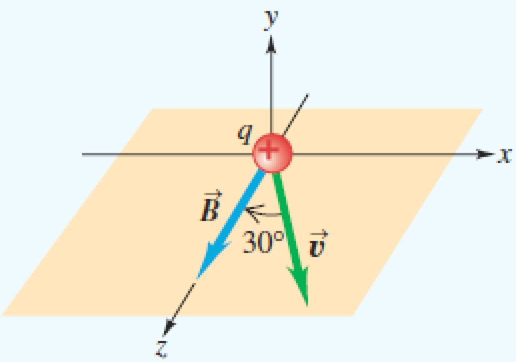


▲ **FIGURE 20.12**

SOLUTION The path depends on the direction of the force (if any) exerted by the magnetic field on the electron. The electron's velocity is not parallel to $\vec{B}$, so the electron experiences a force. To determine the force's direction, we use the right-hand rule. First, we identify the plane containing $\vec{v}$ and $\vec{B}$. (It is perpendicular to the page.) To turn $\vec{v}$ toward $\vec{B}$, we rotate it away from us into the page. Next, we hold our right hand so that the fingers can wrap around a line perpendicular to the plane of $\vec{v}$ and $\vec{B}$. (This line is in the plane of the paper, parallel to the side of the page.) When we curl our fingers in the direction we turned $\vec{v}$, our thumb points toward the top margin of the page. But that is the direction of the force the magnetic field would exert on a *positive* charge. Since the electron is negative, the force exerted on it is in the plane of the paper, directed toward the bottom of the page. Thus, the electron follows path 3.

EXAMPLE 20.1 **A proton beam**

In Figure 20.13, a beam of protons moves through a uniform magnetic field with magnitude 2.0 T, directed along the positive z axis. The protons have a velocity of magnitude 3.0×10^5 m/s in the x-z plane at an angle of 30° to the positive z axis. Find the force on a proton. The charge of the proton is $q = +1.6 \times 10^{-19}$ C.



▶ **FIGURE 20.13**

SOLUTION

SET UP We use the right-hand rule to find the direction of the force. The force acts along the y axis, so we curl the fingers of our right hand around this axis in the direction from $\vec{v}$ toward $\vec{B}$. We find that the force acts in the $-y$ direction.

SOLVE To find the magnitude of the force, we use Equation 20.1:

$$\begin{aligned} F &= qvB\sin\phi \\ &= (1.6 \times 10^{-19}\ \text{C})(3.0 \times 10^5\ \text{m/s})(2.0\ \text{T})(\sin 30°) \\ &= 4.8 \times 10^{-14}\ \text{N}. \end{aligned}$$

REFLECT We could also obtain this result by finding $B_\perp$ and applying Equation 20.2: $F = |q|vB_\perp$. However, since we were given the angle ϕ, Equation 20.1 is more convenient. To check for consistency of units, we recall that $1\ \text{T} = 1\text{N}/(\text{A} \cdot \text{m})$.

Practice Problem: An electron beam moves through a uniform magnetic field with magnitude 3.8 T, directed in the $-z$ direction. The electrons have a velocity of 2.4×10^4 m/s in the y-z plane at an angle of 40° from the $-z$ axis toward the $+y$ axis. Find the force on an electron. *Answer:* $\vec{F} = 9.4 \times 10^{-15}$ N in the $+x$ direction.

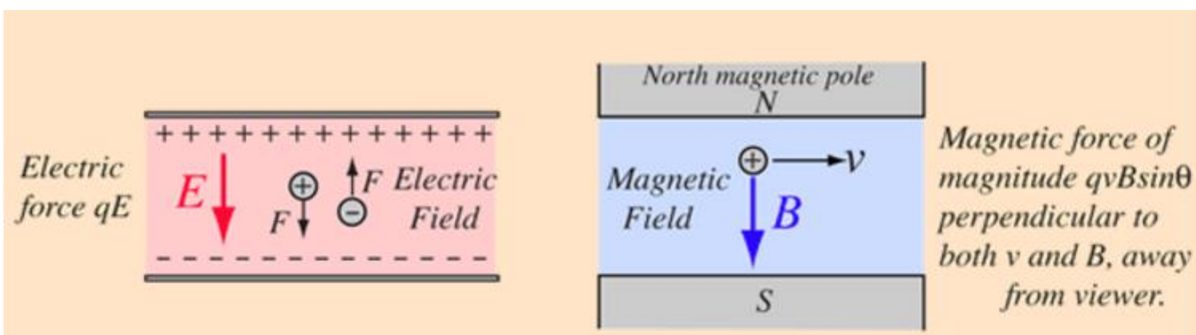
Lorentz Force Law

Both the electric field and magnetic field can be defined from the Lorentz force law:

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$$

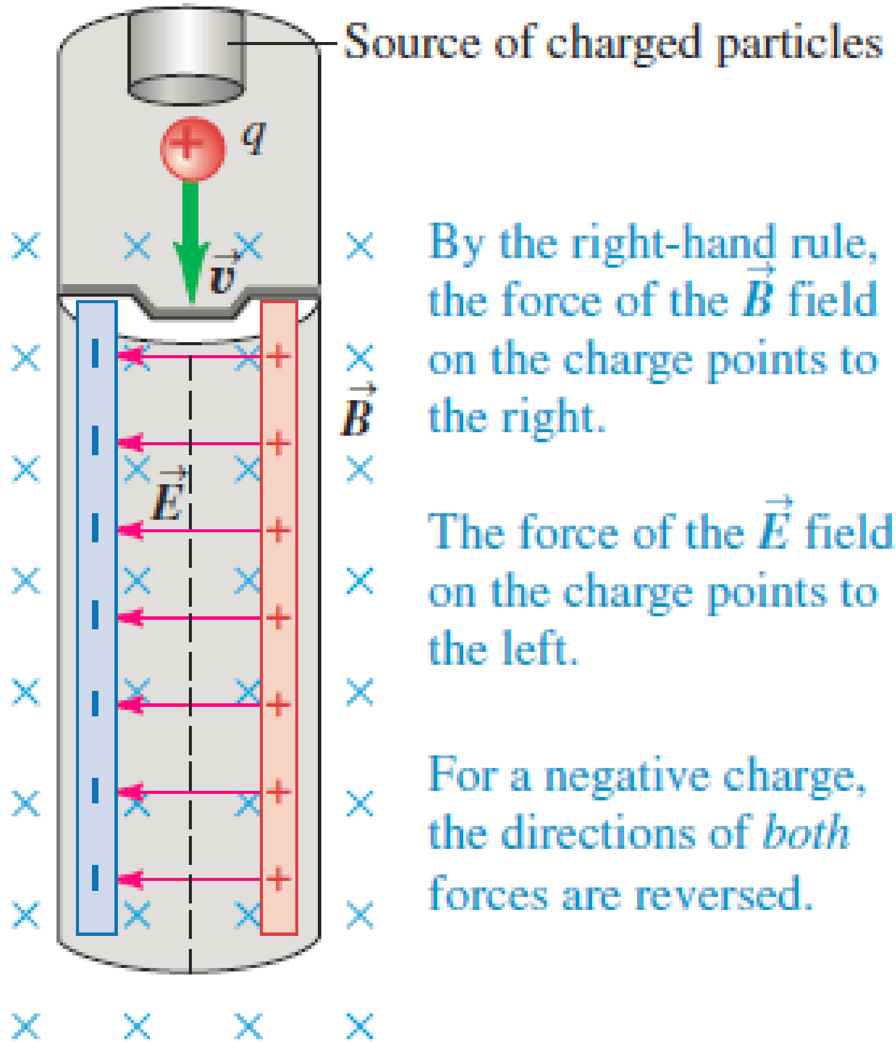
Electric force — Magnetic force

The electric force is straigtforward, being in the direction of the electric field if the charge q is positive, but the direction of the magnetic part of the force is given by the right hand rule.

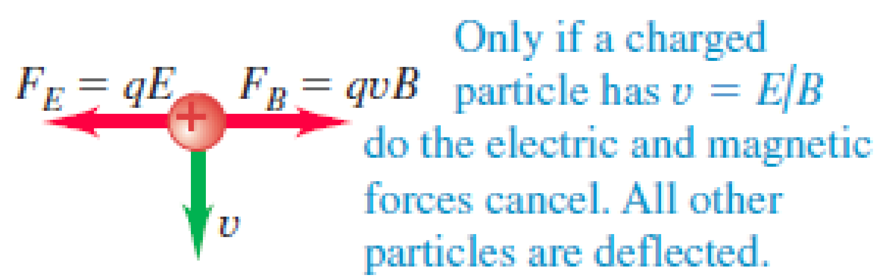


An example of the use of this is the velocity selector

The **velocity selector** uses an arrangement of electric and magnetic fields that lets us select only particles with the desired speed.



(a) Schematic diagram of velocity selector



(b) Free-body diagram for a positive particle

Thomson's e/m Experiment

In one of the landmark experiments in physics at the turn of the 20th century, Sir J. J. Thomson measured the ratio of charge to mass for the electron.

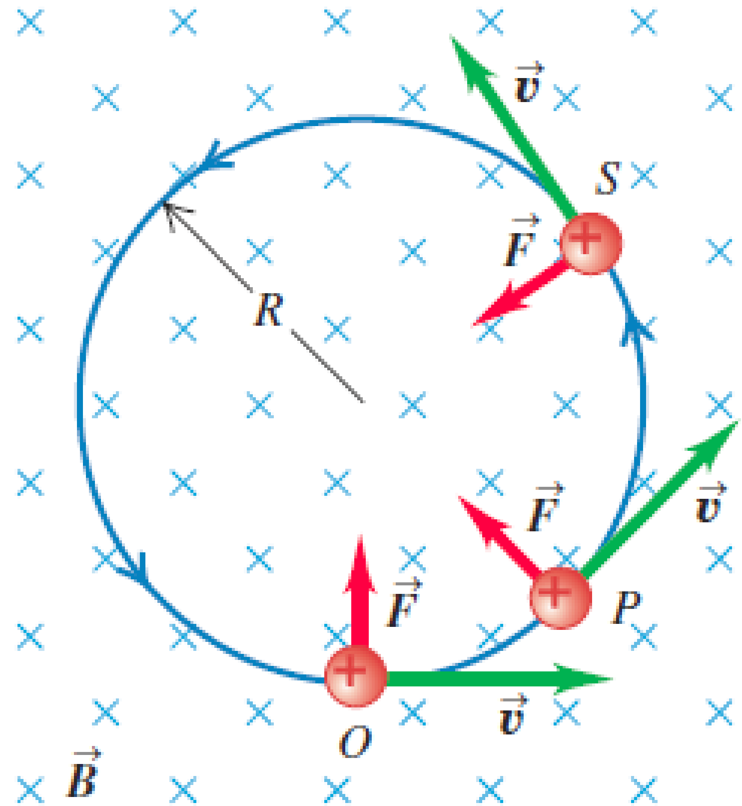
The most significant aspect of Thomson's measurements was that he found that all particles in the beam had the *same value* for this quantity and that the value was independent of the materials used for the experiment.

This independence showed that the particles in the beam, then known as cathode rays, which we now call electrons, are a common constituent of all matter.

Thus, Thomson is credited with the first discovery of a subatomic particle, the electron.

20.3 Motion of Charged Particles in a Magnetic Field

A charge moving at right angles to a uniform $\vec{B}$ field moves in a circle at constant speed because $\vec{F}$ and $\vec{v}$ are always perpendicular to each other.



(a) The orbit of a charged particle in a uniform magnetic field

The force is *always* perpendicular
to so it cannot change the *magnitude* of the velocity, only its direction.

The radial acceleration is v^2/R, and, from Newton's second law,

$$F = |q|vB = m\frac{v^2}{R},$$

where m is the mass of the particle. The radius R of the circular path is

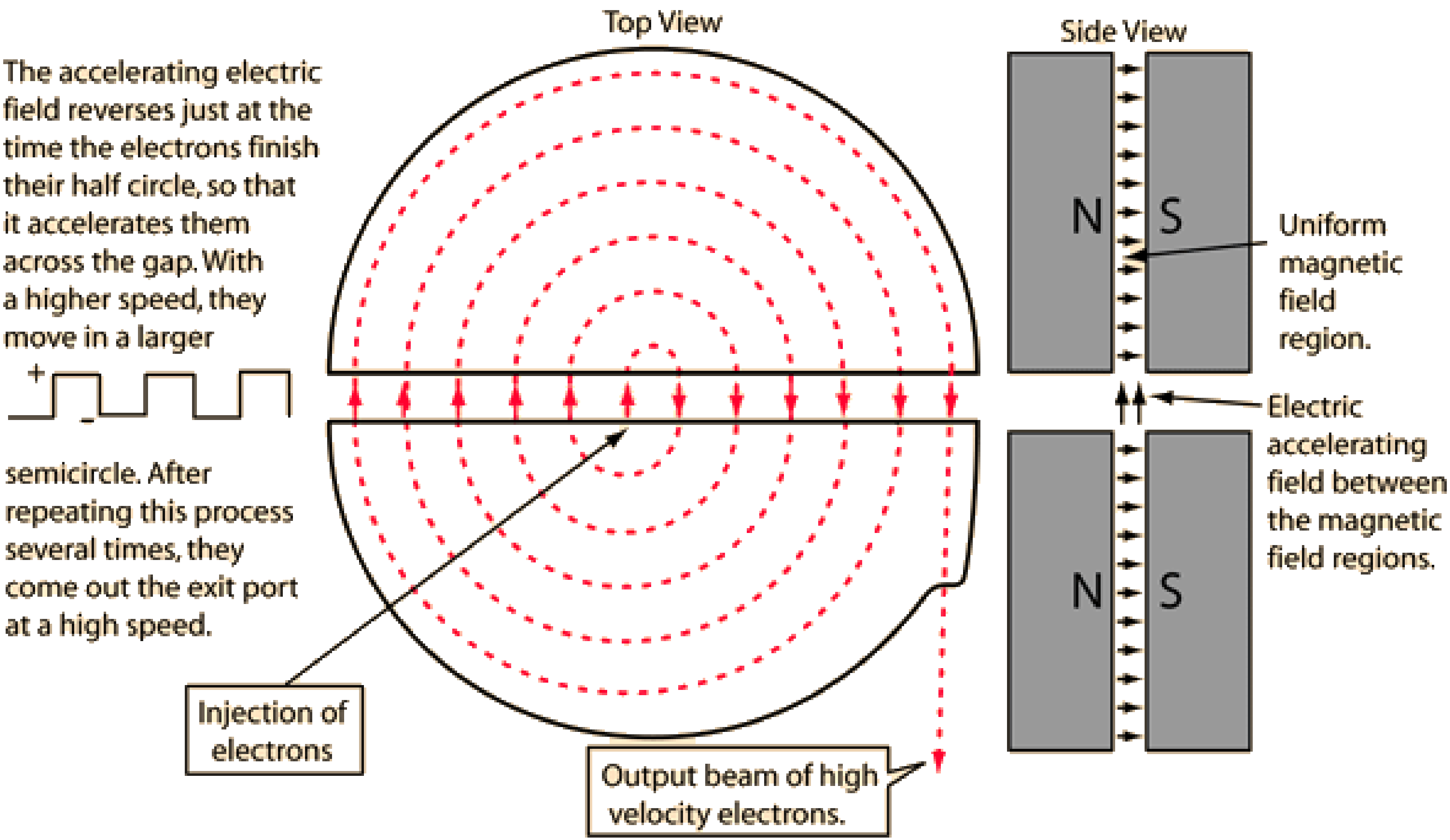
$$R = \frac{mv}{|q|B}. \tag{20.4}$$

This result agrees with our intuition that it's more difficult to bend the paths of fast and heavy particles into a circle, so the radius is larger for fast, massive particles than for slower, less massive particles. Likewise, for a given charge, a larger magnetic field increases the force and pulls the particle into a smaller radius. If the charge q is negative, the particle moves *clockwise* around the orbit in Figure 20.16a.

The angular velocity ω of the particle is given by Equation 9.13: $\omega = v/R$. Combining this relationship with Equation 20.4, we get

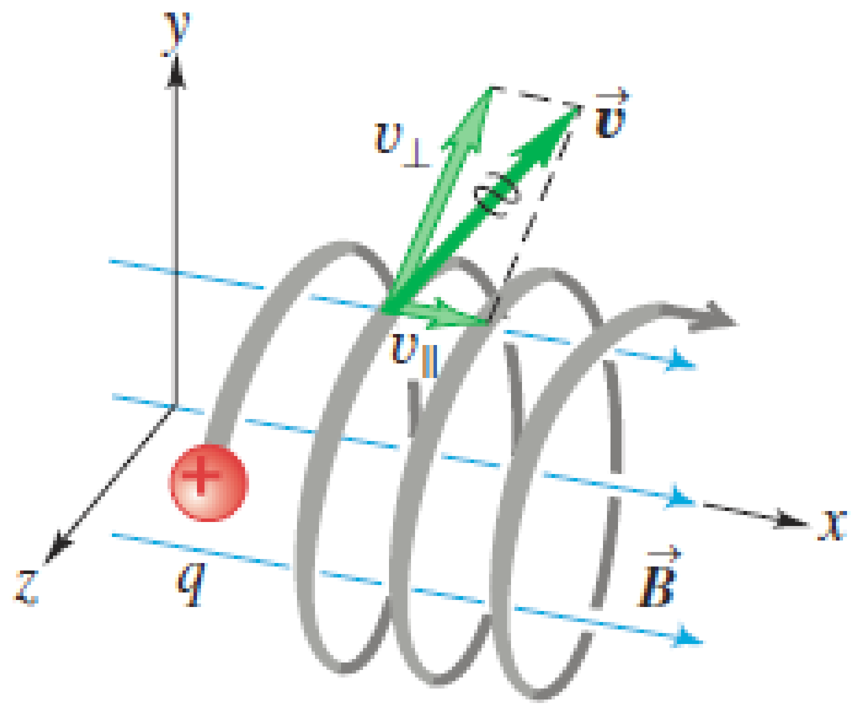
$$\omega = \frac{v}{R} = v\frac{|q|B}{mv} = \frac{|q|B}{m}. \tag{20.5}$$

Cyclotron


Top View
Side View
The accelerating electric field reverses just at the time the electrons finish their half circle, so that it accelerates them across the gap. With a higher speed, they move in a larger
+
-
semicircle. After repeating this process several times, they come out the exit port at a high speed.
N
S
N
S
Uniform magnetic field region.
Electric accelerating field between the magnetic field regions.
Injection of electrons
Output beam of high velocity electrons.

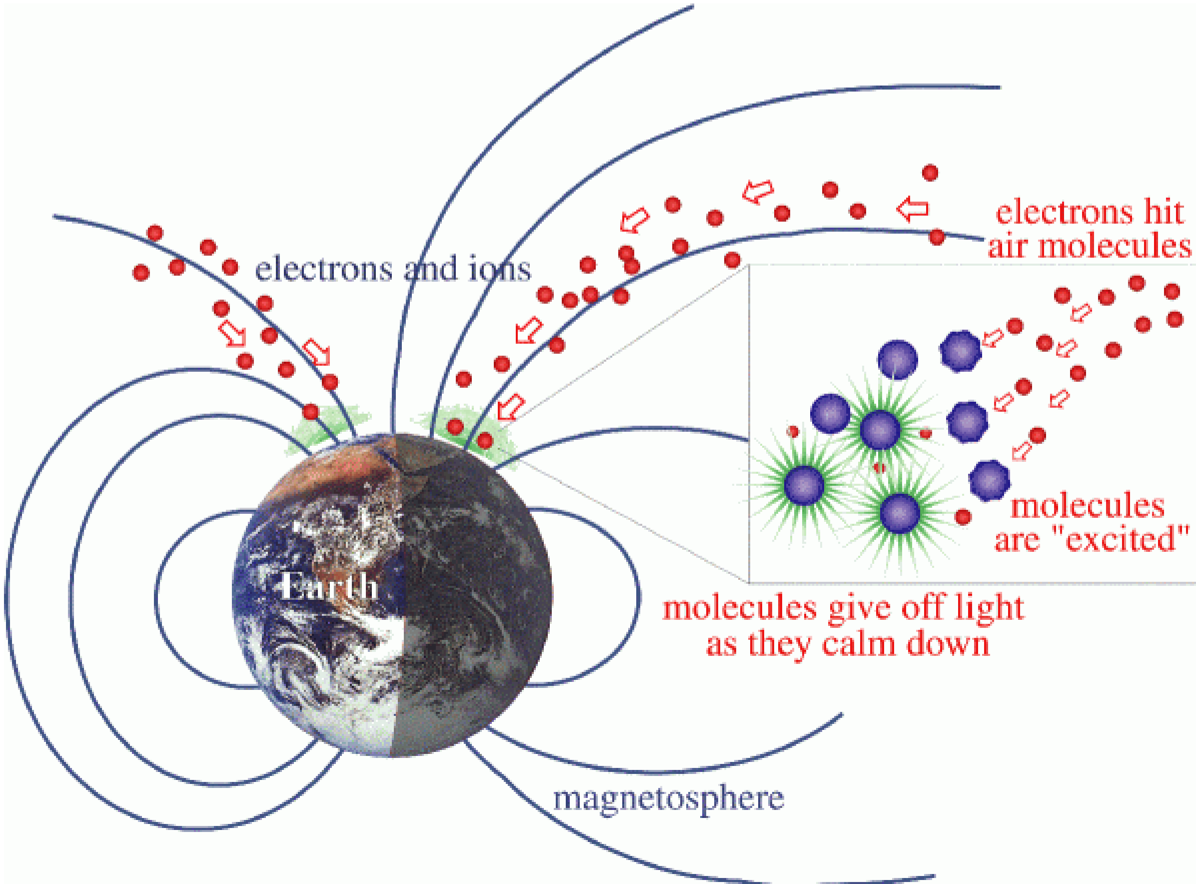
Helical Motion

This particle's motion has components both parallel ($v_{\parallel}$) and perpendicular ($v_{\perp}$) to the magnetic field, so it moves in a spiral.



▲ **FIGURE 20.17** The spiral path of a charged particle whose initial velocity has components parallel and perpendicular to the magnetic field.

Northern lights
(Aurora Borealis)

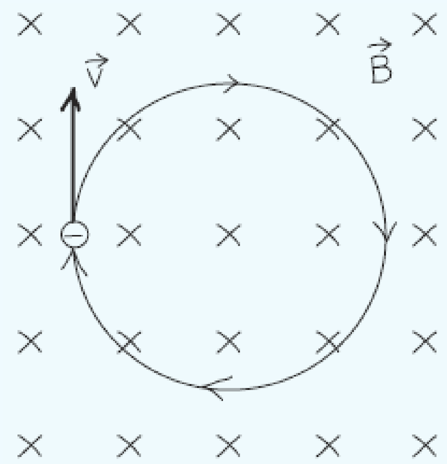


EXAMPLE 20.2 **Electron motion in a microwave oven**

A magnetron in a microwave oven emits microwaves with frequency $f = 2450$ MHz. What magnetic field strength would be required for electrons to move in circular paths with this frequency?

SOLUTION

SET UP Figure 20.19 shows our diagram. Because the electron is negatively charged, the right-hand rule tells us that it circles clockwise. The frequency is $f = 2450\ \text{MHz} = 2.45 \times 10^9\ \text{s}^{-1}$. The corresponding angular velocity is $\omega = 2\pi f = (2\pi)(2.45 \times 10^9\ \text{s}^{-1}) = 1.54 \times 10^{10}\ \text{rad/s}$.



▲ **FIGURE 20.19** Our sketch for this problem.

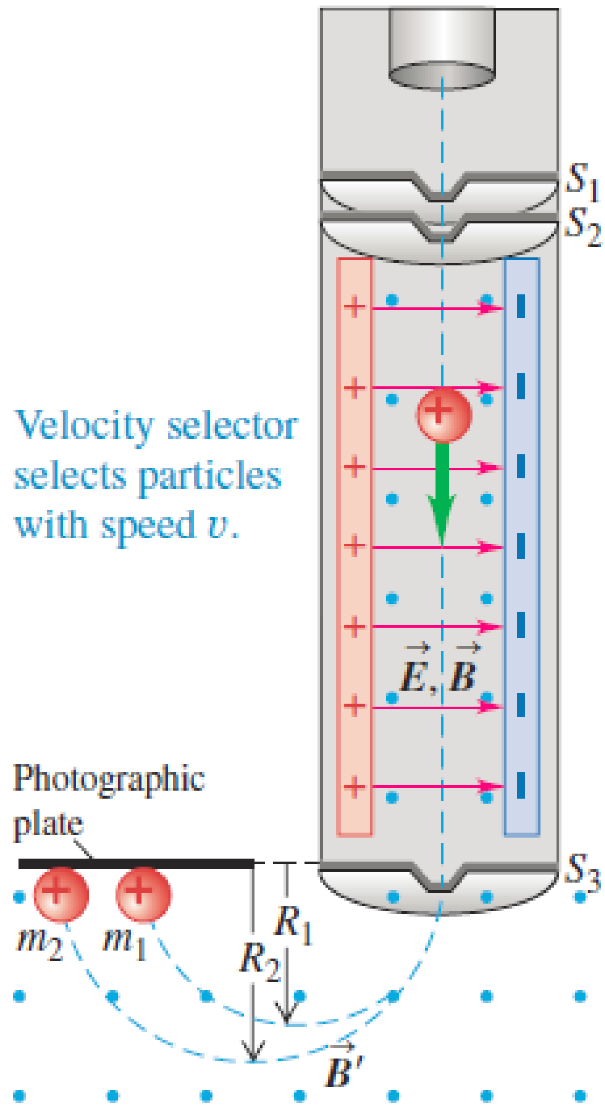
SOLVE From Equation 20.5,

$$B = \frac{m\omega}{|q|} = \frac{(9.11 \times 10^{-31}\ \text{kg})(1.54 \times 10^{10}\ \text{rad/s})}{1.60 \times 10^{-19}\ \text{C}} = 0.0877\ \text{T}.$$

REFLECT This is a moderate field strength, easily produced with a permanent magnet. Electromagnetic waves with this frequency can penetrate several centimeters into food with high water content.

Practice Problem: If the magnetron emits microwaves with frequency 2300 MHz, what magnetic field strength would be required for electrons to move in circular paths with that frequency? *Answer:* $B = 0.0823$ T.

20.4 Mass Spectrometers



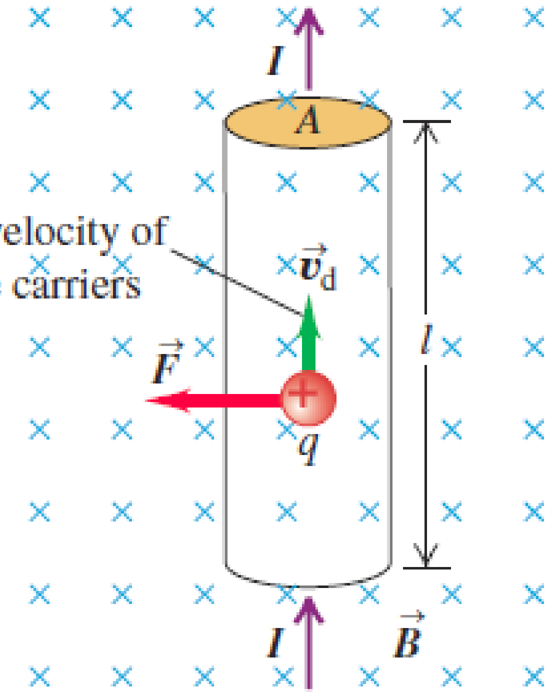
Magnetic field separates particles by mass; the greater a particle's mass, the greater is the radius of its path.

20.5 Magnetic Force on a Current-Carrying Conductor

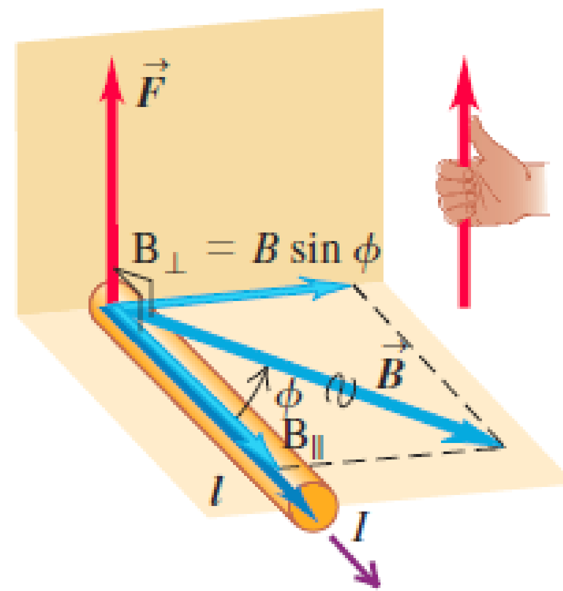
Force on a straight wire carrying a positive current and oriented at an angle ϕ to a magnetic field $\vec{B}$:

Magnitude is $F = I\,lB_{\perp} = I\,l\,\mathrm{B}\sin\phi$

Direction is given by the right-hand rule.



▲ **FIGURE 20.22** Magnetic force on a representative moving positive charge in a current-carrying conductor.



▲ **FIGURE 20.23** The magnetic force on a segment of current-carrying wire in a magnetic field.

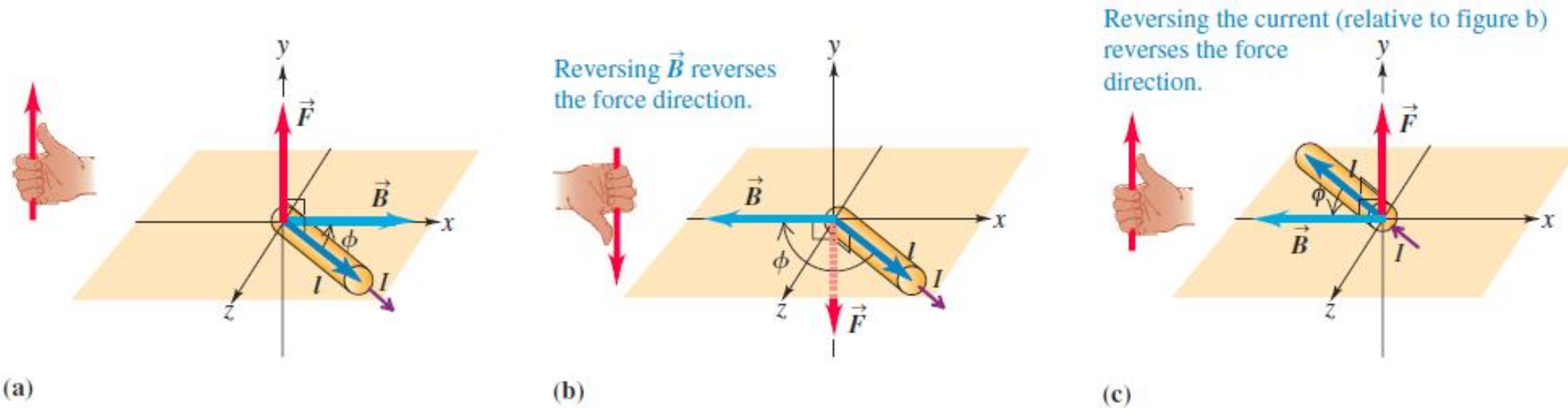
Magnetic force on a current-carrying conductor

The magnetic-field force on a segment of conductor with length l, carrying a current I in a uniform magnetic field $\vec{B}$, is

$$F = IlB_{\perp} = IlB\sin\phi. \qquad (20.7)$$

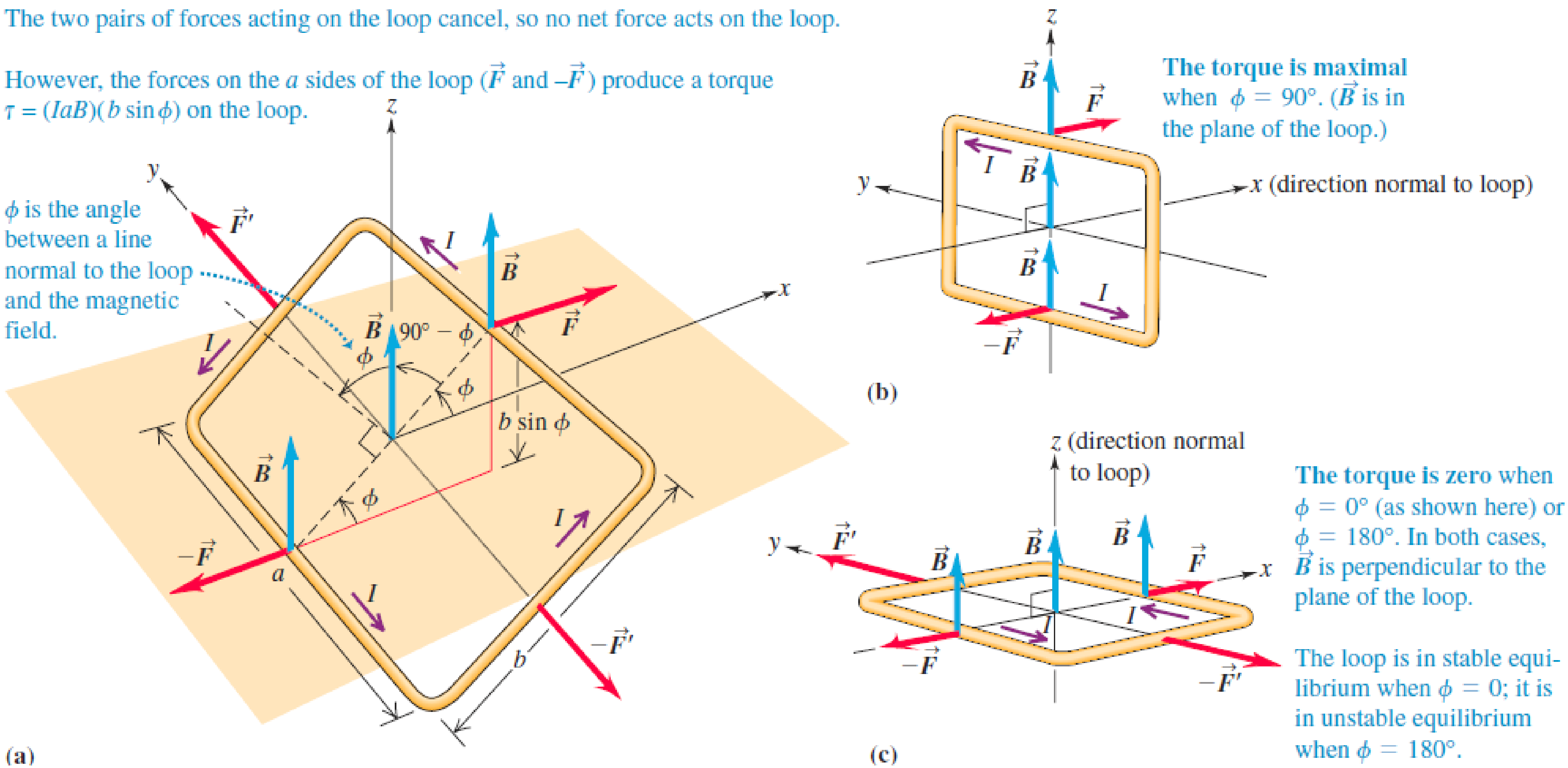
The force is always perpendicular to both the conductor and the field, with the direction determined by the same right-hand rule that we used for a moving positive charge (Figure 20.23).

Figure 20.24 shows the relations among the various directions for several cases.



▲ **FIGURE 20.24** The relation of the direction of the magnetic force on a current-carrying conductor to the directions of the current and the magnetic field.

20.6 Force and Torque on a Current Loop



▲ **FIGURE 20.27** (a) Forces on the sides of a current-carrying loop rotating in a magnetic field. (b), (c) orientations at which the torque on the loop is maximal and zero, respectively.

Use right hand curl rule for direction of force. I is direction of +ve charge flow.

Torque on a current-carrying loop

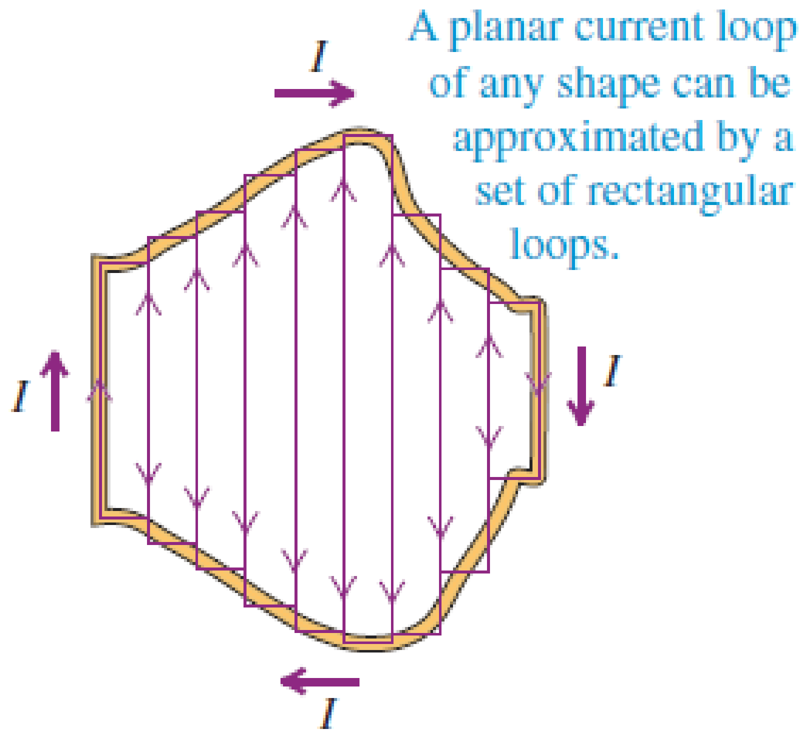
When a conducting loop with area A carries a current I in a uniform magnetic field of magnitude B, the torque exerted on the loop by the field is

$$\tau = IAB\sin\phi, \tag{20.8}$$

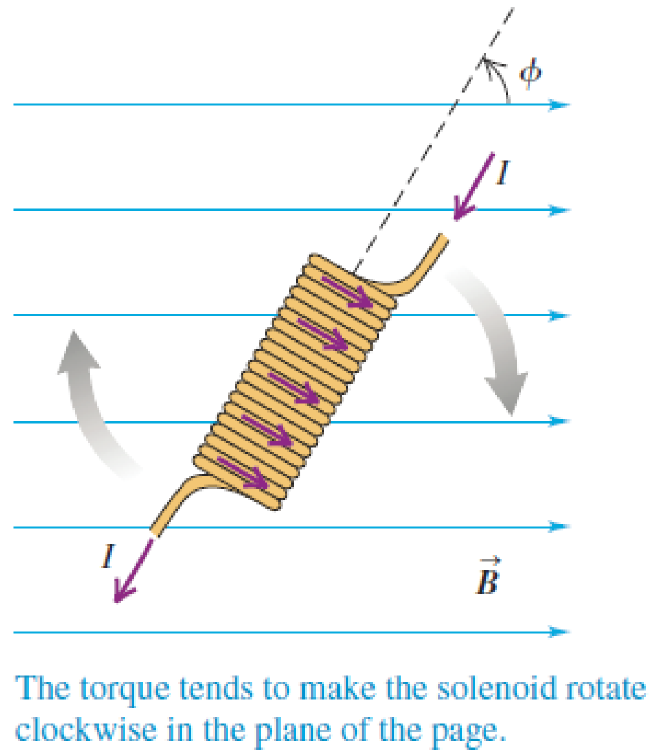
where ϕ is the angle between the normal to the loop and $\vec{B}$.

The torque τ tends to rotate the loop in the direction of *decreasing* ϕ—that is, toward its stable equilibrium position, in which $\phi = 0$ and the loop lies in the x-y plane, perpendicular to the direction of the field $\vec{B}$ (Figure 20.27c). The product IA is called the **magnetic moment** of the loop, denoted by μ:

$$\mu = IA. \tag{20.9}$$



▲ **FIGURE 20.28** An arbitrary planar shape can be approximated to any desired accuracy by a collection of rectangles. The narrower and more numerous the rectangles, the more closely they approximate the shape.



▲ **FIGURE 20.29** A current-carrying solenoid in a uniform magnetic field experiences a torque.

For a solenoid with N turns in a uniform field with magnitude B,

$$\tau = NIAB\sin\phi,$$

where ϕ is the angle between the axis of the solenoid and the direction of the field.

Torque on a Current Loop

The torque on a current-carrying coil, as in a DC motor, can be related to the characteristics of the coil by the "magnetic moment" or "magnetic dipole moment". The torque exerted by the magnetic force (including both sides of the coil) is given by

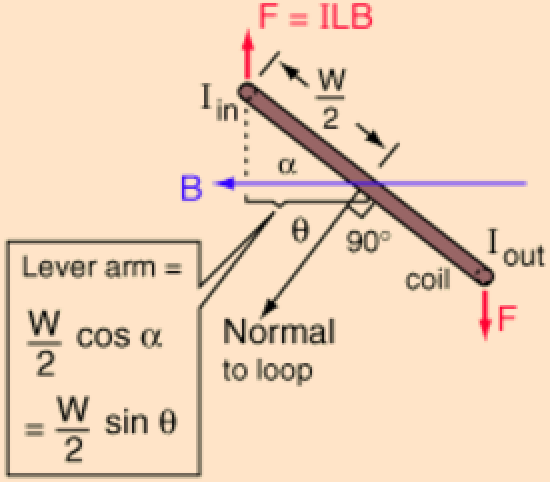
$$\tau = BILW\sin\theta$$

The coil characteristics can be grouped as

$$\mu = IA \quad (or\ \mu = NIA \quad for\ \ n\ \ loops)$$

called the magnetic moment of the loop, and the torque written as

$$\tau = \mu B\sin\theta$$



The direction of the magnetic moment is perpendicular to the current loop in the right-hand-rule direction, the direction of the normal to the loop in the illustration. Considering torque as a vector quantity, this can be written as the vector product

EXAMPLE 20.5 Torque on a circular coil

A circular coil of wire with average radius 0.0500 m and 30 turns lies in a horizontal plane. It carries a current of 5.00 A in a counterclockwise sense when viewed from above. The coil is in a uniform magnetic field directed toward the right, with magnitude 1.20 T. Find the magnetic moment and the torque on the coil. Which way does the coil tend to rotate?

SOLUTION

SET UP Figure 20.31 shows our diagram. The area of the coil is $A = \pi r^2 = \pi(0.0500\ \text{m})^2 = 7.85 \times 10^{-3}\ \text{m}^2$; the angle ϕ between the direction of $\vec{B}$ and the *axis* of the coil (perpendicular to its plane) is 90°.

SOLVE The magnetic moment for one turn of the coil is $\mu = IA$ (Equation 20.9). Therefore, the total magnetic moment for all 30 turns is

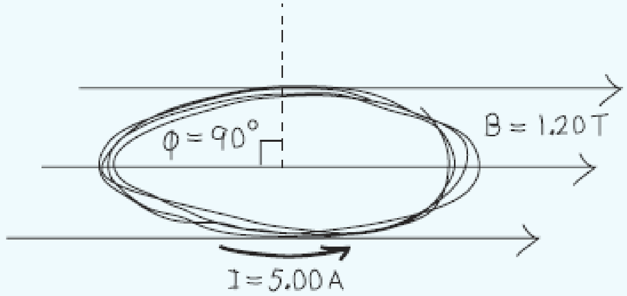
$$\mu_{\text{total}} = 30\mu = 30IA = 30(5.00\ \text{A})(7.85 \times 10^{-3}\ \text{m}^2)$$
$$= 1.18\ \text{A} \cdot \text{m}^2.$$

From Equation 20.8, the torque on each turn of the coil is

$$\tau = IAB\sin\phi = (5.00\ \text{A})(7.85 \times 10^{-3}\ \text{m}^2)(1.20\ \text{T})(\sin 90°)$$
$$= 0.0471\ \text{N} \cdot \text{m},$$

and the total torque on the coil of 30 turns is

$$\tau = (30)(0.0471\ \text{N} \cdot \text{m}) = 1.41\ \text{N} \cdot \text{m}.$$



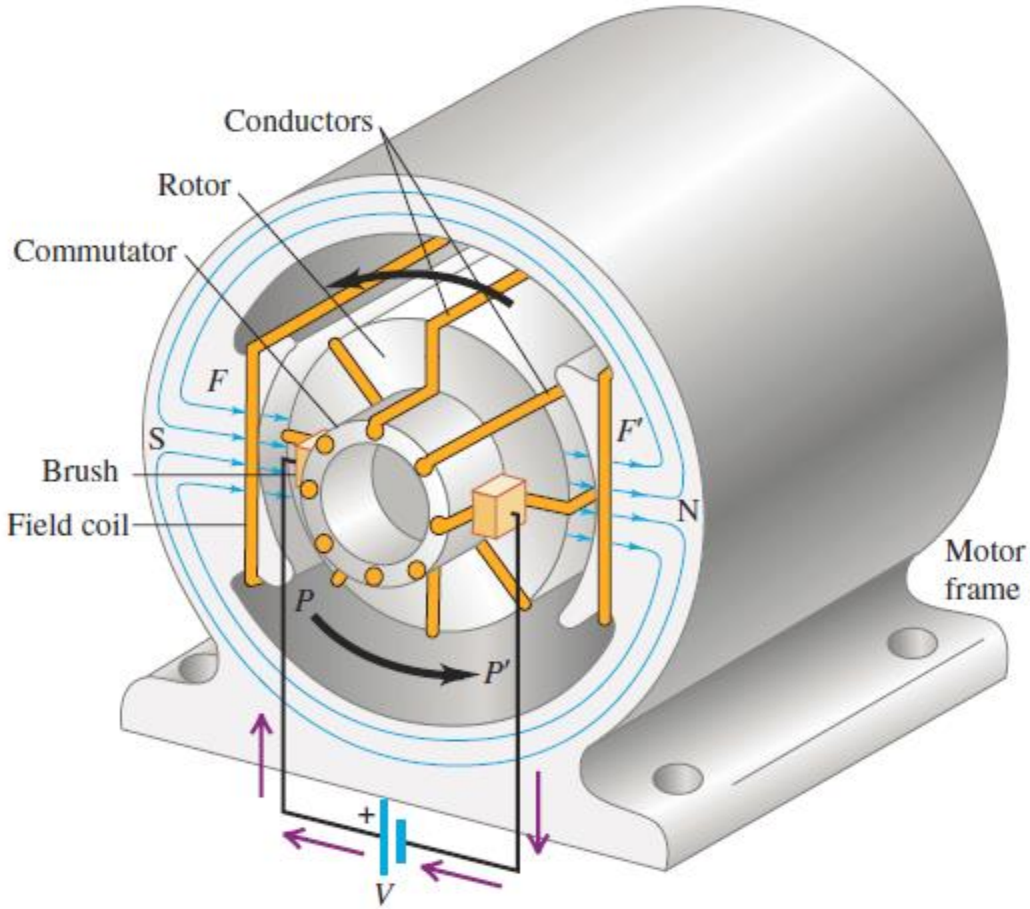
▲ **FIGURE 20.31** Our sketch for this problem.

Using the right-hand rule on the two sides of the coil, we find that the torque tends to rotate the right side down and the left side up.

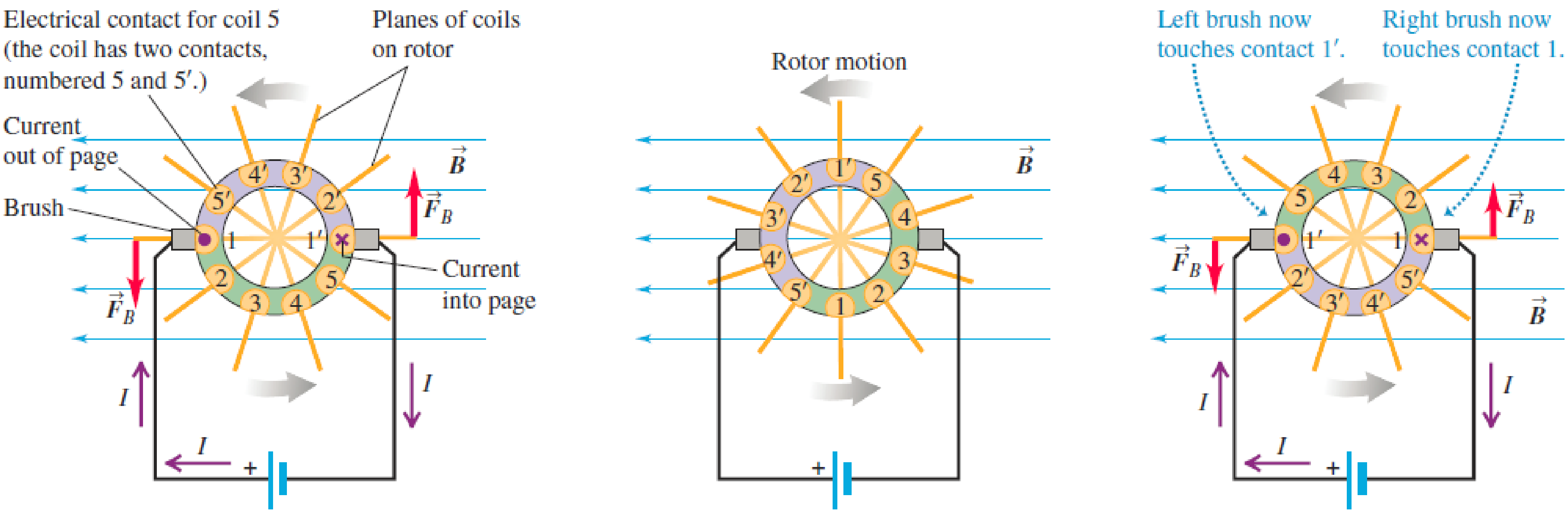
REFLECT The torque tends to rotate the coil toward the stable equilibrium orientation, in which the normal to the plane is parallel to $\vec{B}$.

Practice Problem: Calculate the torque on the coil when it is placed in a magnetic field along the direction of the axis of the coil. *Answer:* $\tau = 0\ \text{N} \cdot \text{m}$.

The Direct-Current Motor



▲ **FIGURE 20.32** Schematic diagram of a dc motor. The rotor rotates on a shaft through its center, perpendicular to the plane of the figure.



The brushes transmit current through contacts 1 and 1′ to coil 1, which is oriented to receive maximal torque from the magnetic field.

When the brushes are between contacts, inertia keeps the rotor turning.

The brushes again contact coil 1, but with opposite polarity, so the torque on the coil is still counterclockwise.

▲ **FIGURE 20.33** By reversing the direction of the current in each coil once per half cycle, the commutator ensures that the torque on the coils always points in the same direction.

DC Motor Operation

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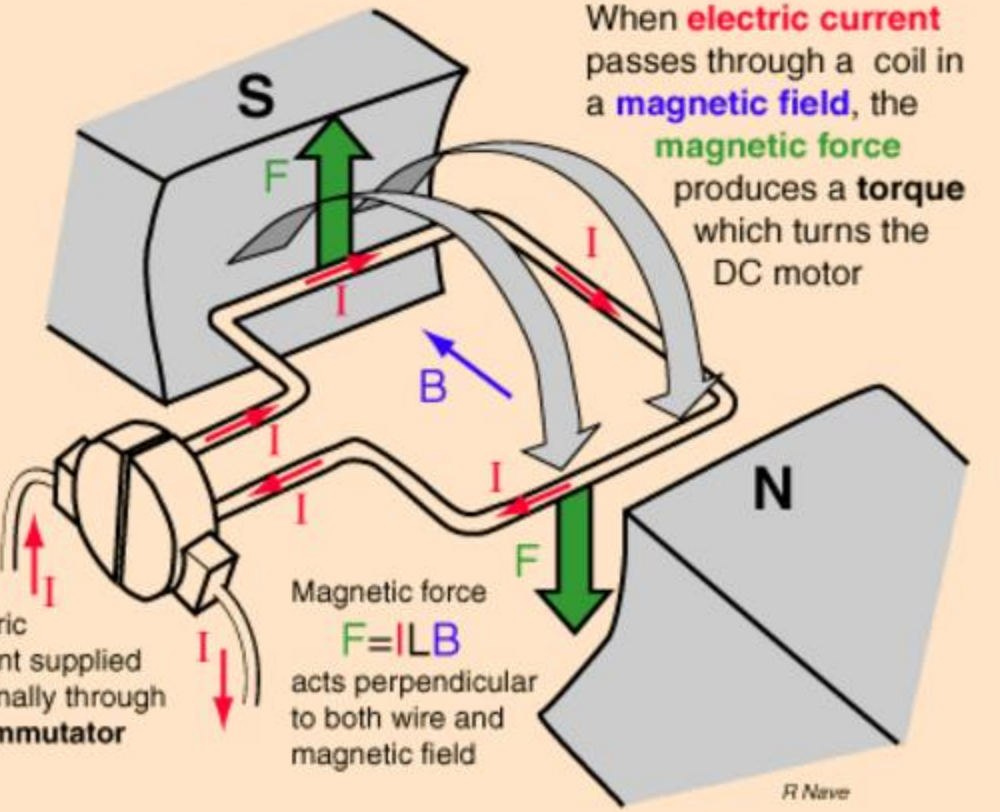

Current in DC Motor

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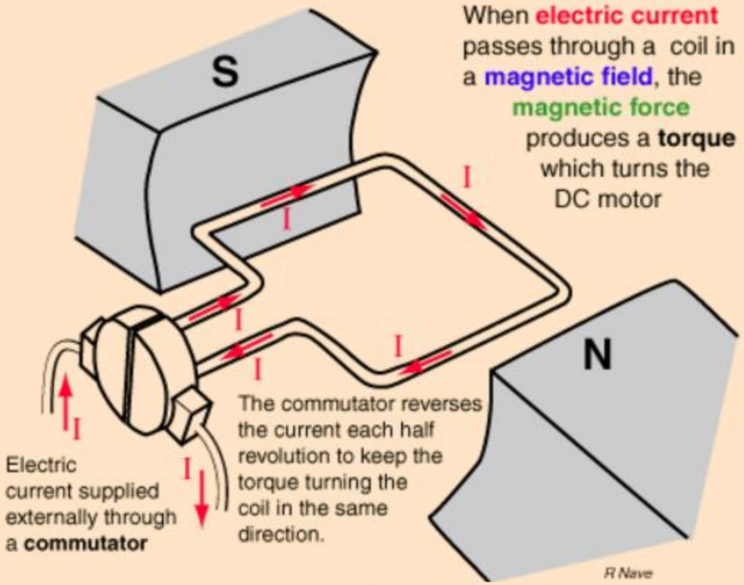

Magnetic Field in DC Motor

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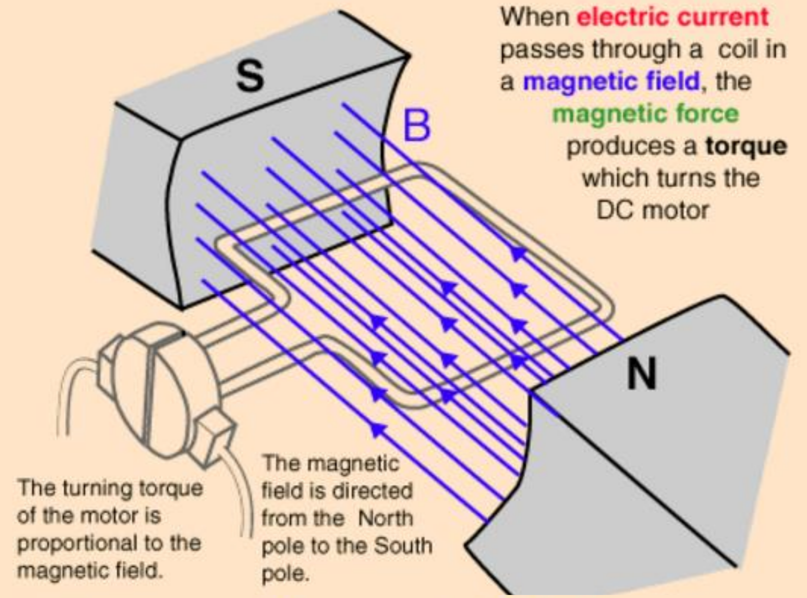

Force in DC Motor

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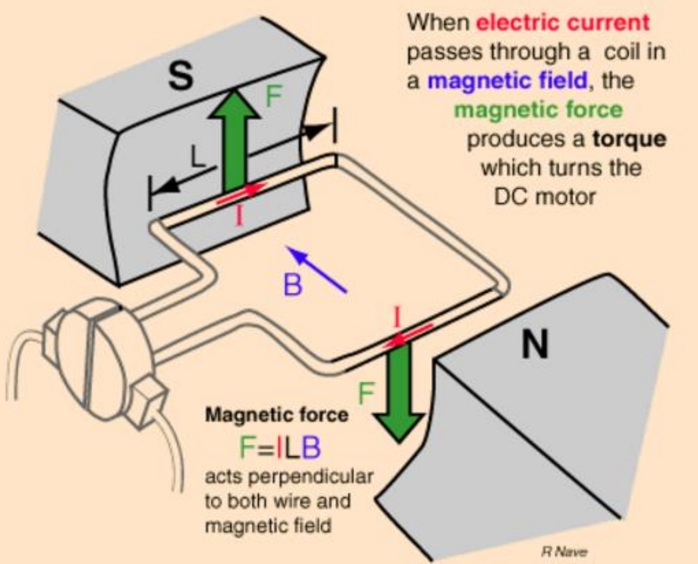

Torque in DC Motor

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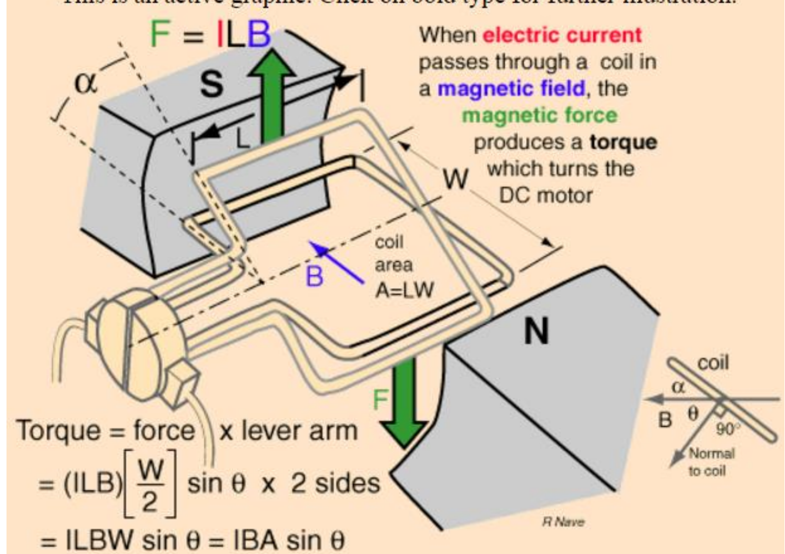
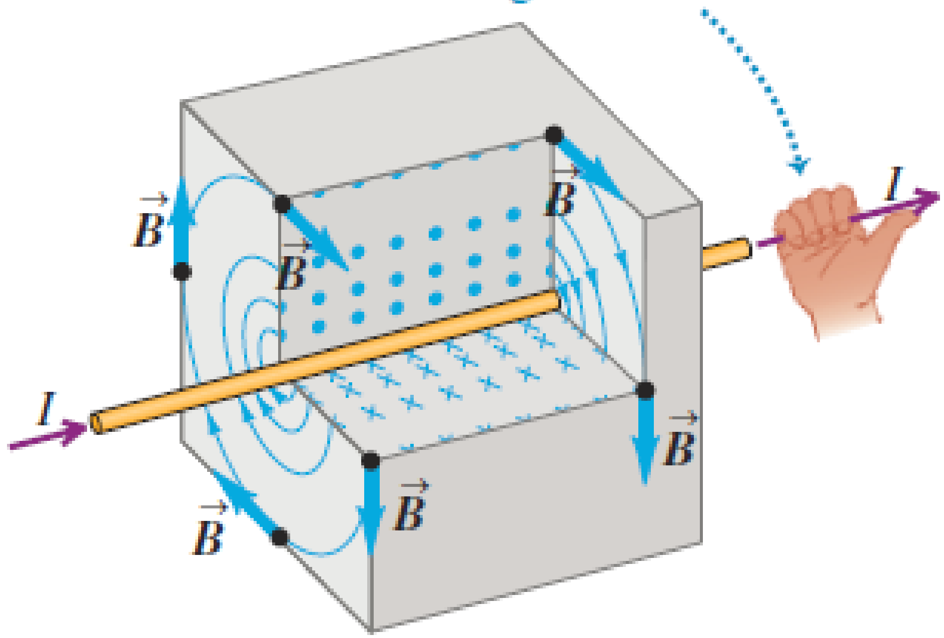
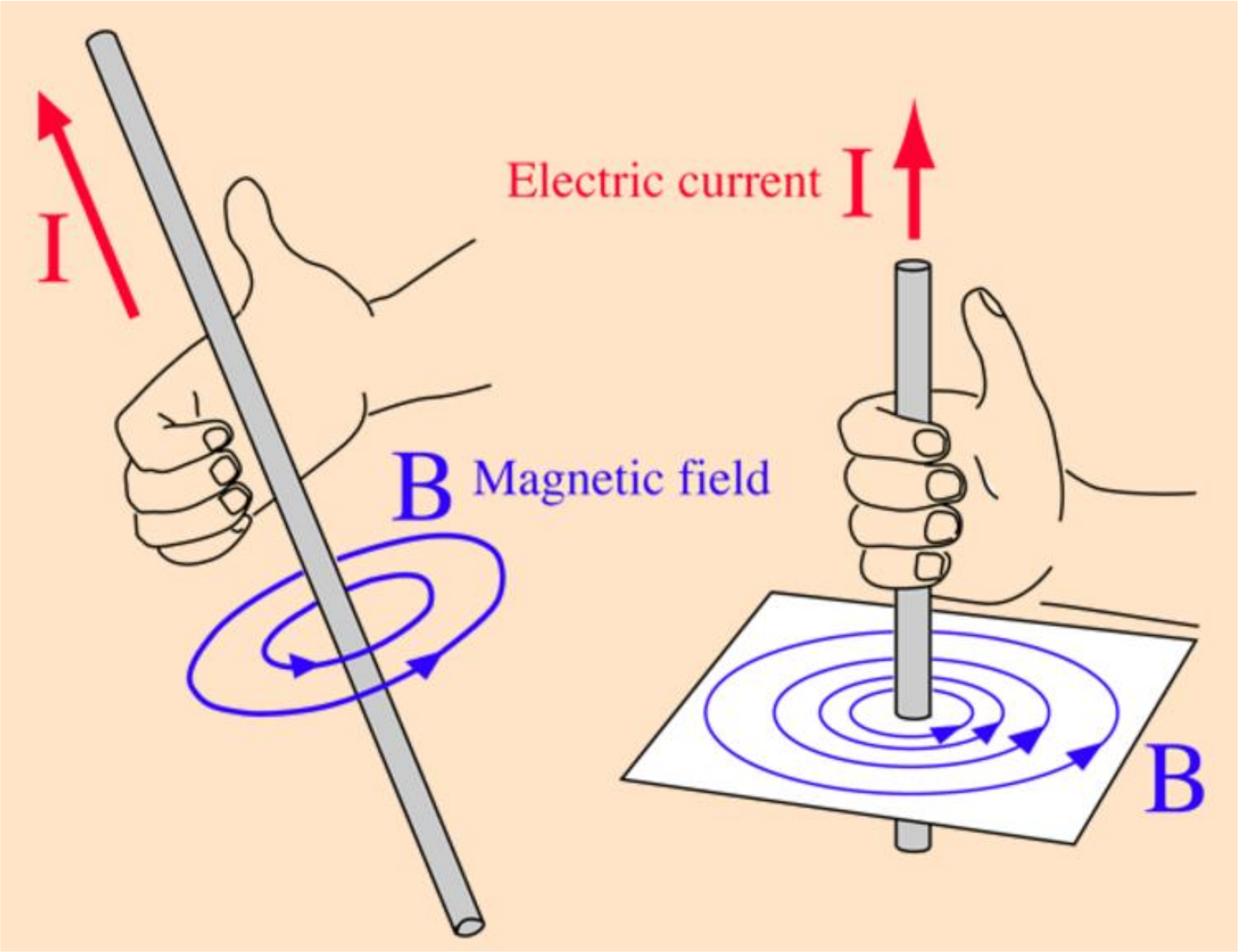


▶ Application **A clever solution.** As you know, a current traveling through a wire generates a magnetic field surrounding the wire. In a normal household extension cord, the two wires run side by side, and their fields add to produce a small net magnetic field. Because many types of sensitive electronic equipment cannot tolerate even these slight magnetic fields, the *coaxial* cable was developed. In a coaxial cable, one of the conductors has the form of a hollow tube, and the other runs through its center. (The cable is called "coaxial" because the conductors have the same axis.) As long as the currents in the two conductors are equal, the two magnetic fields cancel, so the cable produces no net magnetic field.

20.7 Magnetic Field of a Long, Straight Conductor



▲ **FIGURE 20.34** Right-hand rule for the direction of the magnetic field around a long, straight conductor carrying a current.


I
Electric current I
B Magnetic field
B

Magnetic field of a long, straight wire

The magnetic field $\vec{B}$ produced by a long, straight conductor carrying a current I, at a distance r from the axis of the conductor, has magnitude B given by

$$B = \frac{\mu_0 I}{2\pi r}. \tag{20.10}$$

In this equation, μ_0 is a constant called the *permeability of vacuum.* Its numerical value depends on the system of units we use. In SI units, the units of μ_0 are $(\mathrm{T \cdot m/A})$. Its numerical value, which is related to the definition of the unit of current, is defined to be *exactly* $4\pi \times 10^{-7}$:

$$\mu_0 = 4\pi \times 10^{-7}\,\mathrm{T \cdot m/A}.$$

A useful relationship for checks of unit consistency is $1\,\mathrm{T \cdot m/A} = 1\,\mathrm{N/A^2}$. We invite you to verify this equivalence.

EXAMPLE 20.6 Magnetic field from power lines

A long, straight dc power line carries a current of 100 A. A swarm of bees builds a hive next to it. It is hypothesized that bees use the earth's magnetic field as a reference direction when orienting their honeycombs. At what distance from the power line is the magnitude of the magnetic field from the current equal to the magnitude of Earth's magnetic field, about 5.0×10^{-5} T?

SOLUTION

SET UP AND SOLVE We need to find the distance r at which the magnitude of the field B_{power} from the current in the power line equals the magnitude of field B_{earth} from the earth. Setting $B_{\text{power}} = B_{\text{earth}}$ and using Equation 20.10 for B_{power}, we find that

$$\frac{\mu_0 I}{2\pi r} = B_{\text{earth}}.$$

We solve this equation for the distance r from the power line:

$$r = \frac{\mu_0 I}{2\pi B_{\text{earth}}} = \frac{(4\pi \times 10^{-7}\,\text{T}\cdot\text{m/A})(100\,\text{A})}{2\pi(5.0 \times 10^{-5}\,\text{T})} = 0.40\,\text{m}.$$

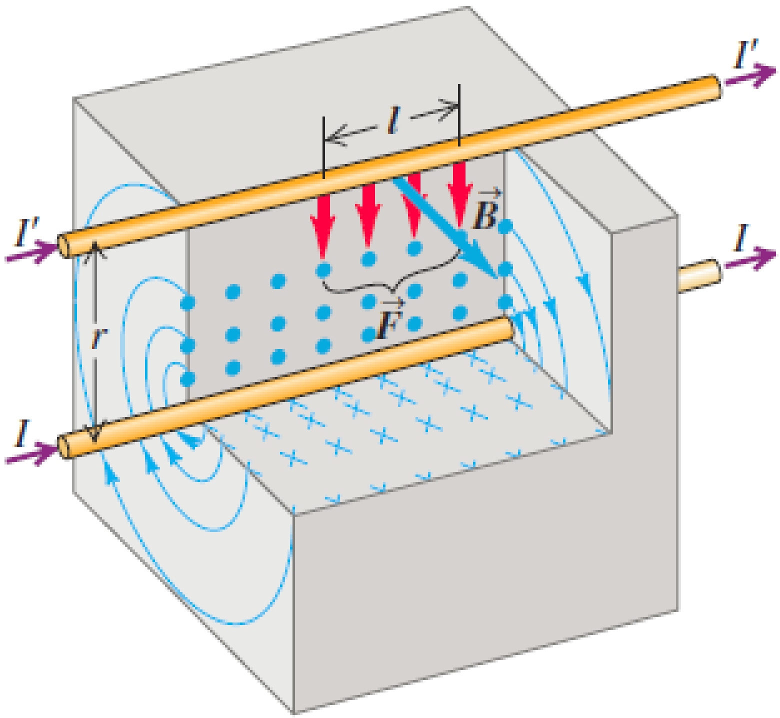
REFLECT The magnetic field of the power line at a distance of half a meter is comparable in magnitude to the earth's field. Depending on the orientation of the power line relative to the earth's field, the current could cause a significant disruption in the bees' perception of direction.

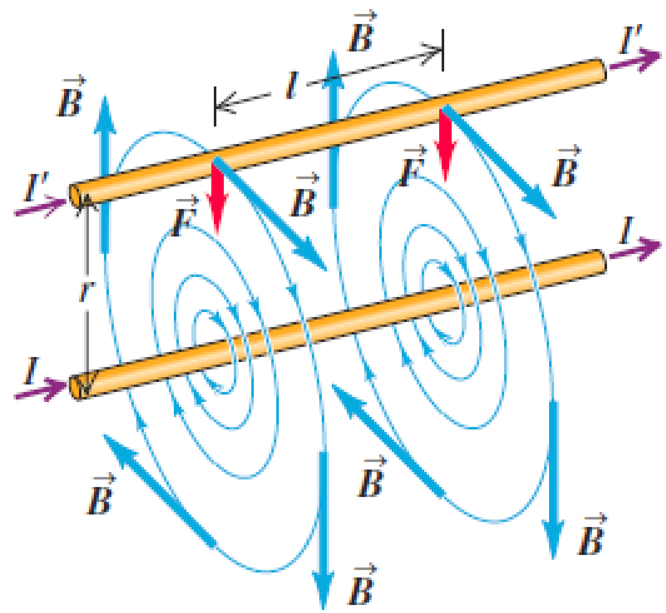
Practice Problem: At what distance from a power line carrying a current of 110 A would it create a magnetic field with the same magnitude as the earth's? *Answer:* $r = 0.44$ m.

20.8 Force between Parallel Conductors

The magnetic field of the lower wire exerts an attractive force on the upper wire. By the same token, the upper wire attracts the lower one.

If the wires had currents in *opposite* directions, they would *repel* each other.





From Equation. 20.10, the magnitude of the $\vec{B}$ vector at points on the upper conductor is

$$B = \frac{\mu_0 I}{2\pi r}.$$

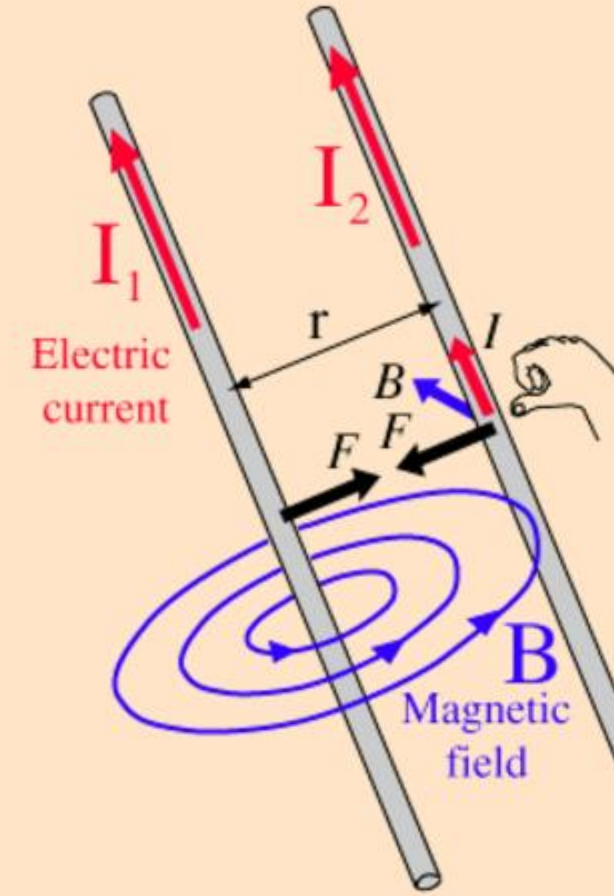
From Equation 20.6, the force on a length l of the upper conductor is

$$F = I'Bl = I'\left(\frac{\mu_0 I}{2\pi r}\right)l = \frac{\mu_0 lII'}{2\pi r},$$

and the force *per unit length, F/l,* is

$$\frac{F}{l} = \frac{\mu_0 II'}{2\pi r}. \tag{20.11}$$

Magnetic Force Between Wires



Magnetic field at wire 2 from current in wire 1:

$$B = \frac{\mu_0 I_1}{2\pi r}$$

Force on a length ΔL of wire 2:

$$F = I_2 \Delta L B$$

Force per unit length in terms of the currents:

$$\frac{F}{\Delta L} = \frac{\mu_0 I_1 I_2}{2\pi r}$$

Definition of the ampere

The forces that two straight, parallel conductors exert on one another form the basis for the official SI definition of the ampere, as follows:

One ampere is that unvarying current which, if present in each of two parallel conductors of infinite length and 1 meter apart in empty space, causes a force of exactly 2×10^{-7} newtons per meter of length on each conductor.

This definition is consistent with the definition of the constant μ_0 as *exactly* $4\pi \times 10^{-7}\ \mathrm{N/A^2}$, as we stated in Section 20.7.

20.9 Current Loops and Solenoids

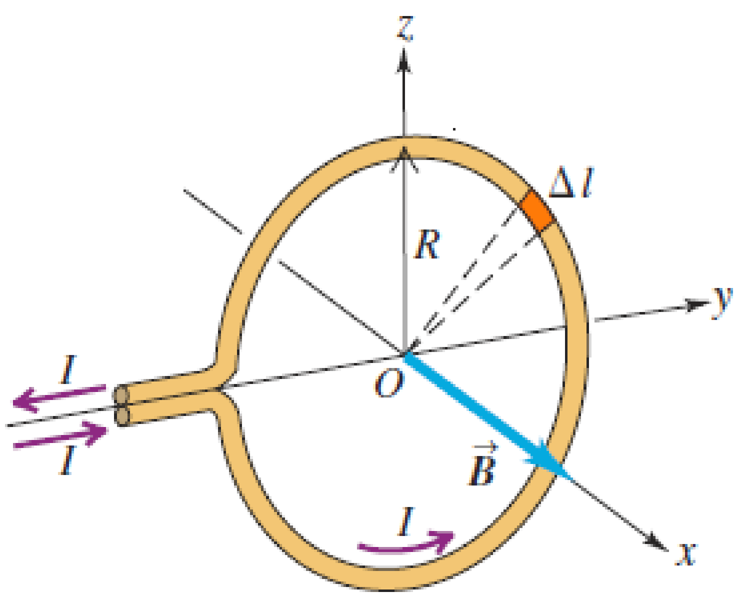


FIGURE 20.38 Magnetic field of a circular loop.

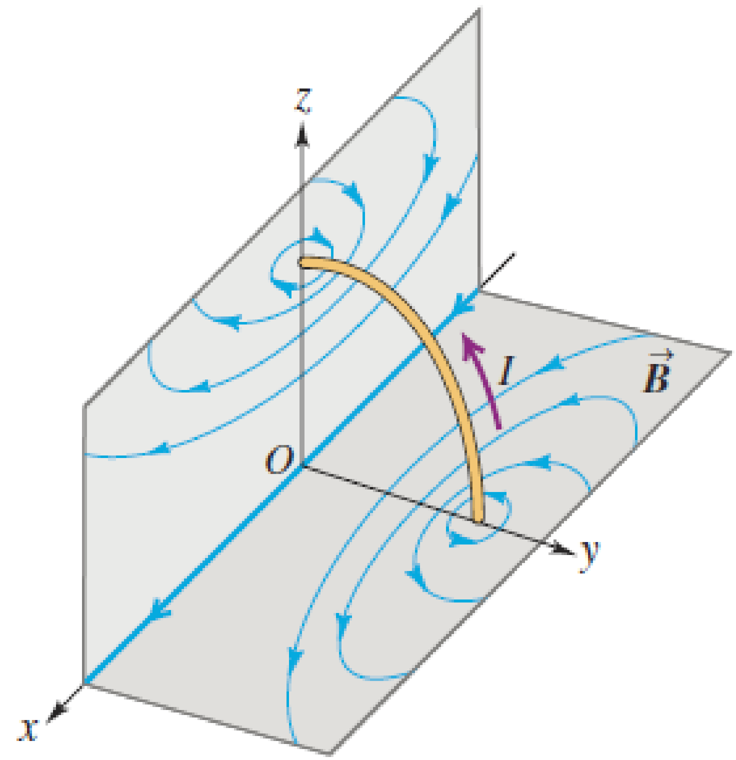


FIGURE 20.39 Magnetic field lines induced by the current in a circular loop. At points on the axis, the $\vec{B}$ field has the same direction as the magnetic moment of the loop.

Magnetic field at center of circular loop

$$B = \frac{\mu_0 I}{2R} \qquad \text{(center of circular loop).} \tag{20.12}$$

If we have a coil of N loops instead of a single loop, and if the loops are closely spaced and all have the same radius, then each loop contributes equally to the field, and the field at the center is just N times Equation 20.12:

$$B = \frac{\mu_0 N I}{2R} \qquad \text{(center of } N \text{ circular loops).} \tag{20.13}$$

EXAMPLE 20.8 A current loop for an electron beam experiment

A coil used to produce a magnetic field for an electron beam experiment has 200 turns and a radius of 12 cm. **(a)** What current is needed to produce a magnetic field with a magnitude of 5.0×10^{-3} T at the center of the coil? **(b)** Figure 20.40 shows an electron being deflected as it moves through the coil. What is the direction of current in the coil?

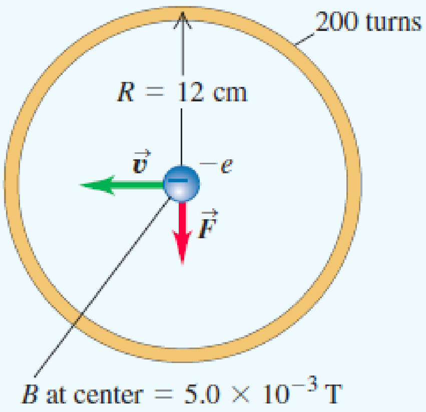


FIGURE 20.40

SOLUTION

SET UP AND SOLVE To find the needed current, solve Equation 20.13 for I:

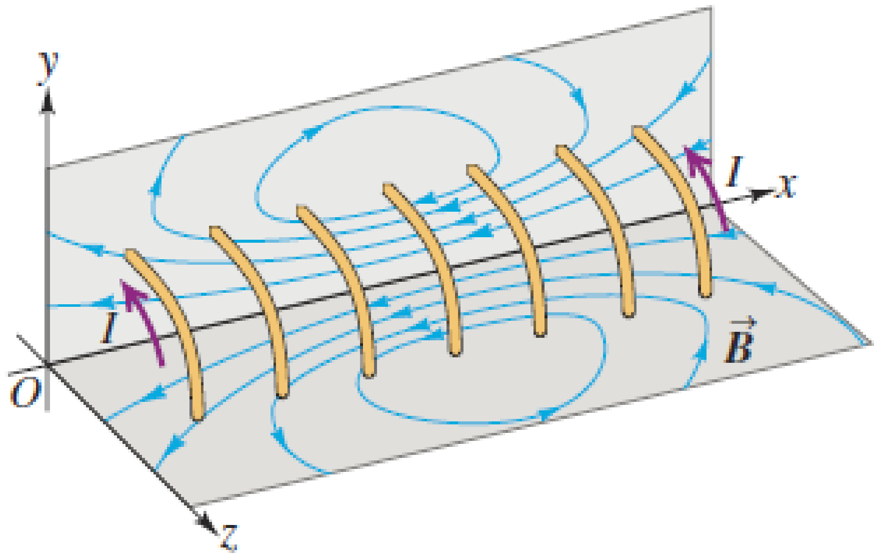
$$I = \frac{2RB}{\mu_0 N} = \frac{2(0.12\text{ m})(5.0 \times 10^{-3}\text{ T})}{(4\pi \times 10^{-7}\text{ T}\cdot\text{m/A})(200)} = 4.8\text{ A}.$$

From the right-hand rule, with the velocity vector of the electron pointing to the left and the force vector toward the bottom of the page, the direction of the magnetic field must be out of the page. (Remember that we're dealing with a *negative* charge.) The direction of the current must be counterclockwise.

REFLECT The current required is directly proportional to the radius of the coil; the greater the distance from the center to the conductor, the greater the current required. And the current required varies inversely with the number of turns; the more turns, the smaller required current.

Practice Problem: A proton moving through the coil to the right is deflected toward the bottom of the page. What is the direction of the current in the coil? *Answer:* Counterclockwise.

Magnetic Field of a Solenoid

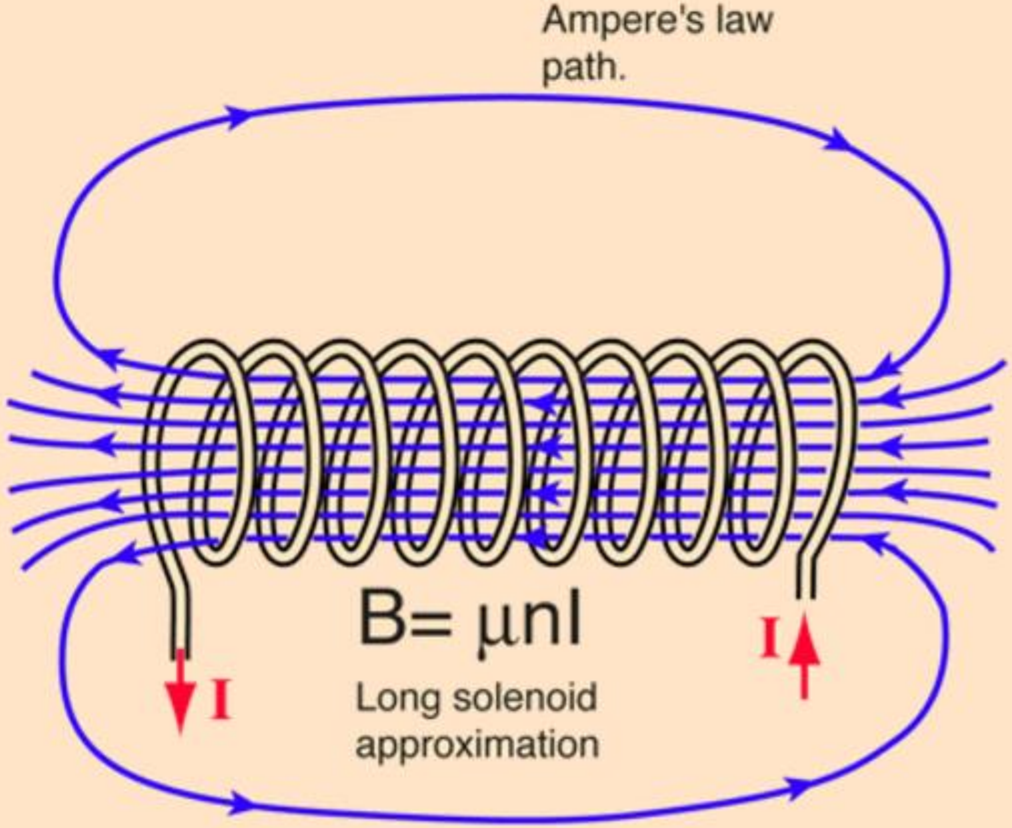


▲ **FIGURE 20.41** Magnetic field lines produced by the current in a solenoid. For clarity, only a few turns are shown.

$B = \mu_0 nl$ (center of long solenoid).

Solenoid

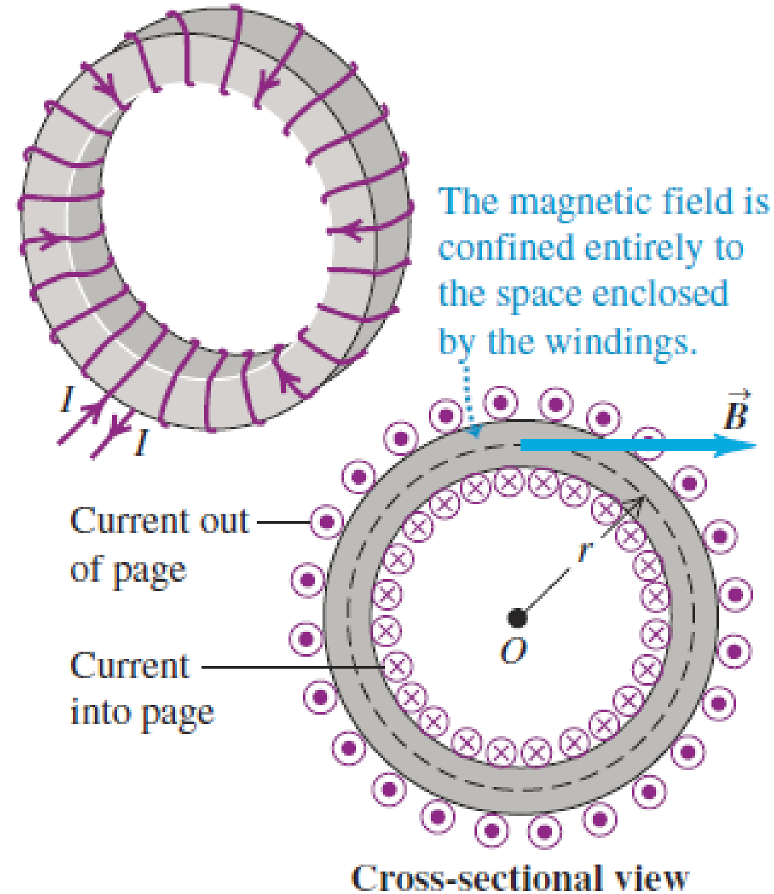
A long straight coil of wire can be used to generate a nearly uniform magnetic field similar to that of a bar magnet. Such coils, called solenoids, have an enormous number of practical applications. The field can be greatly strengthened by the addition of an iron core. Such cores are typical in electromagnets.



The magnetic field is concentrated into a nearly uniform field in the center of a long solenoid. The field outside is weak and divergent.

A variation is the toroidal (doughnut-shaped) solenoid, more commonly called a *toroid,* shown in Figure 20.42.

$$B = \frac{\mu_0 N I}{2\pi r}$$



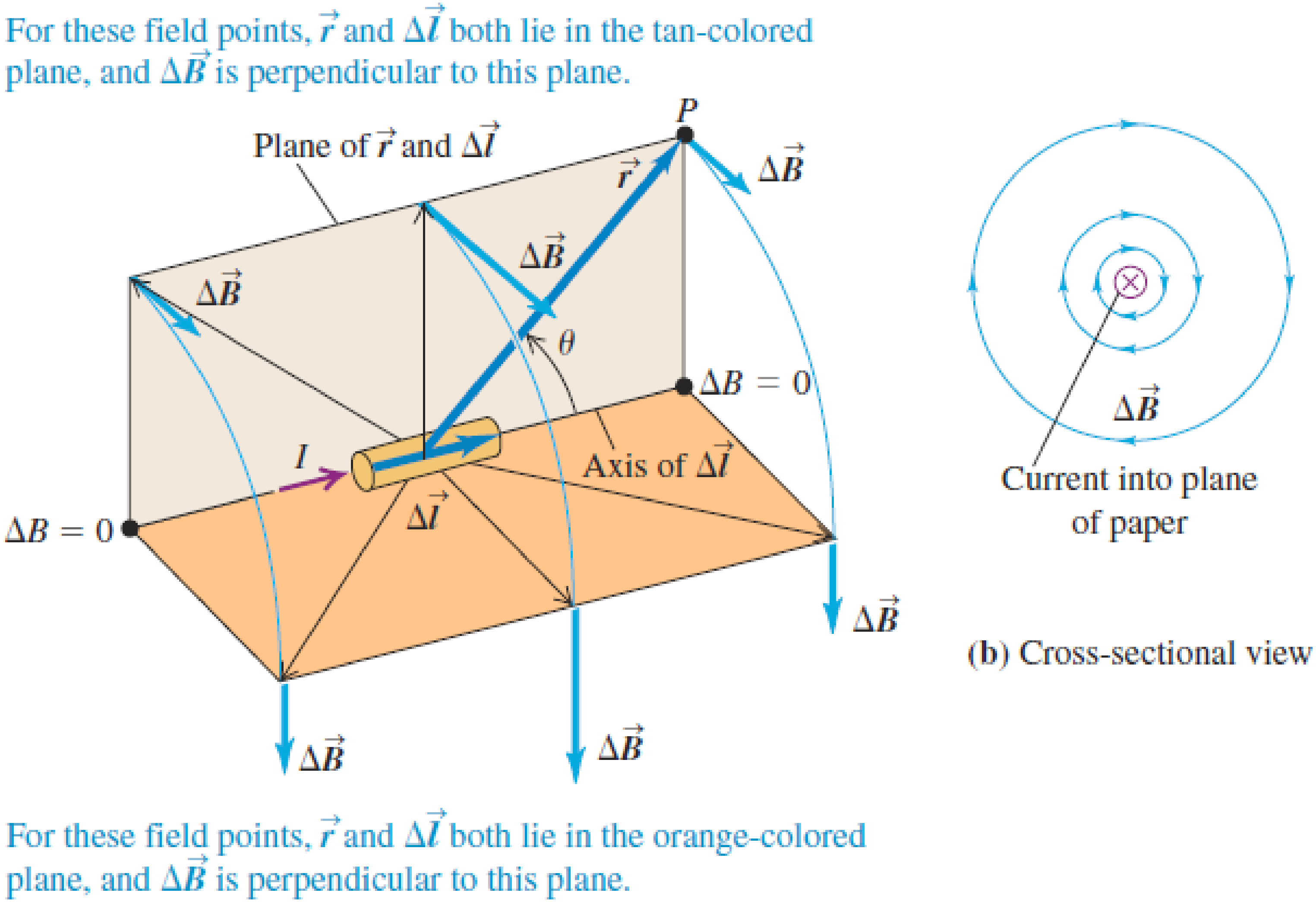
▲ FIGURE 20.42 (a) A toroidal solenoid. For clarity, only a few turns of the winding are shown. (b) Cross-sectional view. The dashed black line represents a possible distance r from the center of the toroid.

20.10 Magnetic Field Calculations

Law of Biot and Savart

The magnitude ΔB of the magnetic field $\Delta\vec{B}$ due to a segment of conductor with length Δl, carrying a current I, is given by

$$\Delta B = \frac{\mu_0}{4\pi}\frac{I\,\Delta l \sin\theta}{r^2}. \qquad (20.16)$$

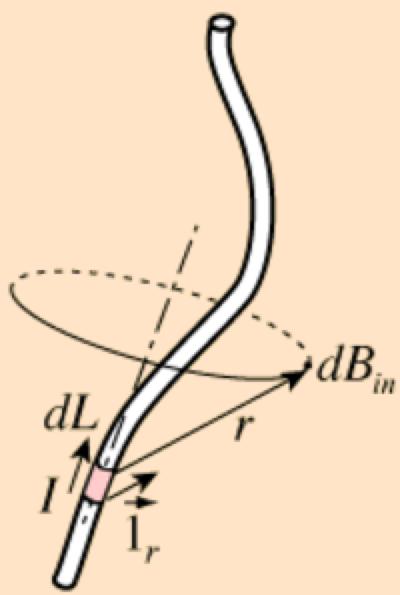


(a) Perspective view

(b) Cross-sectional view

Biot-Savart Law

The Biot-Savart Law relates magnetic fields to the currents which are their sources. In a similar manner, Coulomb's law relates electric fields to the point charges which are their sources. Finding the magnetic field resulting from a current distribution involves the vector product, and is inherently a calculus problem when the distance from the current to the field point is continuously changing.



Magnetic field of a current element

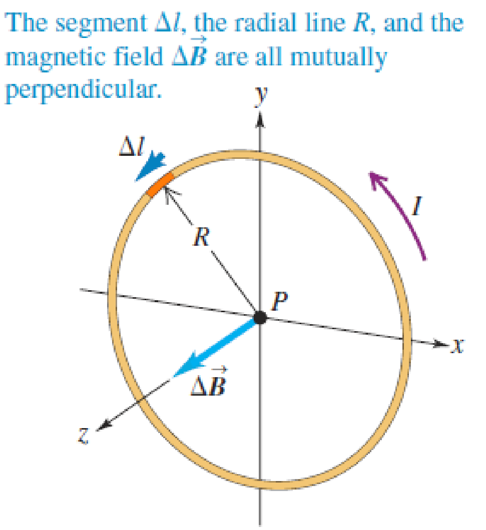
$$d\vec{B} = \frac{\mu_0 I d\vec{L} \, x \, \vec{1}_r}{4\pi r^2}$$

where

$d\vec{L}$ = infinitesmal length of conductor carrying electric current I

$\vec{1}_r$ = unit vector to specify the direction of the the vector distance r from the current to the field point.

For example, we can use the law of Biot and Savart to derive the expression for the magnetic field at the center of a circular conducting loop with radius *R* and current *I* (Equation 20.12).



▲ **FIGURE 20.45** Magnetic field $\Delta\vec{B}$ caused by a segment Δl of a circular conducting loop.

in Figure 20.45. All the segments are at the same distance R from the point P at the center, and each makes a right angle with the line joining it to P. The vectors $\Delta\vec{B}_1$, $\Delta\vec{B}_2$, and so on, due to the various segments, are all in the same direction, perpendicular to the plane of the loop, as shown. In Equation 20.16, $\theta = 90°$, $\sin\theta = 1$, and $r = R$, for every segment. The magnitude of the total $\vec{B}$ field is given by

$$B = \frac{\mu_0 I}{4\pi R^2}(\Delta l_1 + \Delta l_2 + \cdots).$$

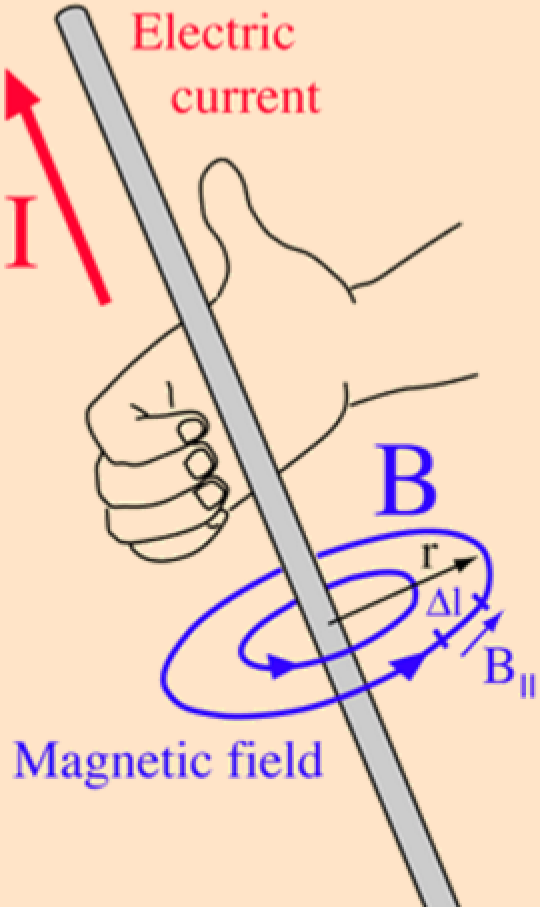
But $\Delta l_1 + \Delta l_2 + \cdots$ is the total distance around the loop—that is, the *circumference* of the loop, $2\pi R$—so

$$B = \frac{\mu_0 I}{4\pi R^2}(2\pi R) = \frac{\mu_0 I}{2R},$$

in agreement with Equation 20.12.

This formulation is strictly valid only when the conductors are surrounded with vacuum. When air or any nonmagnetic material is present, the formulation is in error by only about 0.1% or less. In Section 20.11, we'll show how to modify the formulation to take account of the material around the conductors.

Magnetic Field of Current



The magnetic field of an infinitely long straight wire can be obtained by applying Ampere's law. Ampere's law takes the form

$$\sum B_{||} \Delta l = \mu_0 I$$

and for a circular path centered on the wire, the magnetic field is everywhere parallel to the path. The summation then becomes just

$$\sum B_{||} \Delta l = B 2\pi r$$

$$B = \frac{\mu_0 I}{2\pi r}$$

The constant μ_0 is the permeability of free space.

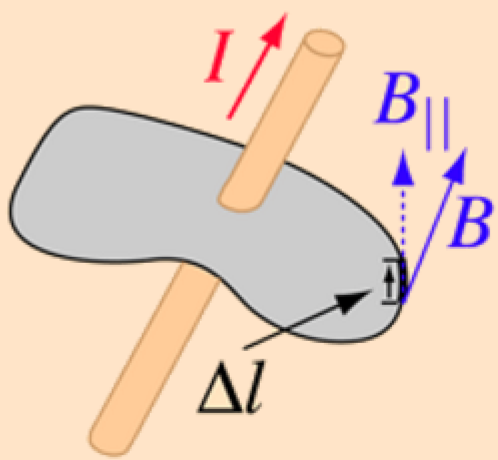
Ampère's Law

Ampère's law
When a path is made up of a series of segments Δs, and when that path links conductors carrying total current I_{encl},

$$\sum B_{\parallel}\,\Delta s = \mu_0 I_{\text{encl}}. \qquad (20.17)$$

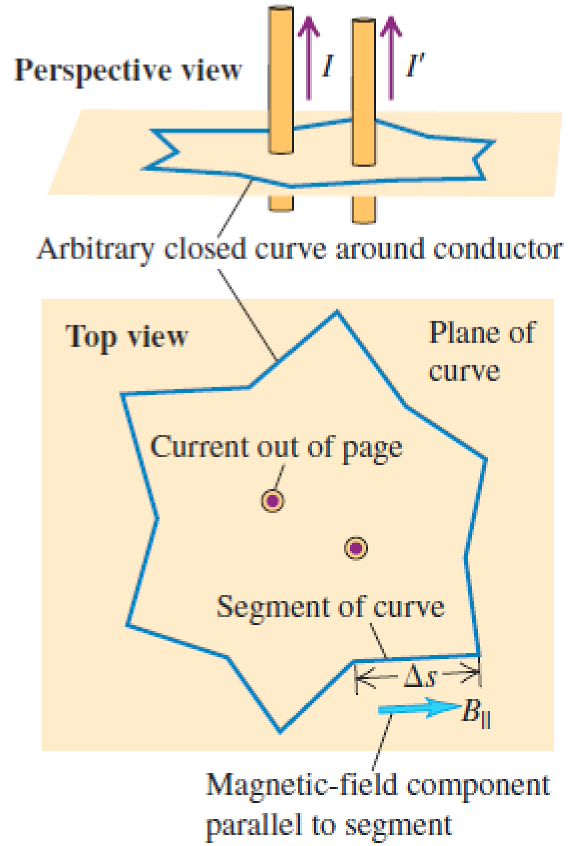
Ampere's Law

The magnetic field in space around an electric current is proportional to the electric current which serves as its source, just as the electric field in space is proportional to the charge which serves as its source. Ampere's Law states that for any closed loop path, the sum of the length elements times the magnetic field in the direction of the length element is equal to the permeability times the electric current enclosed in the loop.



$$\sum B_{||} \Delta l = \mu_0 I$$

In the electric case, the relation of field to source is quantified in Gauss's Law which is a very powerful tool for calculating electric fields.



Ampère's law: If we take the products $B_{\parallel}\Delta s$ for all segments around the curve, their sum equals μ_0 times the total enclosed current:

$$\sum B_{\parallel}\Delta s = \mu_0 I_{\text{encl}}.$$

▲ **FIGURE 20.46** Ampère's law for an arbitrary closed curve of straight segments around a pair of conductors.

If several conductors pass through the surface bounded by the path, the total magnetic field at any point on the path is the vector sum of the fields produced by the individual conductors. Then we evaluate Equation 20.17, using the *total* $\vec{B}$ field at each point and the total current I_{encl} enclosed by the path. The result equals μ_0 times the *algebraic sum* of the currents. We need a sign rule for the currents; here it is: For the surface bounded by our Ampère's-law path, take a line perpendicular to the surface and wrap the fingers of your right hand around this line so that your fingers curl around in the same direction you plan to go around the path when you evaluate the $B_{\parallel}\,\Delta s$ sum. Then your thumb indicates the positive current direction. Currents that pass through the surface in that direction are positive; those in the opposite direction are negative.

20.11 Magnetic Materials

In all of the preceding discussion of magnetic fields caused by currents, we've assumed that the space surrounding the conductors contains only vacuum. If matter is present in the surrounding space, the magnetic field is changed. The atoms that make up all matter contain electrons in motion, and these electrons form microscopic current loops that produce magnetic fields of their own. In many materials, these currents are randomly oriented, causing no net magnetic field.

But in some materials, the presence of an externally caused field can cause the loops to become oriented preferentially with the field so that their magnetic fields *add* to the external field. We then say that the material is *magnetized.*

Paramagnetism

A material showing the behavior we've just described is said to be **paramagnetic.** The magnetic field at any point in such a material is greater by a numerical factor K_m than it would be in vacuum. The value of K_m is different for different materials; it is called the **relative permeability** of a material. For a given material, K_m depends on temperature; values of K_m for common paramagnetic materials at room temperature are typically 1.000002 to 1.0004.

All the equations in this chapter that relate magnetic fields to their sources can be adapted to the situation in which the conductor is embedded in a paramagnetic material by replacing μ_0 everywhere with $K_m\mu_0$. This product is usually denoted as μ; it is called the **permeability** of the material:

$$\mu = K_m\mu_0. \tag{20.18}$$

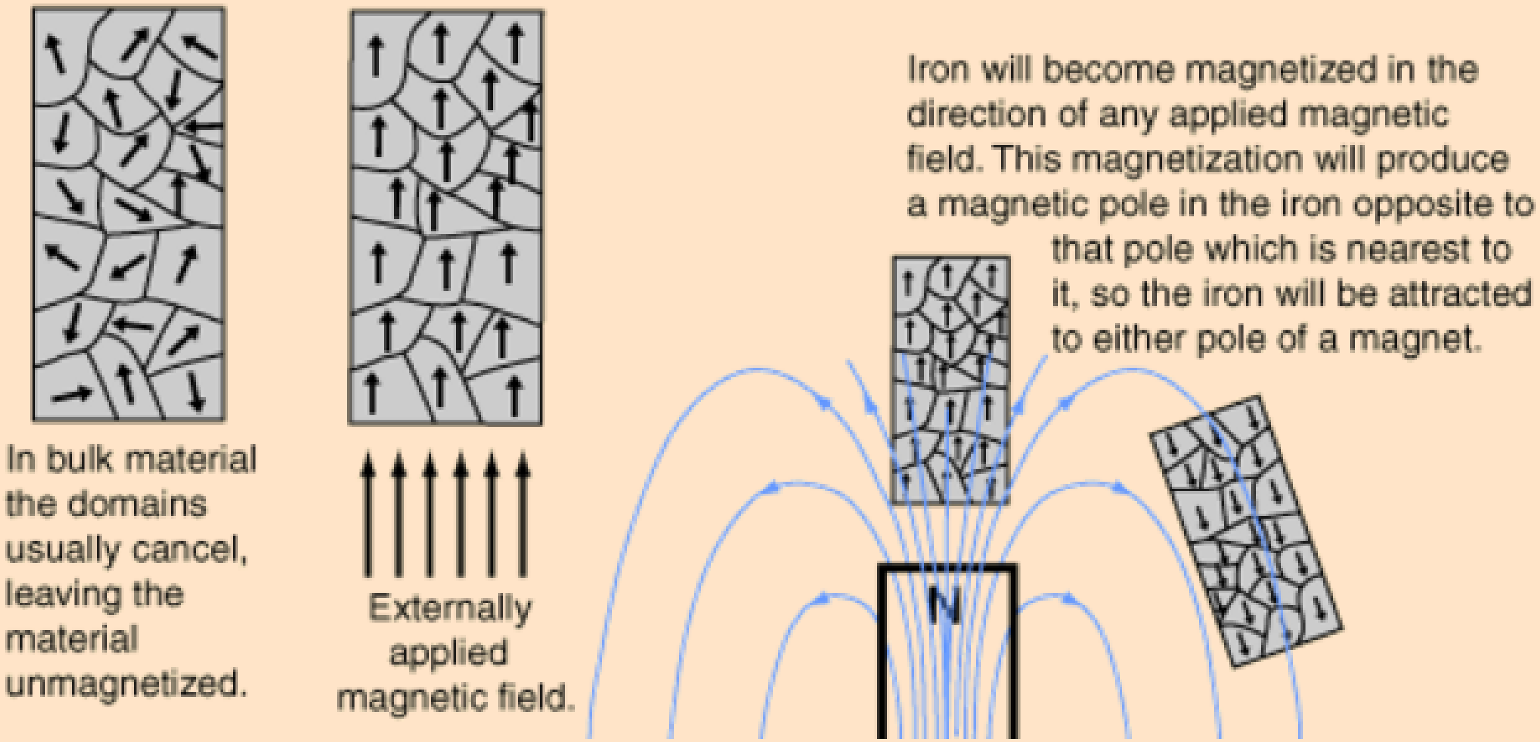
NOTE ► Remember that in this context μ is magnetic permeability, *not* the magnetic moment we defined in Section 20.6. Be careful! ◄

Diamagnetism

The orbital motion of electrons creates tiny atomic current loops, which produce magnetic fields. When an external magnetic field is applied to a material, these current loops will tend to align in such a way as to oppose the applied field. This may be viewed as an atomic version of Lenz's law: induced magnetic fields tend to oppose the change which created them. Materials in which this effect is the only magnetic response are called diamagnetic.

Magnetic Domains

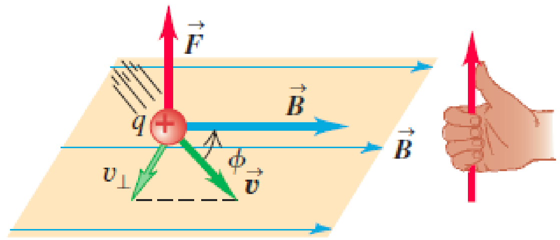
The microscopic ordering of electron spins characteristic of ferromagnetic materials leads to the formation of regions of magnetic alignment called domains.



Summary

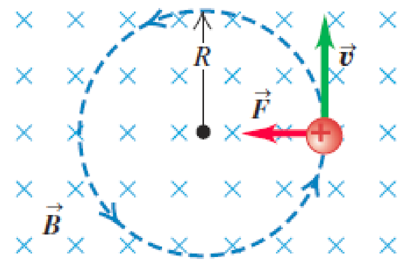
Magnetism; Fields and Forces

(Sections 20.1 and 20.2) A bar magnet has a **north (N) pole** and a **south (S) pole.** Two opposite poles attract each other, and two like poles repel each other. A moving charge creates a **magnetic field** in the surrounding space. A moving charge, or current, experiences a force in the presence of a magnetic field. The direction of the force is given by the right-hand rule for magnetic forces, and the magnitude is given by Equation 20.1.



Motion of Charged Particles in Magnetic Fields

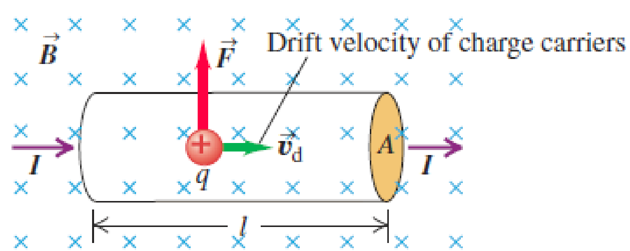
(Sections 20.3 and 20.4) The magnetic force is always perpendicular to $\vec{v}$; a particle moving under the action of a magnetic field alone moves with constant speed. In a uniform field, a particle with initial velocity perpendicular to the field moves in a circle with radius R given by $R = mv/|q|B$ (Equation 20.4). Mass spectrometers use this relationship to determine atomic masses. When a positive ion of known speed undergoes circular motion in a magnetic field of known strength, the mass can be determined by measuring the radius.



The orbit of a positive charge in a uniform magnetic field.

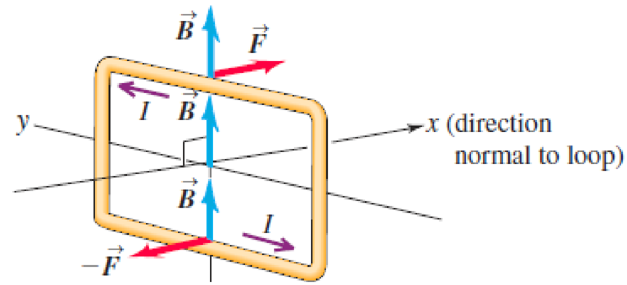
Magnetic Force on a Current-Carrying Conductor

(Section 20.5) When a current-carrying conductor is in the presence of a magnetic field, the field exerts a force on the conductor because each individual charge in the current is acted upon by a force given by $F = |q|vB\sin\phi$ (Equation 20.1). The direction of the force is determined by using the same right-hand rule that we used for a moving positive charge. The magnitude of the force is given by $F = IlB_\perp = IlB\sin\phi$ (Equation 20.7).



Force and Torque on a Current Loop; Direct-Current Motors

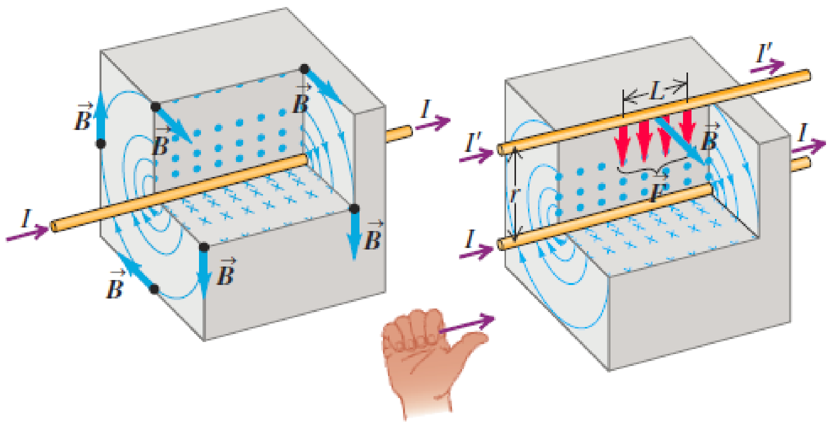
(Section 20.6) A current loop can be represented as connected segments of current-carrying conductors. In the presence of a magnetic field, each segment is acted upon by a force due to the field. In a uniform magnetic field, the total force on a current loop is zero, regardless of its shape, but the magnetic forces create a torque τ given by $\tau = IAB \sin\phi$ (Equation 20.8). A direct-current motor is driven by torques on current-carrying conductors. The key component is the **commutator,** which is used to reverse the direction of the current in order to maintain the torque.



Magnetic Field of a Long, Straight Conductor

(Sections 20.7 and 20.8) Moving charges, and therefore currents, produce magnetic fields. When a current passes through a long, straight wire, the magnetic field lines are circles centered on the wire. Due to this symmetry of the field pattern, the magnitude of the magnetic field is the same at all points on a field line at radial distance r:

$$B = \frac{\mu_0 I}{2\pi r}. \tag{20.10}$$



Current-carrying conductors can exert magnetic forces on each other. Thus, two parallel wires can attract or repel each other, depending on the direction of the currents they carry. For two long, straight parallel wires, the force per unit length is

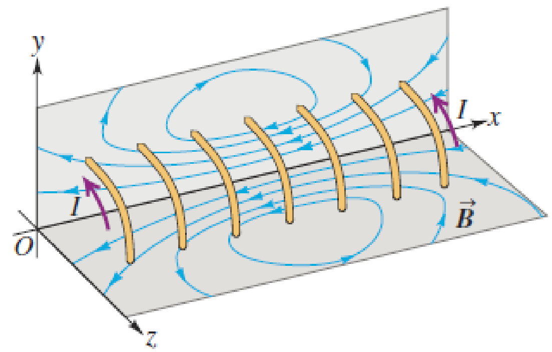
$$\frac{F}{l} = \frac{\mu_0 I I'}{2\pi r}. \tag{20.11}$$

Current Loops and Solenoids

(Section 20.9) Many practical devices depend on the magnetic field produced at the center of a circular coil of wire. If a coil of radius R consists of N loops, the magnetic field at the center is

$$B = \frac{\mu_0 N I}{2R}. \qquad (20.13)$$

A long **solenoid** of many closely spaced windings produces a nearly uniform field in its interior, midway between its ends, having magnitude $B = \mu_0 n I$ (Equation 20.14).



Magnetic Field Calculations

(Section 20.10) The magnetic field magnitude ΔB created by a short current-carrying segment Δl is given by the law of Biot and Savart:

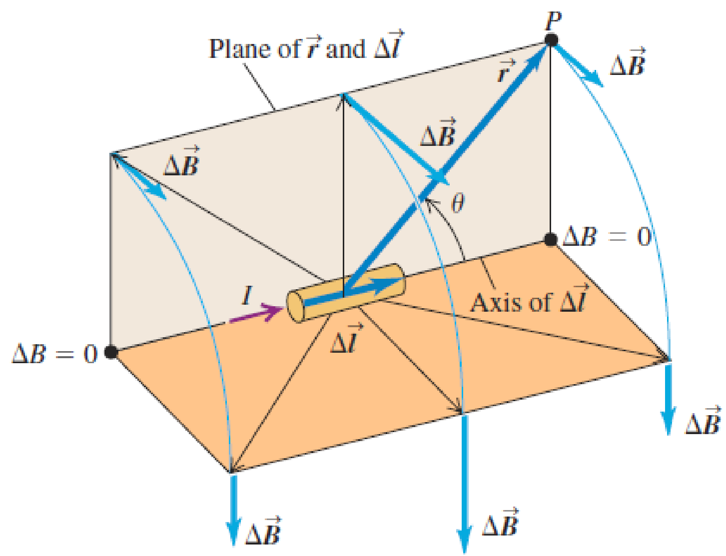
$$\Delta B = \frac{\mu_0}{4\pi}\frac{I\,\Delta l \sin\theta}{r^2}. \qquad (20.16)$$

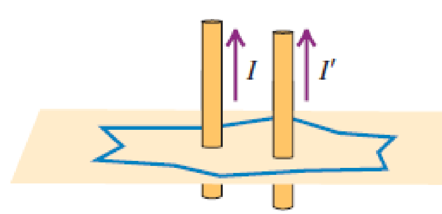
To determine the magnetic field from an extended current-carrying wire, first consider the extended wire as being made of many smaller segments. Then, using the principle of superposition at a particular point in space, compute the vector sum of the magnetic field due to each small segment of current.

When a current I is enclosed by a path made of many small segments Δs, and when at each point the magnetic field has a component $B_\parallel$ parallel to the segments, **Ampère's law** states that

$$\sum B_\parallel \,\Delta s = \mu_0 I_{\text{encl}}. \qquad (20.17)$$

If the products of the magnetic field component $B_\parallel$ and each segment Δs (that is, the components $B_\parallel \,\Delta s$) are summed around *any* path enclosing a total current I_{encl}, the result is $\mu_0 I_{\text{encl}}$.





Magnetic Materials

(Section 20.11) For magnetic materials, the magnetization of the material causes an additional contribution to $\vec{B}$. For paramagnetic and diamagnetic materials, μ_0 is replaced in magnetic-field expressions by $\mu = K_m\mu_0$, where μ is the permeability of the material and K_m is its relative permeability. For **ferromagnetic** materials, K_m is much larger than unity. Some ferromagnetic materials are permanent magnets, retaining their magnetization even after the external magnetic field is removed.