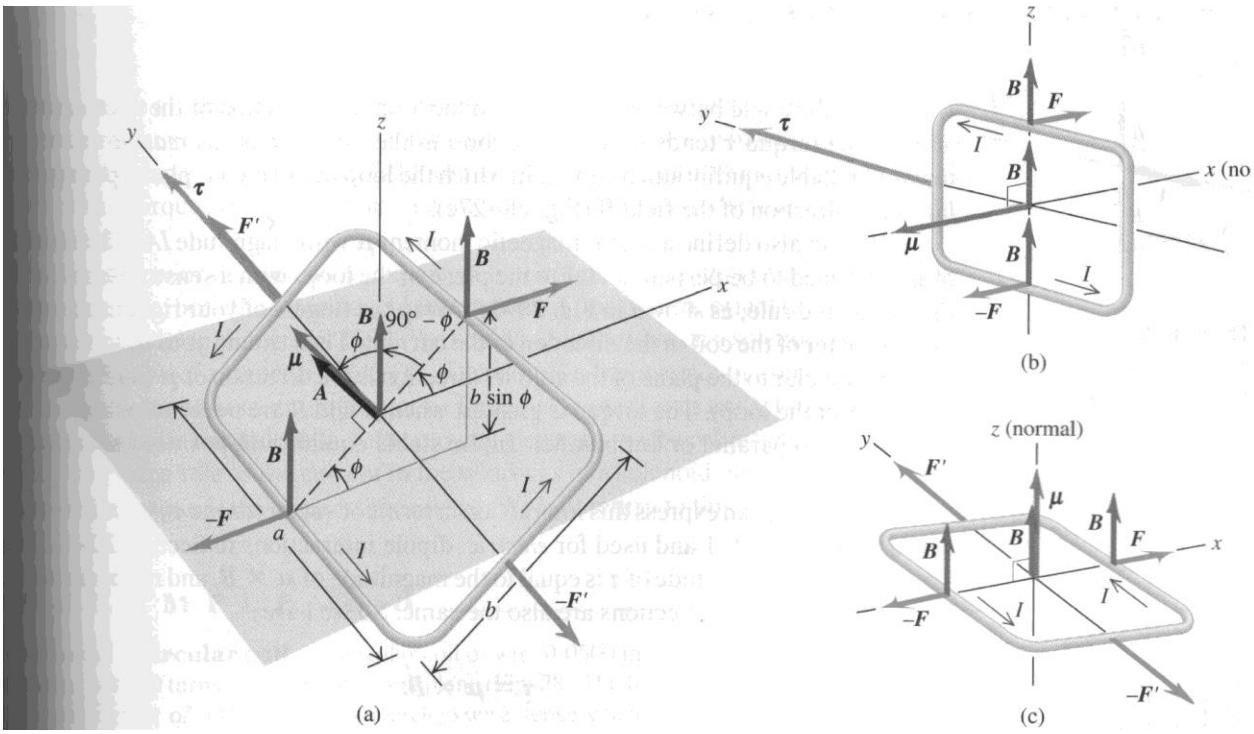


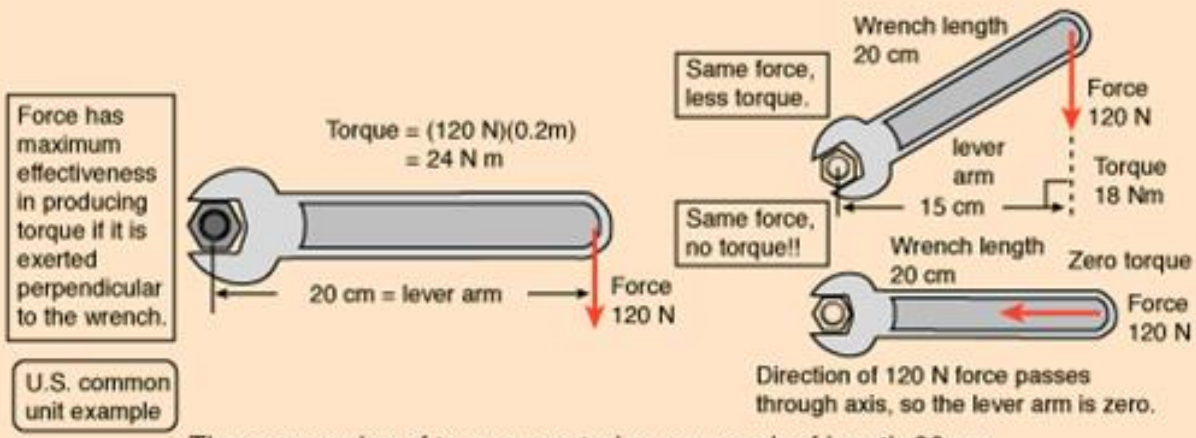
y
τ
F′
z
I
B
x
B
90° − ϕ
F
I
μ
ϕ
ϕ
A
b sin ϕ
B
ϕ
I
−F
a
I
b
−F′
(a)
z
B
F
y
τ
I
B
x (no
μ
B
−F
I
(b)
z (normal)
y
F′
μ
B
B
B
F
x
−F
I
I
−F′
(c)

Torque

A torque is an influence which tends to change the rotational motion of an object. One way to quantify a torque is

Torque = Force applied x lever arm

The lever arm is defined as the perpendicular distance from the axis of rotation to the line of action of the force.



Three examples of torque exerted on a wrench of length 20 cm.

Vectors and Vector Components

$A_x = A \cos \theta$

$A_y = A \sin \theta$

$$A = \sqrt{(A_x{}^2 + A_y{}^2)}$$

$$\theta = tan^{-1}\left(\frac{A_y}{A_x}\right)$$

Vertical and horizontal components

- Hang the spring balance as shown in Figure 19.3. Measure the vertical force needed to support mass m_1 and check that it agrees with the weight of the mass.
- Pull the mass m_1 to the right with a string over the pulley attached to mass m_2 (Figure 19.4) and calculate the horizontal force that is pulling mass m_1 aside. Measure the new force on the spring balance and the angle (θ_1) between this balance and the vertical.
- Repeat for a range of masses m_2 and show your results in a table.
- Try to find a mathematical connection between the forces you measure and the angle.

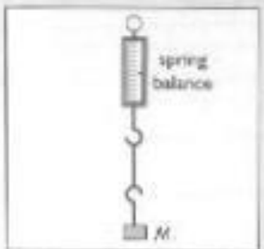
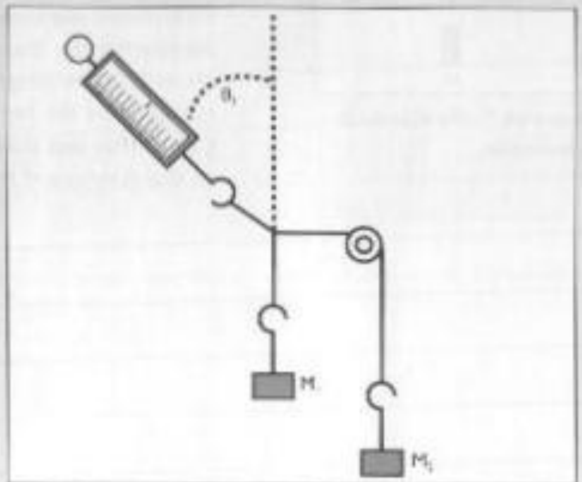
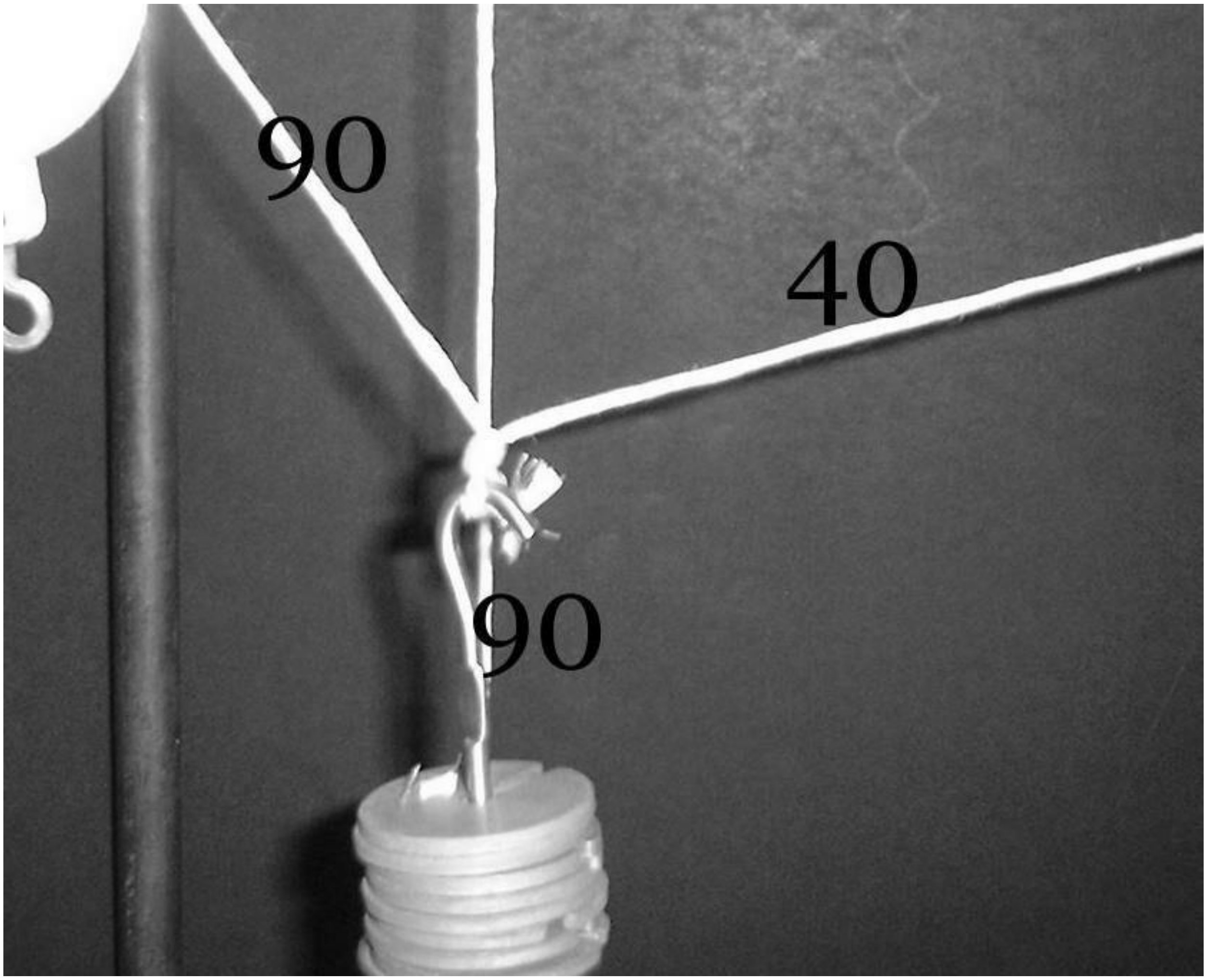
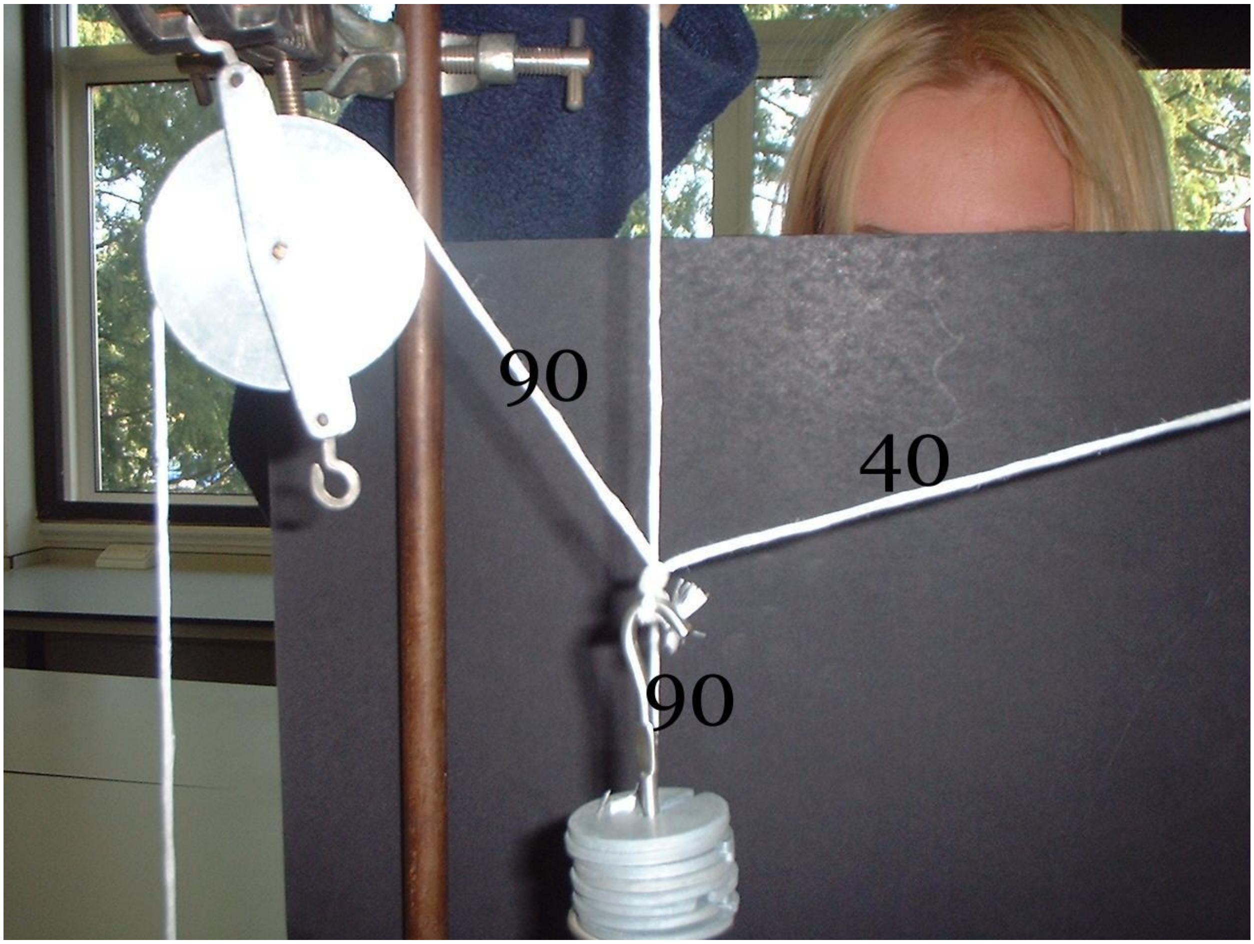


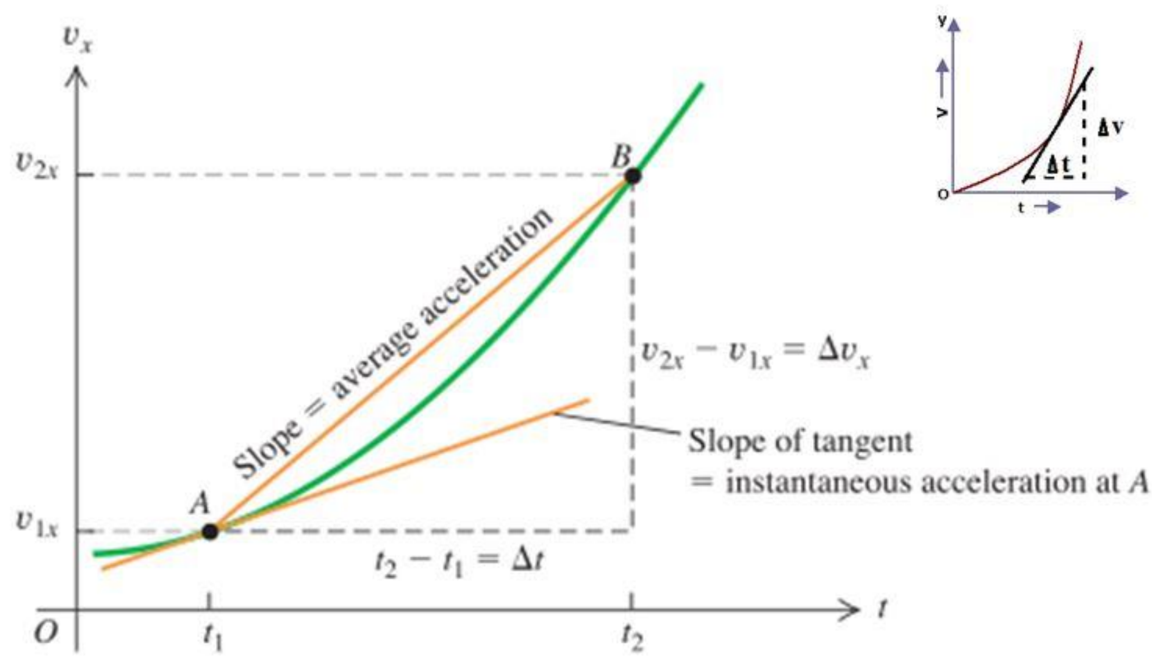
Figure 19.3 What vertical force is needed to support the mass?

Figure 19.4 What is the force on the spring balance now? What is the angle?

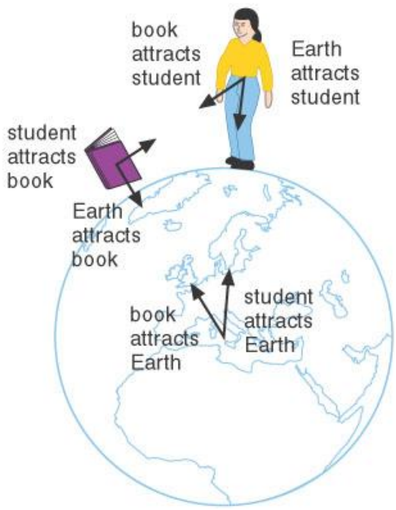


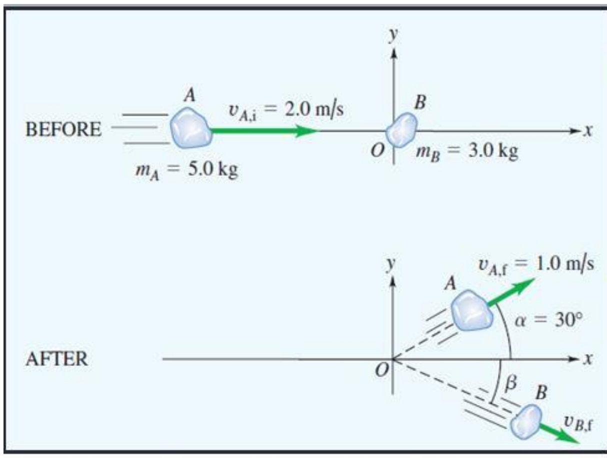

90
40
90


90
40
90


v_x
v_{2x}
v_{1x}
B
A
Slope = average acceleration
$v_{2x} - v_{1x} = \Delta v_x$
Slope of tangent
= instantaneous acceleration at A
$t_2 - t_1 = \Delta t$
O
t_1
t_2
t
y
v
Δv
Δt
o
t

WORK DONE
=
ENERGY TRANSFERRED





X - Component

SOLVE We start by writing expressions for the total x component of momentum before and after the collision. For the x components, we have

$$m_A(v_{A,i,x}) + m_B(v_{B,i,x}) = (5.0\ \text{kg})(2.0\ \text{m/s}) + (3.0\ \text{kg})(0), \quad \text{(before)}$$

$$m_A(v_{A,f,x}) + m_B(v_{B,f,x}) = (5.0\ \text{kg})(1.0\ \text{m/s})(\cos 30°) + (3.0\ \text{kg})(v_{B,f,x}). \quad \text{(after)}$$

Equating these two expressions and solving for $v_{B,f,x}$, we find that

$$v_{B,f,x} = 1.89\ \text{m/s}. \quad (x \text{ component of final velocity of } B)$$

Y - Component

Conservation of the y component of total momentum gives

$$m_A(v_{A,i,y}) + m_B(v_{B,i,y}) = (5.0\ \text{kg})(0) + (3.0\ \text{kg})(0), \quad \text{(before)}$$

$$m_A(v_{A,f,y}) + m_B(v_{B,f,y}) = (5.0\ \text{kg})(1.0\ \text{m/s})(\sin 30°) + (3.0\ \text{kg})(v_{B,f,y}). \quad \text{(after)}$$

Equating these two expressions and solving for $v_{B,f,y}$, we obtain

$$v_{B,f,y} = -0.83\ \text{m/s}. \quad (y \text{ component of final velocity of } B)$$

Equate Both Directions for Magnitude

We now have the x and y components of the final velocity $\vec{v}_{B,f}$ of chunk B. The magnitude of $\vec{v}_{B,f}$ is

$$|\vec{v}_{B,f}| = \sqrt{(1.89\ \text{m/s})^2 + (-0.83\ \text{m/s})^2} = 2.1\ \text{m/s}, \quad \text{(final } speed \text{ of } B)$$

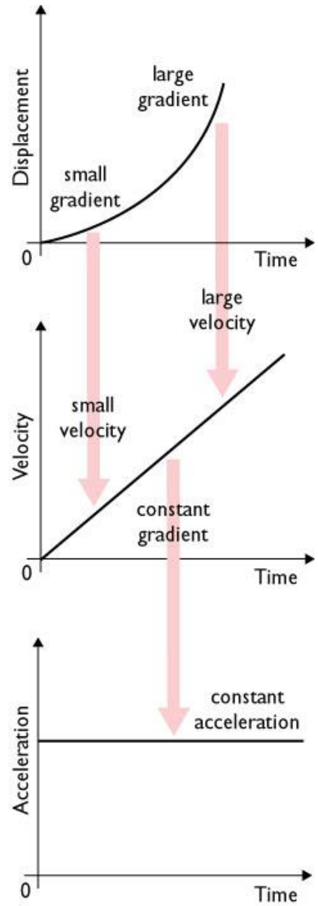
And the Angle......

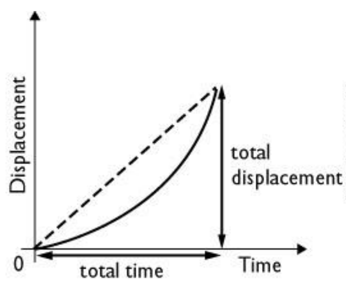
and the angle β of its direction from the positive x axis is

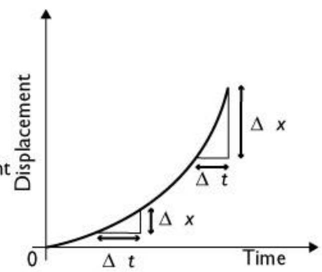
$$\beta = \tan^{-1}\frac{-0.83\ \text{m/s}}{1.89\ \text{m/s}} = -24°.$$

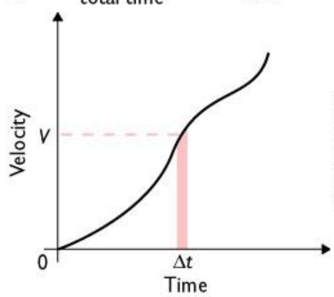
Now Try This One......

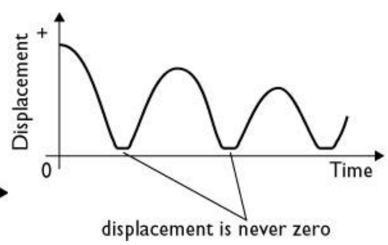
Practice Problem: If chunk B has an initial velocity of magnitude 2.0 m/s in the $+y$ direction instead of being initially at rest, find its final velocity (magnitude and direction). *Answer:* 2.2 m/s, 32°.

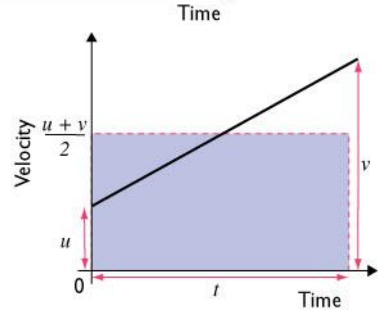

Displacement
large gradient
small gradient
0
Time
large velocity
small velocity
Velocity
constant gradient
0
Time
Acceleration
constant acceleration
0
Time

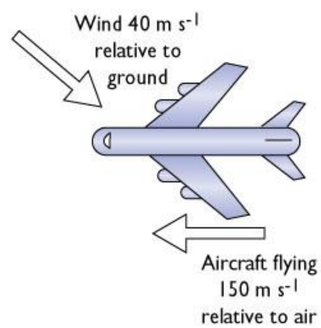

Displacement
total displacement
0
total time
Time

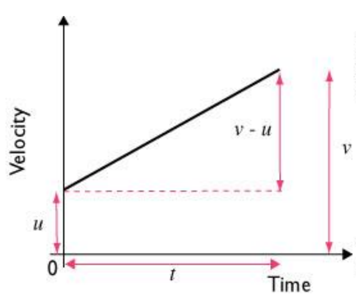

Displacement
Δ x
Δ t
Δ x
Δ t
0
Time

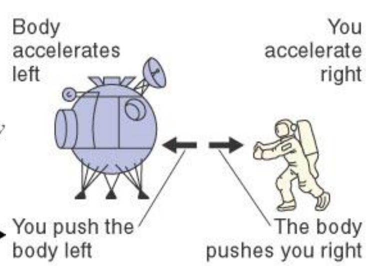

Velocity
V
0
Δt
Time

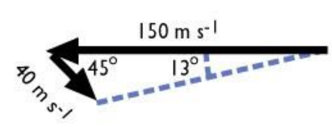

Displacement
+
0
Time
displacement is never zero

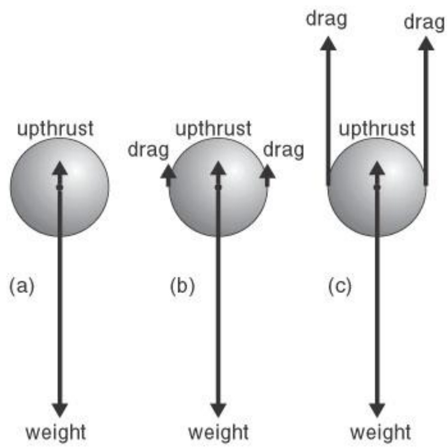

Velocity
u + v
2
u
v
0
t
Time

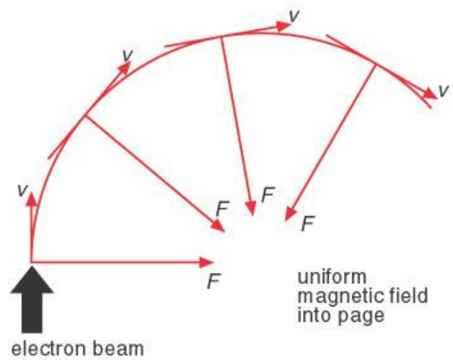

Wind 40 m s-1
relative to
ground
Aircraft flying
150 m s-1
relative to air

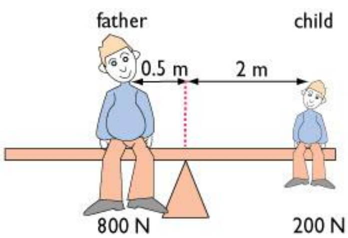

Velocity
v - u
v
u
0
t
Time


Body accelerates left
You accelerate right
You push the body left
The body pushes you right


150 m s-1
45°
13°
40 m s-1







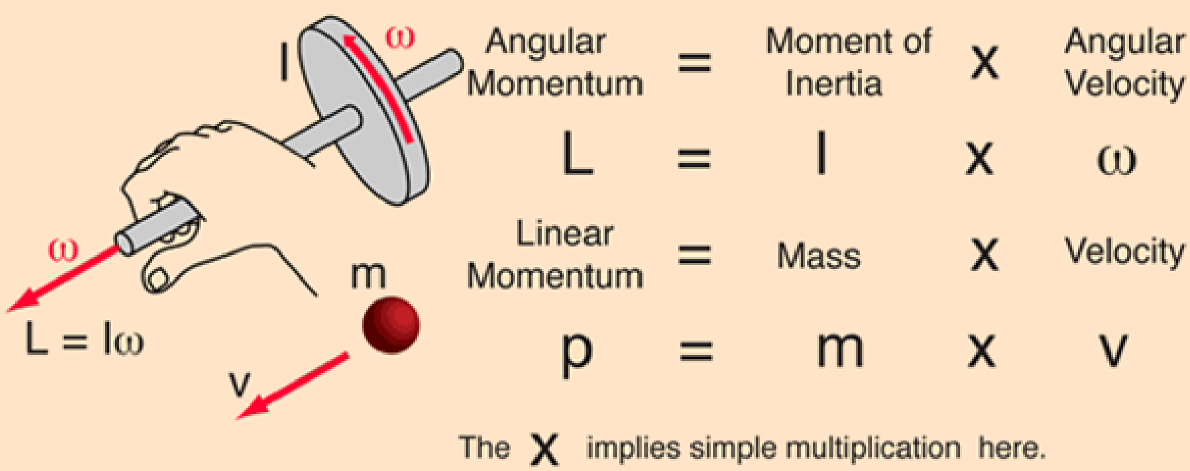
anti-clockwise moment of father = clockwise moment of child

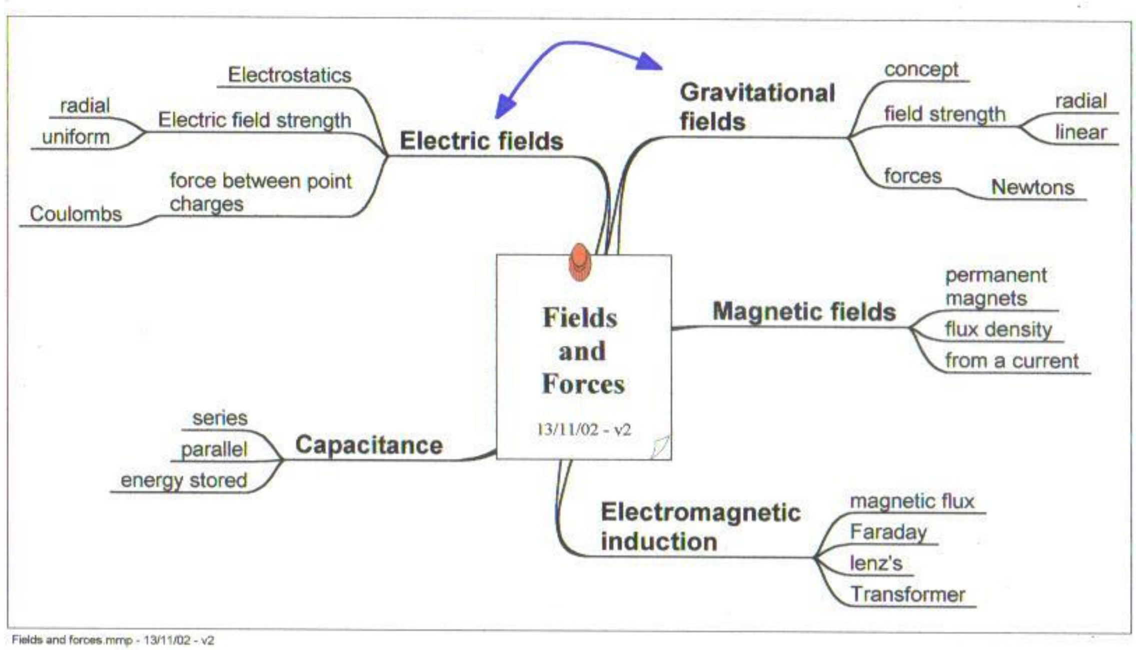
800 N x 0.5 m = 200 N x 2 m

400 N m = 400 N m

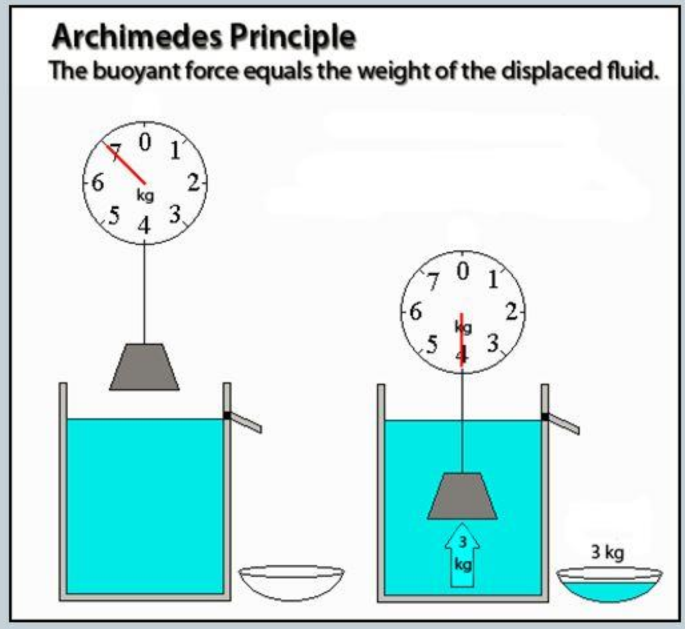
Angular and Linear Momentum

Angular momentum and linear momentum are examples of the parallels between linear and rotational motion. They have the same form and are subject to the fundamental constraints of conservation laws, the conservation of momentum and the conservation of angular momentum .





- When an object is completely or partially immersed in a fluid, the fluid exerts an upward force on the object equal to the weight of the fluid that is displaced by the object.



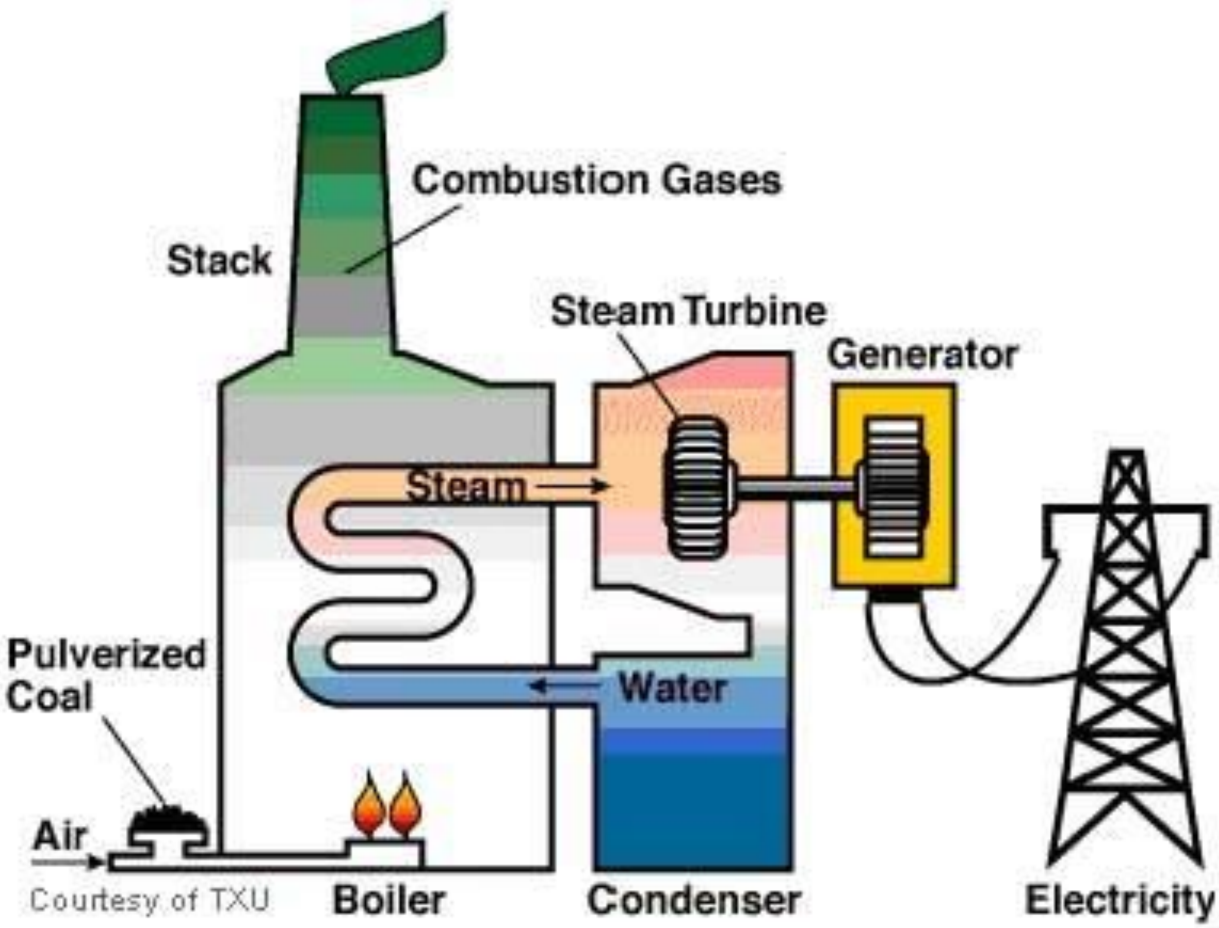


TABLE 13.1 Densities of Some Common Substances

Material	Density (kg/m^3)*	Material	Density (kg/m^3)*
Gases		Concrete	2×10^3
Air (1 atm, 20° C)	1.20	Aluminum	2.7×10^3
Liquids		Iron, steel	7.8×10^3
Benzene	0.90×10^3	Brass	8.6×10^3
Ethanol	0.81×10^3	Copper	8.9×10^3
Water	1.00×10^3	Silver	10.5×10^3
Seawater	1.03×10^3	Lead	11.3×10^3
Blood	1.06×10^3	Gold	19.3×10^3
		Platinum	21.4×10^3
Solids		Mercury	13.6×10^3
Glycerin	1.26×10^3	White-dwarf star	10^{10}
Ice	0.92×10^3	Neutron star	10^{18}

Density, Pressure and Fluids

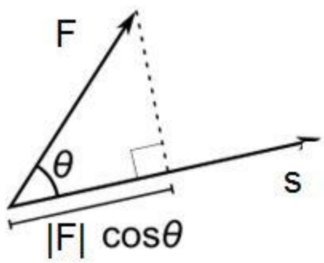
$$\rho = \frac{m}{V}$$

$$P = \frac{F_{\perp}}{A}$$

$$P_{abs} = P_{atm} + P_{gauge}$$

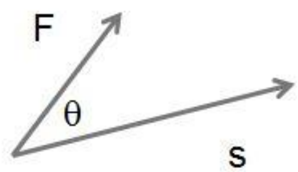
$$P_{gauge} = \rho g h$$

WORK = FORCE COMPONENT IN THE DIRECTION OF MOTION TIMES THE DISTANCE



θ Is the angle between the two vectors

F has a component |F| cos θ in the direction of s
So W = |F| cos θ |s|

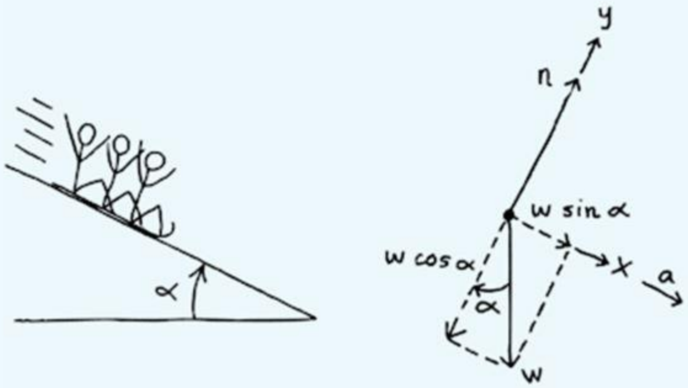


Mathematically this is a dot product.
Such that

$W = \mathbf{F}.\mathbf{s} = |F|\,|s| \cos\theta$

Two ways of looking at the same thing
(note the product of the two vectors is a scalar)

2 $\Sigma\vec{F} = m\vec{a}$



(a) The situation **(b)** Free-body diagram for toboggan

▲ **FIGURE 5.8** Our diagrams for this problem.

SOLVE There is only one x component of force, so

$$\sum F_x = w \sin\alpha.$$

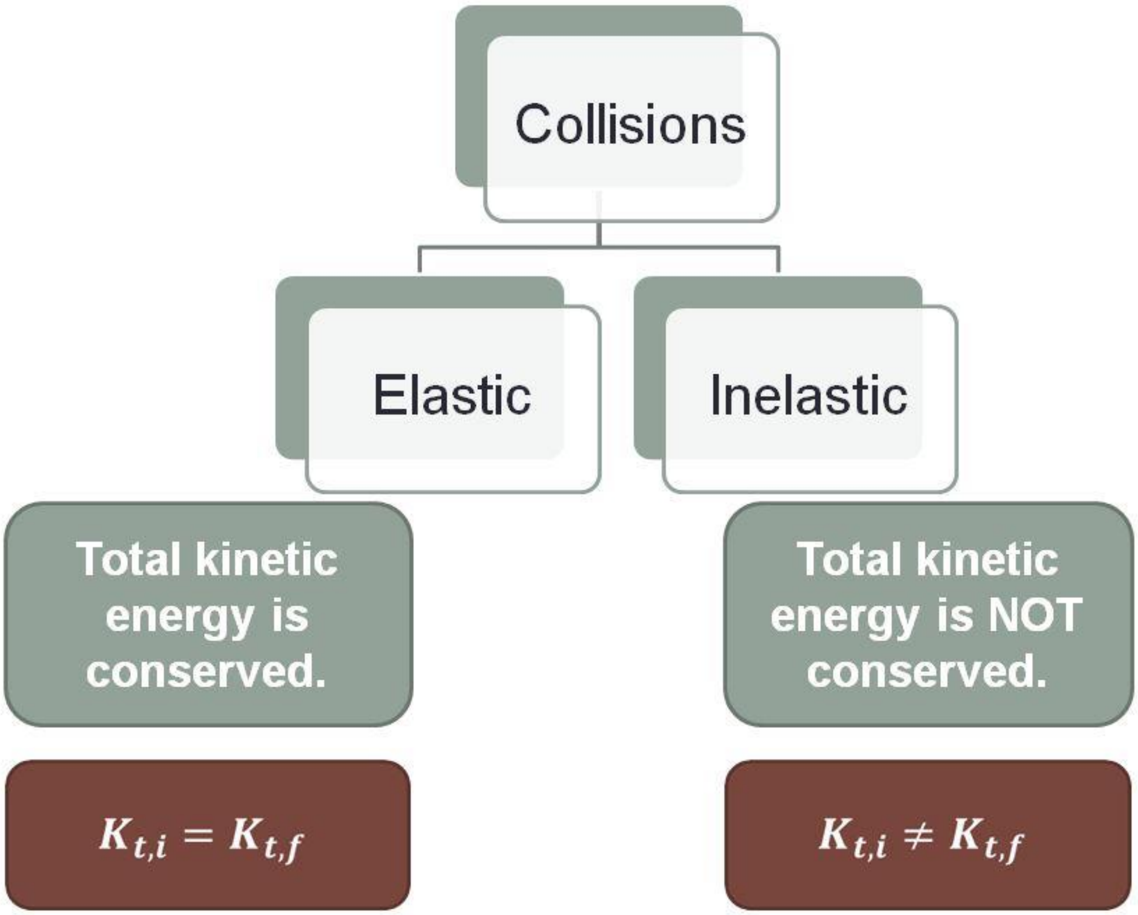
From $\sum F_x = ma_x$, we have

$$w \sin\alpha = ma_x,$$

and since $w = mg$, the acceleration is

$$a_x = g \sin\alpha.$$

The y-component equation gives $\sum F_y = n + (-mg\cos\alpha)$. We know that the y component of acceleration is zero, because there is no motion in the y direction. So $\sum F_y = 0$ and $n = mg\cos\alpha$.8

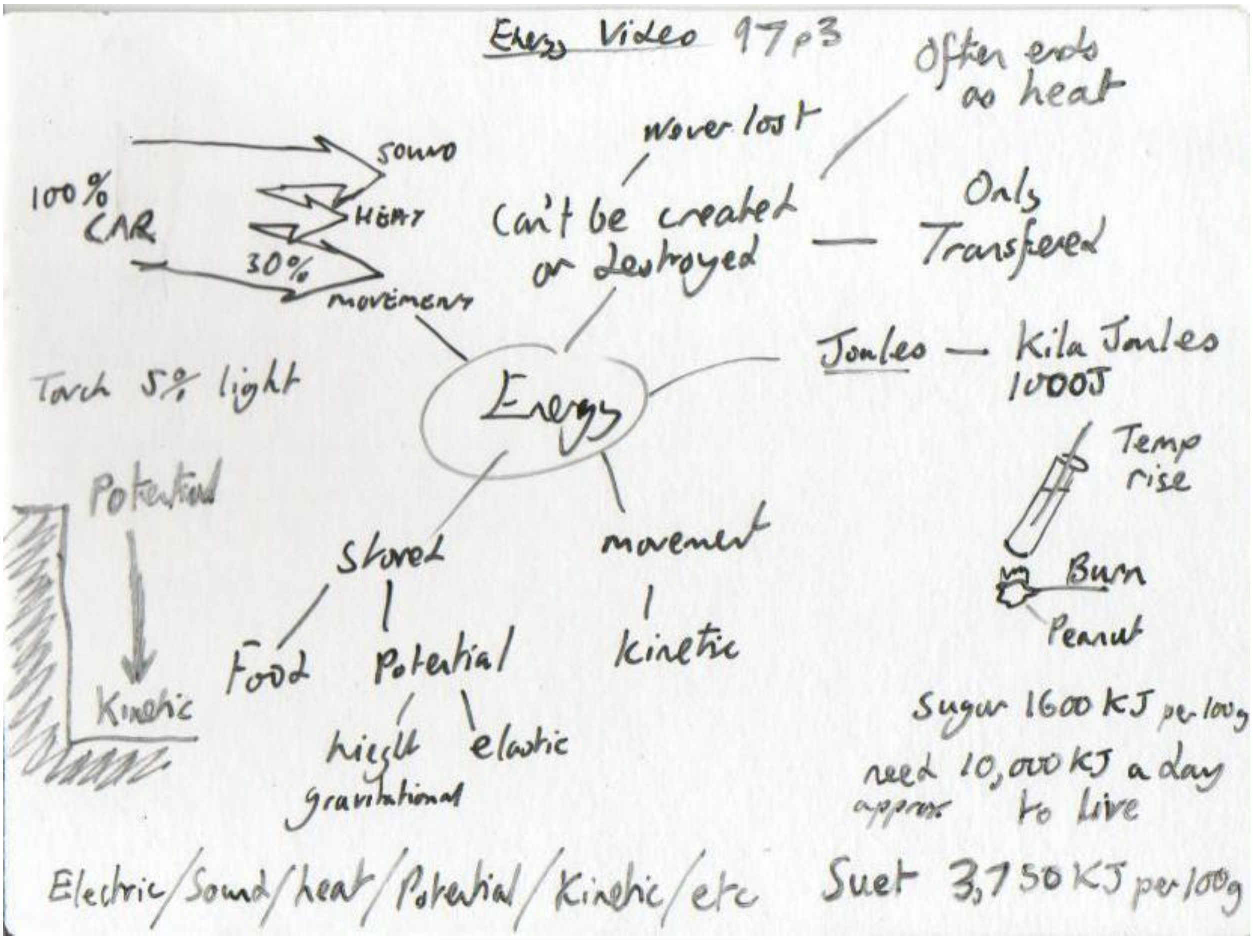


Energy and Power

$$K.E._{LIN} = \frac{1}{2}mv^2 \qquad U_g = mgh$$

$$F = ma = kx \qquad U_e = \frac{1}{2}kx^2$$

$$W = F\,s = F\,\Delta x \qquad P = \frac{W}{t} = Fv$$



Assuming no air resistance

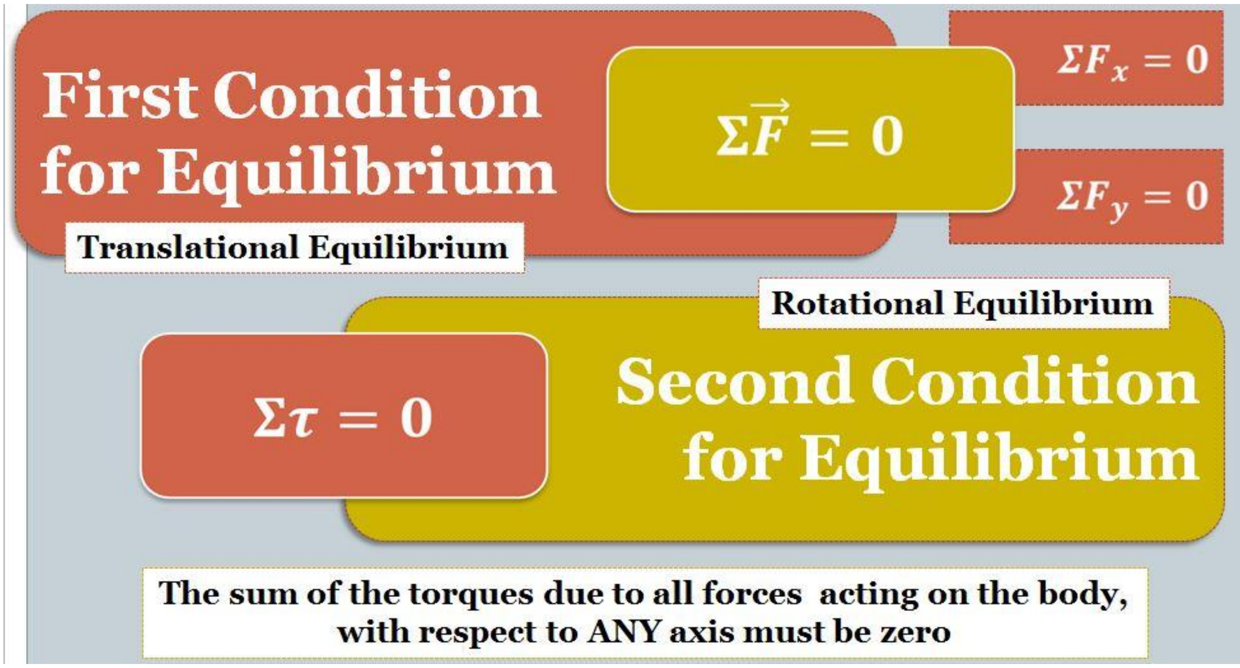
Horizontally: **constant velocity**

$$v_x = v_{0x} = \frac{\Delta x}{t}$$

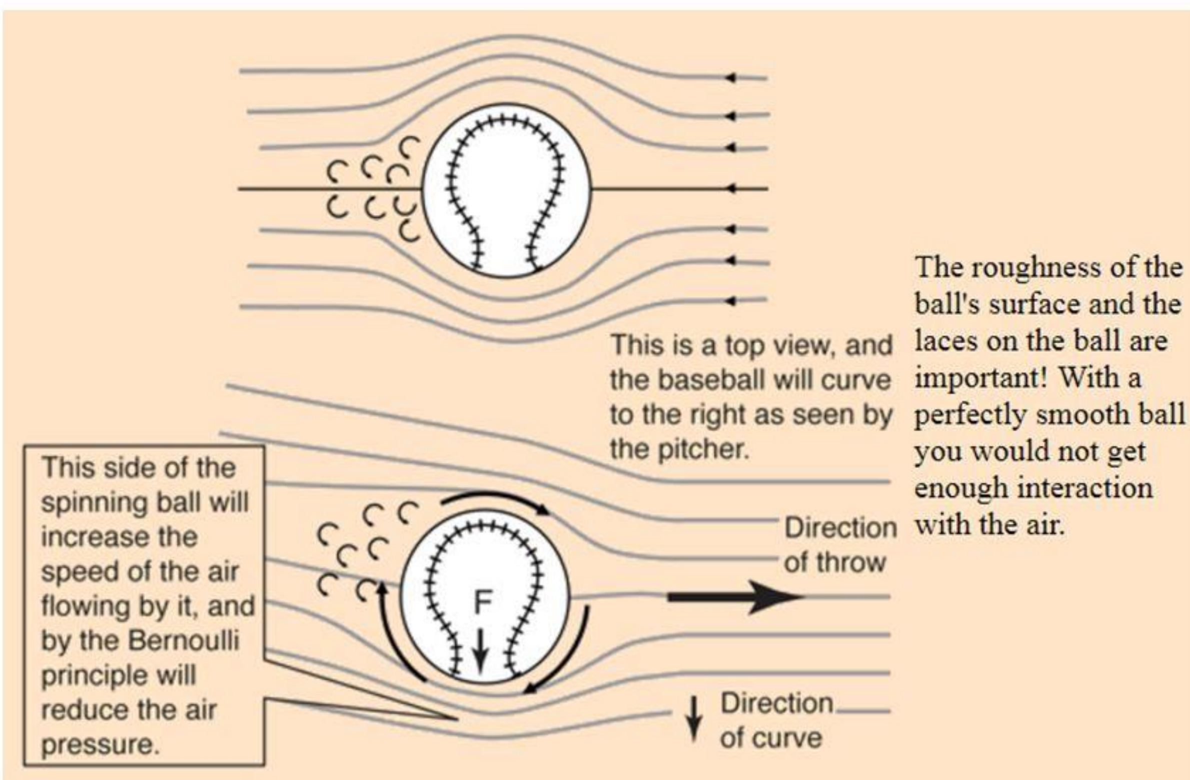
Vertically: **free fall; constant acceleration *g***

$$v_y = v_{0y} + a_y t \qquad \Delta y = \left(\frac{v_{0y}+v_y}{2}\right)t$$

$$\Delta y = v_{0y}t + \frac{1}{2}a_y t^2 \qquad v_y^2 = v_{0y}^2 + 2a\Delta y$$



Faster flow = less pressure



Free-body force diagrams – 1

Figure 16.1 Situation diagram for a man near the Earth

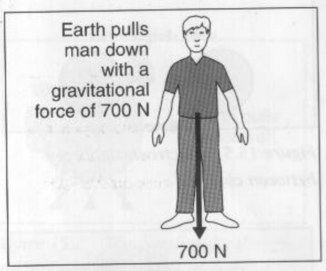


Figure 16.2 Free-body force diagram for a man near the Earth

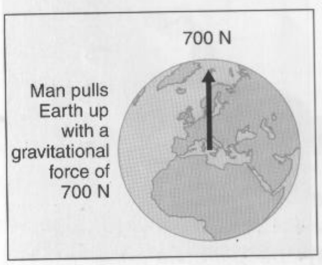


Figure 16.3 Free-body force diagram for the Earth near a man

Situation diagrams and free-body force diagrams

You know that forces always occur in pairs, one force acting on one body, and one force on another body. If you draw both these forces in the same diagram it is confusing because, in all but the simplest situations, it is unclear which forces act on which body. Free-body force diagrams are a neat and effective way to show the forces on a body; they are diagrams of a *single* body, showing the forces on that body *only*.

Figure 16.1 shows a situation diagram for a man near the Earth. Figures 16.2 and 16.3 show two *free-body force diagrams*, one for the man and one for the Earth. Each free-body force diagram shows a single body, and the forces on that body alone. Free-body force diagrams may look like situation diagrams, but there is one really important difference. In the man's free-body force diagram, he is on his own, without the Earth. That does not mean that the Earth doesn't exist, just that the diagram is only about what is happening to the man.

If you need to draw a free-body force diagram, it sometimes helps first to sketch a rough situation diagram with all the bodies, and then to draw a separate free-body force diagram for each body.

Describing forces

The labels on the forces in the free-body force diagrams for the man and the Earth describe the forces fully. These descriptions consist of five parts:

The Earth pulls the man down with a gravitational force of 700 N

The description states

- what is exerting the force
- on what the force is exerted
- the direction of the force
- the type of the force
- the size of the force.

Similarly,

The man pulls the Earth up with a gravitational force of 700 N

Descriptions like this help identify **Newton III pairs of forces**.

Identifying Newton III pairs of forces

From this work on free-body force diagrams, you should be able to see that the forces in a Newton III pair of forces have the same line of action – if you extend the force lines on the diagram of the situation, then they will pass through each other. The forces also act for the same time. In addition, you should realise that the two forces of a Newton III pair have to be the same type of force. If A pulls on B with a gravitational force, then B pulls on A with a gravitational force also. This information allows a complete statement of Newton's third law:

> While a body A exerts a force on a body B, body B exerts a force on body A. The forces are equal, opposite and of the same type; they have the same line of action and act for the same time.

If you have described one force fully, then you can identify the Newton III pair to this force from its description. Simply exchange the two bodies in the statement and reverse the direction of the force. For example:

The Earth pulls the man down with a gravitational force of 700 N

The man pulls the Earth up with a gravitational force of 700 N

So if

Freda pushes Jo left with a contact force of 35 N

Jo pushes Freda right with a contact force of 35 N

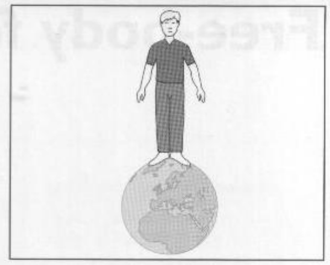


Figure 16.4 Situation diagram for a man standing on the Earth

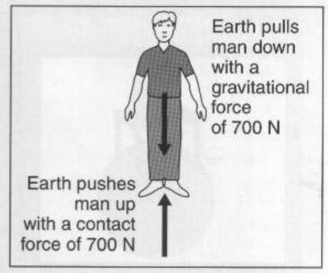


Figure 16.5 Free-body force diagram for a man standing on the Earth. The man is in equilibrium because the two forces on him are equal and opposite

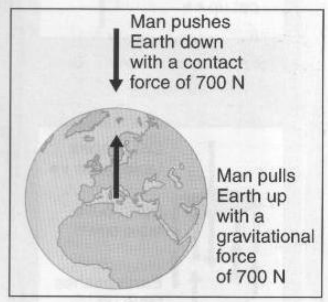


Figure 16.6 Free-body force diagram for the Earth. The Earth is in equilibrium because the two forces on it are equal and opposite

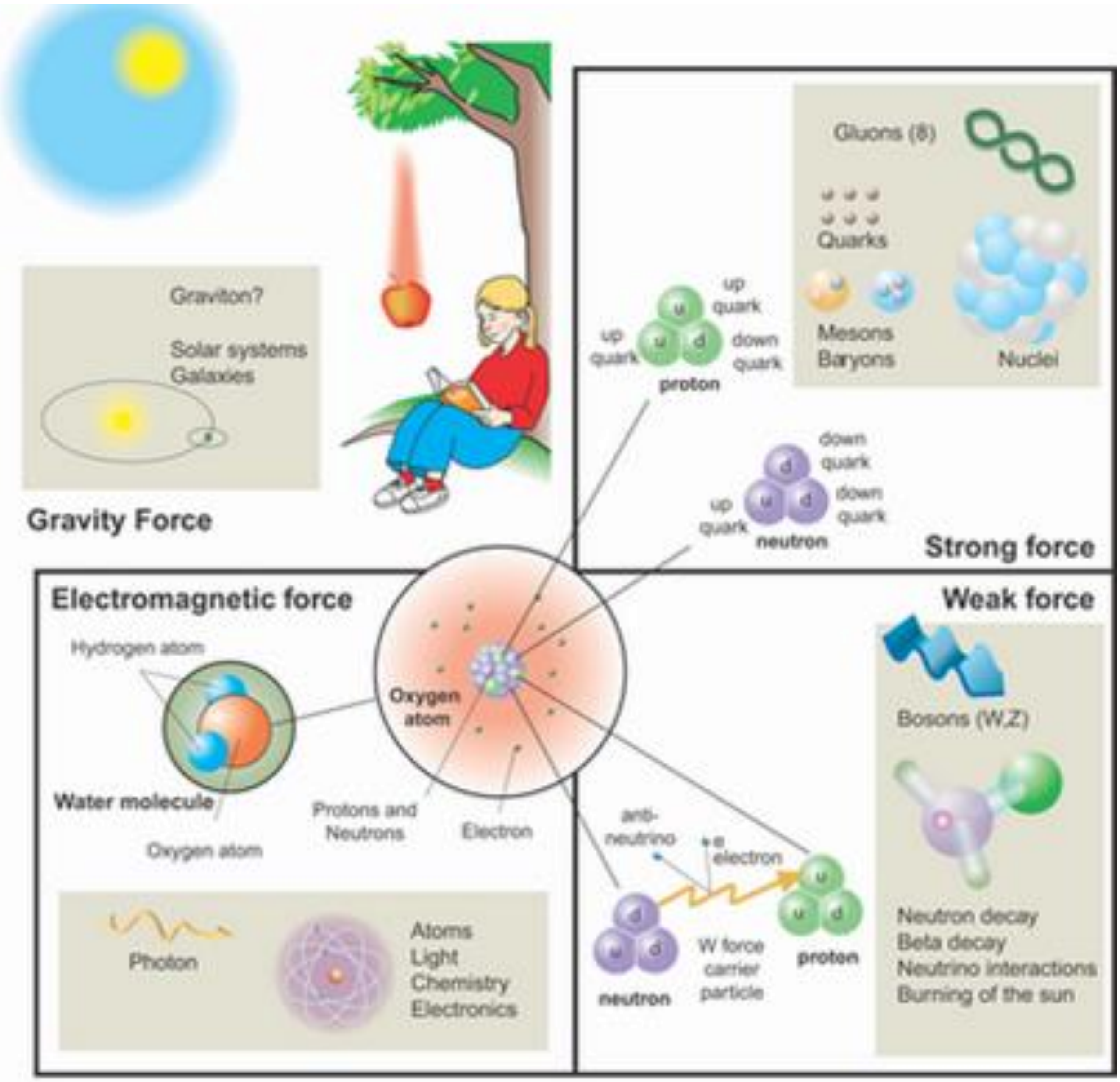
Standing on a planet

Two bodies often exert more than one pair of forces on each other. Think about a body resting on the surface of a planet. Figure 16.4 shows the diagram of this situation for a man on the Earth, and Figures 16.5 and 16.6 show the two free-body force diagrams for the man and the Earth.

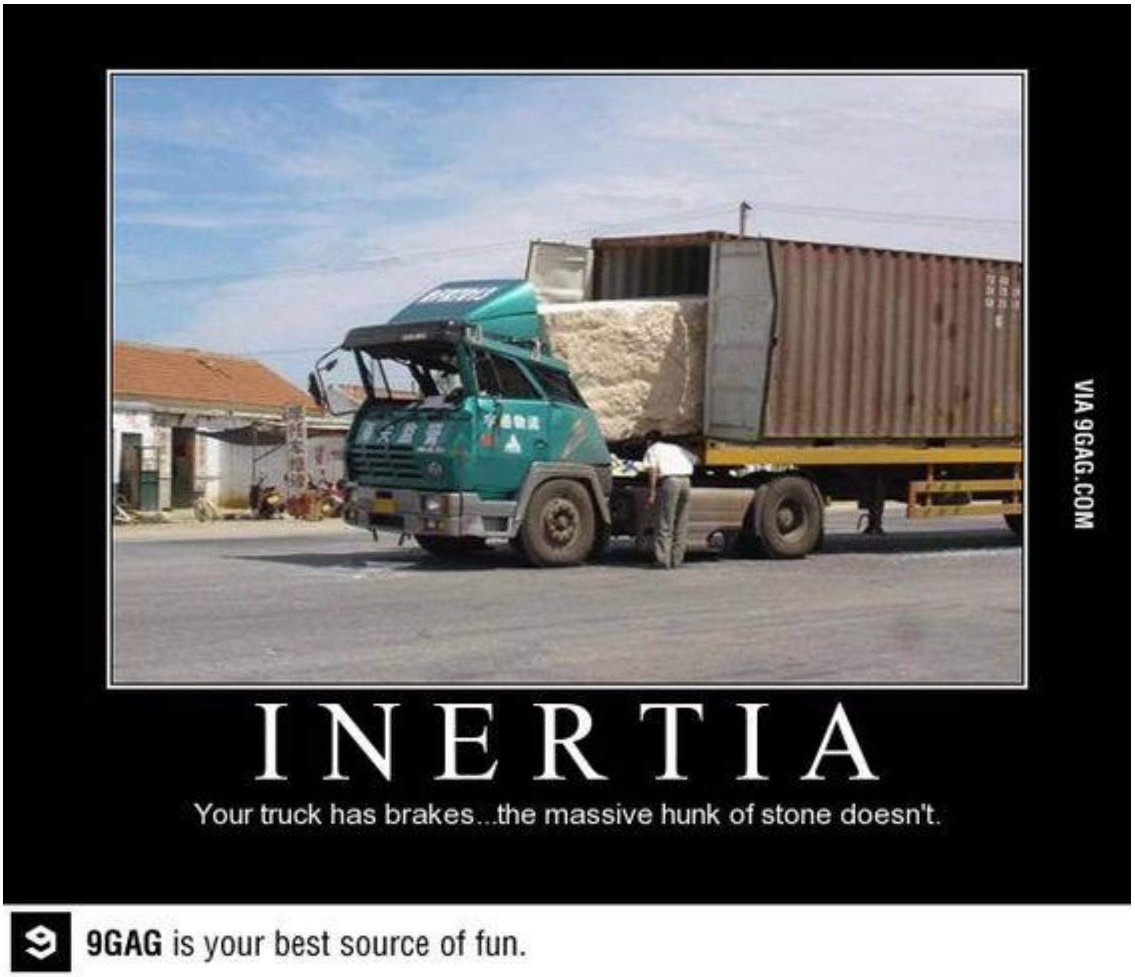
As before, there is a pair of gravitational forces between the man and the Earth, one on the man and one on the Earth. But there are also contact forces, again one on the man and one on the Earth.

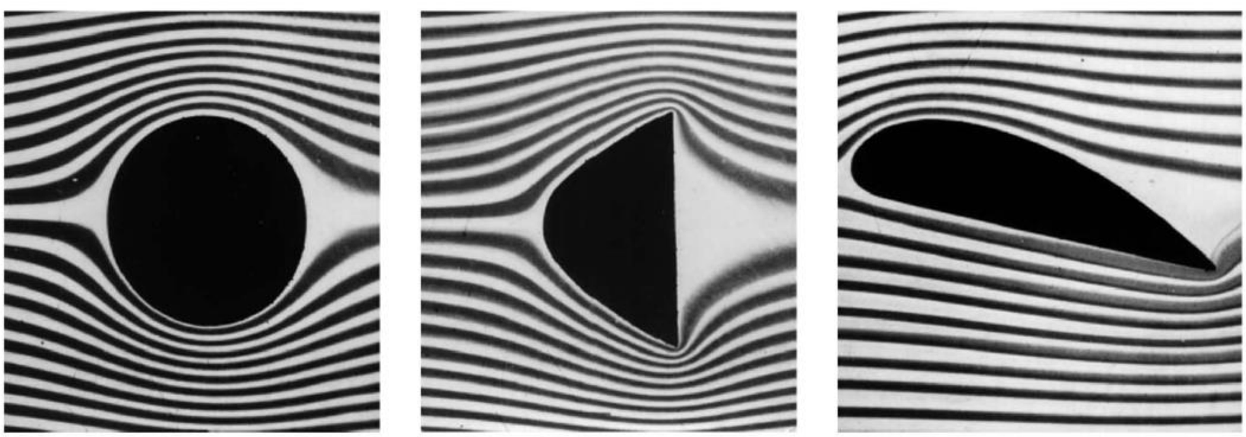
First think about the forces on the man. The Earth pulls the man down with a gravitational force. The Earth also pushes the man up with a contact force.

Now think about the forces on the Earth. The man pulls the Earth up with a gravitational force, and also pushes the Earth down with a contact force.

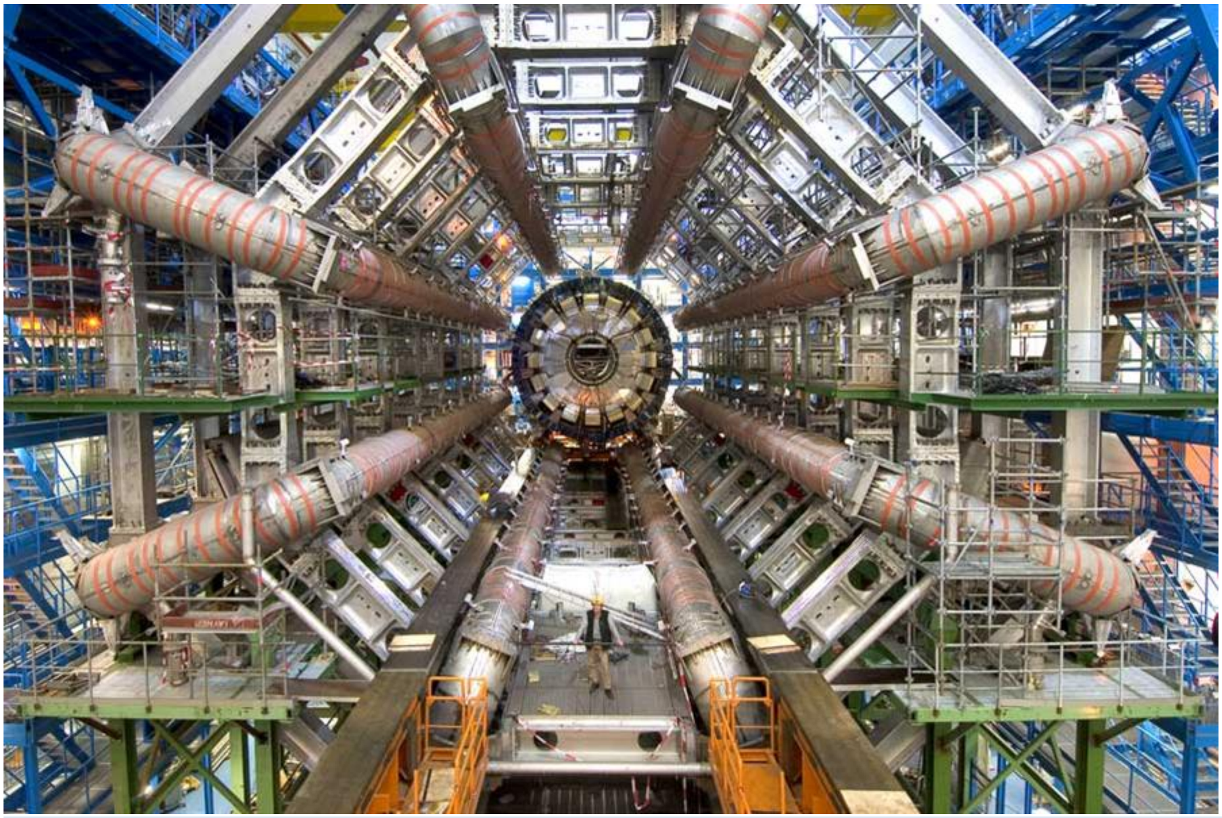

Gravity Force
Graviton?
Solar systems
Galaxies
Strong force
Gluons (8)
Quarks
Mesons
Baryons
Nuclei
up quark
up quark
down quark
proton
down quark
down quark
up quark
neutron
Electromagnetic force
Hydrogen atom
Water molecule
Oxygen atom
Oxygen atom
Protons and Neutrons
Electron
Photon
Atoms
Light
Chemistry
Electronics
Weak force
Bosons (W,Z)
anti-neutrino
e electron
neutron
W force carrier particle
proton
Neutron decay
Beta decay
Neutrino interactions
Burning of the sun


A free-body diagram of a man dragging a crate.
The diagram shows *all* the forces acting on the man, and *only* forces acting on the man.
$\vec{n}$
Man (treated as particle)
$\vec{T}$
$\vec{f}$
$\vec{w}$


VIA 9GAG.COM
INERTIA
Your truck has brakes...the massive hunk of stone doesn't.
9GAG is your best source of fun.



(a) (b) (c)



Linear Equations of Motion

$\Delta x = v_x t$

$v_x = \frac{\Delta x}{\Delta t}$

$a = \frac{\Delta v}{\Delta t}$

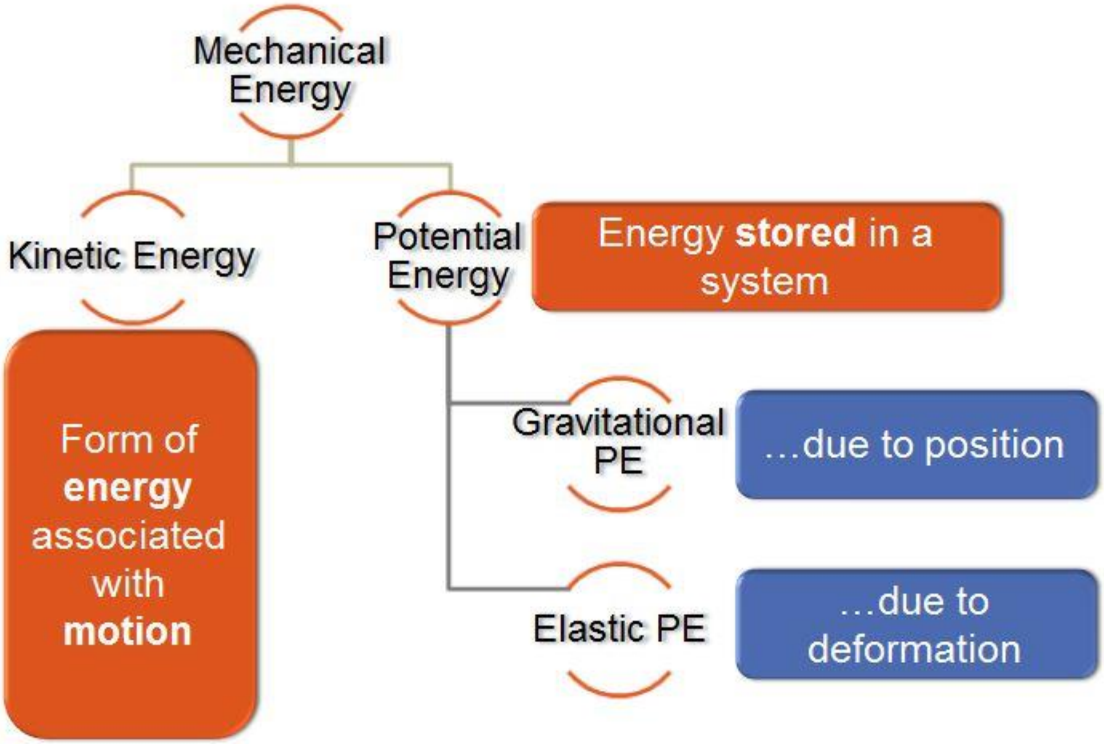
$v_y = v_0 + at$

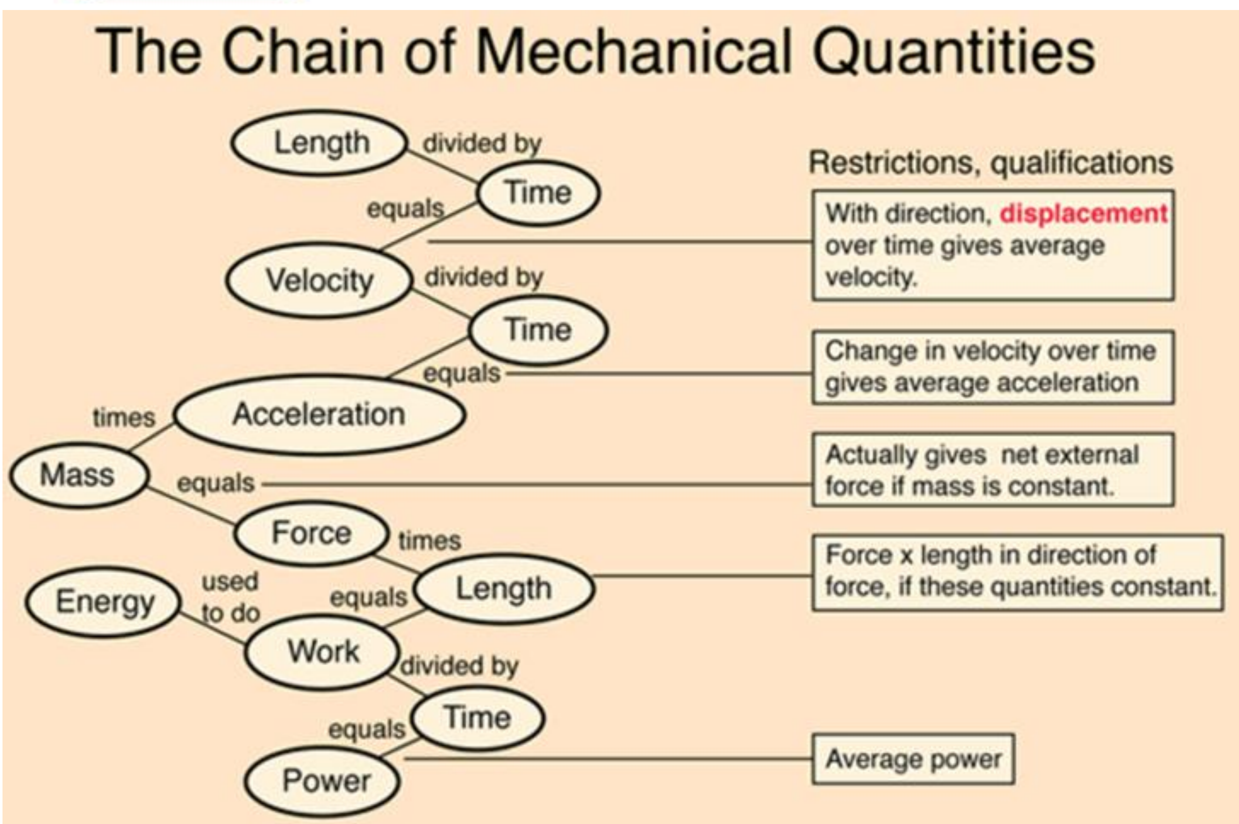
$\Delta y = \left(\frac{v_0 + v}{2}\right) t$

$\Delta y = v_0 t + ½ a t^2$

$v^2 = {v_0}^2 + 2a\,\Delta x$

Range = $\frac{{v_0}^2 sin2\theta_0}{g}$


Mechanical Energy
Kinetic Energy
Potential Energy
Energy **stored** in a system
Form of **energy** associated with **motion**
Gravitational PE
…due to position
Elastic PE
…due to deformation


The Chain of Mechanical Quantities
Length
divided by
Time
equals
Velocity
divided by
Time
equals
Acceleration
times
Mass
equals
Force
times
Length
Energy
used to do
equals
Work
divided by
Time
equals
Power
Restrictions, qualifications
With direction, displacement over time gives average velocity.
Change in velocity over time gives average acceleration
Actually gives net external force if mass is constant.
Force x length in direction of force, if these quantities constant.
Average power

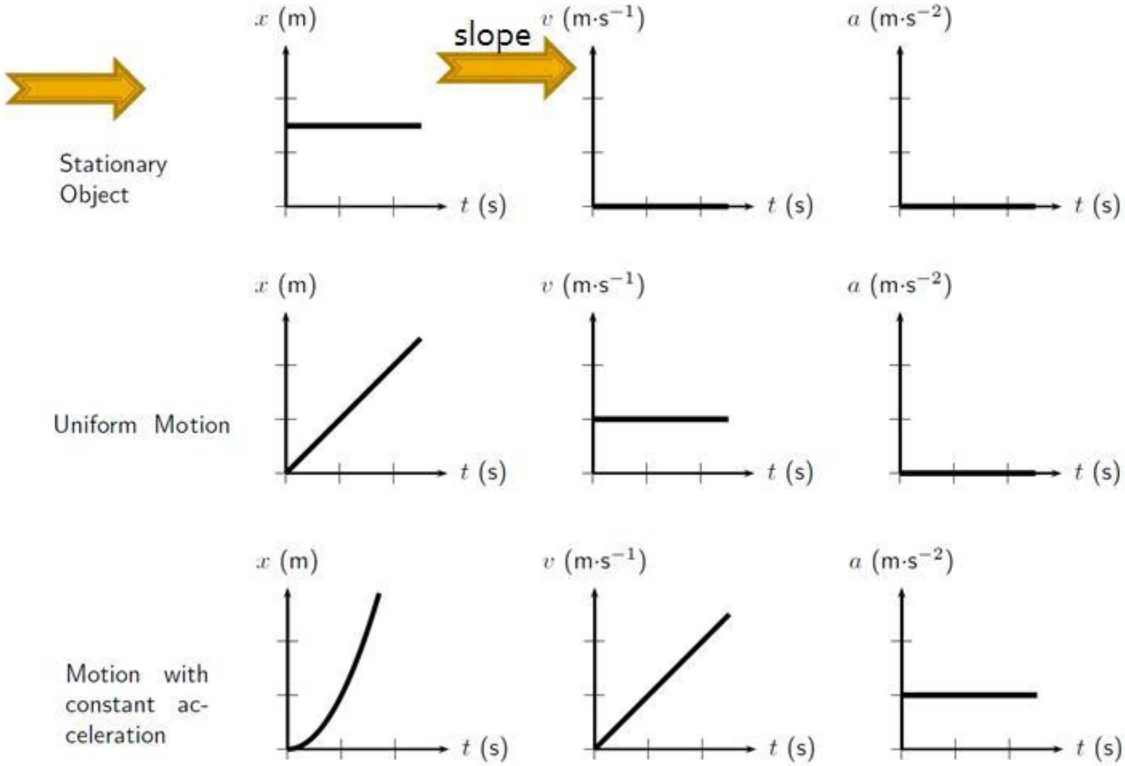
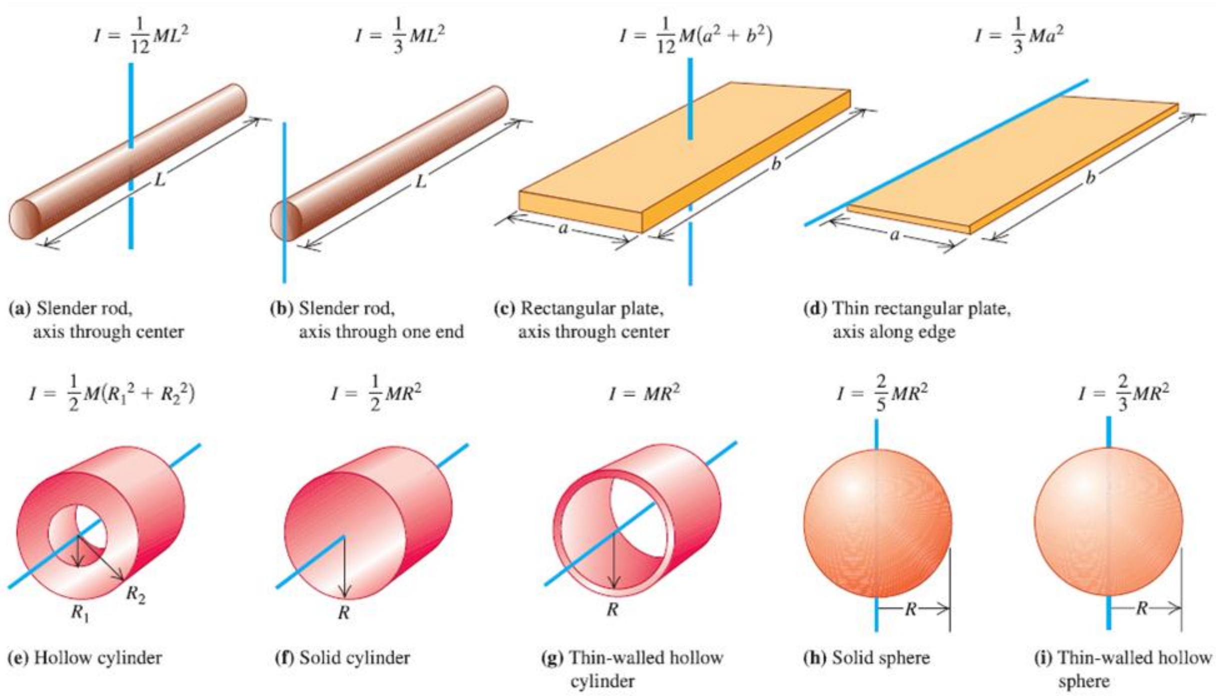
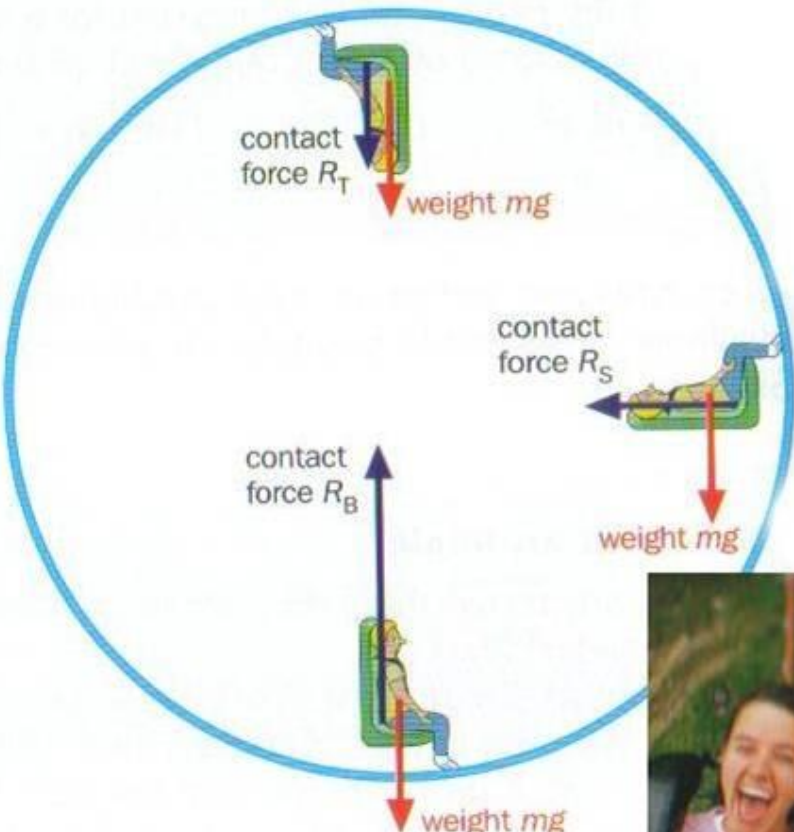


TABLE 9.2 Moments of inertia for various bodies



Momentum Equation	**Linear Kinetic Energy**	**Impulse**
$p = mv$	$K.E._{linear} = \frac{1}{2}mv^2$	$J = F\Delta t$

$$R_T + mg = \frac{m v^2}{r}$$



$$R_S = \frac{m v^2}{r}$$

$$R_B - mg = \frac{m v^2}{r}$$

Variable Mass Applications

The generalization of Newton's 2nd Law to apply to variable mass systems takes the form

$$F_{net\ external} = \frac{d(mv)}{dt} = m\frac{dv}{dt} + v\frac{dm}{dt}$$

The term involving the derivative of the mass is responsible for the thrust in rocket propulsion and must be included in any problem where the mass changes.

Newton's first law

Every object continues either at rest or in constant motion in a straight line, unless it is forced to change that state by forces acting on it.

Newton's First Law: Vector form

$$\sum \vec{F} = 0 \qquad \vec{a} = 0$$

$$\Delta\vec{v} = 0$$

$$\vec{v} = \text{constant}$$

Newton's Second Law

- **The net force acting on an object is equal to its mass times its acceleration**

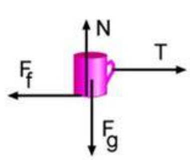
Vector form

$$\Sigma\vec{F} = m\vec{a}$$

Component form

$$\Sigma F_x = ma_x \qquad \Sigma F_y = ma_y$$

1. $\Sigma\vec{F} = 0$
2. $\Sigma\vec{F} = m\vec{a}$
3. $\vec{F}_{AB} = -\vec{F}_{BA}$

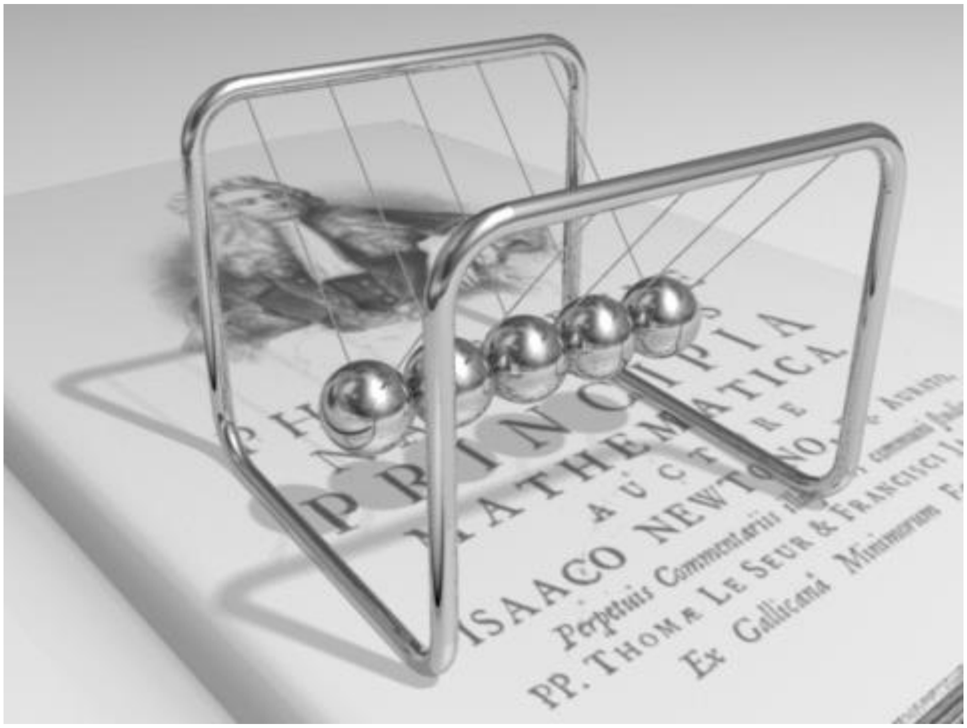


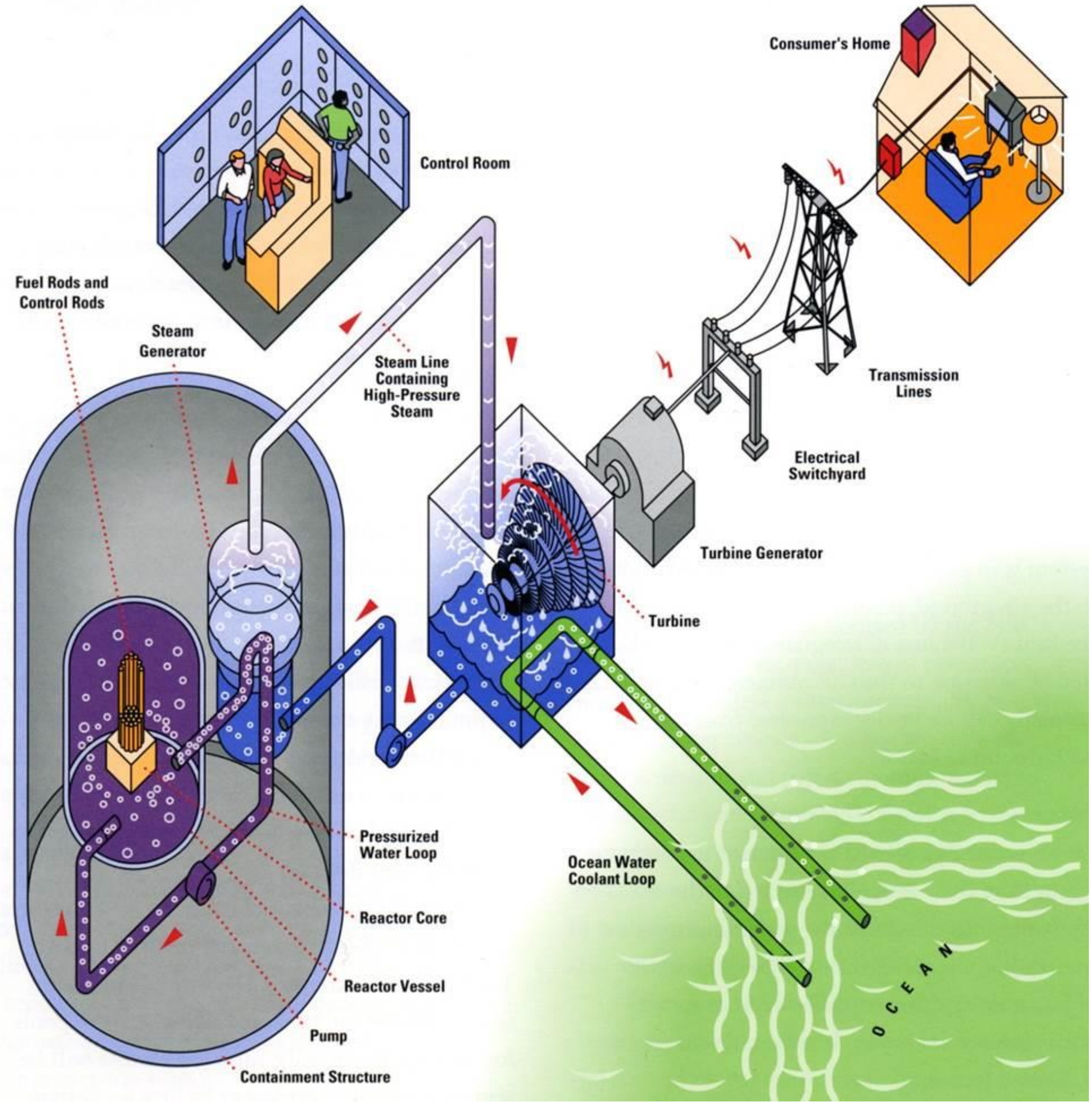
Newton's third law

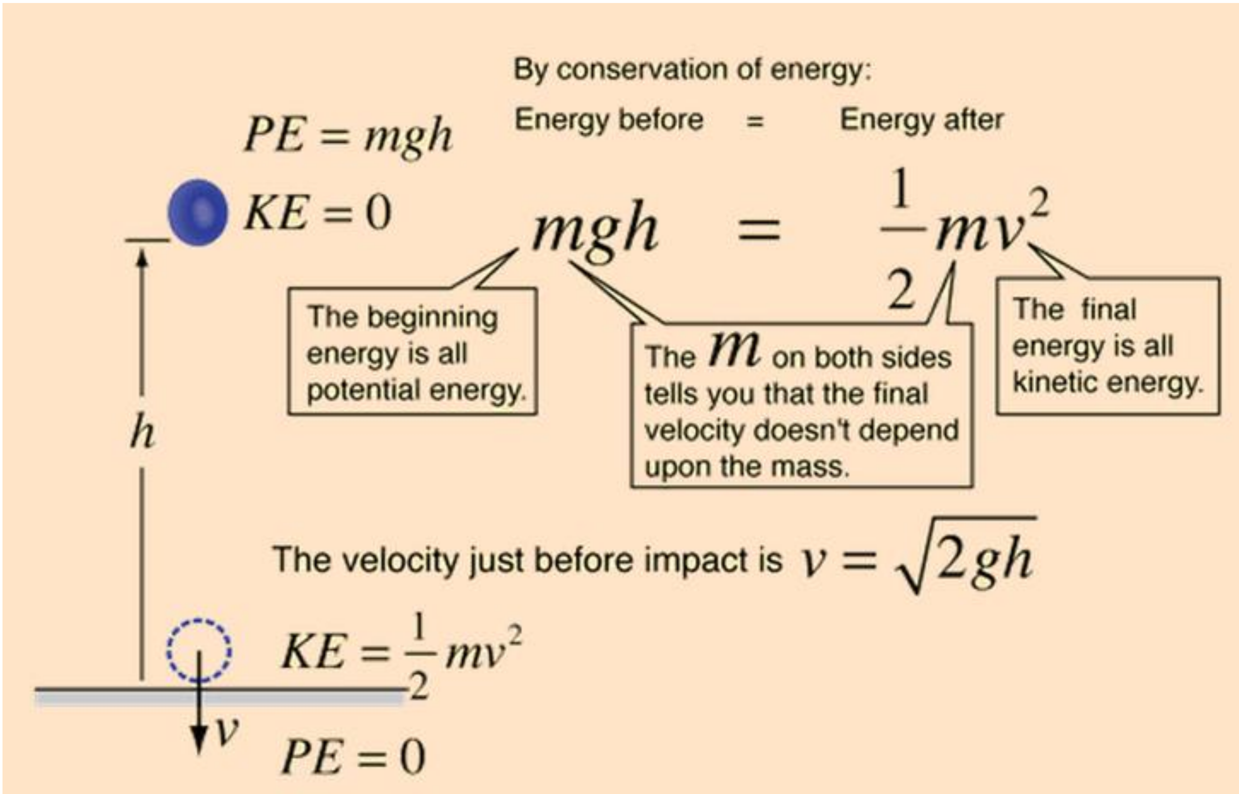
- **For every action (force) there is a reaction (force) equal in magnitude and opposite in direction.**

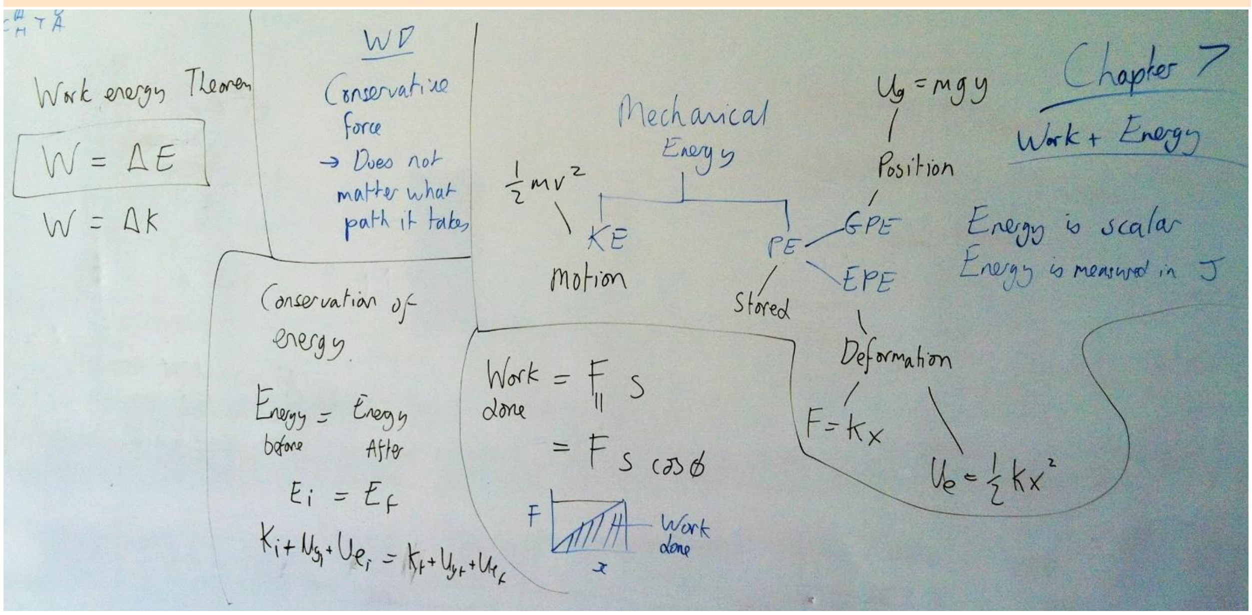
$$\vec{F}_{AonB} = -\vec{F}_{BonA}$$
$$\vec{F}_{AB} = -\vec{F}_{BA}$$

- Forces always act in pairs (of action and reaction)

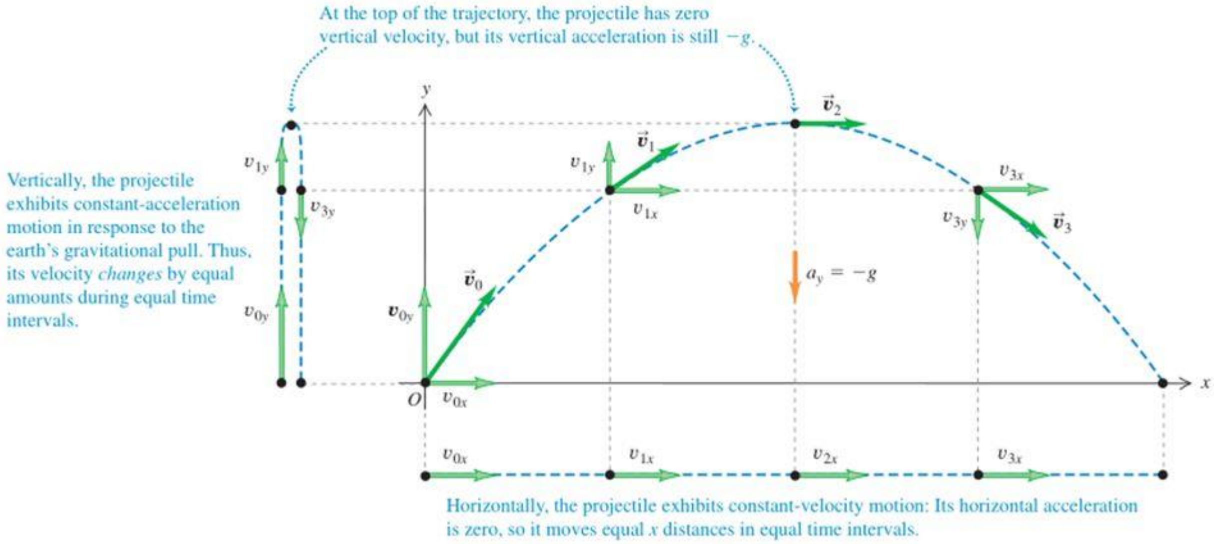

PRINCIPIA
MATHEMATICA
ISAACO
Perpetuis Commentariis
PP. THOMÆ LE SEUR & FRANCISCI
Ex Gallicanâ Minimorum


Consumer's Home
Control Room
Fuel Rods and
Control Rods
Steam
Generator
Steam Line
Containing
High-Pressure
Steam
Transmission
Lines
Electrical
Switchyard
Turbine Generator
Turbine
Pressurized
Water Loop
Ocean Water
Coolant Loop
Reactor Core
Reactor Vessel
Pump
Containment Structure
OCEAN


By conservation of energy:
Energy before = Energy after
$PE = mgh$
$KE = 0$
$mgh = \frac{1}{2}mv^2$
The beginning energy is all potential energy.
The m on both sides tells you that the final velocity doesn't depend upon the mass.
The final energy is all kinetic energy.
h
The velocity just before impact is $v = \sqrt{2gh}$
$KE = \frac{1}{2}mv^2$
v
$PE = 0$


Chapter 7
Work + Energy
Work energy Theorem
$W = \Delta E$
$W = \Delta K$
WD
Conservative force
→ Does not matter what path it takes
Mechanical Energy
$\frac{1}{2}mv^2$
KE
motion
PE
Stored
$U_g = mgy$
Position
GPE
EPE
Deformation
$F = kx$
$U_e = \frac{1}{2}kx^2$
Energy is scalar
Energy is measured in J
Conservation of energy
Energy before = Energy After
$E_i = E_f$
$K_i + U_{g_i} + U_{e_i} = K_f + U_{g_f} + U_{e_f}$
Work done $= F_{\parallel} s$
$= F s \cos\theta$
F
Work done

- Motion is resolved into a horizontal component and a vertical component.

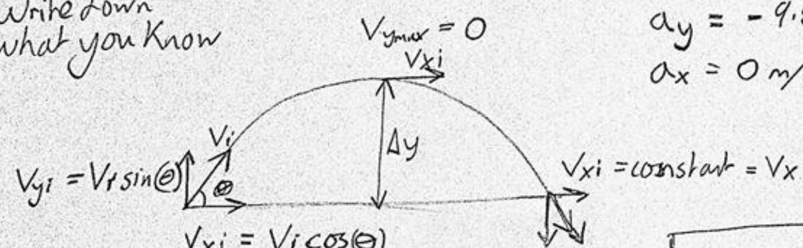


Projetiles

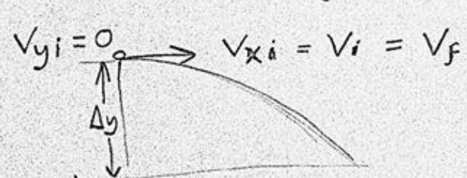
① Write down what you know

$a_y = -9.8\ m/s^2$
$a_x = 0\ m/s^2$

DEAL WITH HORIZONTAL AND VERTICAL SEPARATLY



If launched horizontaly



② Write down what you want to find

③ Choose the correct equation.

④ Rearange and solve carefully

EQUATIONS

Horizontal (x)

$$\Delta x = v \Delta t$$

Vertical (y)

$$V_f = V_i + a \Delta t$$
$$\Delta y = V_i \Delta t + \frac{1}{2} a \Delta t^2$$
$$V_f^2 = V_i^2 + 2 a \Delta y$$

Early records
about 78 rpm

LP $33_{1/3}$ rpm
Single 45 rpm

CD 7200 rpm

$MoI = I = mr^2$

$W_d = \tau \Delta\theta$

(Shape Dependent -See MoI Table)

$\Sigma\tau = I\alpha$

$\tau = Fd\,cos\theta$

$P = \tau\omega$

$L = I\omega$

Rotational Dynamics (Linking Linear with Rotational)

$$a = \frac{g}{1 + \frac{M}{2m}}$$

$$\alpha = \frac{\frac{g}{R}}{1 + \frac{M}{2m}}$$

Rotational motion equations

$$\omega = \frac{\Delta\theta}{\Delta t}$$

$$\alpha = \frac{\Delta\omega}{\Delta t}$$

$$\omega = \omega_0 + \alpha t$$

$$\Delta\theta = \left(\frac{(\omega_0 + \omega)}{2}\right) t$$

$$\Delta\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\Delta\theta = \omega t - \frac{1}{2}\alpha t^2$$

$$\omega^2 = {\omega_0}^2 + 2\alpha\Delta\theta$$

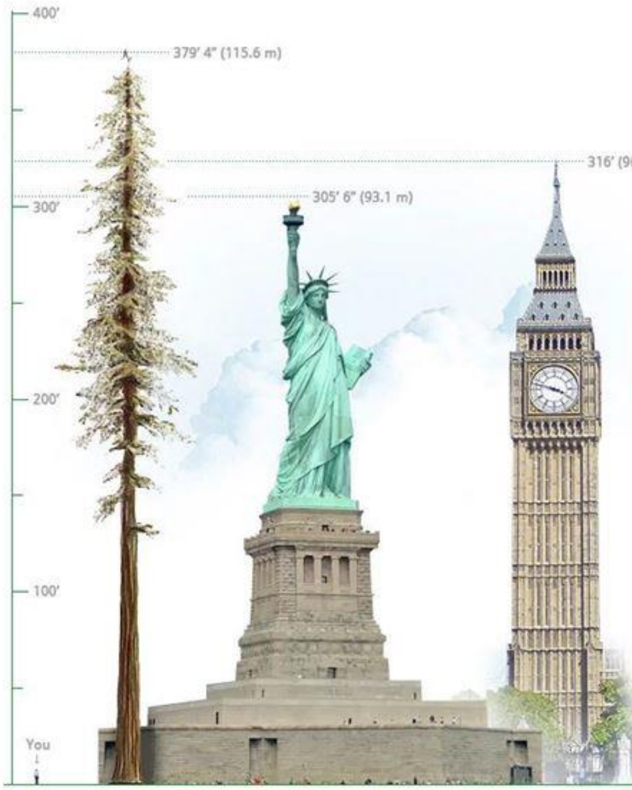
$$v_{linear} = \omega r$$

$$a_{tan} = r\alpha$$

$$a_{rad} = \omega^2 r$$

$$a_{linear} = \sqrt{{a_{tan}}^2 + {a_{rad}}^2}$$

$$K.E._{rotational} = \frac{1}{2} I\omega^2$$



Height comparison of the Hyperion tree (iltwmt.com)