Nuclear Stability

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C1 Stable and unstable nuclei

Nuclear matter.	Electrostatic repulsion between protons. The strong nuclear force between nucleons. Comparative ranges. Variation of nuclear radius with nucleon number; $r = r_0A^{1/3}$. Density of nuclear matter.
N-Z curve for nuclides.	Region of stability; relevance to α, β^+ and β^- decay. Decay chains; principle of radioactive dating.
Nuclear decays.	Decay of n and p within the nucleus. Energy spectra for α, β^+ and β^- particles and subsequent γ ray emissions. (Knowledge of $E = hf$ is *not* required.) The neutrino and antineutrino.
Energy and the nucleus.	Nuclear masses in terms of u, the unified atomic mass unit. Nuclear decay energies in MeV. the principle of conservation of mass-energy: 1u = 930 MeV. (Use of $c^2\Delta m = \Delta E$ *not* required.) Binding energy per nucleon.

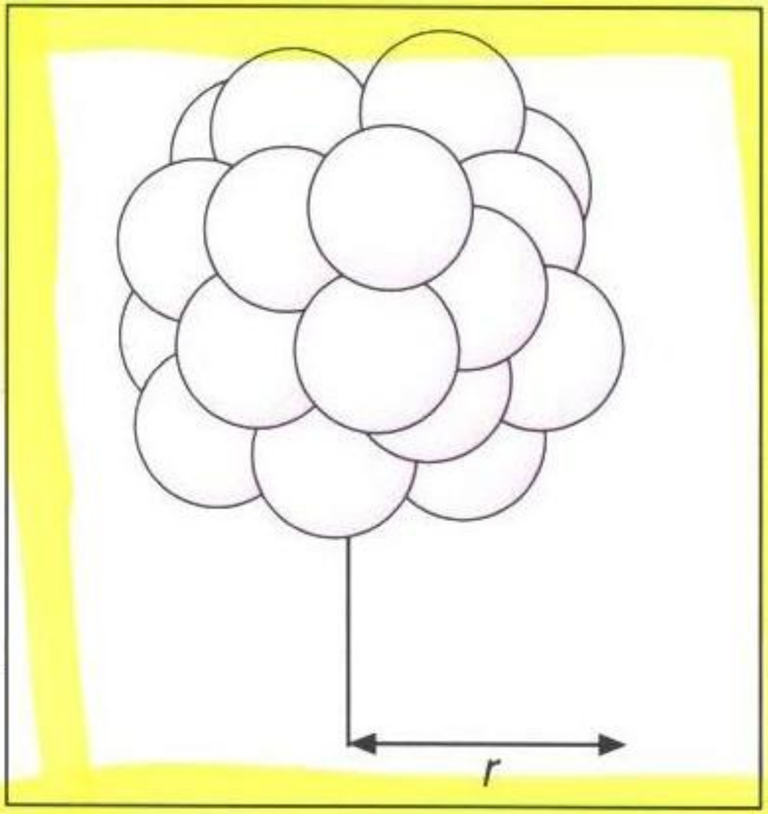


Figure NP3 A larger nucleus: the volume of this larger nucleus is $4/3\ \pi r^3$

$$\text{So } 4/3\ \pi r^3 = A\ 4/3\ \pi r_0^{\ 3}$$

$$r^3 = A\ r_0^{\ 3}$$

$$\therefore\ r = r_0\ A^{1/3}$$

This electrostatic force between the protons does not succeed in making the protons in the nucleus fly apart – so there must be another force that holds them together. This force is called the *strong nuclear force*.

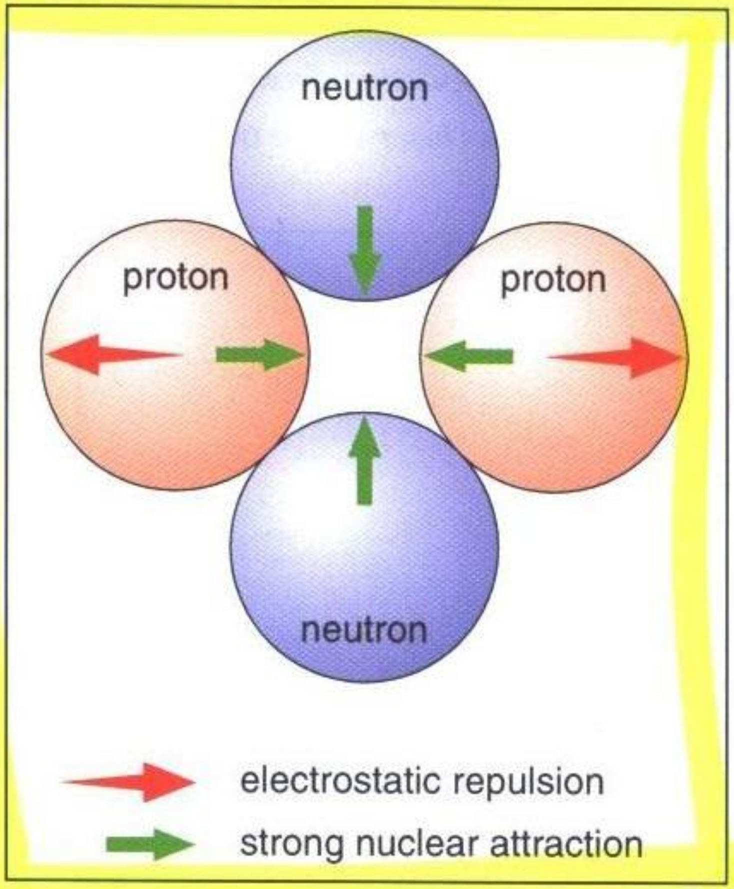
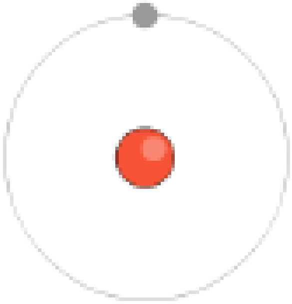
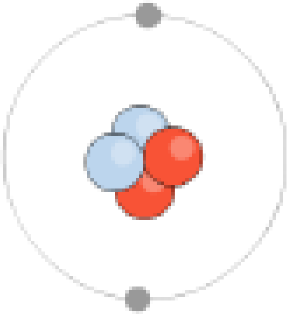


Figure NP4* *Repulsion and attraction: the strong nuclear force holds the nucleus together, against the electrostatic repulsion

When it was proposed that all of an atom's positive charge was contained in a very small nucleus scientists were faced with the challenge of explaining how these nuclei were held together. The 'standard model' of matter explains nuclear stability by considering the balance between the electrostatic repulsion of the protons and the strong nuclear force.



Hydrogen



Helium

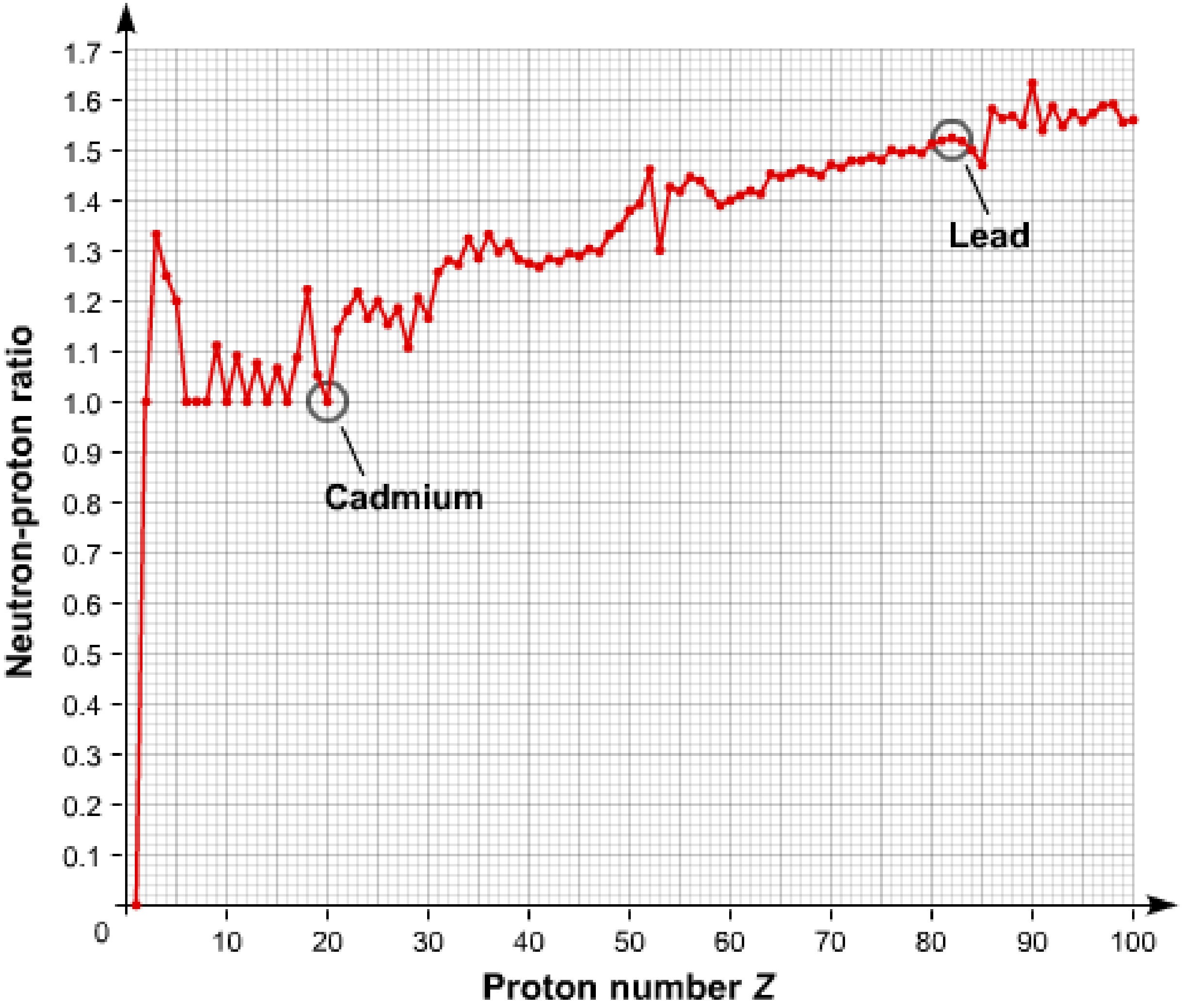
Adding protons or neutrons to a nucleus alters the balance between the attractive and repulsive forces and can result in instability. Adding protons or neutrons increases the attractive strong nuclear force because there are more nucleons. However adding protons also increases the repulsive force between charged particles.

The strong nuclear force is short-range while the electrostatic repulsion is long-range. Hence, adding a proton adds to the attractive forces between neighbours but introduces extra repelling forces between this proton and *all* the other protons.

The neutrons act as a glue –
The strong nuclear force

As we go up
We get more
Neutrons than
protons

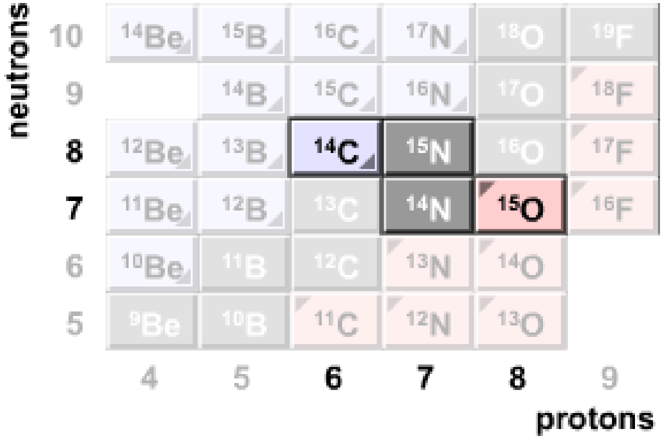
Same number
of protons and
neutrons



These results show that as the atomic number increases the neutron-to-proton ratio increases. This means that equal increases in the number of protons result in larger increases in the number of neutrons.

It is generally accepted that nuclei larger than bismuth, which has an atomic number of 83, are unstable if the nuclei have more than 1.5 times as many neutrons as protons. These large, unstable nuclei will undergo radioactive decay, releasing alpha particles, beta particles, or gamma radiation. Very large nuclei also decay by splitting into two parts approximately equal in size. This is called **nuclear fission**.

The idea that atoms can increase their stability by achieving a certain neutron-to-proton ratio can be used to explain beta and positron decay.



neutrons						
10	^{14}Be	^{15}B	^{16}C	^{17}N	^{18}O	^{19}F
9		^{14}B	^{15}C	^{16}N	^{17}O	^{18}F
8	^{12}Be	^{13}B	^{14}C	^{15}N	^{16}O	^{17}F
7	^{11}Be	^{12}B	^{13}C	^{14}N	^{15}O	^{16}F
6	^{10}Be	^{11}B	^{12}C	^{13}N	^{14}O	
5	^{9}Be	^{10}B	^{11}C	^{12}N	^{13}O	
	4	5	6	7	8	9

In the beta decay of carbon-14, one neutron has been changed into a proton and a fast-moving electron has been expelled from the nucleus. As a result, the neutron-to-proton ratio has been reduced and a stable isotope of nitrogen is formed.

The decay of oxygen-15 into nitrogen-15 involves the emission of a **positron**. This happens when a proton changes into a neutron and emits a positively charged beta particle called a positron. As a result of this decay, the neutron-to-proton ratio has increased and a stable isotope is formed.

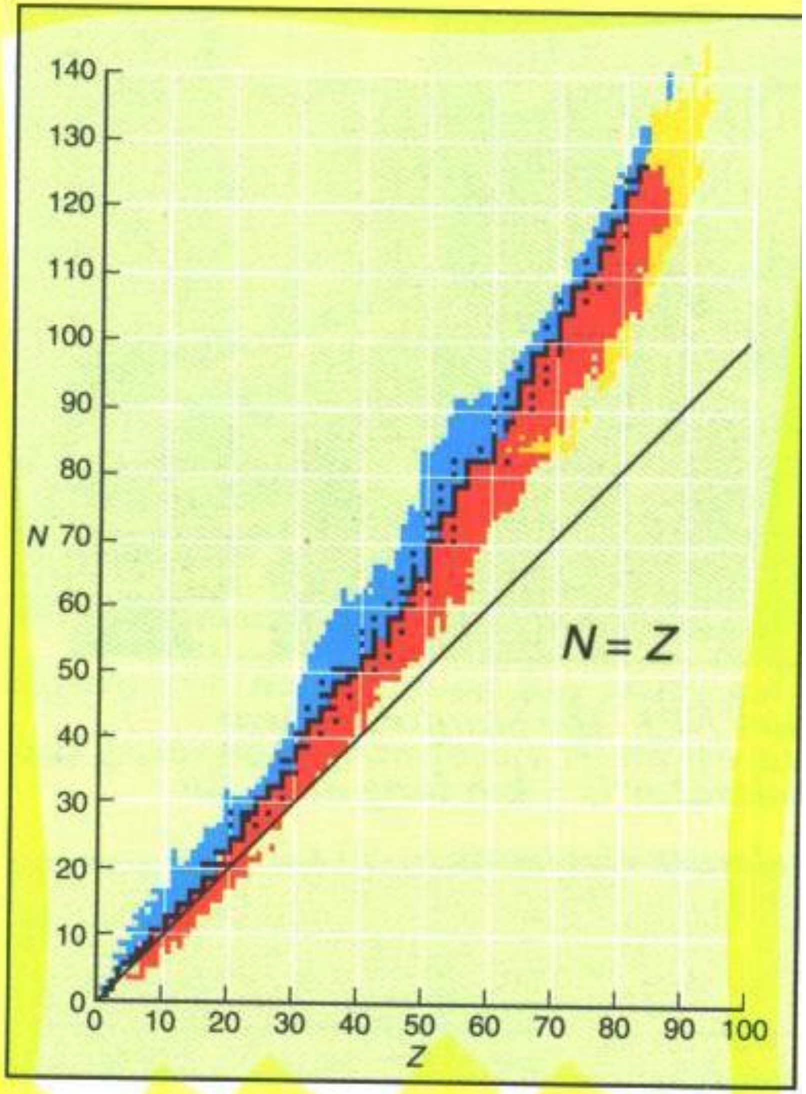


Figure NP5 N–Z trend line: shows a characteristic pattern when the number of neutrons in a nucleus is plotted against the number of protons

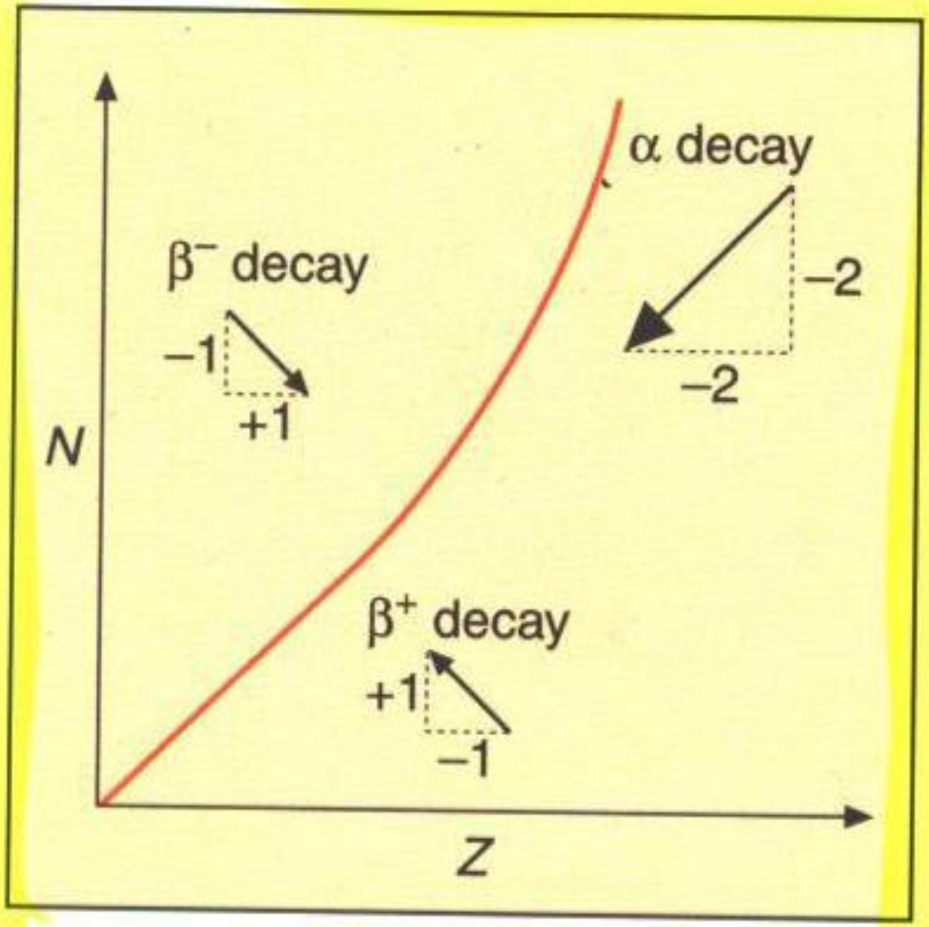
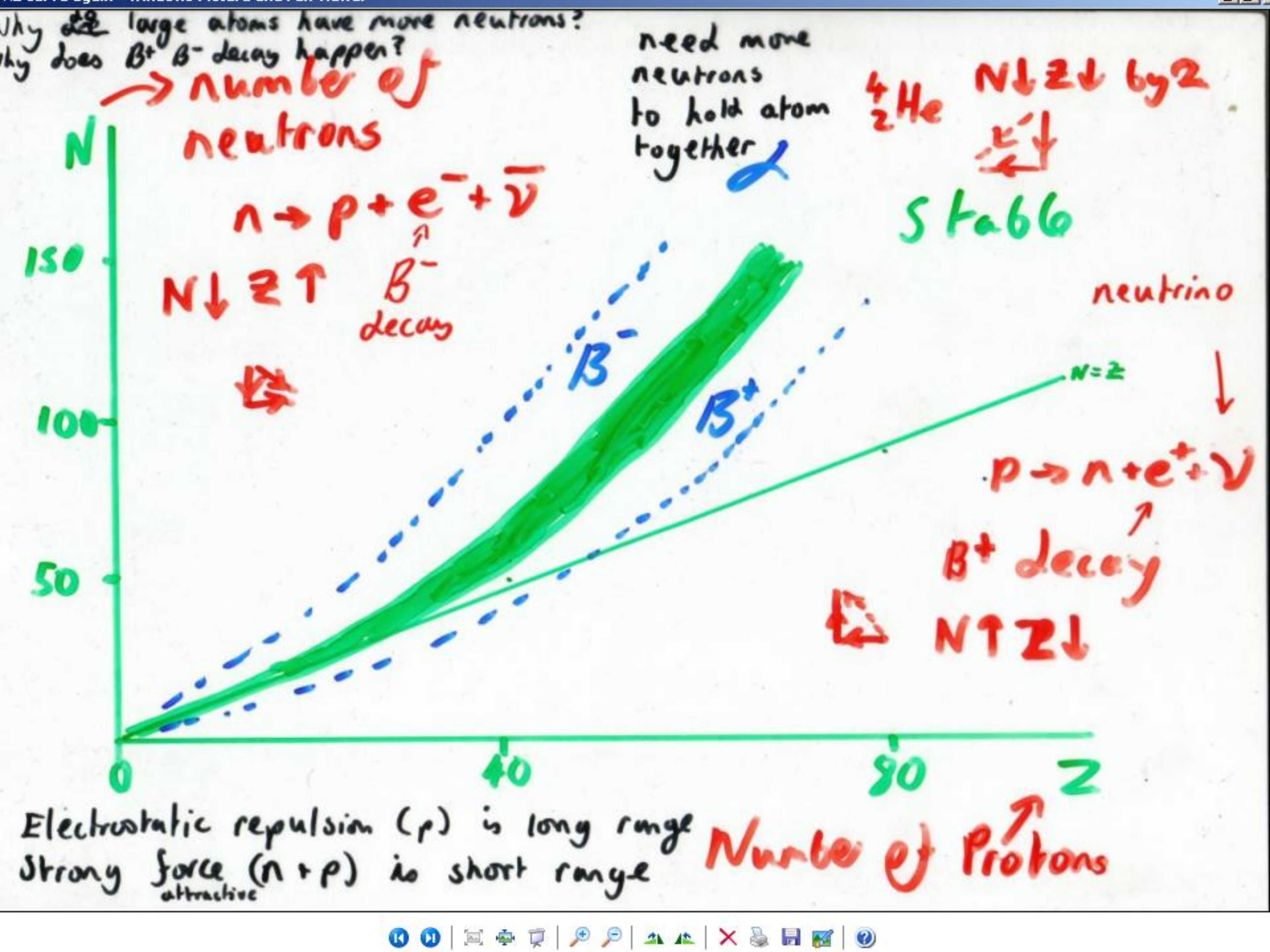


Figure NP6 Radioactive decay brings the nuclide plot closer to the trend line



Why ~~do~~ large atoms have more neutrons?
hy does β+ β- decay happen?

need more neutrons to hold atom together

$n \to p + e^- + \bar{\nu}$

N↓ Z↑ β- decay

^4_2He N↓ Z↓ by 2

neutrino

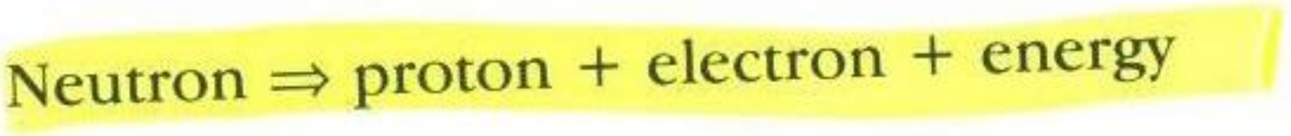
$p \to n + e^+ + \nu$

β+ decay

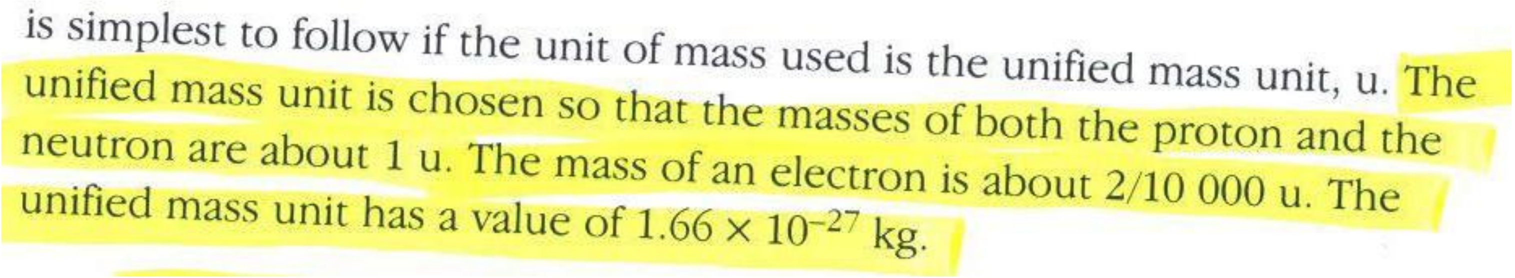
N↑ Z↓

Electrostatic repulsion (p) is long range
Strong force (n + p) is short range
attractive

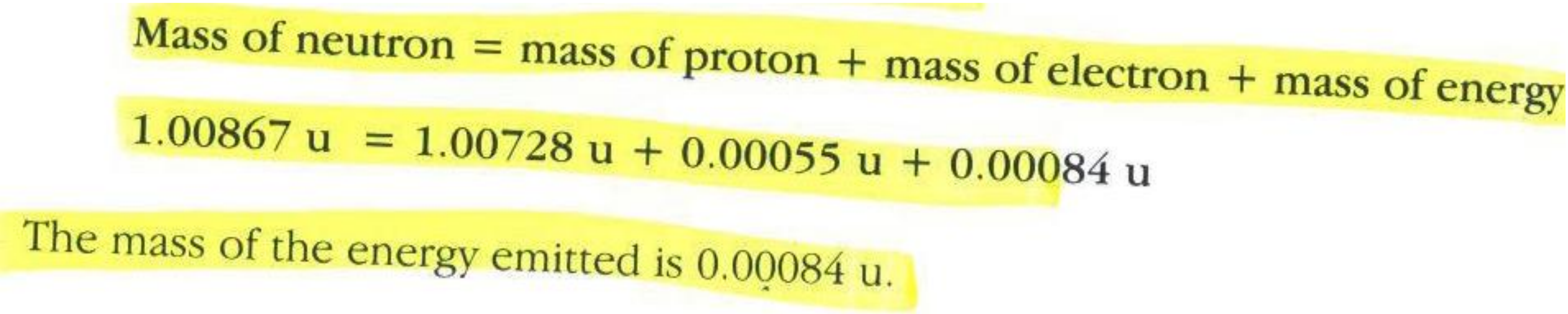
β- decay



Neutron ⇒ proton + electron + energy



is simplest to follow if the unit of mass used is the unified mass unit, u. The unified mass unit is chosen so that the masses of both the proton and the neutron are about 1 u. The mass of an electron is about 2/10 000 u. The unified mass unit has a value of 1.66×10^{-27} kg.



Mass of neutron = mass of proton + mass of electron + mass of energy

1.00867 u = 1.00728 u + 0.00055 u + 0.00084 u

The mass of the energy emitted is 0.00084 u.

$1\ \text{eV} = 1.6 \times 10^{-19}\ \text{J}$

Mass of neutron = mass of proton + mass of electron + mass of energy

938.06 MeV = 936.77 MeV + 0.51 MeV + 0.78 MeV

The emmited electron should have KE of 0.782 MeV.

Most have less than this.

What takes away the missing energy?

$$n = p + e^{-} + \bar{\nu}$$

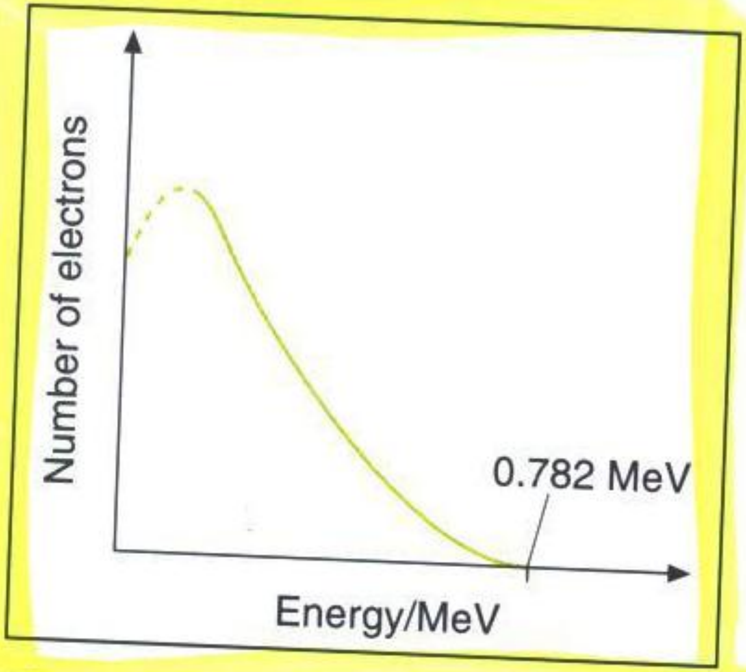


Figure NP10 The energy spectrum of β^{-} particles

The neutrino and antineutrino

β^- and β^+ decays are the first two indications that there are particles other than the proton, neutron and electron. They reveal the neutrino, ν, and the antineutrino, $\bar{\nu}$. There are other particles that make up matter.

electron + positron ⇒ energy of a photon

0.51 MeV + 0.51 MeV ⇒ 1.02 MeV

In the next section, you will meet the antiproton. This, like the proton, has a mass of 930 MeV. When proton meets an antiproton:

$p + \bar{p} \Rightarrow$ energy

930 MeV + 930 MeV ⇒ 1860 MeV

Binding energy

Another way of analysing the stability of atomic nuclei is to look at the energy involved in holding the nuclei together.

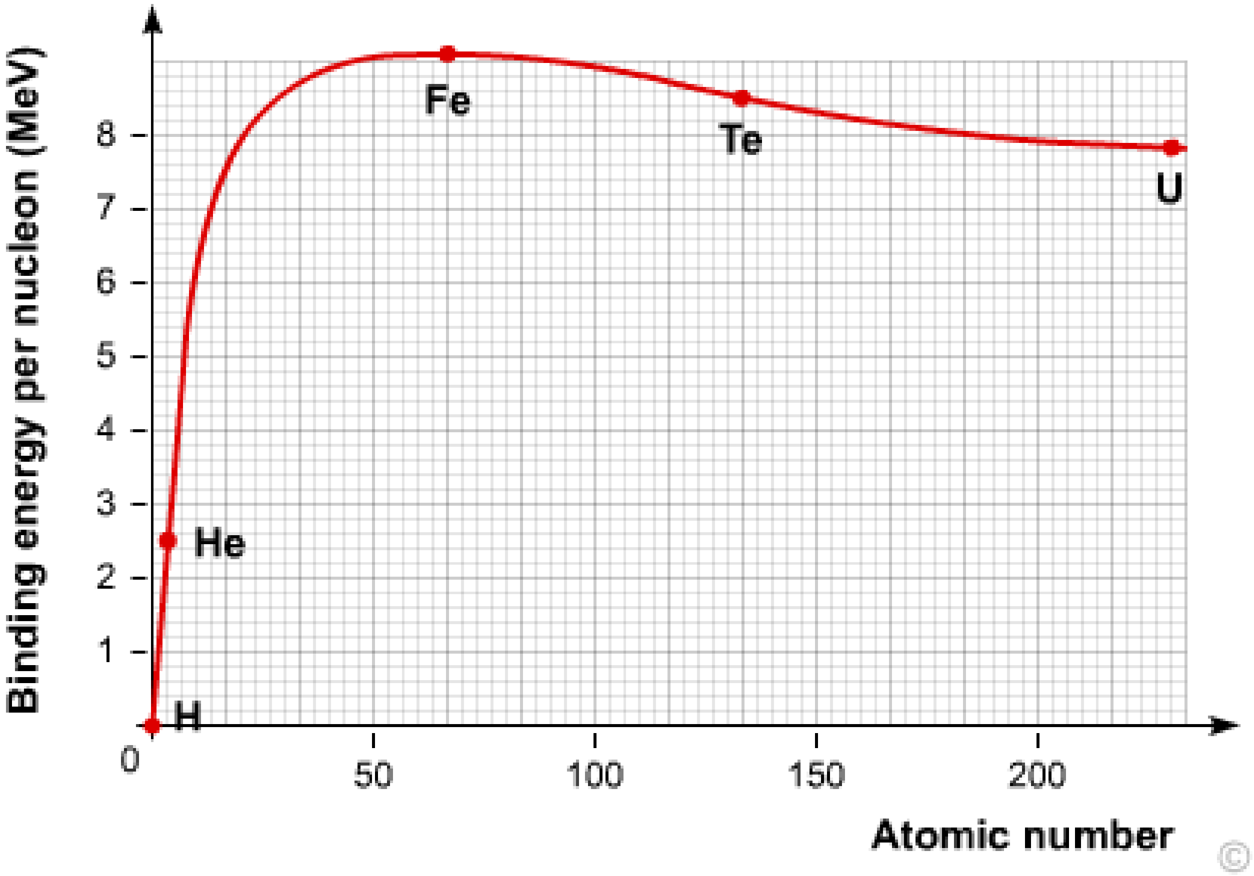
To assemble a nucleus by bringing its constituent nucleons together would require energy. When a nucleus is formed it is held together by what is called the **binding energy**. The source of this binding energy was explained by Albert Einstein, who examined the difference between the mass of the constituents of an nucleus and the mass of the nucleus itself. The mass of protons and neutrons are as shown below.

Mass of proton	Mass of neutron
1.6726×10^{-27} kg	1.6749×10^{-27} kg

In both the uranium-235 and caesium-138 isotopes the total mass of the constituent nucleons is greater than the measured mass of the nucleus. The difference between the total mass of the constituents and the measured mass of the nuclei is called the **mass defect** of the nucleus. The mass defect arises because there is a reduction in the total energy of the system when the nucleons combine to form a nucleus. This reduction in energy is equal to the binding energy of the system.

Binding energy per nucleon

The higher the number of nucleons in a particular nucleus, the larger the resulting binding energy simply because there are more nucleons. To compare the effect of the binding energy for different nuclei it is fairer to calculate the binding energy *per nucleon*. Higher values of binding energy per nucleon mean that those nuclei are more stable.

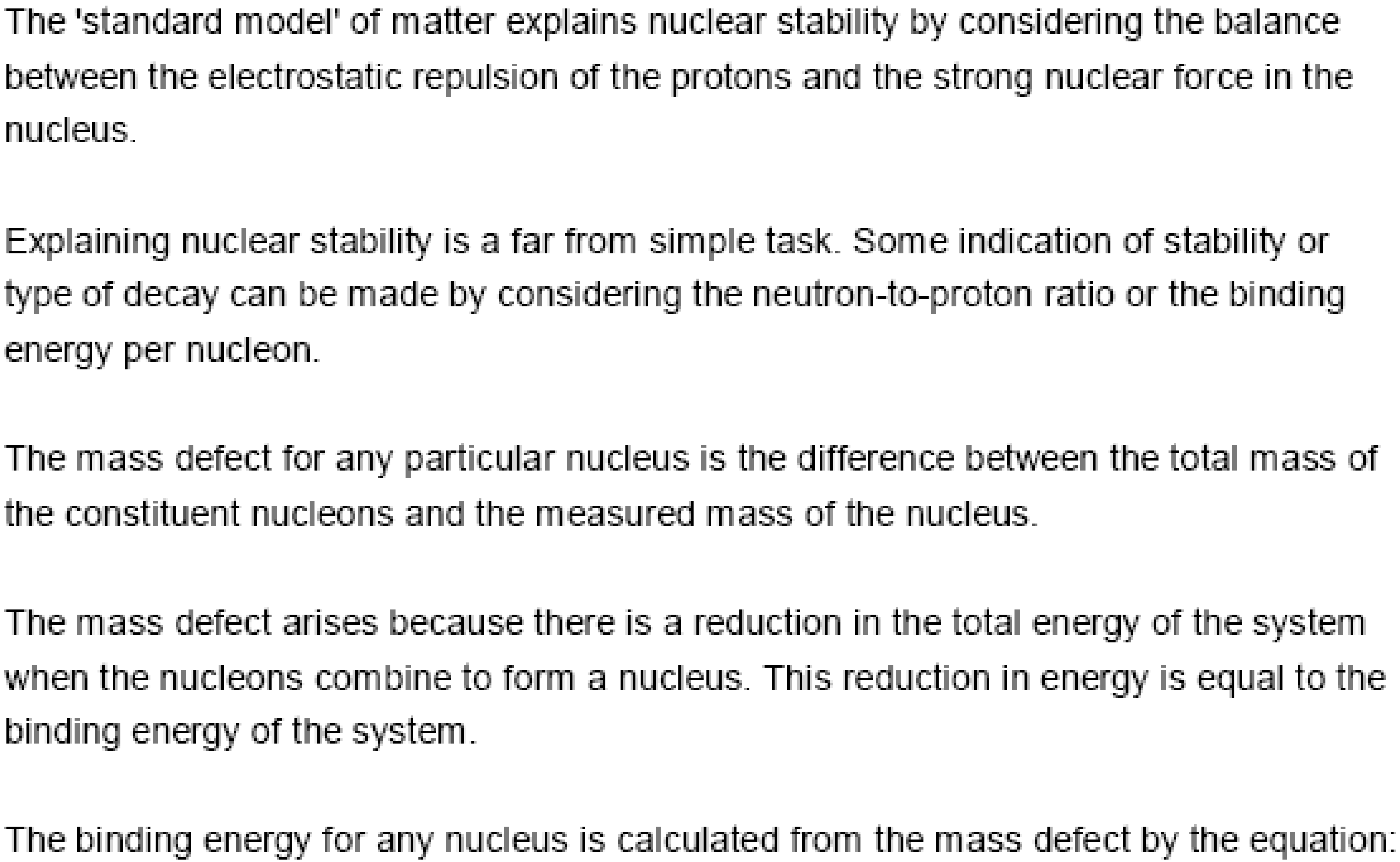


Nuclear units

So far in this unit we have considered masses in kilograms and energy in joules. These are the standard units for these physical quantities. However, the very small sizes of the quantities involved make the mathematical manipulation difficult, so physicists often use **atomic mass units** (amu) to describe the mass and **electron volts** (eV) to quantify energies on a nuclear scale.

	Mass	Energy
Unit	amu	eV
Conversion	1 amu = 1.66×10^{-27} kg	1 eV = 1.6×10^{-19} J

Summary



The 'standard model' of matter explains nuclear stability by considering the balance between the electrostatic repulsion of the protons and the strong nuclear force in the nucleus.

Explaining nuclear stability is a far from simple task. Some indication of stability or type of decay can be made by considering the neutron-to-proton ratio or the binding energy per nucleon.

The mass defect for any particular nucleus is the difference between the total mass of the constituent nucleons and the measured mass of the nucleus.

The mass defect arises because there is a reduction in the total energy of the system when the nucleons combine to form a nucleus. This reduction in energy is equal to the binding energy of the system.

The binding energy for any nucleus is calculated from the mass defect by the equation:

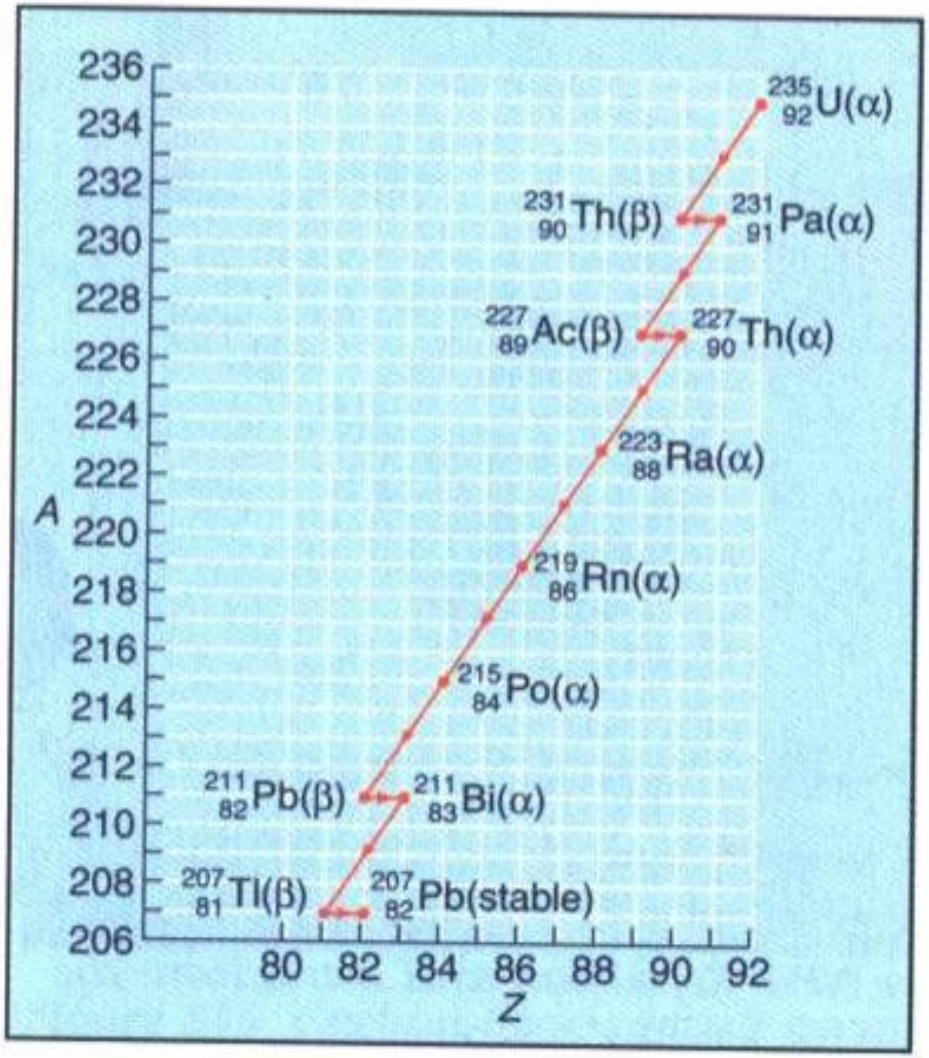


Figure NP7 Decay chain

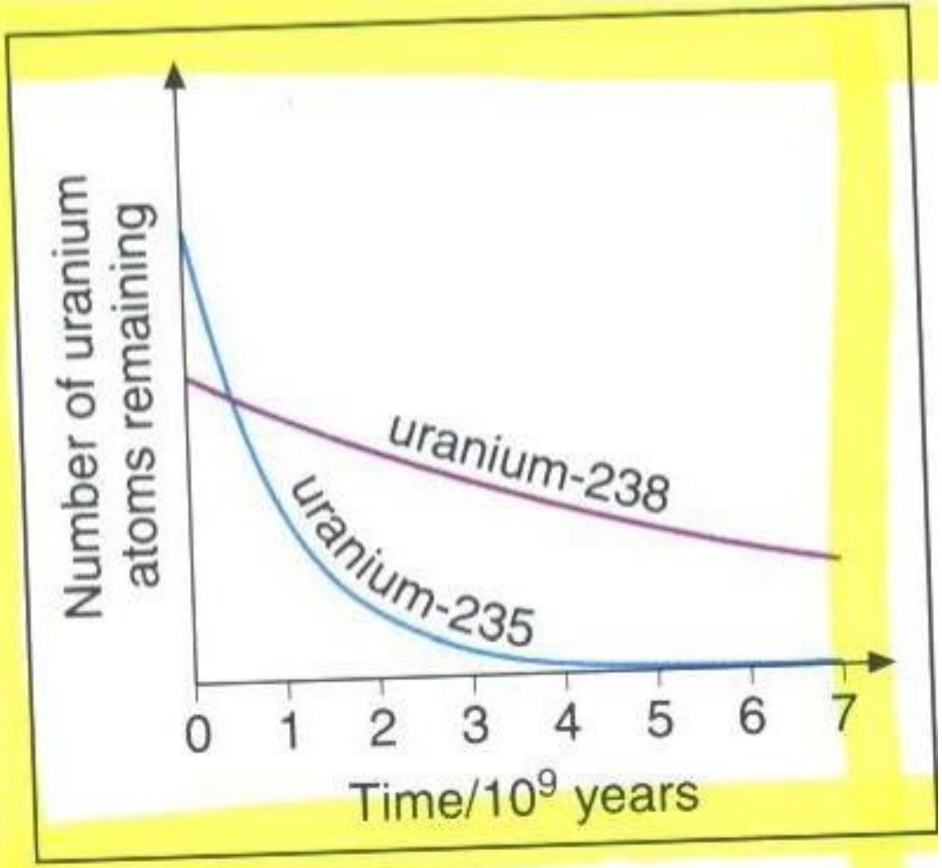


Figure NP9 ^{235}U has a shorter half-life than ^{238}U

Figure NP8 Carbon-14 dating showed that the shroud of Turin dates from the Middle Ages

Other types of radioactive dating can be used to estimate the age of older objects. Present astrophysical theories indicate that both ^{235}U and ^{238}U are produced in the ratio 0.7 : 1 when some stars (supernovae) explode. The ratio of these isotopes on Earth is 138 : 1. Figure NP9 shows why this ratio has increased with time.