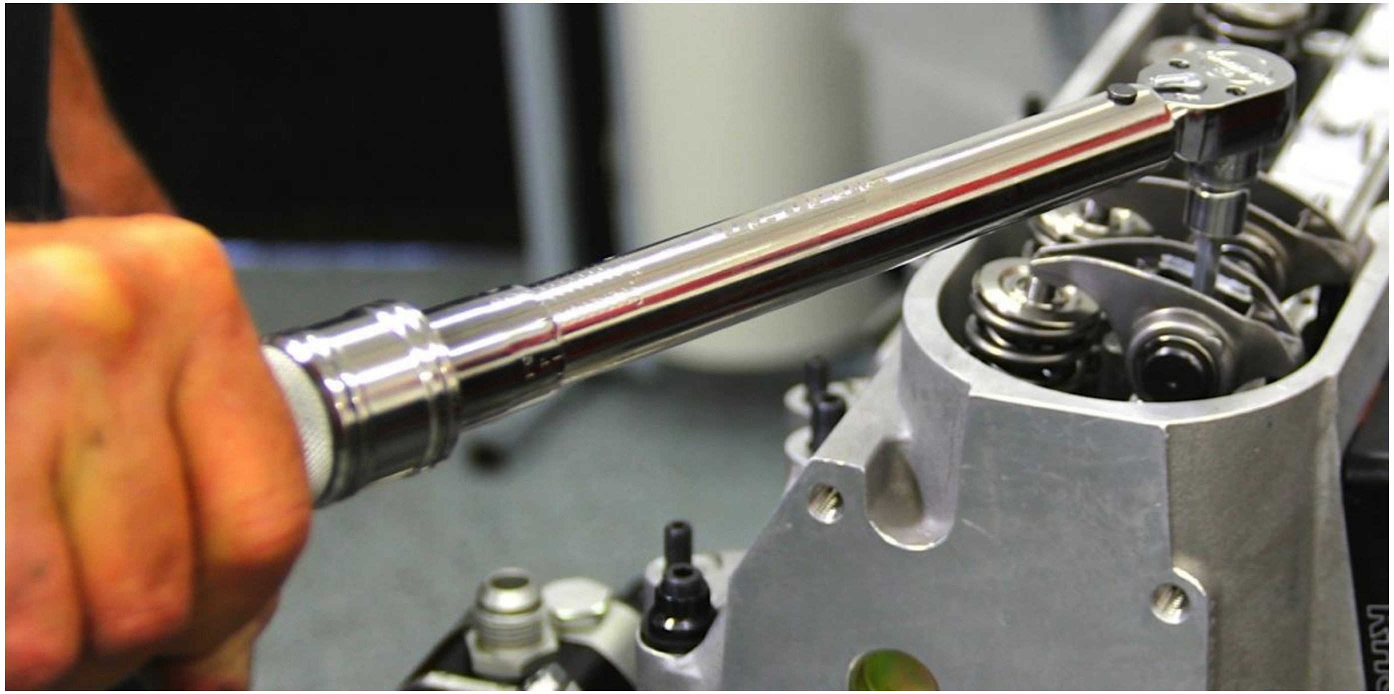
Dynamics of Rotational Motion

Chapter 10



10 June 2022

Goals for Chapter 10

- To study torque.
- To relate angular acceleration and torque.
- To examine rotational work and include time to study rotational power.
- To understand angular momentum.
- To examine the implications of angular momentum conservation.
- To study how torques add a new variable to equilibrium.
- To see the vector nature of angular quantities.

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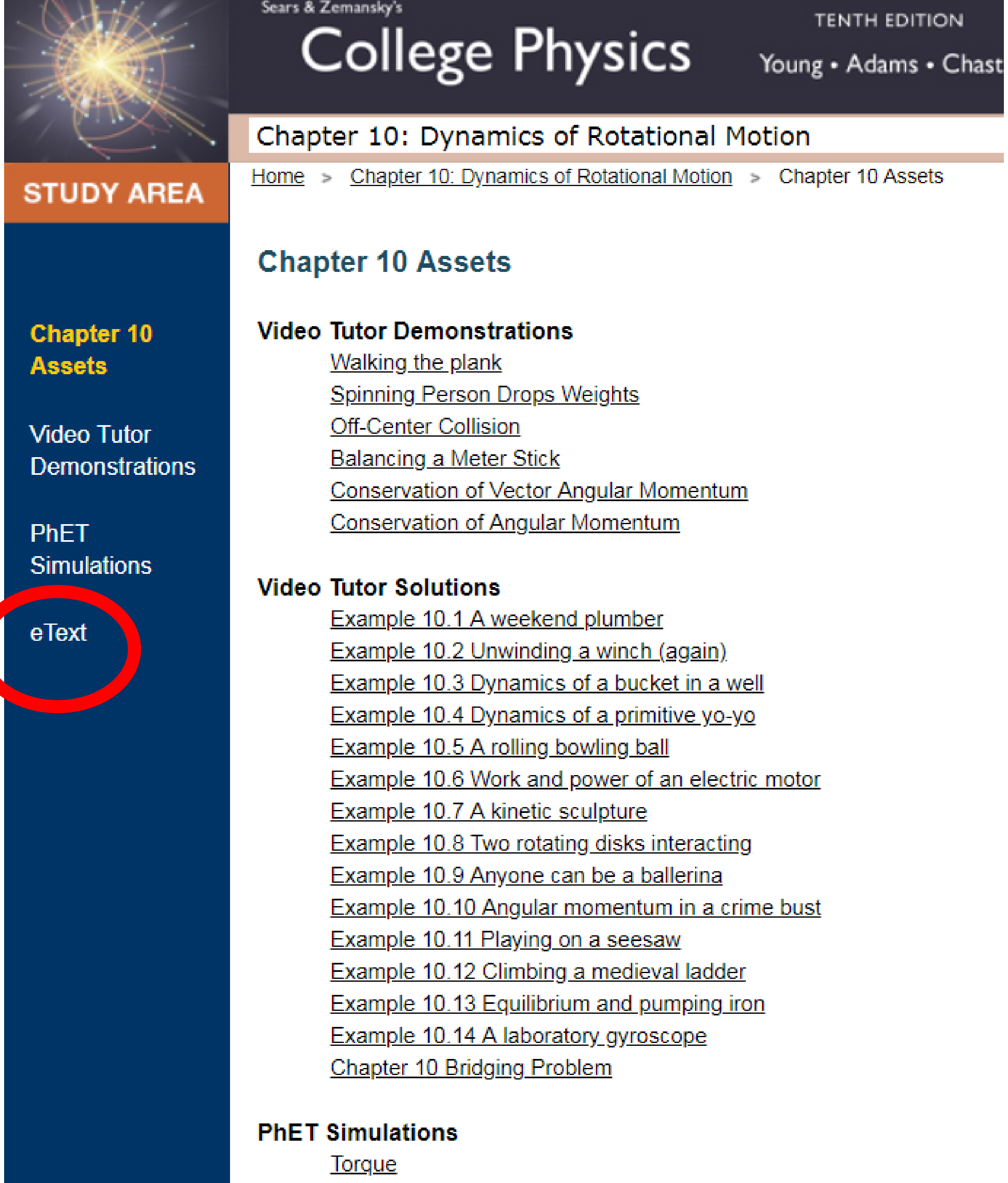
Use the study area as we go through the slides

VTD Walking the plank

VTS Ex 10.1

PhET Torque



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Chapter 10: Dynamics of Rotational Motion

STUDY AREA

Home > Chapter 10: Dynamics of Rotational Motion > Chapter 10 Assets

Chapter 10 Assets

Video Tutor Demonstrations

PhET Simulations

eText

Chapter 10 Assets

Video Tutor Demonstrations

- Walking the plank
- Spinning Person Drops Weights
- Off-Center Collision
- Balancing a Meter Stick
- Conservation of Vector Angular Momentum
- Conservation of Angular Momentum

Video Tutor Solutions

- Example 10.1 A weekend plumber
- Example 10.2 Unwinding a winch (again)
- Example 10.3 Dynamics of a bucket in a well
- Example 10.4 Dynamics of a primitive yo-yo
- Example 10.5 A rolling bowling ball
- Example 10.6 Work and power of an electric motor
- Example 10.7 A kinetic sculpture
- Example 10.8 Two rotating disks interacting
- Example 10.9 Anyone can be a ballerina
- Example 10.10 Angular momentum in a crime bust
- Example 10.11 Playing on a seesaw
- Example 10.12 Climbing a medieval ladder
- Example 10.13 Equilibrium and pumping iron
- Example 10.14 A laboratory gyroscope
- Chapter 10 Bridging Problem

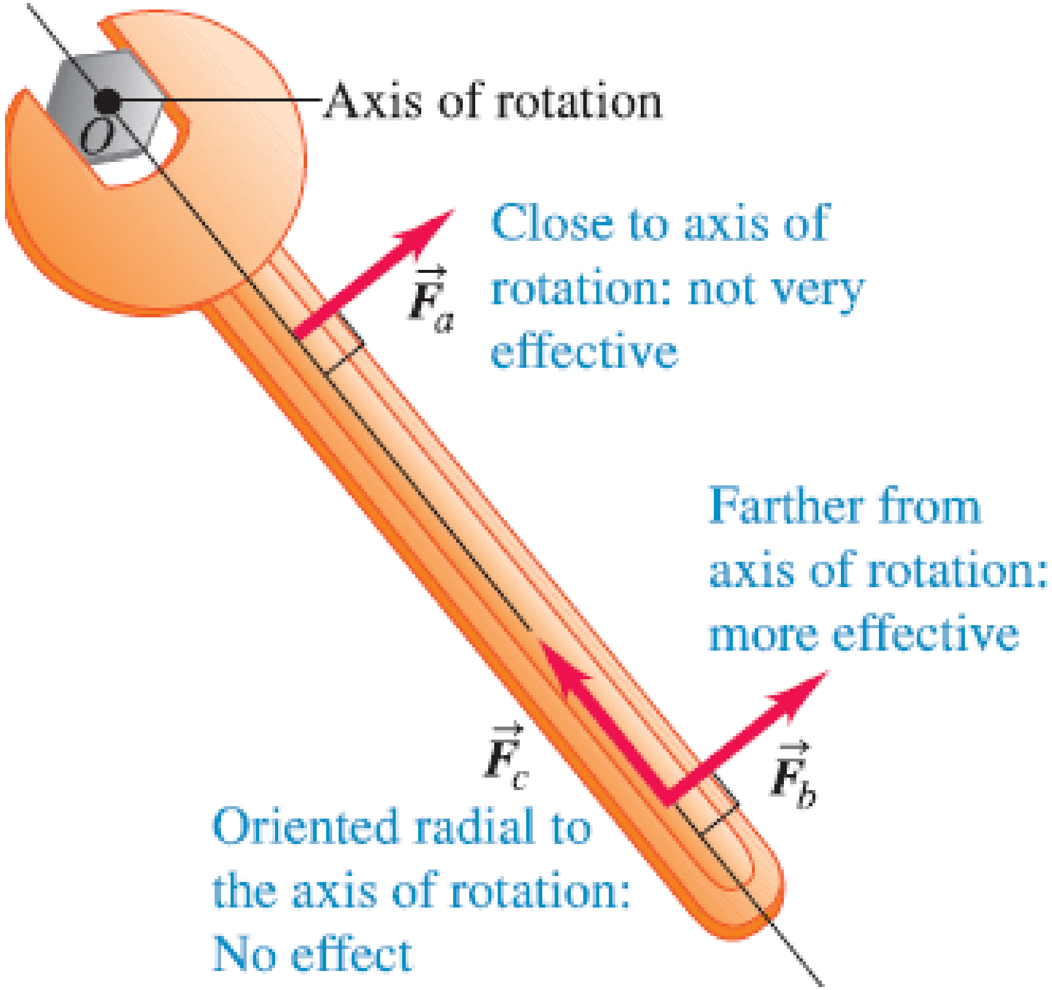
PhET Simulations

- Torque

Torque (a.k.a. moment) of a force

Torque (a.k.a. moment) of a force

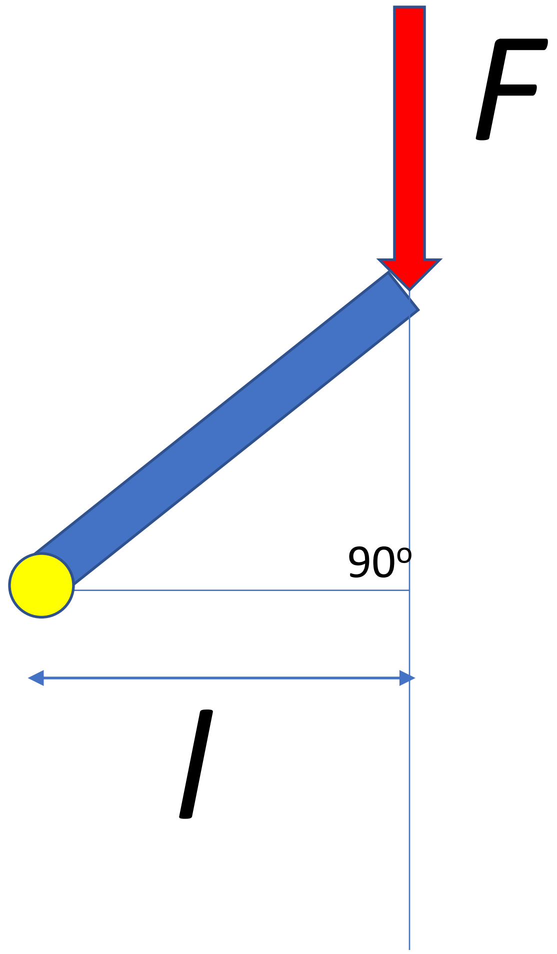
$$\tau = Fl$$



Torque

The tendency of a force to cause or change rotational motion about a certain axis.

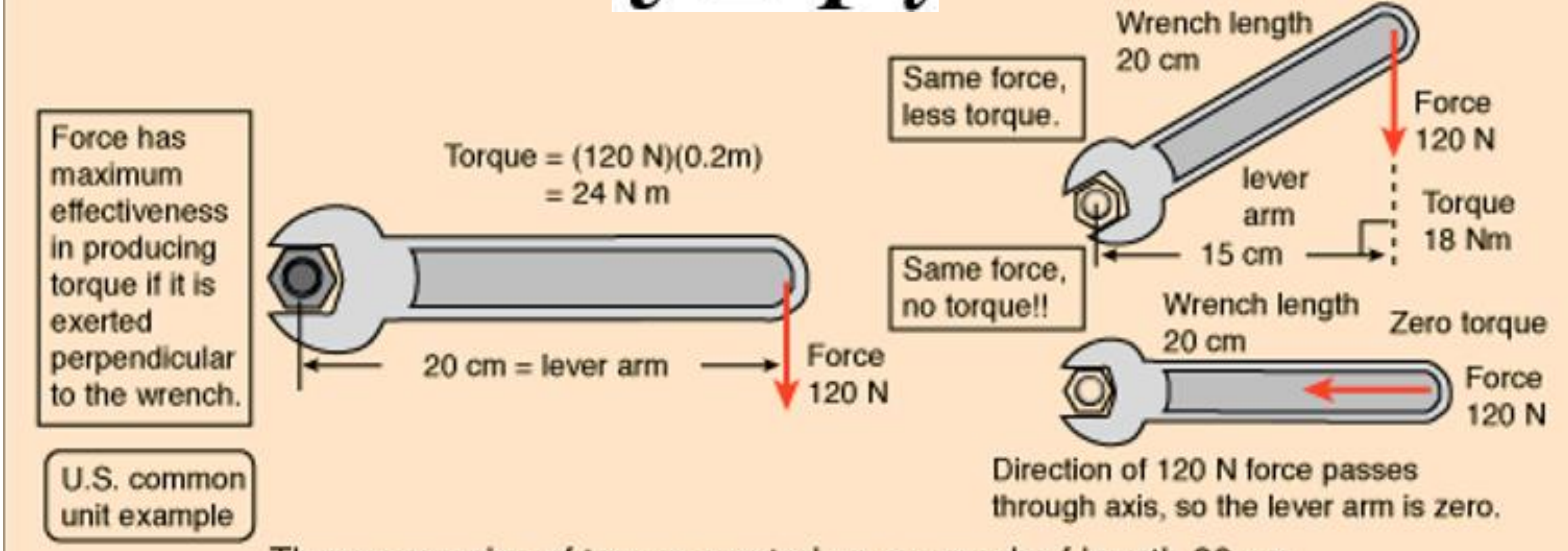
$$\tau = Fl$$



- **l is called the lever arm. This is the perpendicular distance from the line of force to the axis of rotation.**
- If Force and distance to axis are perpendicular, then its just Fl.
- Positive sense of rotation is counter-clockwise

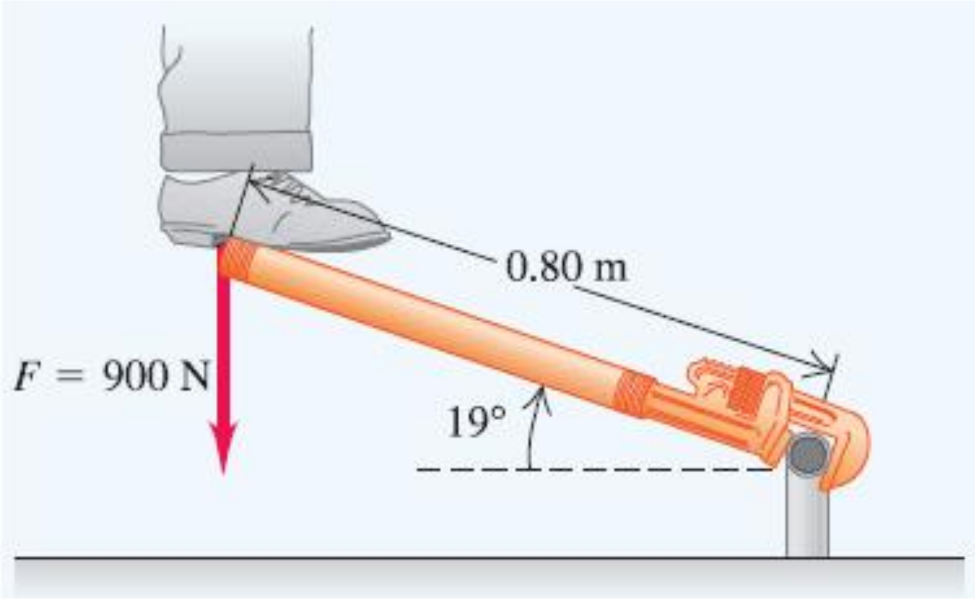
The lever arm is the perpendicular distance from the line of force to the axis of rotation.

$$\tau = Fl$$

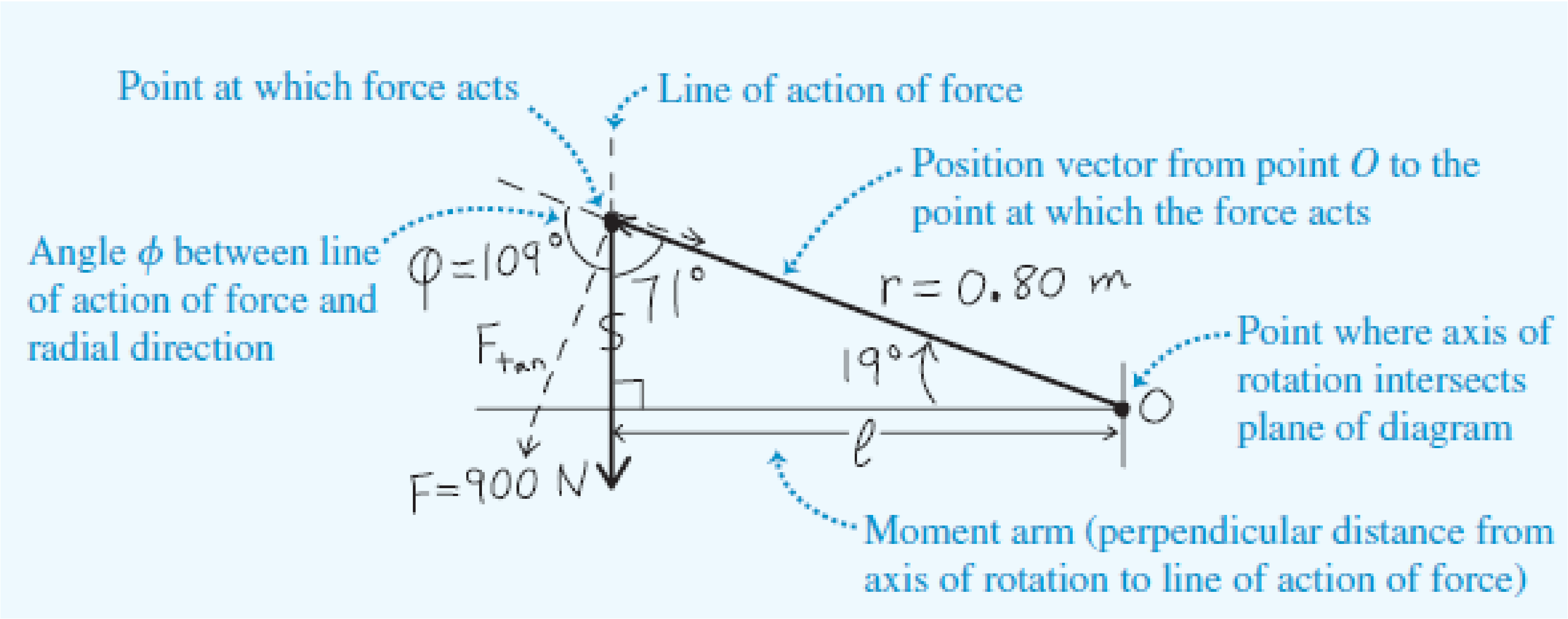


Three examples of torque exerted on a wrench of length 20 cm.

Calculate the torque acting on the pipe shown due to the 900-N force.



VTS Ex 10.1



Torque = Force x perpendicular Distance

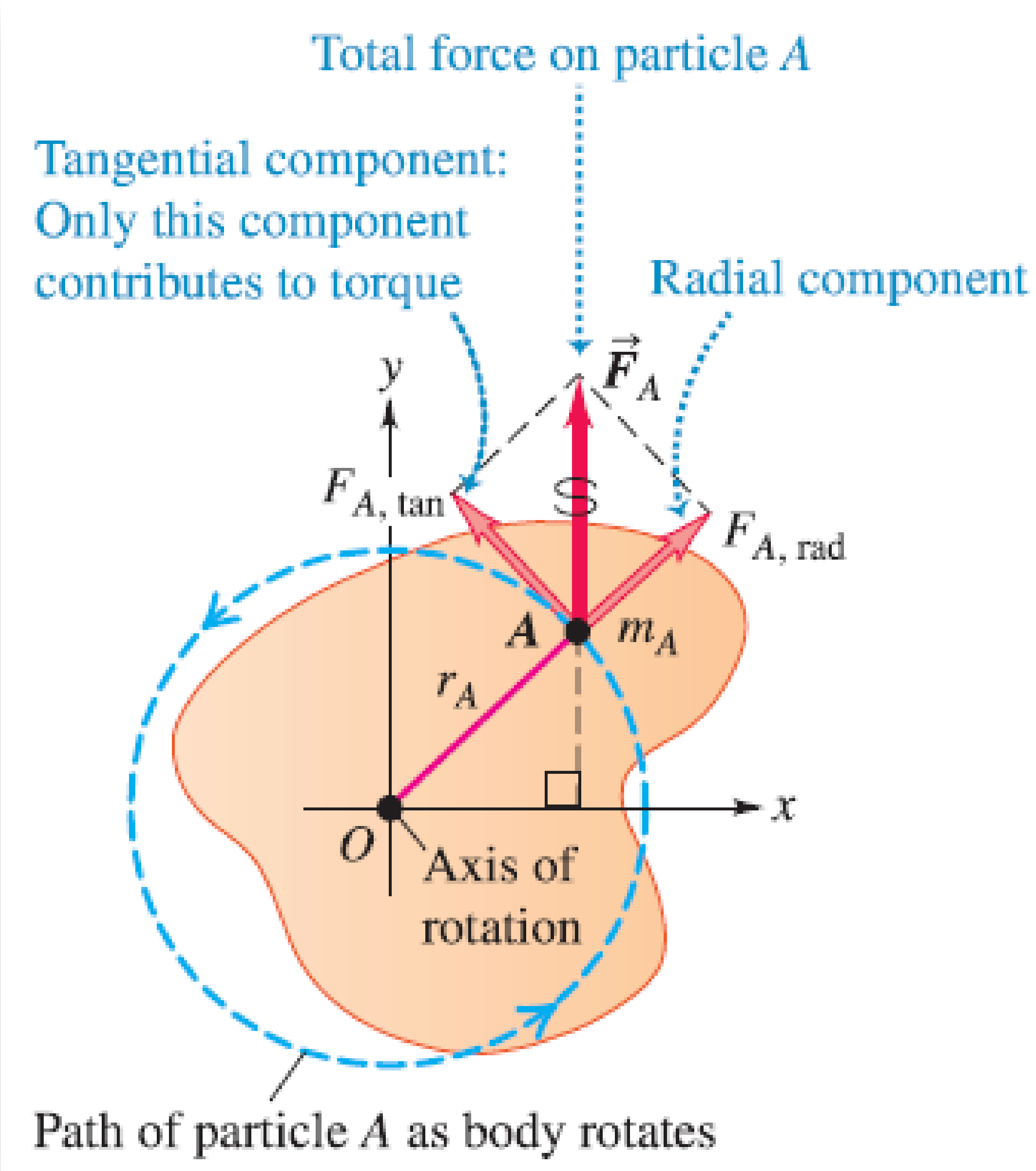
= 900 x (lever (or moment arm))

= 900 * L

= 900 * 0.8 cos 19

= 680 Nm

Torque and Angular Acceleration



Torque and Angular Acceleration

- For rotational motion: $F_{tan} = ma_{tan.}$ (Newton's 2nd Law)
- But $a_{tan} = r * \alpha$,
- Thus $F_{tan} = m\ \ r\ \alpha$
- $F_{tan}\ r = m\ \ r\ \alpha\ r$
- $F_{tan}\ r = \tau = (m * r^2)\ \alpha = I\ \alpha$
- Since: $I = m * r^2$ (Which is what we had before)

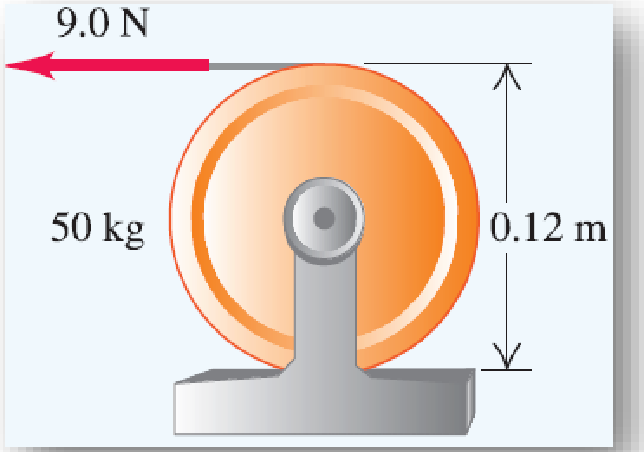
Torque and Angular Acceleration

$$\Sigma\tau = I\alpha$$

Net Torque or Sum of all the Torque is equal to the
Moment of Inertia * Angular Acceleration.
REMEMBER: Only F_{tan} contributes to this and not F_{rad}

A cable is wrapped several times around a uniform solid cylinder with diameter 0.12 m and mass 50 kg that can rotate freely about its axis.

The cable is pulled by a force with magnitude 9.0 N. If the cable unwinds without stretching or slipping, find the magnitude of its acceleration.



VTS Ex 10.2

Solution…

- The Force is at a tangent on the cylinder; thus the Moment Arm is the radius = 0.06m. The Torque is the Force * Distance:

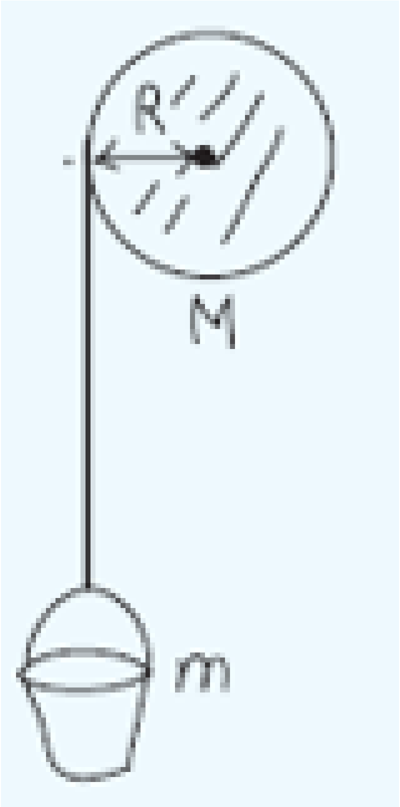
Torque = F * d = 9.0 * 0.06 = 0.54 Nm

The MoI_{Disc} = ½ mr^2 = ½ * 50 * 0.06^2 = 0.09 kgm^2

Angular Accel. = $\alpha = \tau / I$ = 0.54 / 0.09 = 6.0 rad/s^2

Linear Acceleration = a = R * α = 0.06 * 6.0 = 0.36m/s^2

You will get these equations in the exam. They are used **JUST** for the falling rock on rotating winch. See end of slides for derivation.



Falling mass (m) from solid cylinder (M) only

$$a = \frac{g}{1 + \frac{M}{2m}}$$

Linear acceleration

$$\alpha = \frac{a}{r}$$

angular acceleration

Work, Energy and Torque

$$W = \tau \Delta \theta$$

The flywheel of a motor has a mass of 300.0 kg and a moment of inertia of 580 kg m^2. The motor develops a constant torque of 2000.0 N m, and the flywheel starts from rest.

(a) What is the angular acceleration of the flywheel?

(b) How much work is done by the motor during the first 4.00 revolutions?

(c) What is its angular velocity after it makes 4.00 revolutions?

First we calculate the angular acceleration:

$\tau = I\,\alpha$, $I = 580\ kgm^2$ and $\tau = 2000N$.

Therefore: $\alpha = \frac{\tau}{I} = \frac{2000}{580} = 3.45\ rad\ s^{-2}$

Next the Work Done:

$W = \tau\,\Delta\theta = 2000 * 8\pi = 16{,}000\pi$ or 50,625.48 Joules

Solution cont…

- Angular Velocity after the 4 revolutions:

As ω changes with time we could solve that way…..but we have 2 ways in reality with equations of motion…..

$\Delta\theta = \omega_0 t + \frac{1}{2}\alpha t^2$ *or* $\omega^2 = \omega_o{}^2 + 2\alpha\Delta\theta$

$8\pi = 0 + \frac{1}{2} * 3.45 * t^2$ *or* $\omega^2 = 0 + (2 * 3.45 * 8\pi)$

$\omega^2 = 173.416$

t = ***3.817*** seconds. $\omega = 13.17\ rads^{-1}$

Now: $\omega = \omega_0 + \alpha t$

$\omega = 0 + 3.45 * 3.817$

$\omega = 13.17 rads^{-1}$

Both Calculations give the Same Answer !!

There is more than one way to solve section c
A student in a class looked at energy considerations.
Work done = energy transferred.
The energy transferred is the KE.

$$W = \frac{1}{2} I\omega^2$$

$$50{,}625 = \frac{1}{2}\ \ 580\ \ \omega^2$$

So $\omega = 13.2$ rad s^{-1}

Angular Momentum of a rigid body

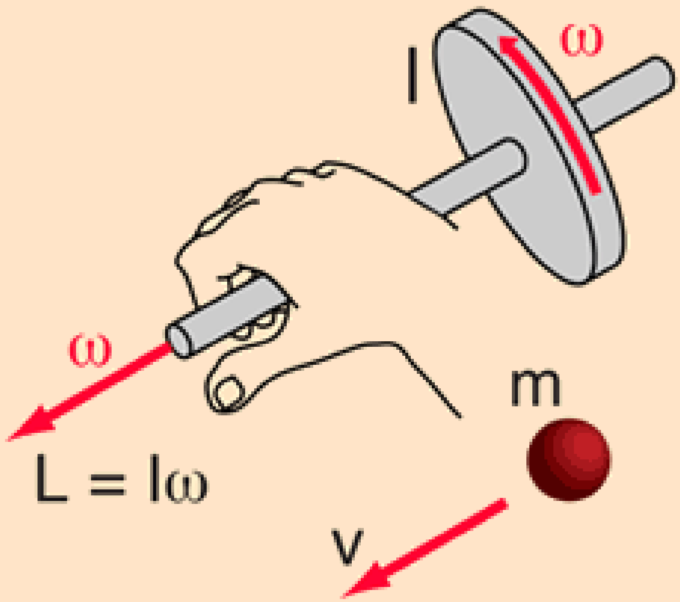
When a rigid body with moment of inertia I (with respect to a specified symmetry axis) rotates with angular velocity ω about that axis, the angular momentum L of the body with respect to the axis is

the product of the moment of inertia I about the axis and the angular velocity ω.

$$L = I\omega$$

Angular and Linear Momentum

Angular momentum and linear momentum are examples of the parallels between linear and rotational motion. They have the same form and are subject to the fundamental constraints of conservation laws, the conservation of momentum and the conservation of angular momentum .



Angular Momentum	=	Moment of Inertia	x	Angular Velocity
L	=	I	x	ω
Linear Momentum	=	Mass	x	Velocity
p	=	m	x	v

The X implies simple multiplication here.

Torque and Angular Momentum

We can now combine equations to make new ones

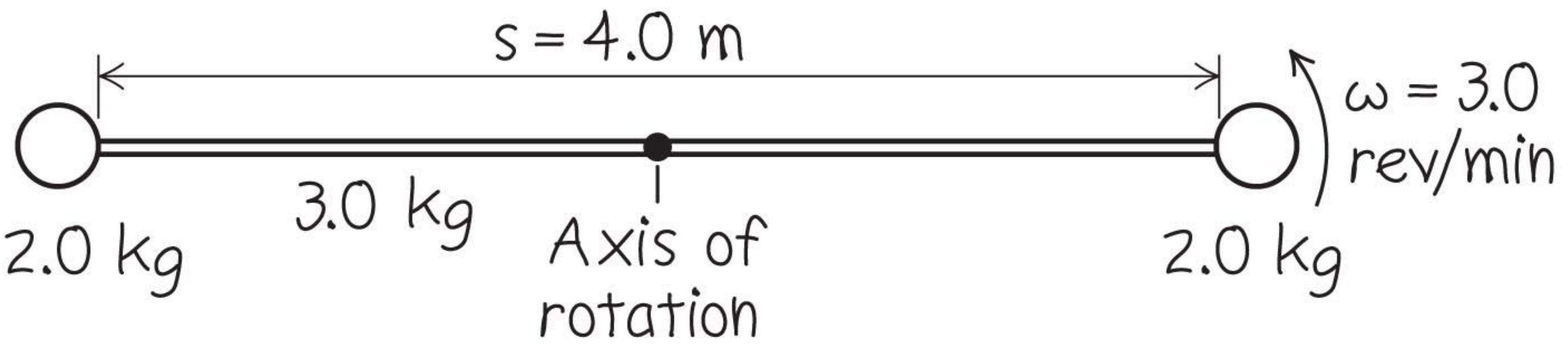
$$\sum \tau = I\alpha = I\frac{\Delta\omega}{\Delta t} = \frac{\Delta L}{\Delta t}$$

(This can also be done for the linear equations $\sum F = ma = m\frac{\Delta v}{\Delta t} = \frac{\Delta p}{\Delta t}$)

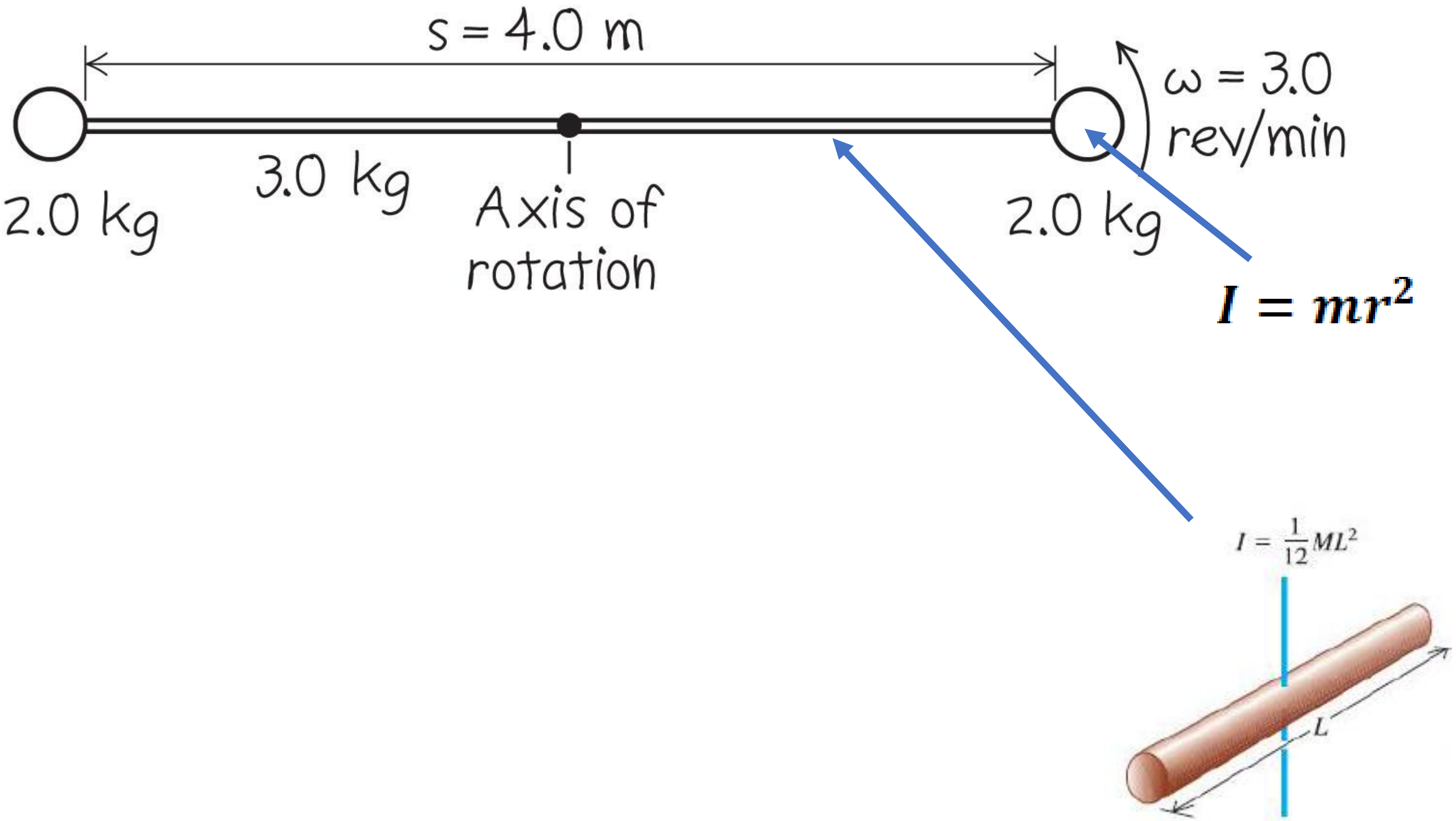
Mobile Example

VTS Ex 10.7

A part of a mobile suspended from the ceiling of an airport terminal building consists of two metal spheres, each with mass 2.0 kg, connected by a uniform metal rod with mass 3.0 kg and length of 4.0 m. The assembly is suspended at its midpoint by a wire and rotates in a horizontal plane, making 3.0 revolutions per minute. Find the angular momentum and kinetic energy of the assembly.



Find the angular momentum (L) and kinetic energy of the assembly.



Solution…

- First we have to find the MoI through the midpoint of this assembly.

$$L = I\omega \qquad \text{and} \qquad K.E._{ang} = \tfrac{1}{2} I \omega^2$$

So firstly for the spheres (treat as point particles):

$I_{sphere} = mr^2 = 2 * 2^2 = 8\ kgm^2$

So for 2 spheres we have $2 * 8 = 16\ kgm^2$

Now the rod:

$I_{rod} = 1/12\ ML^2 = 1/12 * 3 * 4^2 = 4\ kgm^2$

So the grand total of MoI is: $16 + 4 = 20\ kgm^2$

Convert to Rads: $\omega = 3rpm * (2\pi/60) = 0.31 rad/s$

Solution continued…

- Now we can simply use the Angular Momentum and Angular K.E. Equations…

$$\boldsymbol{Momentum_{ang} = L = I\omega = 20 * 0.31 = 6.2\ kgm^2s^{-1}}$$

$$\boldsymbol{K.E._{ang} = \tfrac{1}{2}\, I\, \omega^2 = \tfrac{1}{2} * 20 * 0.31^2 = 0.96\ kgm^2s^{-2}}$$

Angular Momentum Is Conserved

- The first figure shows the figure skater with a large moment of inertia.
- In the second figure, she has made the moment much smaller by bringing her arms in.
- Since *L* is constant, ω must increase.

VTD Conservation of angular momentum

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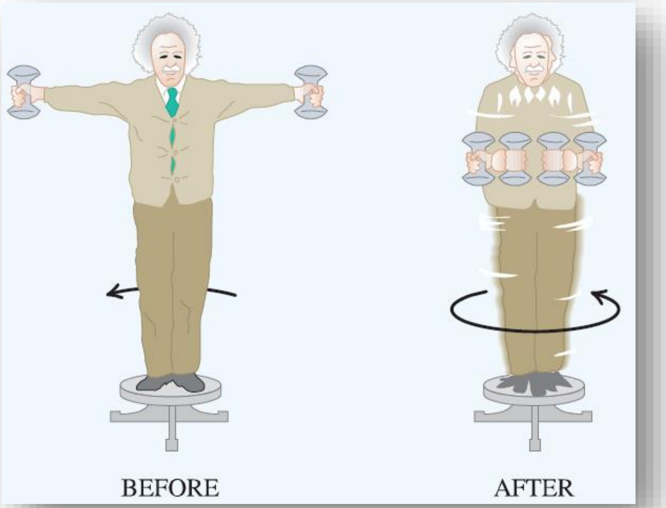
Conservation of angular momentum (L)

The **total** angular momentum of an *isolated system* is constant.

$$\sum L_i = \sum L_f$$

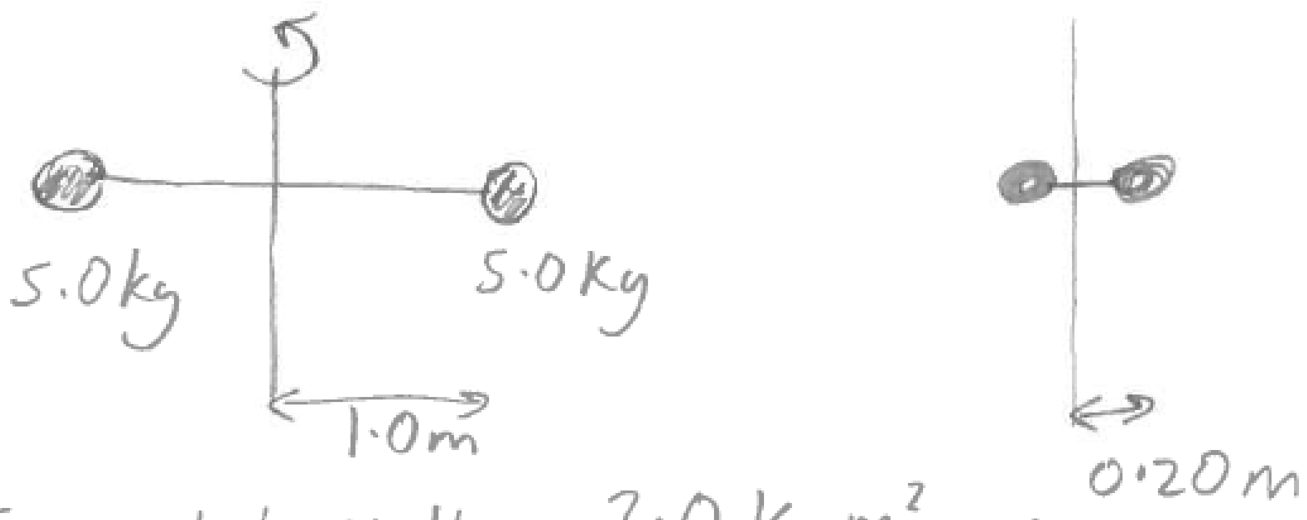


An acrobatic physics professor stands at the center of a turntable, holding his arms extended horizontally, with a 5.0 kg dumbbell in each hand. He is set rotating about a vertical axis, making one revolution in 2.0 s.
His moment of inertia (without the dumbbells) is 3.0 kg m^2 when his arms are outstretched, and drops to 2.2 kg m^2 when his arms are pulled in close to his chest.
The dumbbells are 1.0 m from the axis initially and 0.20 m from it at the end. Find the professor's new angular velocity if he pulls the dumbbells close to his chest, and compare the final total kinetic energy with the initial value.

VTS Ex 10.9

ANS $\omega_f = 5\,\pi$ rad/s

T = period = 2·0 s



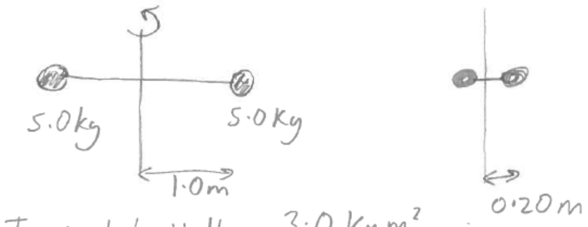
MoI = I without dumbbell outstretched = 3·0 kg m^2

I without dumbbell close to chest = 2·2 kg m^2

Find New angular velocity when close to chest

T = period = 2.0 s

one revolution in 2.0 s.



no I = I without dumbbell outstretched $= 3.0\ kg\,m^2$

I without dumbbell close to chest $= 2.2\ kg\,m^2$

Find New angular velocity when close to chest

Solution…

- We should know that by Conservation of Momentum:

$$\mathbf{L} = \boldsymbol{Ii\omega_i} = \boldsymbol{If\omega_f}$$

For each case we need I_{total} so we do this for both cases:

$$I_{total} = I_{professor} + I_{dumbells}$$

$$I_{initial} = 3 + (2 \ * 5 * 1^2) = 13\ kgm^2$$

$$I_{final} = 2.2 + (2 \ * 5 * 0.2^2) = 2.6\ kgm^2$$

$\boldsymbol{\omega_i}$ = 1 Revolution in 2 seconds = $\frac{\mathbf{2\pi}}{\mathbf{2}}$ = π rad/s

We Now Use The Conservation of Momentum…..

Solution Cont…

- Using out initial equation…

$$I_i\omega_i = If\omega_f$$

$$13 \ * \ \pi = 2.6 \ * \ \omega_f$$

$$13\ \pi\ /2.6 = \omega_f = 5\ \pi \text{ rad/s or } 15.71 \text{ rad/s}$$

This is an Angular Velocity increase of **5 times** !!

For the Angular Kinetic Energy:

$$\text{K.E.}_{\text{initial}} = ½\ I_i\omega_i^2 = ½ * 13 * \pi^2 = 64\ Joules$$

$$\text{K.E.}_{\text{final}} = ½\ I_f\omega_f^2 = ½ * 2.6 * 5\pi^2 = 320\ Joules$$

Where has this Gain in Angular K.E. come from ?

Equilibrium of a rigid body

First Condition for Equilibrium

Translational Equilibrium

$$\Sigma\vec{F} = 0$$

$$\Sigma F_x = 0$$

$$\Sigma F_y = 0$$

Rotational Equilibrium

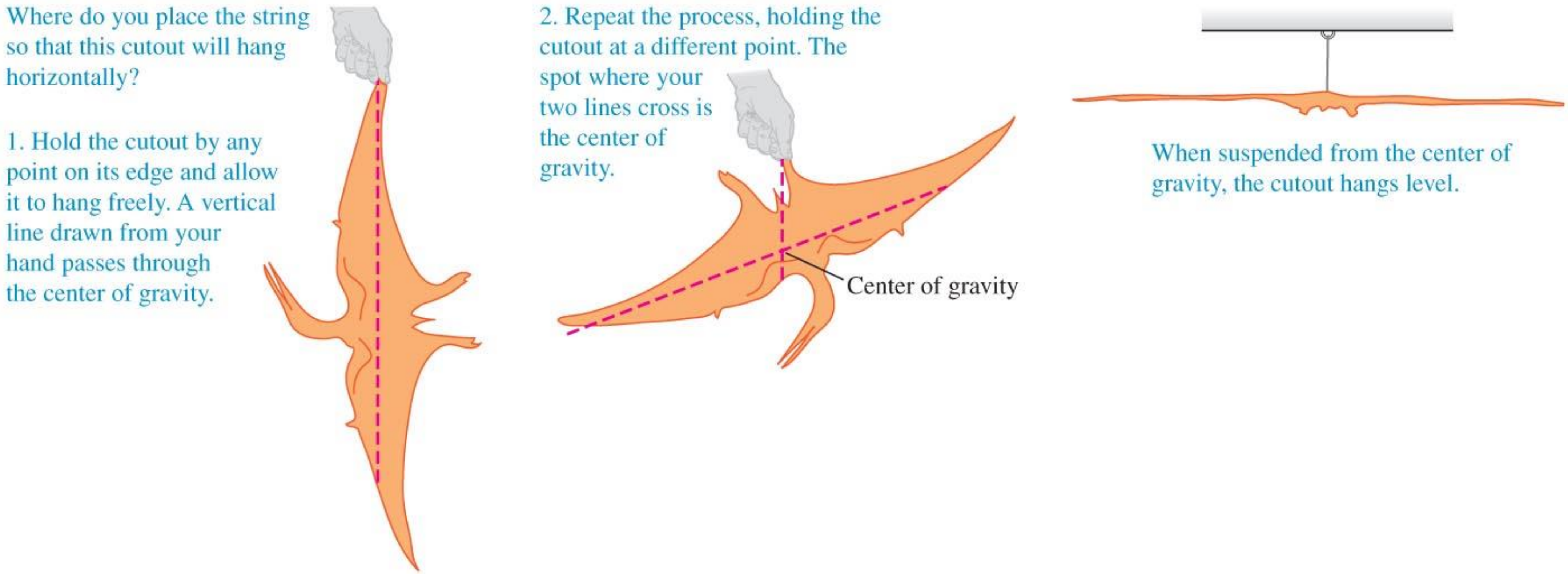
Second Condition for Equilibrium

$$\Sigma\tau = 0$$

The sum of the torques due to all forces acting on the body, with respect to ANY axis must be zero

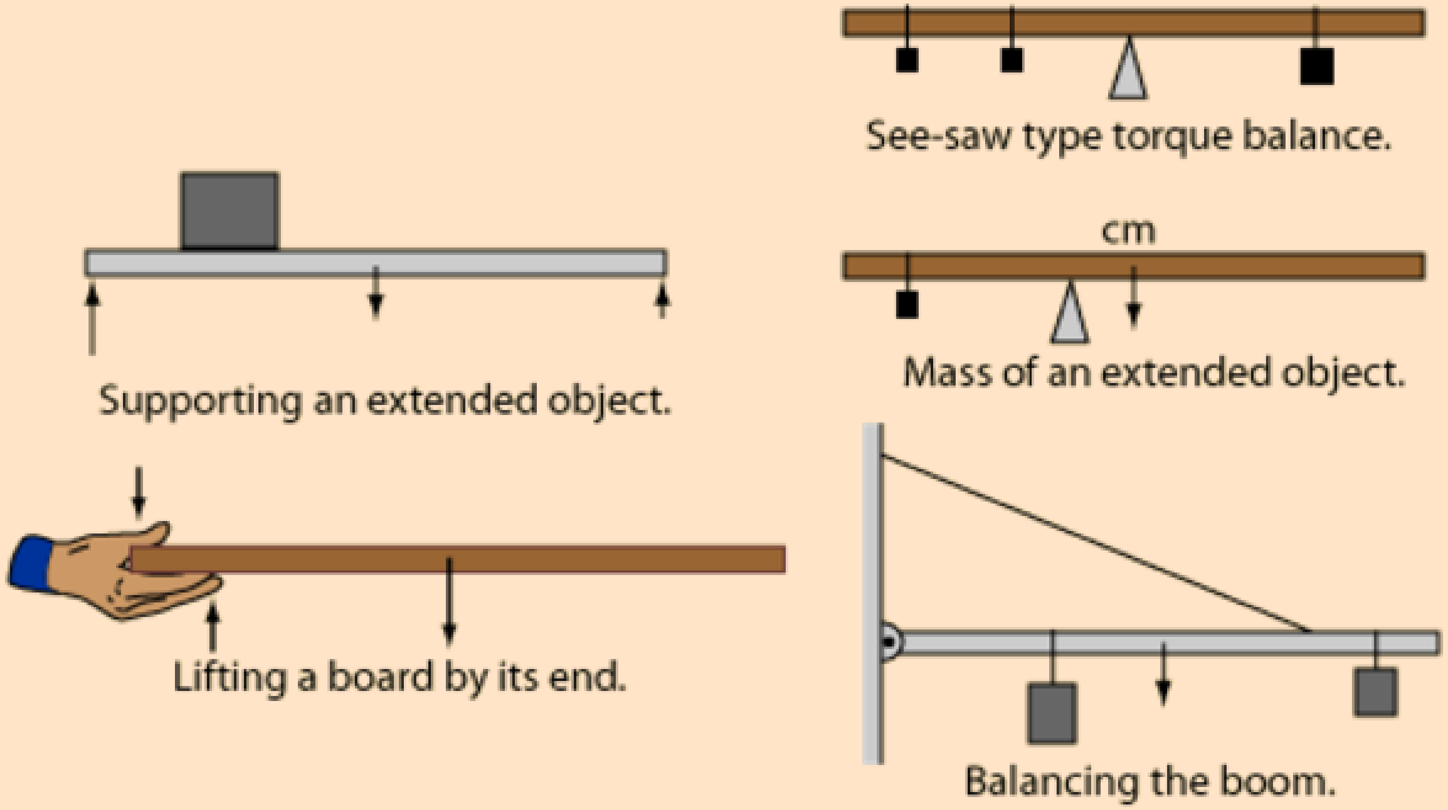
Center of gravity

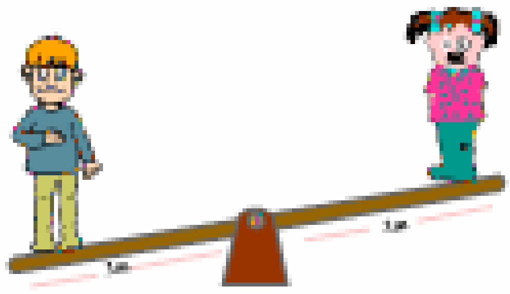
$$\Sigma F_x = 0 \text{ and } \Sigma F_y = 0 \qquad \Sigma\tau = 0.$$

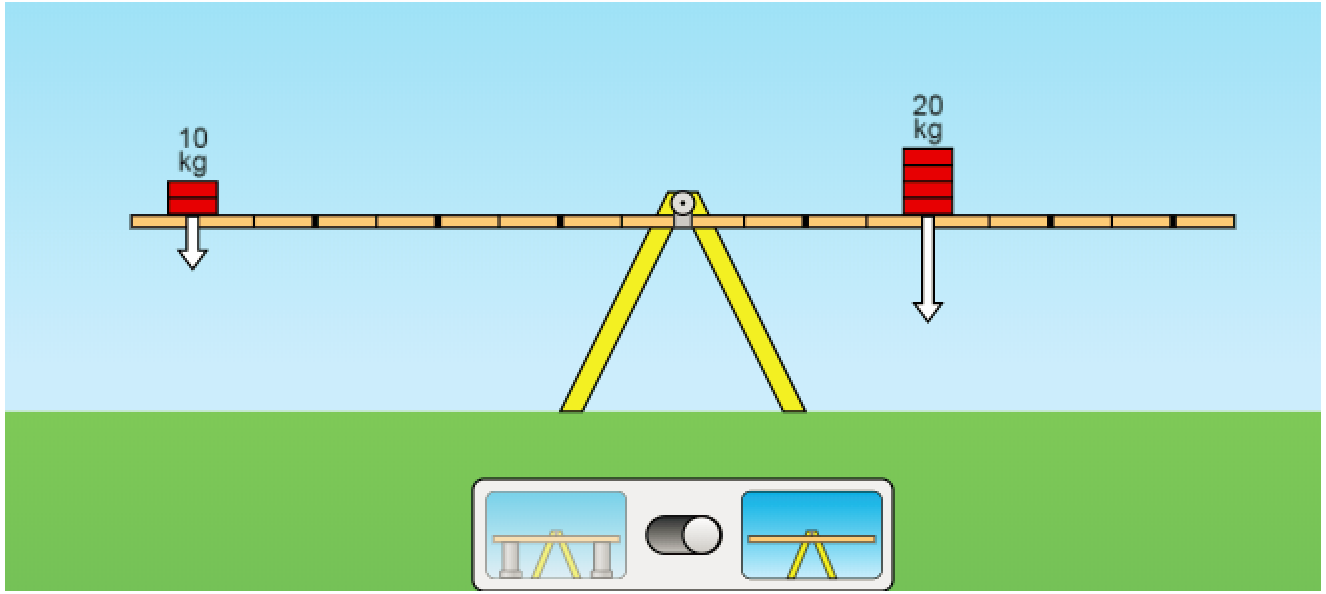


Torque Equilibrium Examples

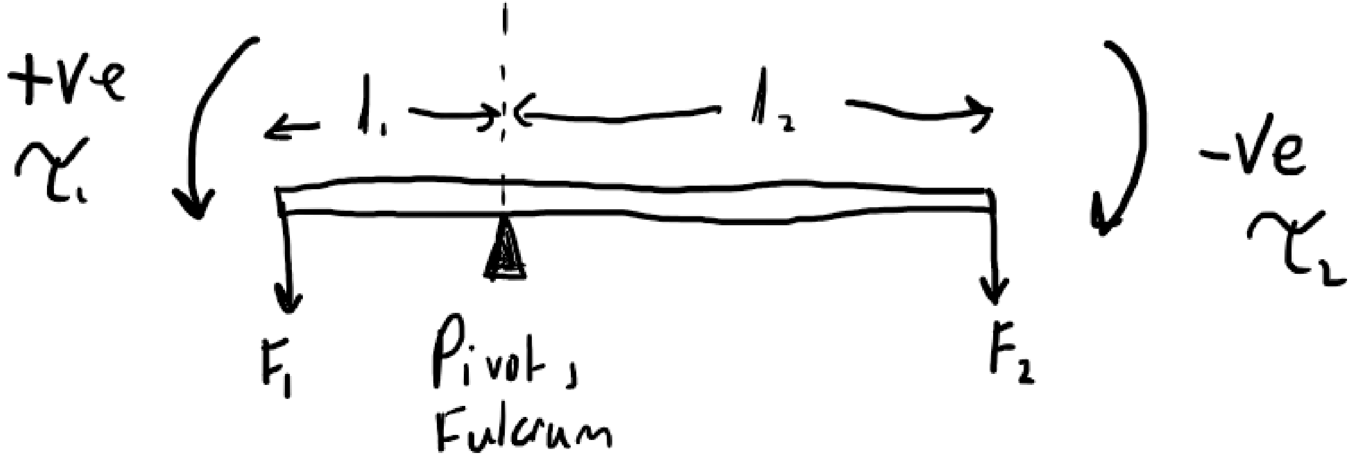
Most equilibrium problems require the application of force as well as torque for their solution, but the examples below illustrate equilibrium of torque.







https://phet.colorado.edu/sims/html/balancing-act/latest/balancing-act_en.html



In equilibrium (balanced)

$$\sum \tau = 0$$

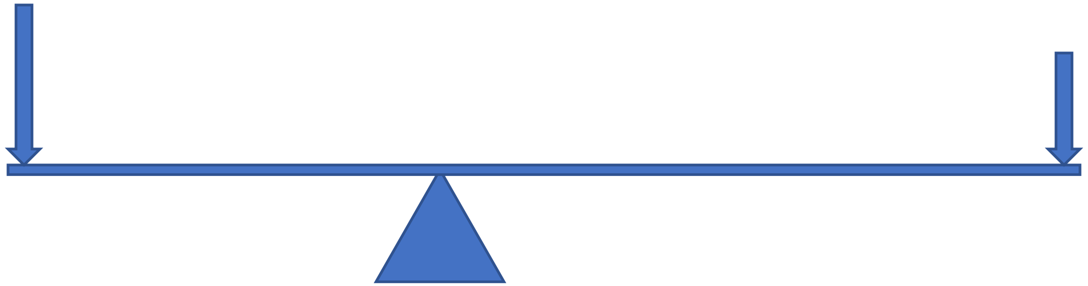
$$\tau_1 + \tau_2 = 0$$

$$(+F_1 l_1) + (-F_2 l_2) = 0$$

$$F_1 l_1 = F_2 l_2$$

$$\tau \circlearrowleft = \tau \circlearrowright$$

You and a friend play on a seesaw. Your mass is 90 kg, and your friend's mass is 60 kg. The seesaw board is 3.0 m long and has negligible mass. Where should the pivot be placed so that the seesaw will balance when you sit on the left end and your friend sits on the right end?

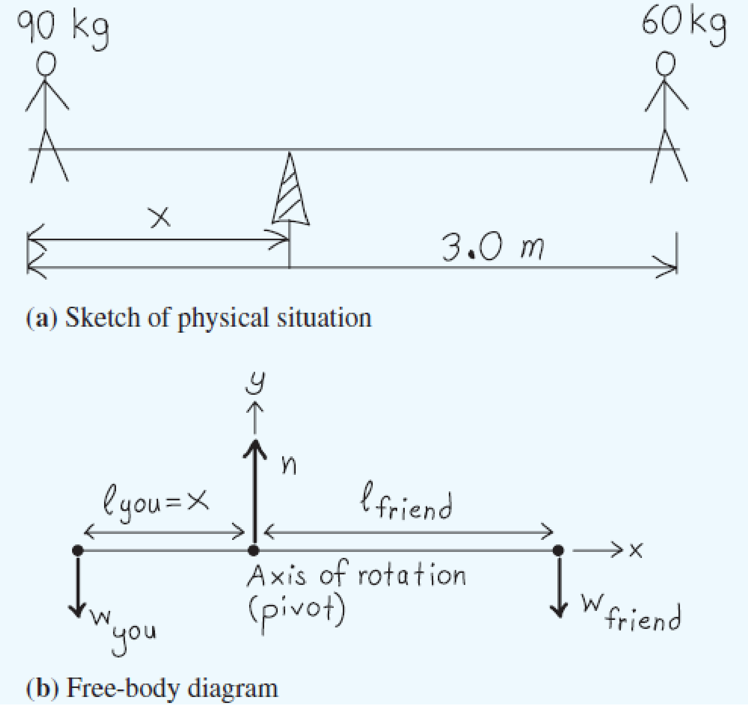


VTS Ex 10.11

ANS 1.2 meters from the left end

Solution…

A small diagram will help us here:



(a) Sketch of physical situation

(b) Free-body diagram

First we need the Torque

for ourselves:

τ_{me} = **Mass * Gravity * Distance**

τ_{me} = **90 * g * x**

Now we need to do the same for our friend:

τ_{friend} = **60 * g * (3.0 – x)**

In Equilibrium, $\tau_{me} + \tau_{friend} = 0$ or $\tau_{me} = -\tau_{friend}$

Solution cont…

$\tau_{us} = 90 * g * x$ ***and*** $\tau_{friend} = 60 * g * (3.0 - x)$

Lets use: $\tau_{us} + \tau_{friend} = 0$

Remember that ***anti-clockwise*** rotation is ***+VE:***

$$90 * g * x \quad + \quad -(60 * g * (3 - x)) = 0$$

Dividing both sides by **'g'** gives:

$$90x - (180 - 60x) = 0$$

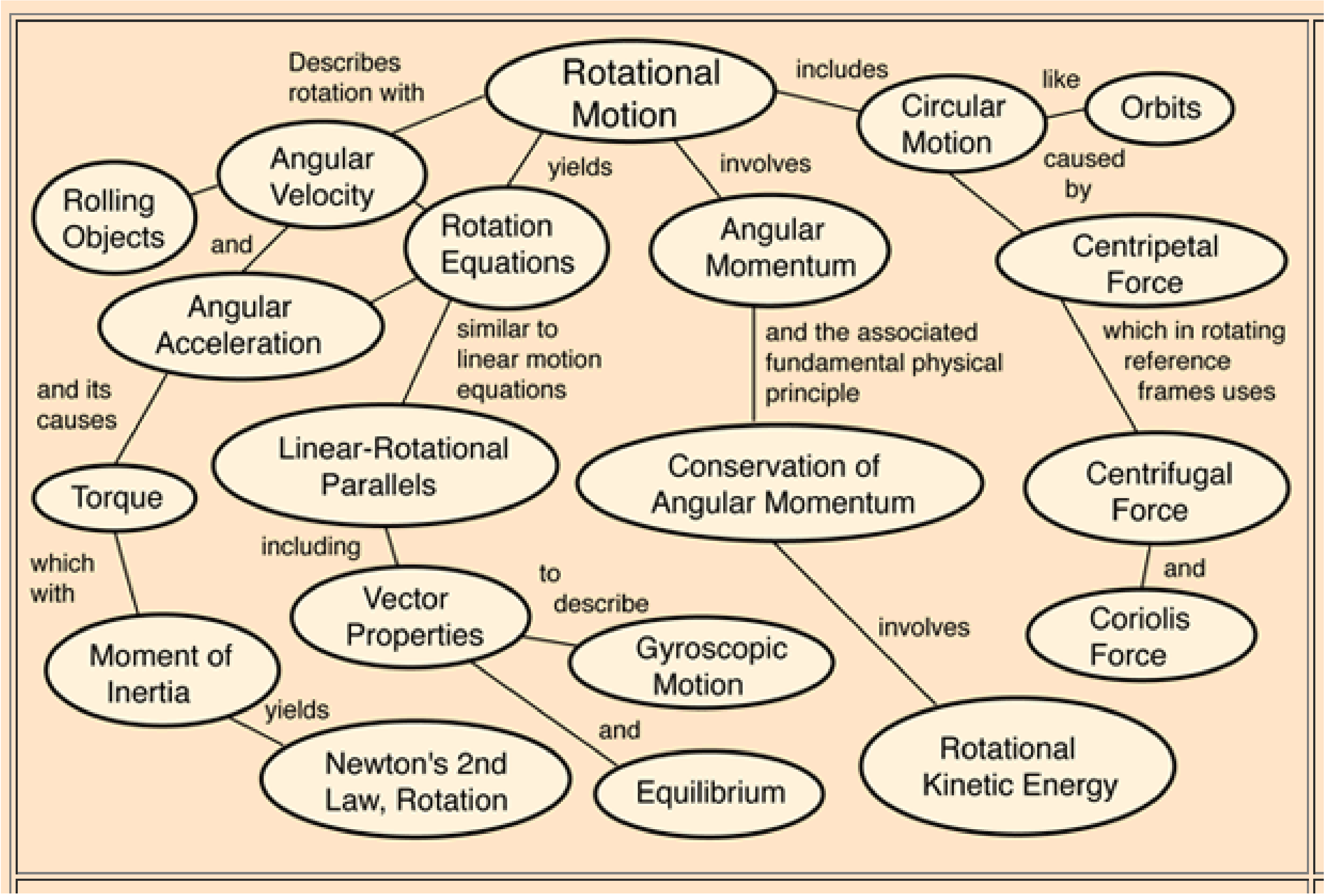
$$150x - 180 = 0$$

x = 180/150 = 1.2 metres from the left end

VTD Walking the plank

VTD Off-Center Collision

VTD Balancing a meter stick



http://hyperphysics.phy-astr.gsu.edu/hbase/circ.html#rotcon

	Linear Motion	Rotational Motion	
Position	x	θ	Angular position
Velocity	v	ω	Angular velocity
Acceleration	a	α	Angular acceleration
Motion equations	$x = \bar{v} t$	$\theta = \bar{\omega} t$	Motion equations
	$v = v_0 + at$	$\omega = \omega_0 + \alpha t$	
	$x = v_0 t + \frac{1}{2} a t^2$	$\theta = \omega_0 t + \frac{1}{2} \alpha t^2$	
	$v^2 = v_0^2 + 2ax$	$\omega^2 = \omega_0^2 + 2\alpha\theta$	
Mass (linear inertia)	m	I	Moment of inertia
Newton's second law	$F = ma$	$\tau = I \alpha$	Newton's second law
Momentum	$p = mv$	$L = I \omega$	Angular momentum
Work	Fd	$\tau\theta$	Work
Kinetic energy	$\frac{1}{2} m v^2$	$\frac{1}{2} I \omega^2$	Kinetic energy
Power	Fv	$\tau\omega$	Power

Linear	Angular
Δx	$\Delta \theta$
v	ω
a	α
F	τ
m	I
p	L

$$\tau = Fl$$

$$\Sigma\tau = I\alpha$$

$$W = \tau\Delta\theta$$

$$L = I\omega$$

$$\sum L_i = \sum L_f$$

$$\Sigma\tau = 0$$

What do all these equations mean?
How did we use them?

Extra information and examples to try yourself, or do in class, if we have time

Chapter 10 Assets

Video Tutor Demonstrations

- Walking the plank
- Spinning Person Drops Weights
- Off-Center Collision
- Balancing a Meter Stick
- Conservation of Vector Angular Momentum
- Conservation of Angular Momentum

Video Tutor Solutions

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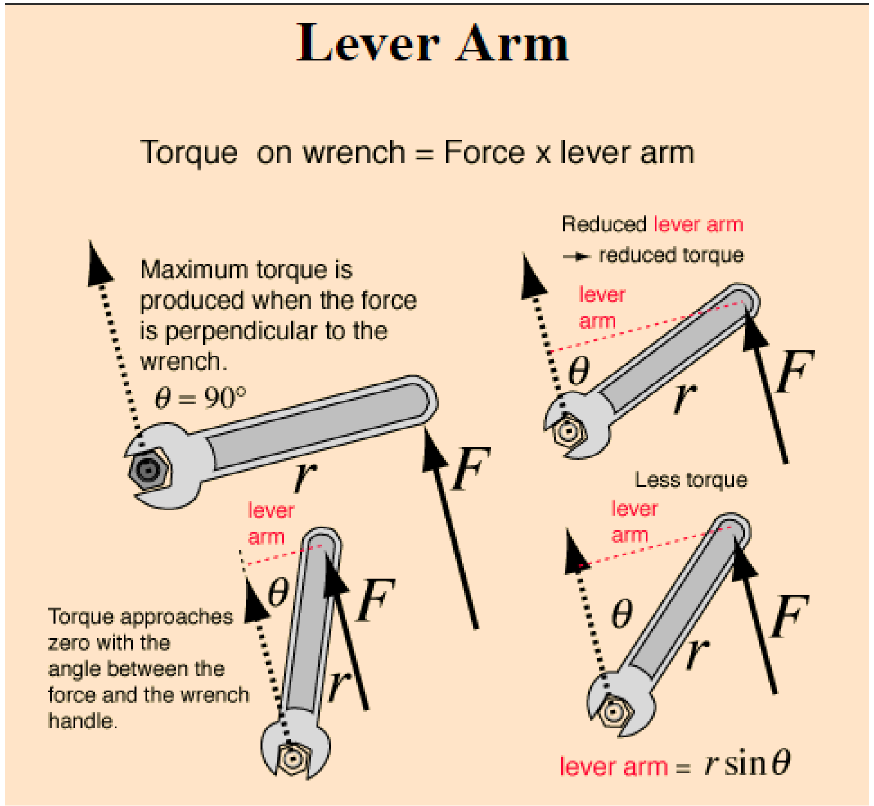
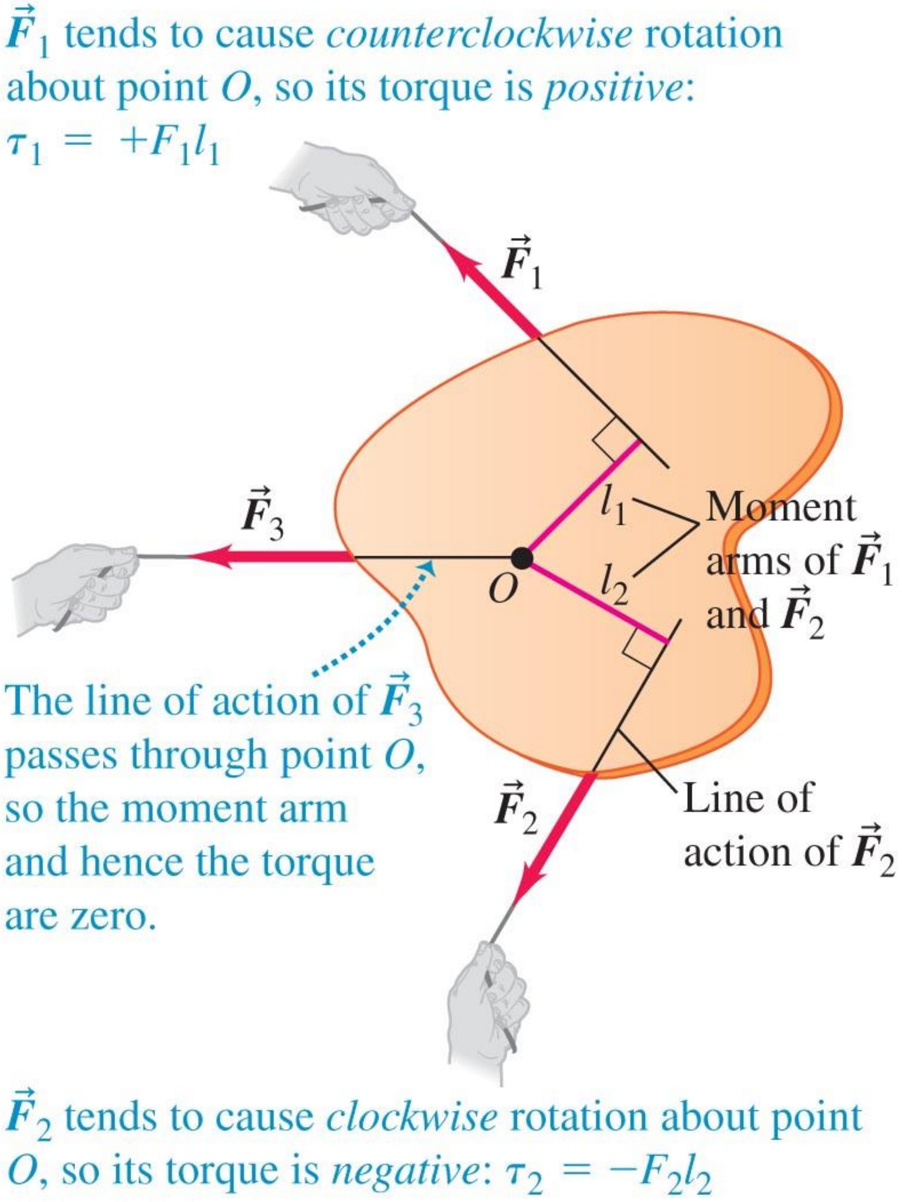
- Example 10.14 A laboratory gyroscope
- Chapter 10 Bridging Problem

PhET Simulations

- Torque

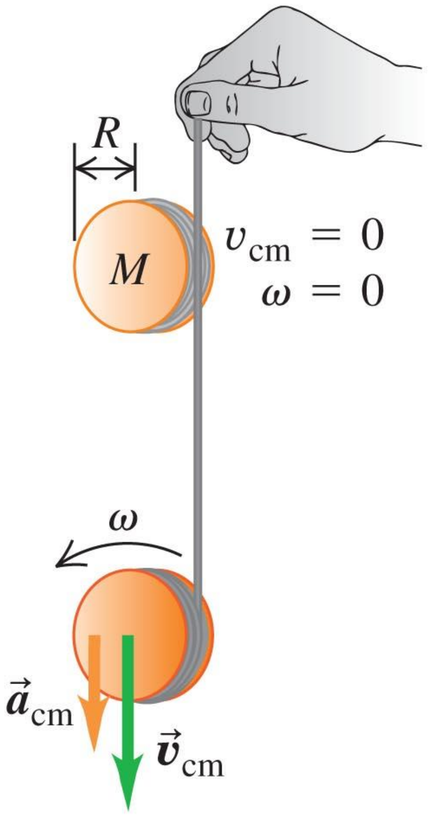


- A counterclockwise force is designated as positive (+).
- A clockwise force is designated as negative (−).

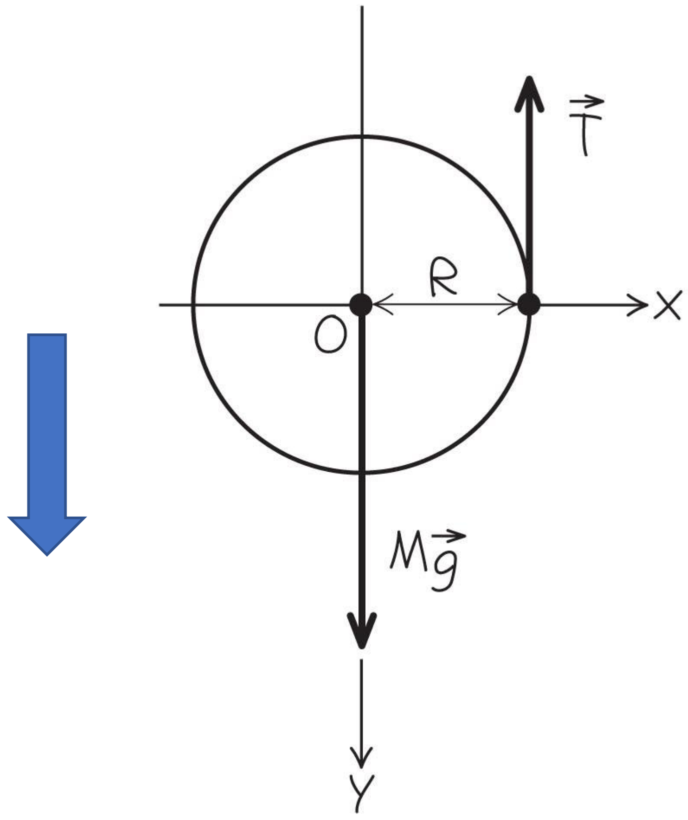


The Yo-Yo Rotates on a Moving Axis

- Show this via mastering physics > VTS Ex 10.4
- Refer to worked example on page 334.



(a) The yo-yo



(b) Our free-body diagram

JUST LINEAR

$\Sigma F = Mg - T = Ma$

(taking down as the positive direction)

JUST ANGULAR

$\Sigma\tau = TR = I\alpha = (\frac{1}{2} MR^2)\alpha$

Combining linear and angular

In this question taking down as positive
Velocity is through the center of mass

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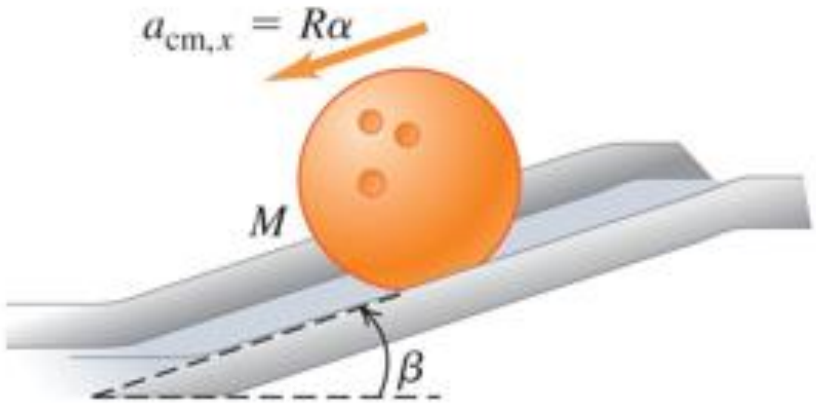
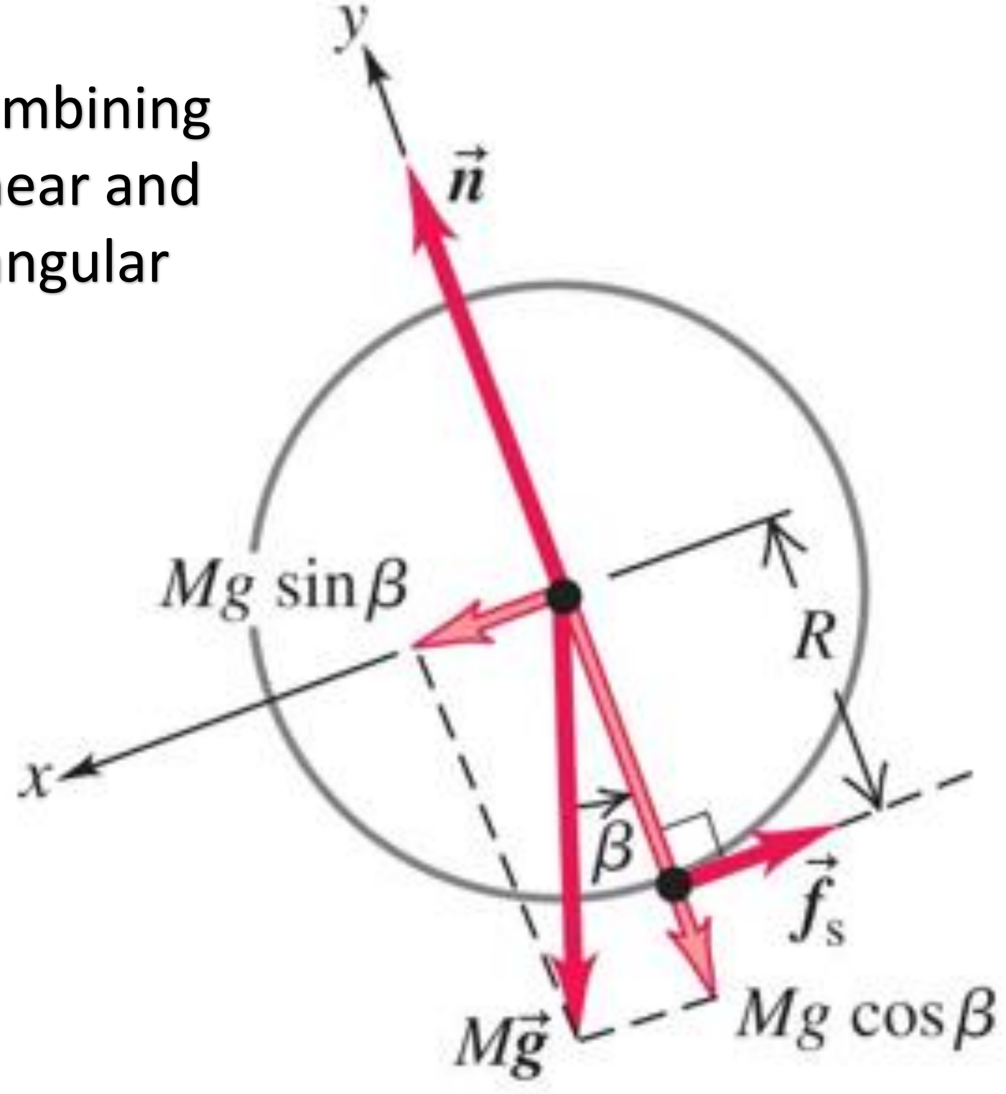
A Bowling Ball (Solid sphere) Rotates on a Moving Axis – Ex10.5



Combining linear and angular



$$\sum F_x = Mgsin\beta - f_s = Ma$$

$$\sum \tau = f_s R = I\alpha = (\frac{2}{5} MR^2)\alpha$$

Example 2

Combining linear and angular

A 2.00 kg stone is tied to a thin, light wire wrapped around the outer edge of the uniform 10.0 kg cylindrical pulley. The diameter of the pulley is 50.0 cm. The system is released from rest, and there is no friction at the axle of the pulley. Find

(a) the acceleration of the stone,

(b) the tension in the wire, and

(c) the angular acceleration of the pulley.

VTS Ex 10.3

JUST LINEAR BUCKET

$F = mg - T = ma$ (taking down as the positive direction)

JUST ANGULAR WINCH

$\tau = Fl = TR$

$\tau = I\alpha$

$I = \frac{1}{2} MR^2$

Gives

$TR = \frac{1}{2} MR^2 \alpha$

also

$a = R\alpha$

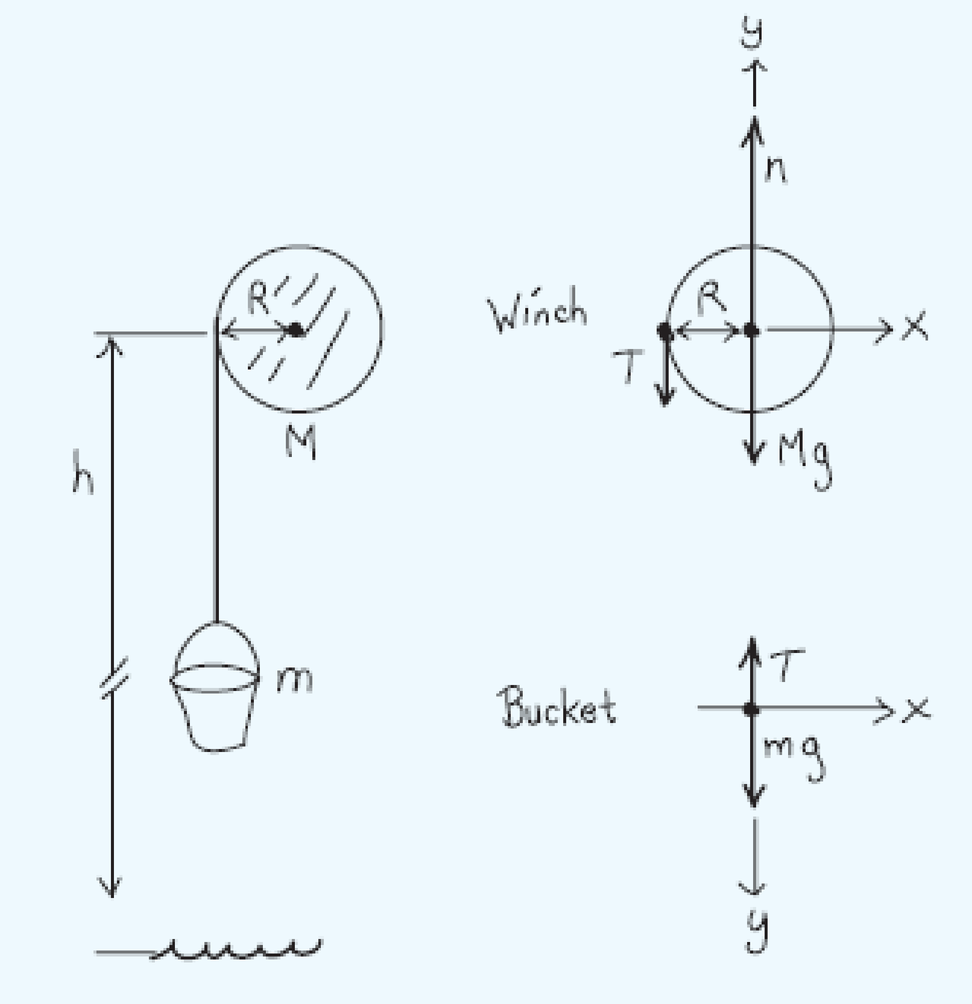
Sub into above equation and then divide through by R,

$T = \frac{1}{2} Ma$

COMBINE EQUATIONS (LINK LINEAR AND ANGULAR)

$$mg - \frac{1}{2} Ma = ma$$

Combining linear and angular



(a) Diagram of situation (b) Free-body diagrams mg – ½ Ma = ma then leads to these equations

- Solving for **a** we get:

$$a = \frac{g}{1+\frac{M}{2m}} \qquad (4)$$

Using the fact that **a** is also = **Rα**, we solve for **α**:

$$\alpha = \frac{\frac{g}{R}}{1+\frac{M}{2m}} \qquad (5)$$

Finally we can substitute Eq.4 into **F = ma** instead of **a**.

$$T = \frac{mg}{1+\frac{2m}{M}} \qquad (6)$$

We Can Now Use These To Solve Our Problem...........

- Linear Acceleration of Stone:

$$a = \frac{g}{1+\frac{M}{2m}} \text{ (from page 300) (4)}$$

where M is wheel mass, m is stone mass.

$$a = \frac{10}{1+\frac{10}{2*2}} = \frac{10}{3.5} = 2.86\, ms^{-2}$$

- Tension in wire:

$$T = \frac{mg}{1+\frac{2m}{M}} = \frac{2*10}{1+\frac{2*2}{10}} = 14.29\, Newtons \;\; (6)$$

This is less than **mg** as the stone accelerates downwards!!

Solution continued…

- Angular acceleration of the pulley:

$$\alpha = \frac{\frac{g}{R}}{1+\frac{M}{2m}} = \frac{\frac{10}{0.34}}{1+\frac{10}{2*2}} = \frac{29.41}{3.5} = 8.40\ rads^{-2}\ (From\ Page\ 300)\ (5)$$

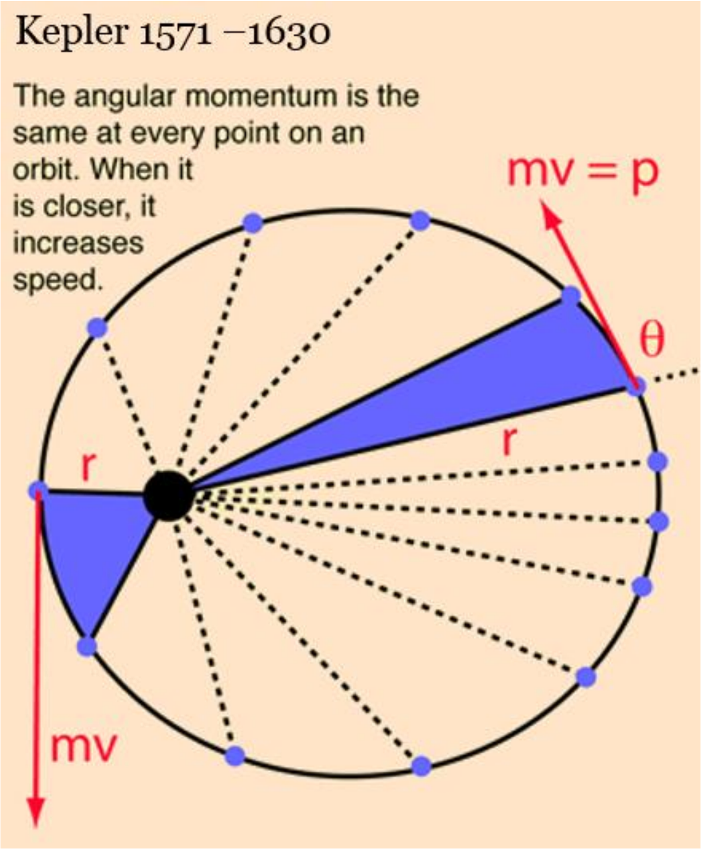
The Radius of 0.34 comes from R = $(R_1^2 + R_2^2)$ for a ***partially hollow*** cylinder !!

We can also solve for final linear velocity, v:

We know: $v = \sqrt{2ah} = \sqrt{\frac{2gh}{1+\frac{M}{2m}}}$

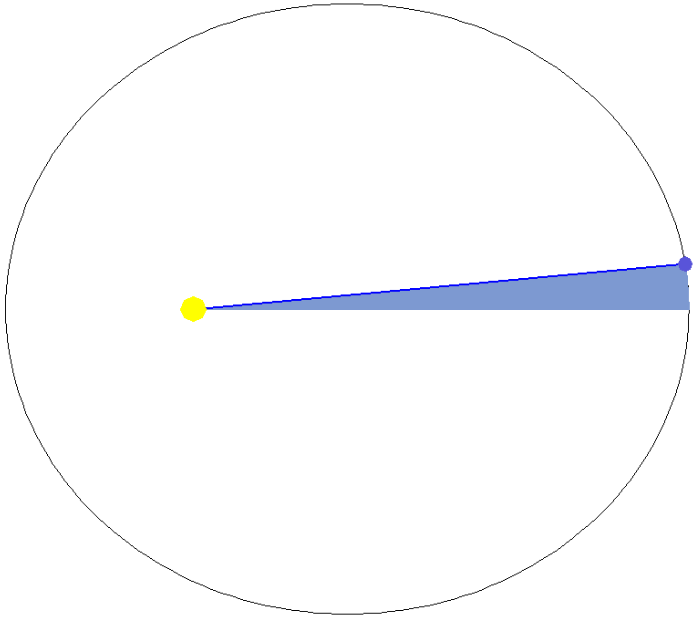
Now We Have A Full Set of Working Equations !!

Angular momentum of a particle



A line joining a planet and the Sun sweeps out equal areas during equal intervals of time

Halley's Comet visible from Earth every 74–79 years.



$$L = I\omega$$

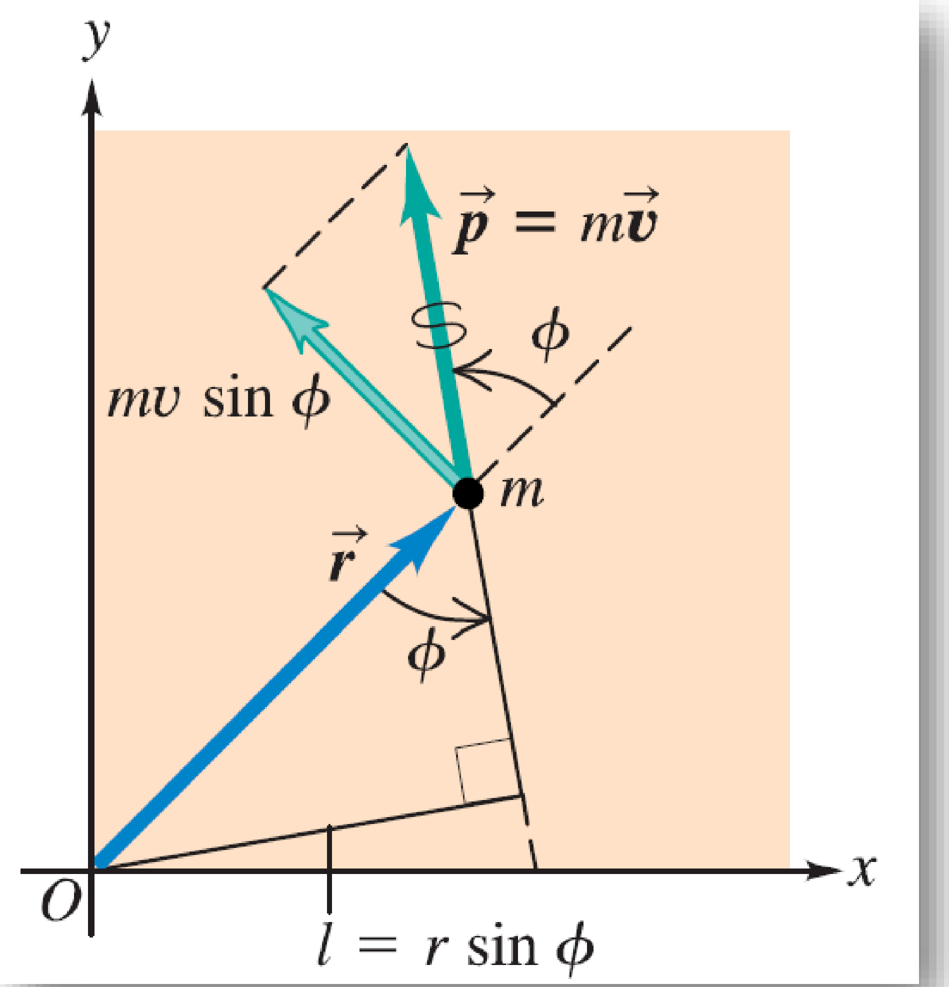
$$L = mr^2\omega$$

$$L = mr^2\frac{v}{r}$$

$$L = mvr$$

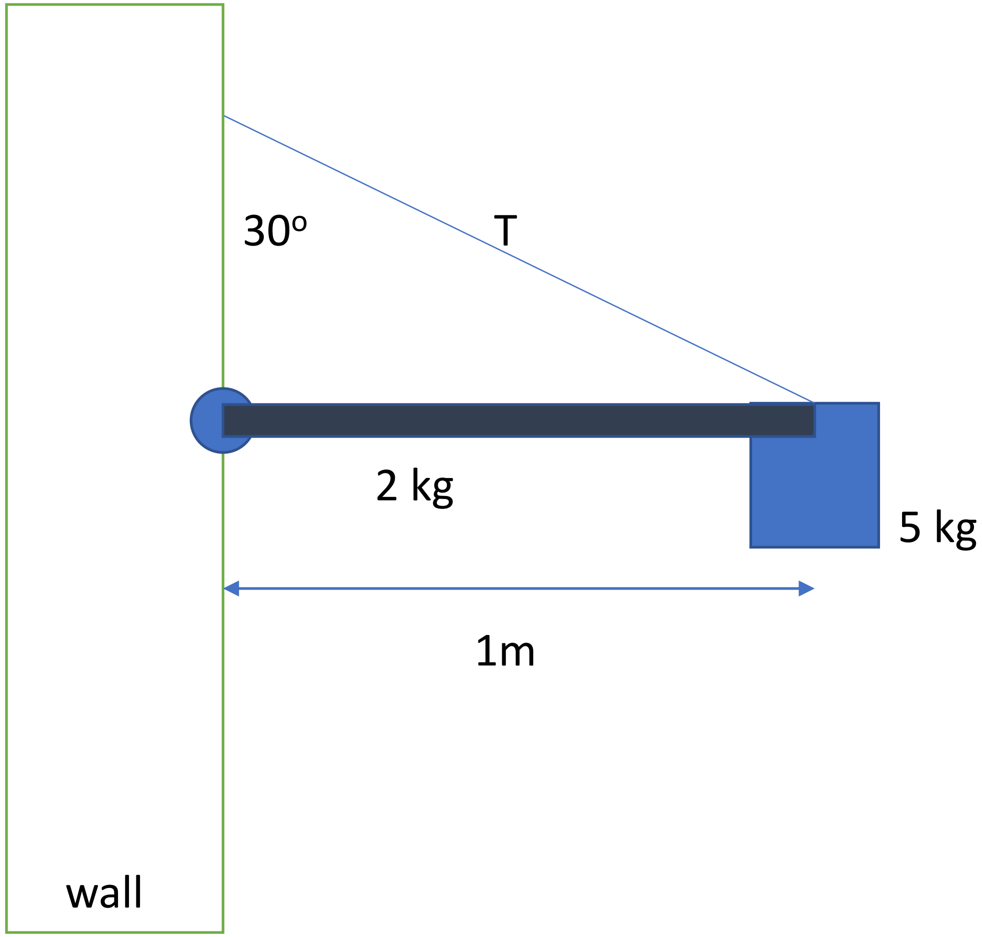
If at an angle

$$L = m(v \sin \phi)r$$



What is the minimum tension needed to hold up the sign?

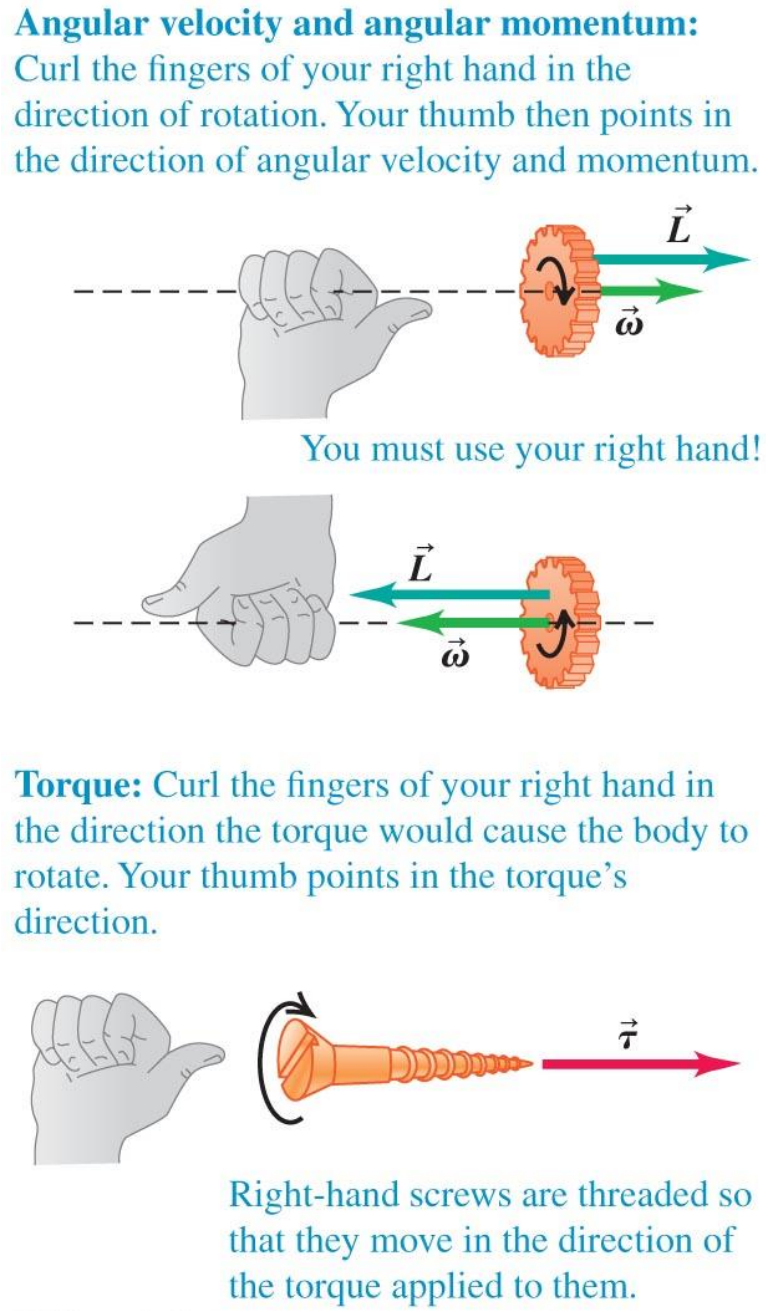
$$\tau = Fl$$



ANS T=68N

Angular Quantities Are Vectors – Figure 10.29

- The "right-hand rule" gives us a vector's direction.



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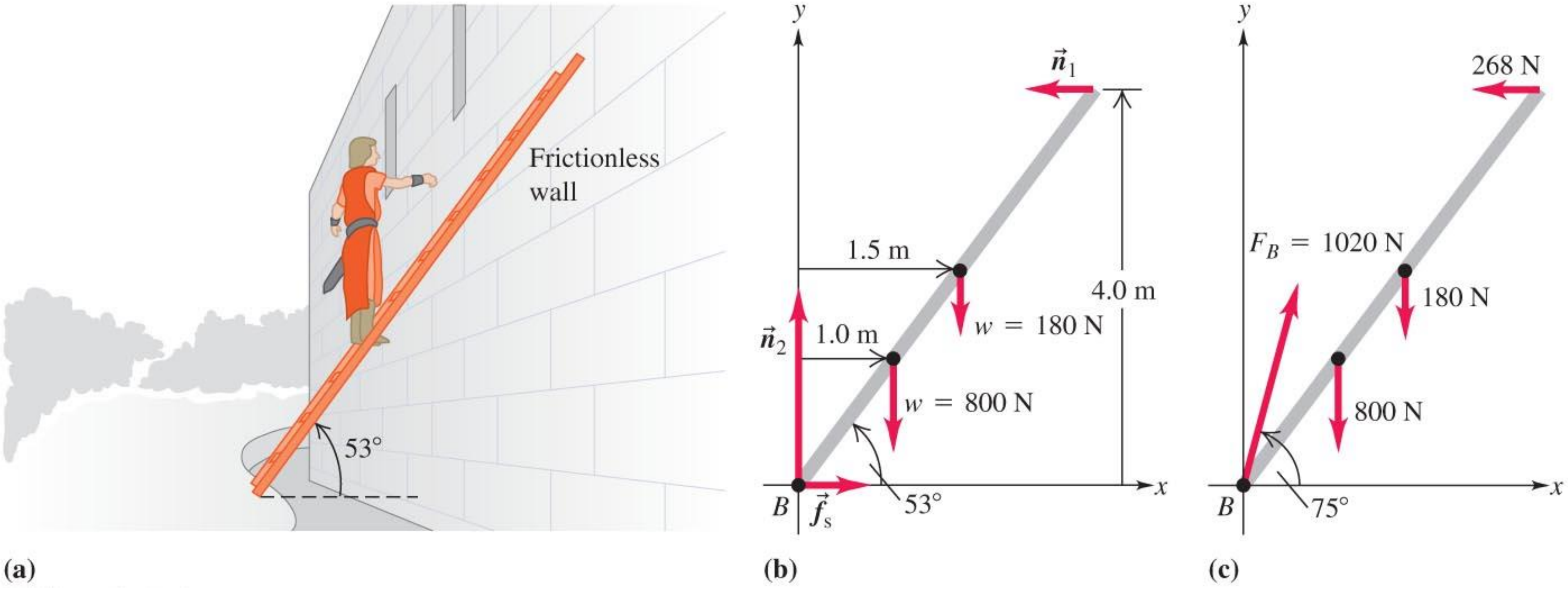

Other worked examples in text book and self study area