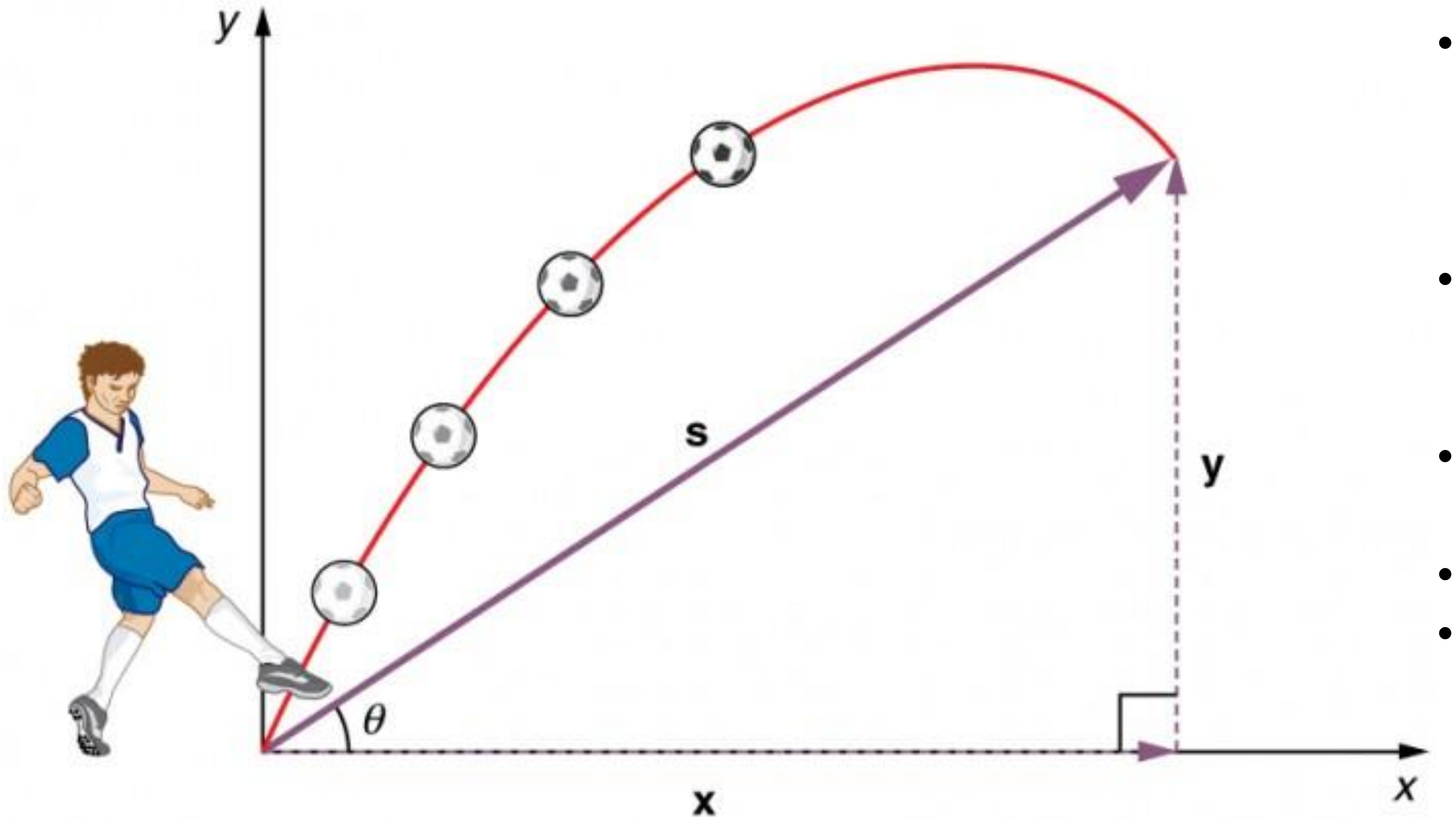
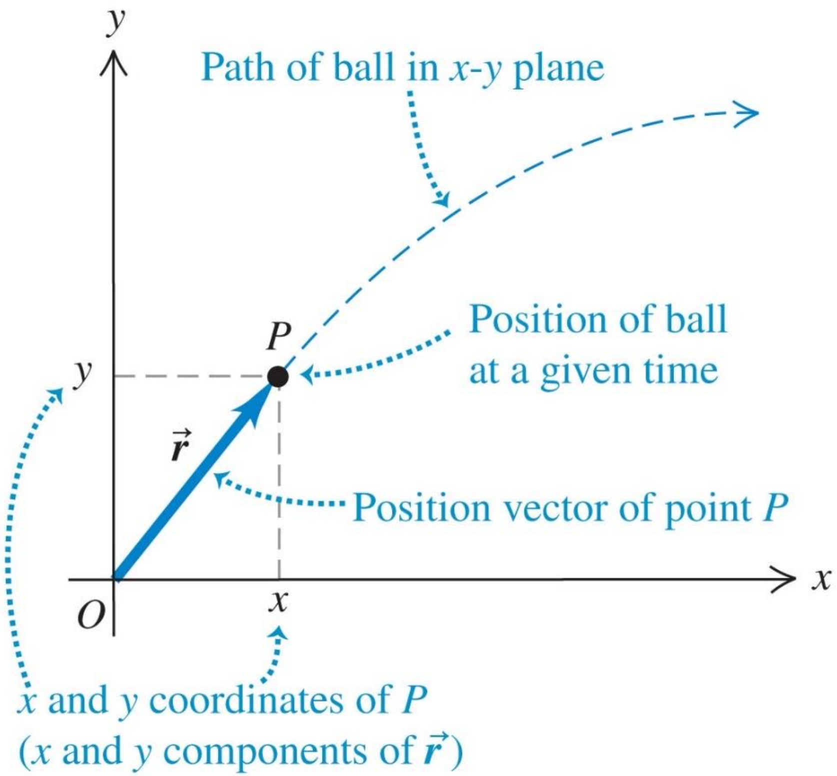
Chapter 3 Motion in a plane



- Summary > Just like up and down but now also side motion.
- Vertical and horizontal are independent of each other.
- Same equations of motion
- Example of monkey and gun
- Relative velocity..

Velocity in a Plane

- Vectors in terms of Cartesian *x*- and *y*- coordinates may now also be expressed in terms of magnitude and angle.



(a)

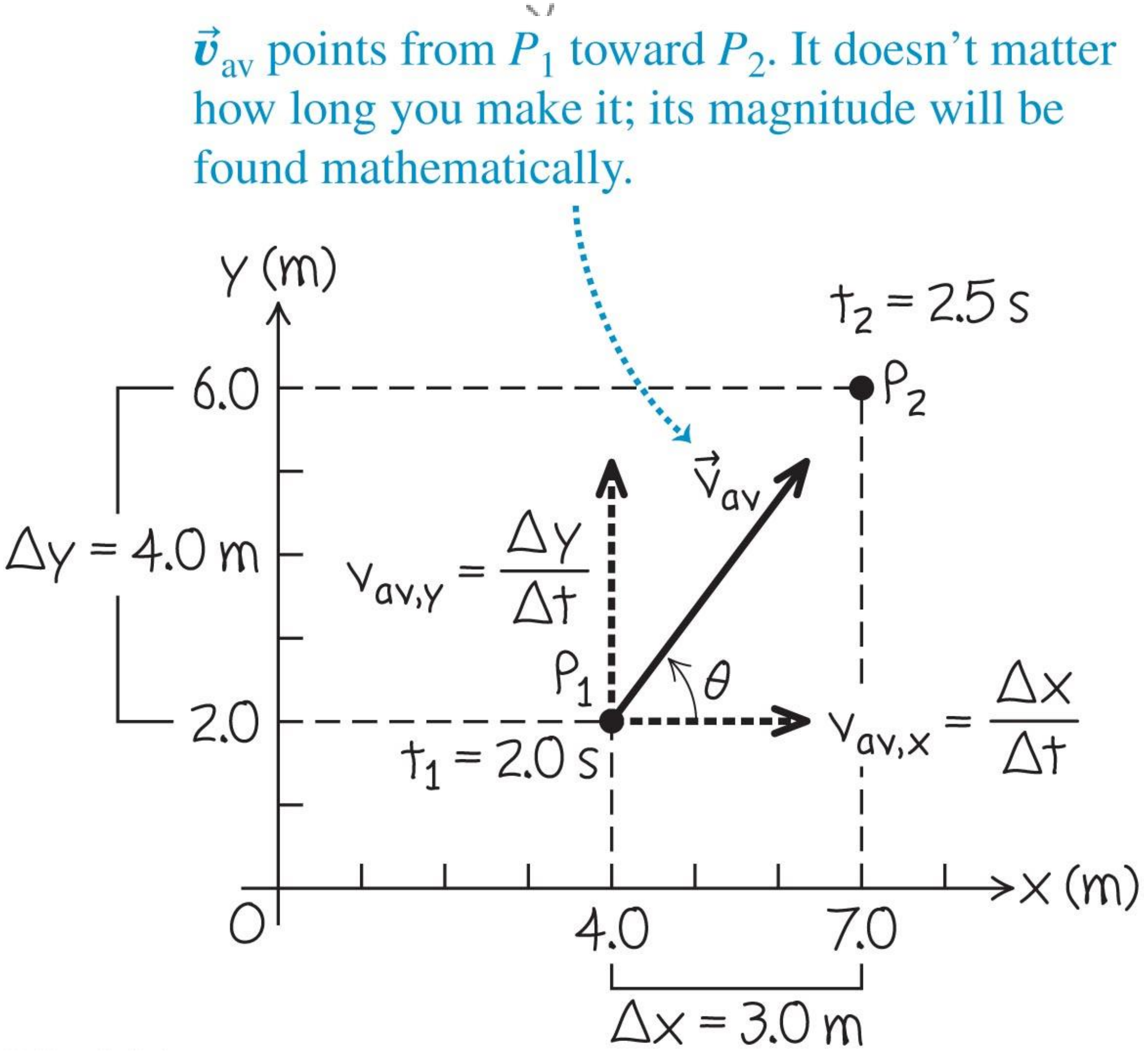
distance of point P from the origin is the magnitude of vector $\vec{r}$

$$r = |\vec{r}| = \sqrt{x^2 + y^2}$$

Example VTS Ex 3.1

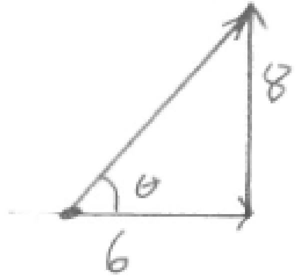
You are operating a radio-controlled model car on a vacant tennis court. The surface of the court represents the *x*-*y* plane, and you place the origin at your own location. At time t_1 = 2.00 s the car has *x* and y coordinate. (4.0 m, 2.0 m), and at time t_2 = 2.50. s it has coordinates (7.0 m, 6.0 m). For the time interval from t_1 to t_2, find **(a)** the components of the average velocity of the car and **(b)** the magnitude and direction of the average velocity.

The Motion of a Model Car



$V_{x\,av} = \frac{\Delta x}{\Delta t} = \frac{3}{0.5} = 6$ m/s

$V_y = \frac{\Delta y}{\Delta t} = \frac{4}{0.5} = 8$ m/s

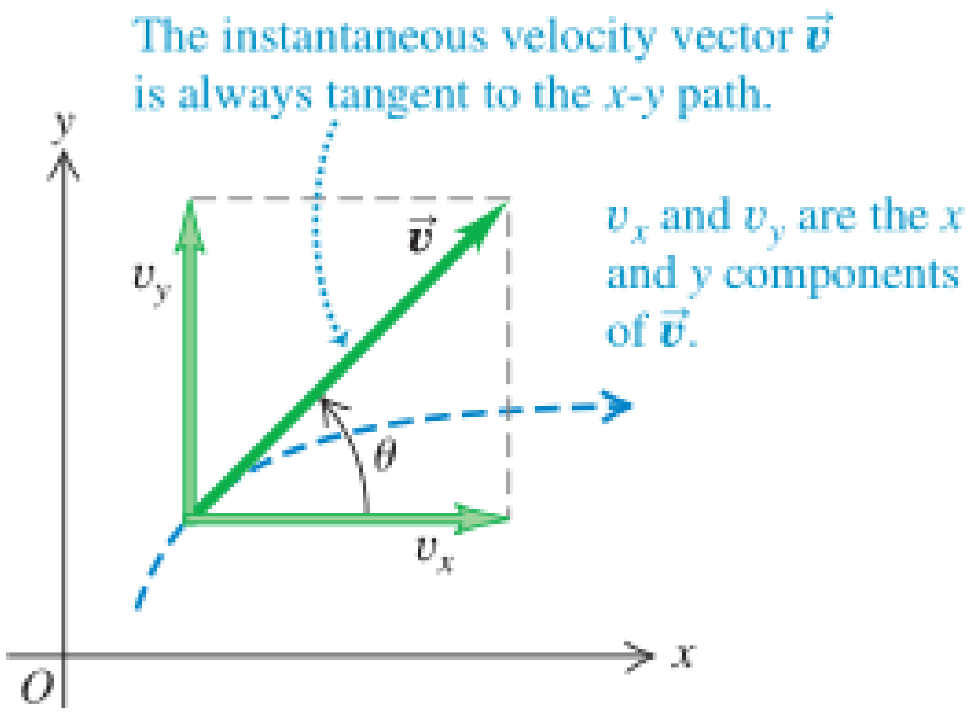


$V_{AV} = \sqrt{V_x^2 + V_y^2} = 10$ m/s

$\theta = \tan^{-1} \frac{8}{6} = 53^{\circ}$

Instantaneous Velocity

- Instantaneous velocity is always tangent to the path of the particle



▲ **FIGURE 3.2** The two velocity components for motion in the x-y plane.

Definition of instantaneous velocity $\vec{v}$ in a plane
The instantaneous velocity is the limit of the average velocity as the time interval Δt approaches zero:

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t}. \quad (3.3)$$

1. • A meteor streaking through the night sky is located with radar. At point A its coordinates are $(5.00\text{ km}, 1.20\text{ km})$, and 1.14 s later it has moved to point B with coordinates $(6.24\text{ km}, 0.925\text{ km})$. Find (a) the x and y components of its average velocity between A and B and (b) the magnitude and direction of its average velocity between these two points.

b) Magnitude   v=1.12 km/s

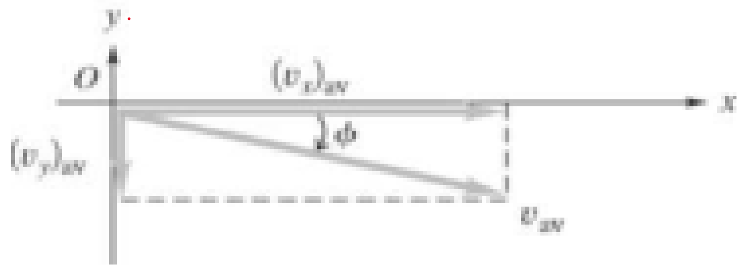
Solutions to Problems

A

3.1. Set Up: Since distances are given in km, use km/s as units for velocity.

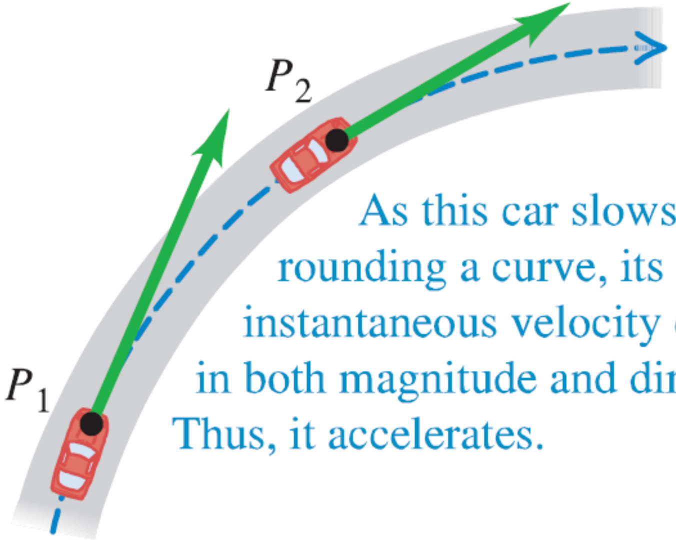
Solve: (a) $(v_x)_{av} = \frac{\Delta x}{\Delta t} = \frac{6.24 \text{ km} - 5.00 \text{ km}}{1.14 \text{ s}} = 1.09 \text{ km/s}$; $(v_y)_{av} = \frac{\Delta y}{\Delta t} = \frac{0.925 \text{ km} - 1.20 \text{ km}}{1.14 \text{ s}} = -0.24 \text{ km/s}$

(b) $\vec{v}_{av}$ and its components are shown in the figure below.

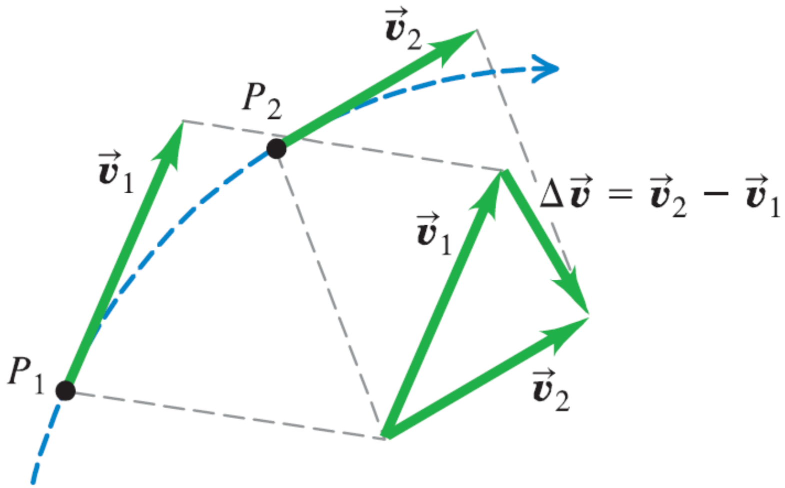


$v_{av} = \sqrt{(v_x)_{av}^2 + (v_y)_{av}^2} = 1.12 \text{ km/s}$; $\tan\phi = \frac{(v_y)_{av}}{(v_x)_{av}}$ and $\phi = 12.4°$, below the $+x$ axis.

3.2 Average Acceleration in a Plane



As this car slows while rounding a curve, its instantaneous velocity changes in both magnitude and direction. Thus, it accelerates.



To find the car's average acceleration between P_1 and P_2, we first find the change in velocity $\Delta\vec{v}$ by subtracting $\vec{v}_1$ from $\vec{v}_2$. (Notice that $\vec{v}_1 + \Delta\vec{v} = \vec{v}_2$.)

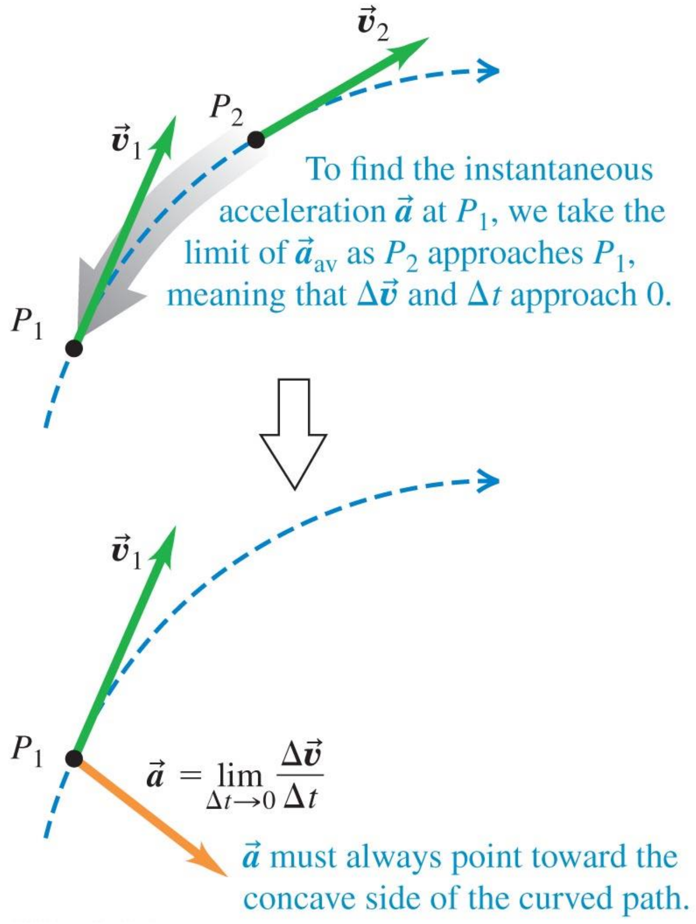
Definition of average acceleration $\vec{a}_{av}$

As an object undergoes a displacement during a time interval Δt, its average acceleration is its change in velocity, $\Delta\vec{v}$, divided by Δt:

$$\text{Average acceleration} = \vec{a}_{av} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} = \frac{\Delta\vec{v}}{\Delta t}. \qquad (3.4)$$

Accelerations in a Plane

- Acceleration must now be considered during change in magnitude AND/OR change in direction.



Average acceleration of a particle over displacement $\Delta\vec{r}$

$$\vec{a}_{av} = \frac{\Delta\vec{v}}{\Delta t}$$

Instantaneous acceleration of particle at point $\vec{r}$

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t}$$

Example 2

Definition of average acceleration $\vec{a}_{av}$
As an object undergoes a displacement during a time interval Δt, its average acceleration is its change in velocity, $\Delta\vec{v}$, divided by Δt:

$$\text{Average acceleration} = \vec{a}_{av} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} = \frac{\Delta\vec{v}}{\Delta t}. \qquad (3.4)$$

Let's look again at the radio-controlled model car in the previous example. Suppose that at time t_1 = 2.00 s the car has components of velocity v_x = 1.0 m/s and v_y = 3.0 m/s and that at time t_2 = 2.50 s the components are v_x = 4.0 m/s and v_y = 3.0 m/s.

Find (a) the components and (b) the magnitude and direction of the average acceleration during this interval.

Find acceleration in the x direction
Find acceleration the y direction
Use Pythagoras to find the magnitude of the average acceleration
Use tan to find the angle.

VTS Ex 3.2

Let's look again at the radio-controlled model car in the previous example. Suppose that at time t_1 = 2.00 s the car has components of velocity v_x = 1.0 m/s and v_y = 3.0 m/s and that at time t_2 = 2.50 s the components are v_x = 4.0 m/s and v_y = 3.0 m/s.

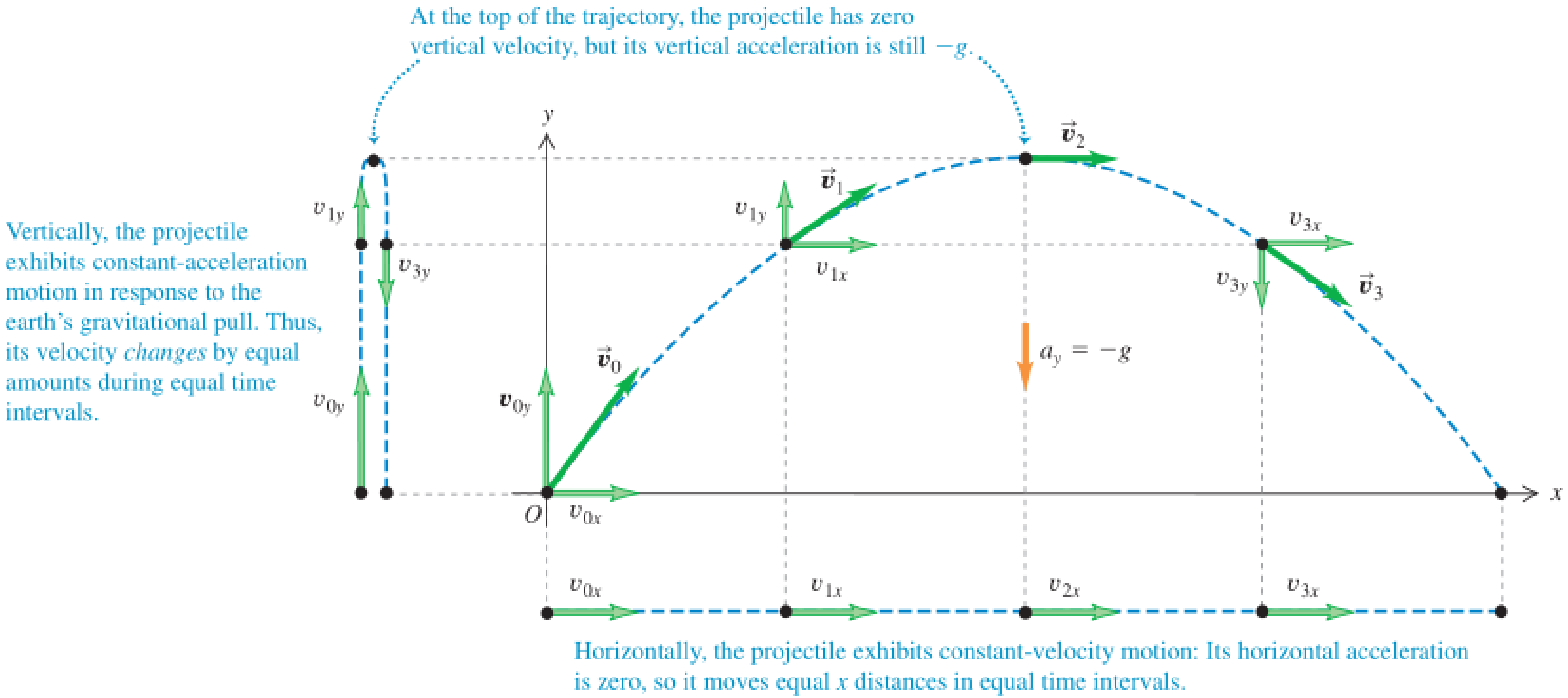
Find (a) the components and (b) the magnitude and direction of the average acceleration during this interval.

Video Tutor Demonstrations
To get us thinking

VTD Ball fired upwards from a moving cart
VTD Ball fired upward from an accelerating cart
VTD Ball fired from cart on incline

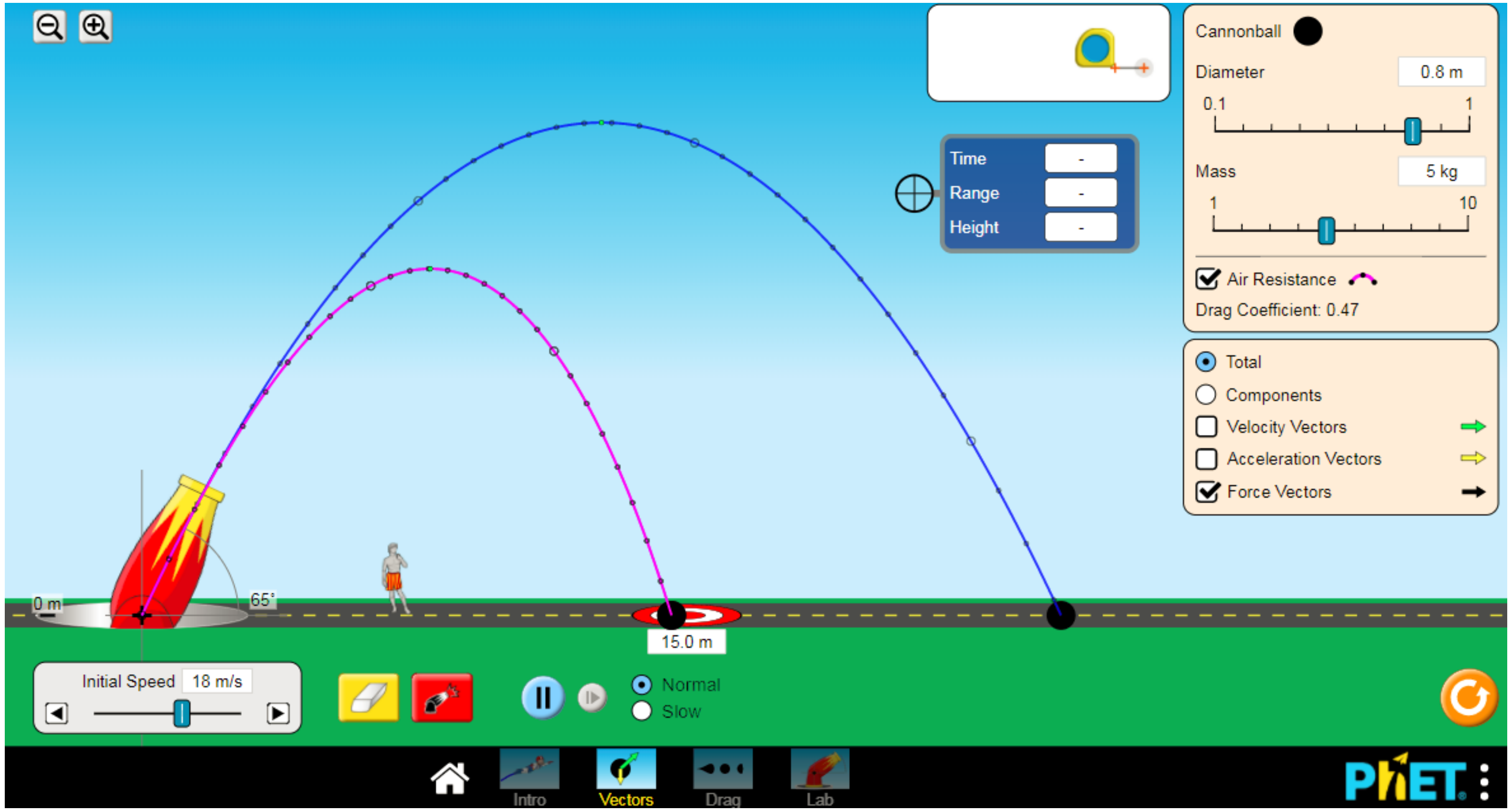
3.3 A 2D special case: Projectile motion

- Motion is resolved into a horizontal component and a vertical component.



▲ **FIGURE 3.11** Independence of horizontal and vertical motion. In the vertical direction, a projectile behaves like a freely falling object; in the horizontal direction, it moves with constant velocity.

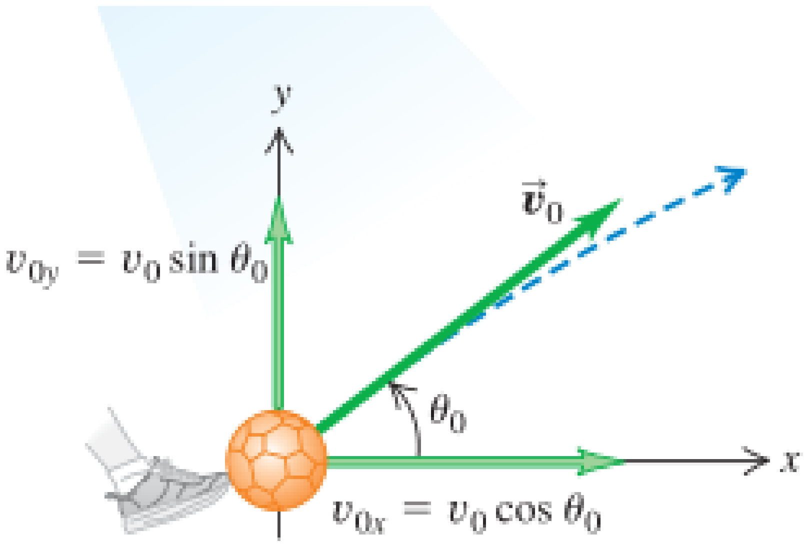
PhET projectile motion



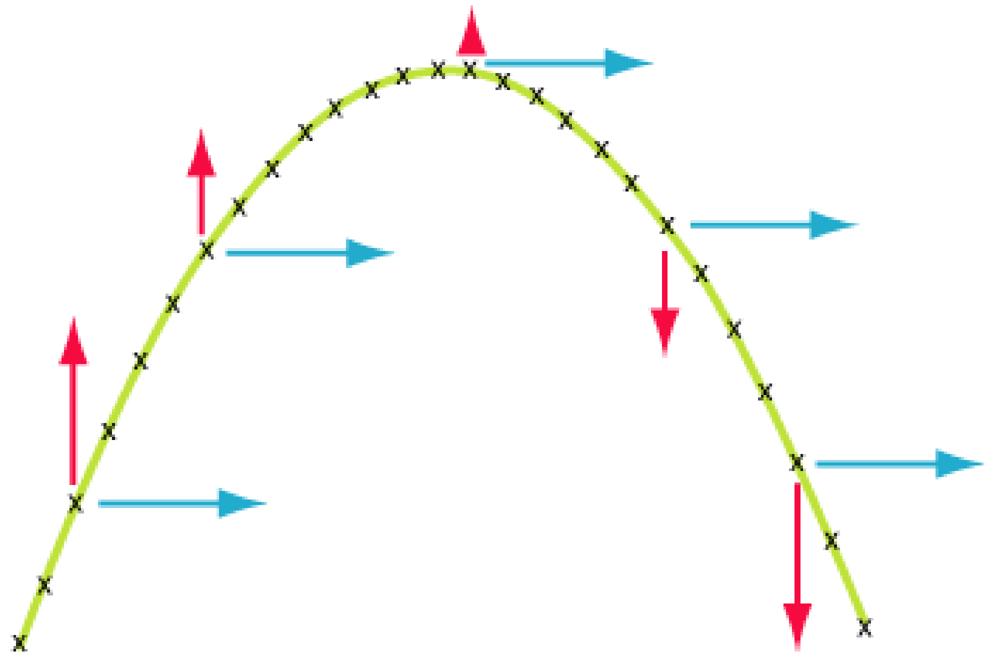
https://phet.colorado.edu/sims/html/projectile-motion/latest/projectile-motion_en.html

Projectiles move in TWO dimensions

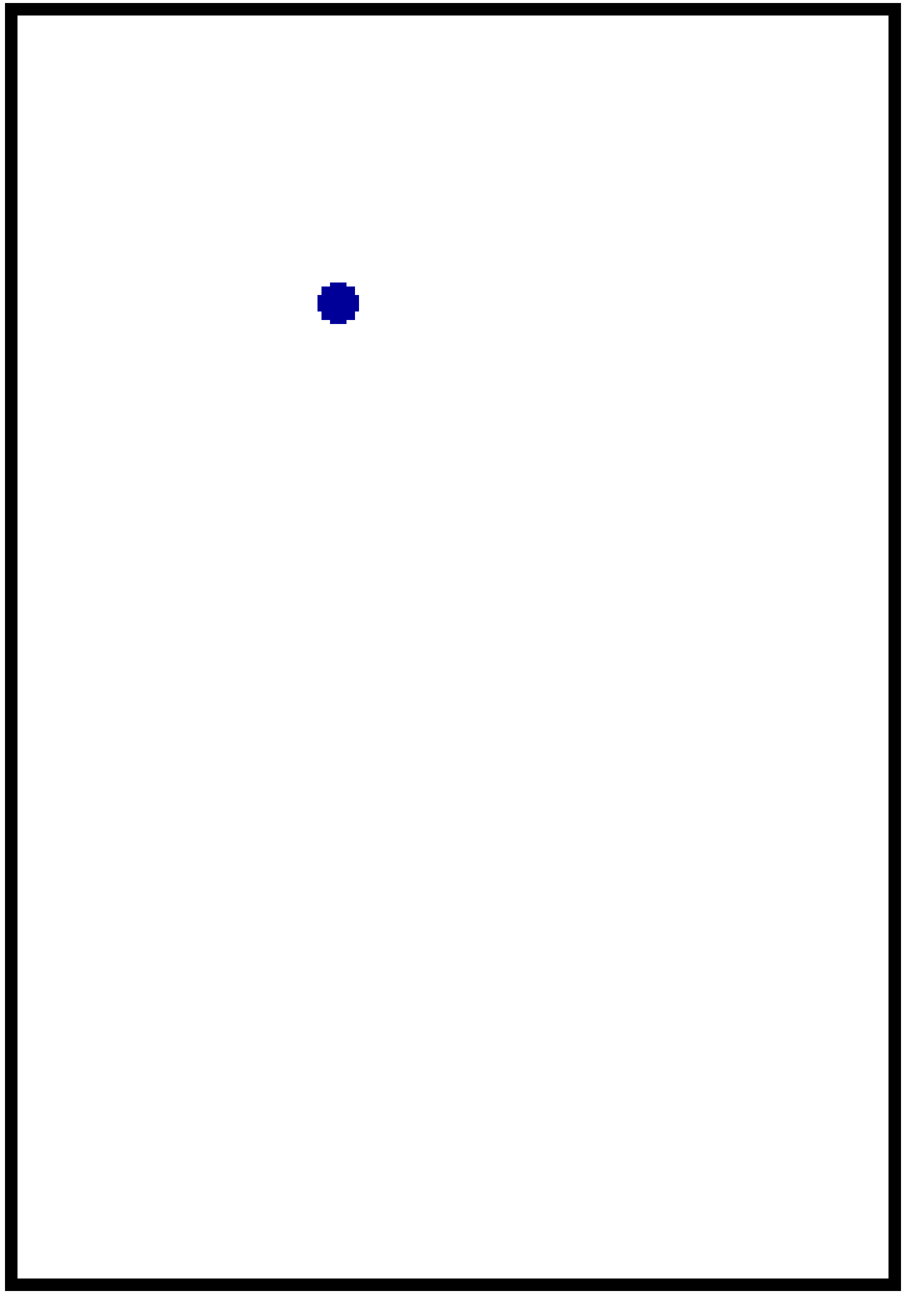
Since a projectile moves in 2-dimensions, it therefore has 2 components just like a resultant vector.

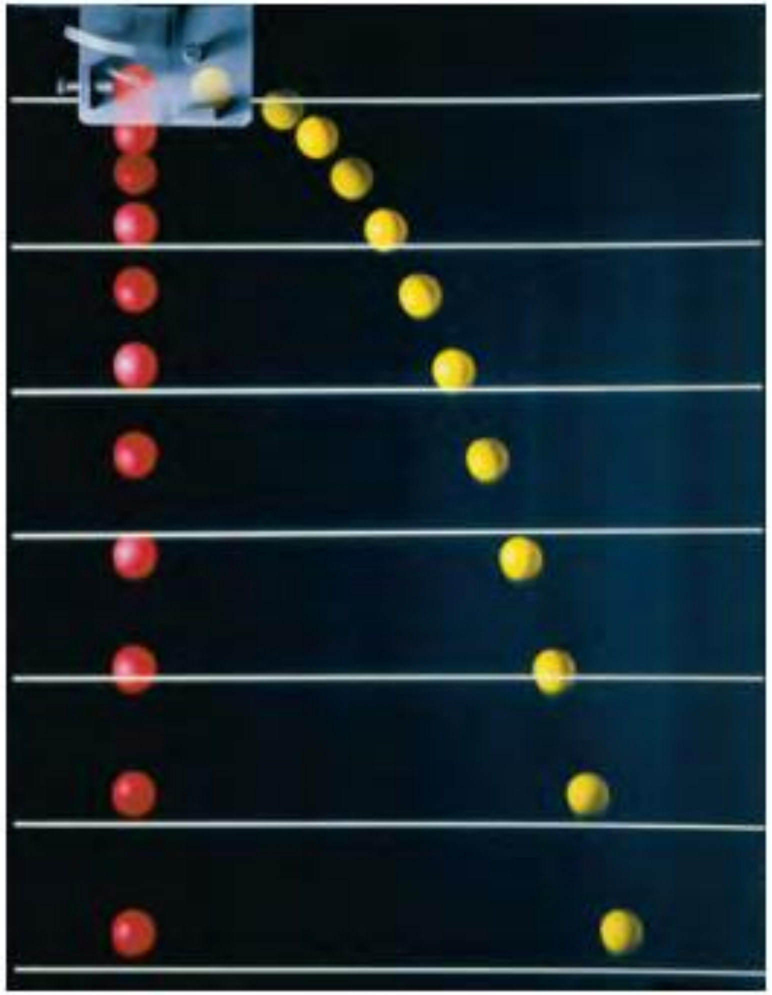


▲ **FIGURE 3.12** The initial velocity of a projectile, showing the components and the launch angle θ_0.



- The force of gravity is what causes the object to curve downward in a parabolic flight path. Its path through space is called its **trajectory**.



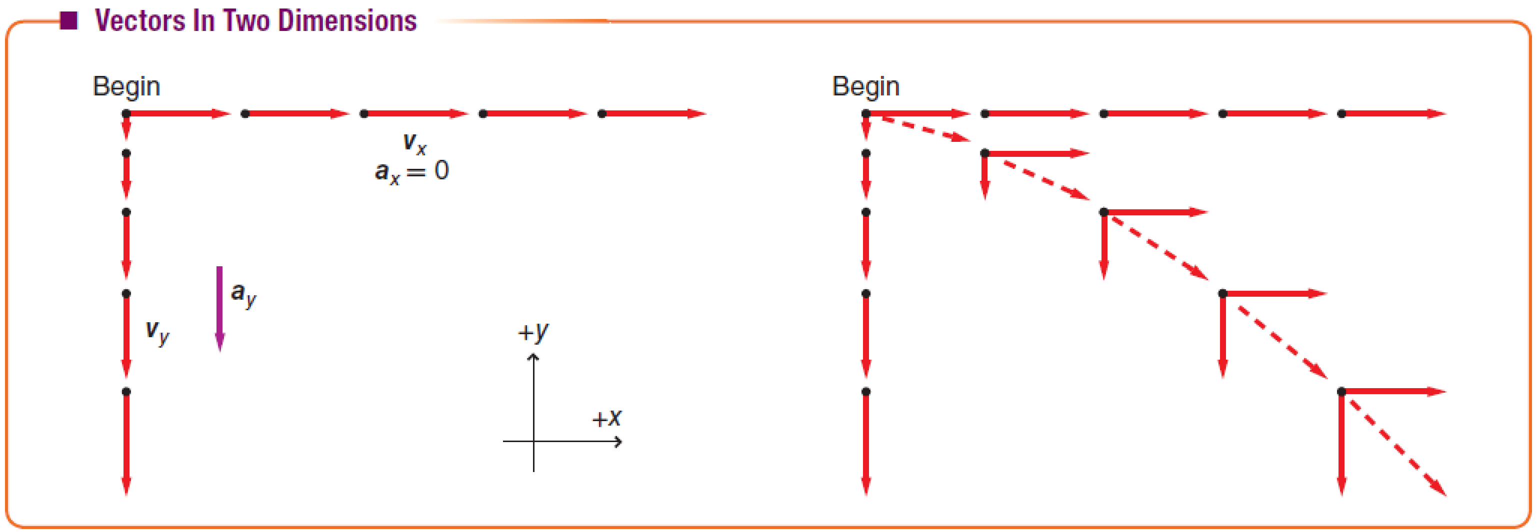


▲ **FIGURE 3.10** Independence of horizontal and vertical motion: At any given time, both balls have the same y position, velocity, and acceleration, despite having different x positions and velocities. Successive images are separated by equal time intervals.

http://www.youtube.com/watch?v=NJoptugkoYk

Horizontal and vertical motions are independent of each other.

(you are expected to know the properties of each motion, and explain what happens to the displacement, velocity, and acceleration vectors through the projectile's trajectory)



VTD Dropped and thrown balls

VTD Independent horizontal and vertical motion

A 2D special case: Projectile motion

Assuming no air resistance

Horizontally: **constant velocity**

$$v_x = v_{0x} = \frac{\Delta x}{t}$$

Vertically: **free fall; constant acceleration *g***

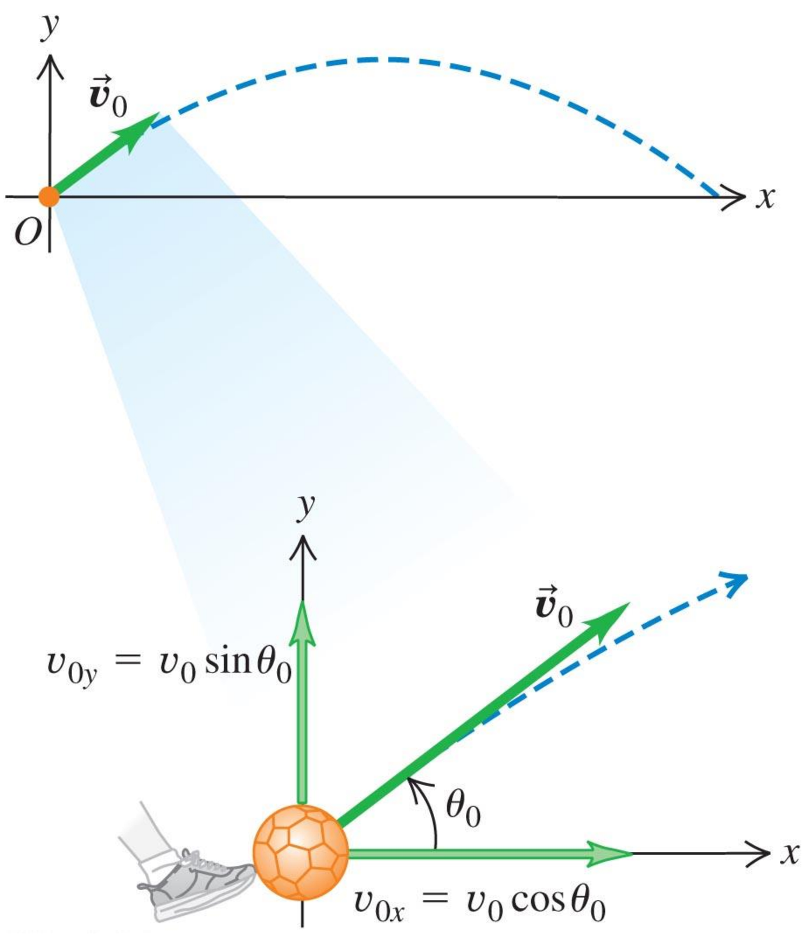
$$v_y = v_{0y} + a_y t$$

$$\Delta y = \left(\frac{v_{0y}+v_y}{2}\right) t$$

$$\Delta y = v_{0y} t + \tfrac{1}{2} a_y t^2$$

$$v_y^2 = v_{0y}^2 + 2a\Delta y$$

Determination of Key Items



Don't forget:

Initial velocity is a 2D vector.

Vector. $\rightarrow$ $\vec{v}_0 = \vec{v}_{0x} + \vec{v}_{0y}$

Components. $\rightarrow$ $\begin{cases} v_{oy} = v_0 \sin\theta_0 \\ v_{ox} = v_0 \cos\theta_0 \end{cases}$

Initial speed. $\rightarrow$ $v_0 = \sqrt{v_{0x}^2 + v_{0y}^2}$

Launch angle. $\rightarrow$ $\tan\theta_0 = {v_{0y}}/{v_{0x}}$

$$v_x = v_{0x} = \frac{\Delta x}{t}$$

$$v_y = v_{0y} + a_y t \qquad \Delta y = \left(\frac{v_{0y}+v_y}{2}\right) t$$

$$\Delta y = v_{0y} t + \tfrac{1}{2} a_y t^2 \qquad v_y^2 = v_{0y}^2 + 2a\Delta y$$

- **Where is the origin?**

I use the launch point

- **List known and unknown quantities.**

Highest point $v_y = 0$
$a_y = -9.8\ m/s^2$
Time to go up = time to come down.
IvI is same for same height.

- **Read the question carefully to find out what it is asking.**
- **Use your equations plus other known values**
- **Keep the same origin throughout the question.**
- **Estimate and check your answer**

EXAMPLE 3.3 **Paintball gun**

The contestant in Figure 3.14 shoots a paintball horizontally at a speed of 75.0 m/s from a point 1.50 m above the ground. The ball misses its target and hits the ground. **(a)** For how many seconds is the ball in the air? **(b)** Find the maximum horizontal displacement (which we'll call the *range* of the ball). Ignore air resistance.

VTS Ex 3.3

$$v_x = v_{0x} = \frac{\Delta x}{t}$$

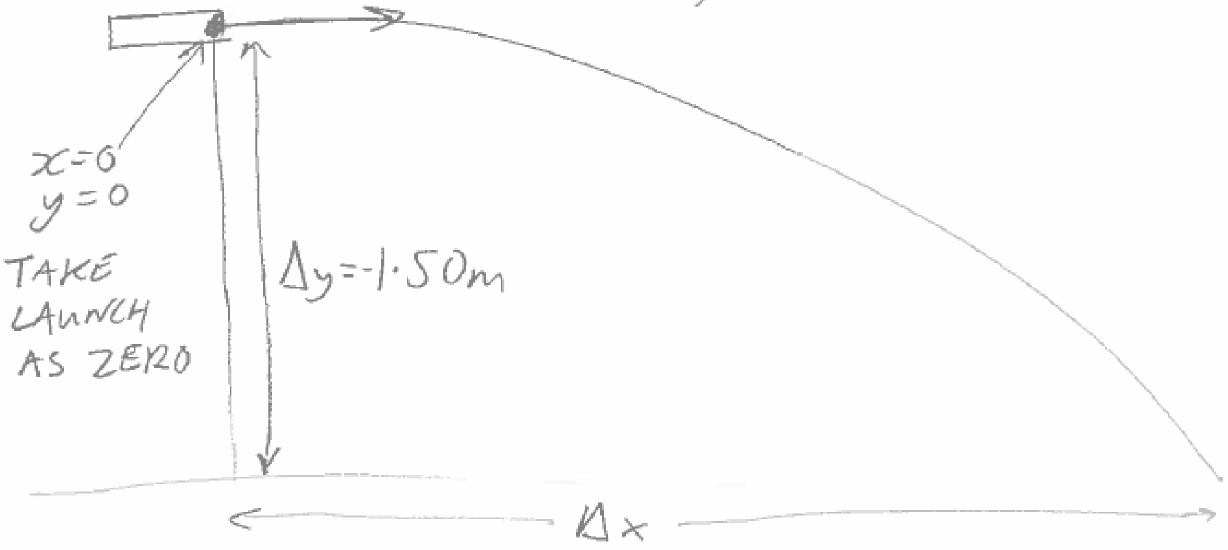
$$v_y = v_{0y} + a_y t \qquad \Delta y = \left(\frac{v_{0y}+v_y}{2}\right) t$$

$$\Delta y = v_{0y} t + \tfrac{1}{2} a_y t^2 \qquad v_y^2 = v_{0y}^2 + 2a\Delta y$$

The contestant in Figure 3.14 shoots a paintball horizontally at a speed of 75.0 m/s from a point 1.50 m above the ground. The ball misses its target and hits the ground. **(a)** For how many seconds is the ball in the air? **(b)** Find the maximum horizontal displacement (which we'll call the *range* of the ball). Ignore air resistance.

$V_{oy} = 0$

$V_{ox} = 75.0\ m/s$



(a) Ⓨ

$t = ?$

$V_{oy} = 0$

$a = -9.8\ m/s^2$

$\Delta y = -1.50\ m$

(negative because goes in negative direction)

$\Delta y = v_0 t + \frac{1}{2} a t^2$

$-1.50 = 0 + \frac{1}{2}(-9.8)t^2$

$t = 0.55\ s$

(b) Ⓧ

$\Delta x = V_x t$

$= 75\,(0.55)$

$= 41\ m$

A tennis player hits a ball at ground level, giving it an initial velocity of 24 m/s at 57° above the horizontal.

(a) What are the horizontal and vertical components of the ball's initial velocity?

(b) How high above the ground does the ball go?

$$v_x = v_{0x} = \frac{\Delta x}{t}$$

$$v_y = v_{0y} + a_y t \qquad \Delta y = \left(\frac{v_{0y}+v_y}{2}\right) t$$

$$\Delta y = v_{0y} t + \tfrac{1}{2} a_y t^2 \qquad v_y^2 = v_{0y}^2 + 2a\Delta y$$

(c) How long does it take the ball to reach its maximum height?

(d) For how long a time is the ball in the air?

(e) What are the ball's velocity and acceleration at its highest point?

(f) When this ball lands on the court, how far is it from the place where it was hit

(g) What is the final velocity just before it hits the ground?

$$v_x = v_{0x} = \frac{\Delta x}{t}$$

$$v_y = v_{0y} + a_y t \qquad \Delta y = \left(\frac{v_{0y}+v_y}{2}\right) t$$

$$\Delta y = v_{0y} t + \tfrac{1}{2} a_y t^2 \qquad v_y^2 = v_{0y}^2 + 2a\Delta y$$

A tennis player hits a ball at ground level, giving it an initial velocity of 24 m/s at 57° above the horizontal.

a) Components
b) High
c) Time to max
d) How long in the air
e) Highest point values
f) Range
g) Final velocity
h) Extra – velocity at 3 s

$$v_x = v_{0x} = \frac{\Delta x}{t}$$

$$v_y = v_{0y} + a_y t \qquad \Delta y = \left(\frac{v_{0y}+v_y}{2}\right)t$$

$$\Delta y = v_{0y}t + \frac{1}{2}a_y t^2 \qquad v_y^2 = v_{0y}^2 + 2a\Delta y$$

Solution

$$v_x = v_{0x} = \frac{\Delta x}{t}$$

$$v_y = v_{0y} + a_y t \qquad \Delta y = \left(\frac{v_{0y}+v_y}{2}\right) t$$

$$\Delta y = v_{0y} t + \tfrac{1}{2} a_y t^2 \qquad v_y^2 = v_{0y}^2 + 2a\Delta y$$

a) x and y- components:

x-component = 24 * cos 57 = 13.07 m/s

y-component = 24 * sin 57 = 20.13 m/s

b) Maximum Height (i.e. final velocity =0)

$V^2 = V_0^2 + 2a\Delta y$ (using the y-component)

$0^2 = 20.13^2 + (2 * -10 * \Delta y)$

$-405.22 = -20 * \Delta y$

$\Delta y = 20.26$ metres

Solution cont...

$$v_x = v_{0x} = \frac{\Delta x}{t}$$

$$v_y = v_{0y} + a_y t \qquad \Delta y = \left(\frac{v_{0y}+v_y}{2}\right)t$$

$$\Delta y = v_{0y}t + \tfrac{1}{2}a_y t^2 \qquad v_y^2 = v_{0y}^2 + 2a\Delta y$$

c) Time to maximum height: (Use y-component)

$V_y = 0 = 20.13 – 10t$

10 t = 20.13, thus t = 2.013 seconds

d) 2 x time to maximum height = 4.026 seconds

e) $v_x = v_{ox}$ = 13.07 m/s $\qquad v_y$ = 0 m/s

(f) When this ball lands on the court, how far is it from the place where it was hit?

$\Delta x = v_x \Delta t$ = 13.07 X 4.026 = 52.6 m

Solution cont...

$$v_x = v_{0x} = \frac{\Delta x}{t}$$

$$v_y = v_{0y} + a_y t \qquad \Delta y = \left(\frac{v_{0y}+v_y}{2}\right)t$$

$$\Delta y = v_{0y}t + \tfrac{1}{2}a_y t^2 \qquad v_y^2 = v_{0y}^2 + 2a\Delta y$$

(g) What is the final velocity just before it hits the ground?

v_x= 13.07 m/s
v_y = - 20.13 m/s (equal in magnitude and opposite to initial)

v = $\sqrt{13.07^2 + 20.13^2}$ = 24.00 m/s

Direction $\alpha = tan^{-1}\left(\frac{20.13}{13.07}\right)$= 57 deg

θ = 360 – 57 = 303 deg

Solution cont…

$$v_x = v_{0x} = \frac{\Delta x}{t}$$

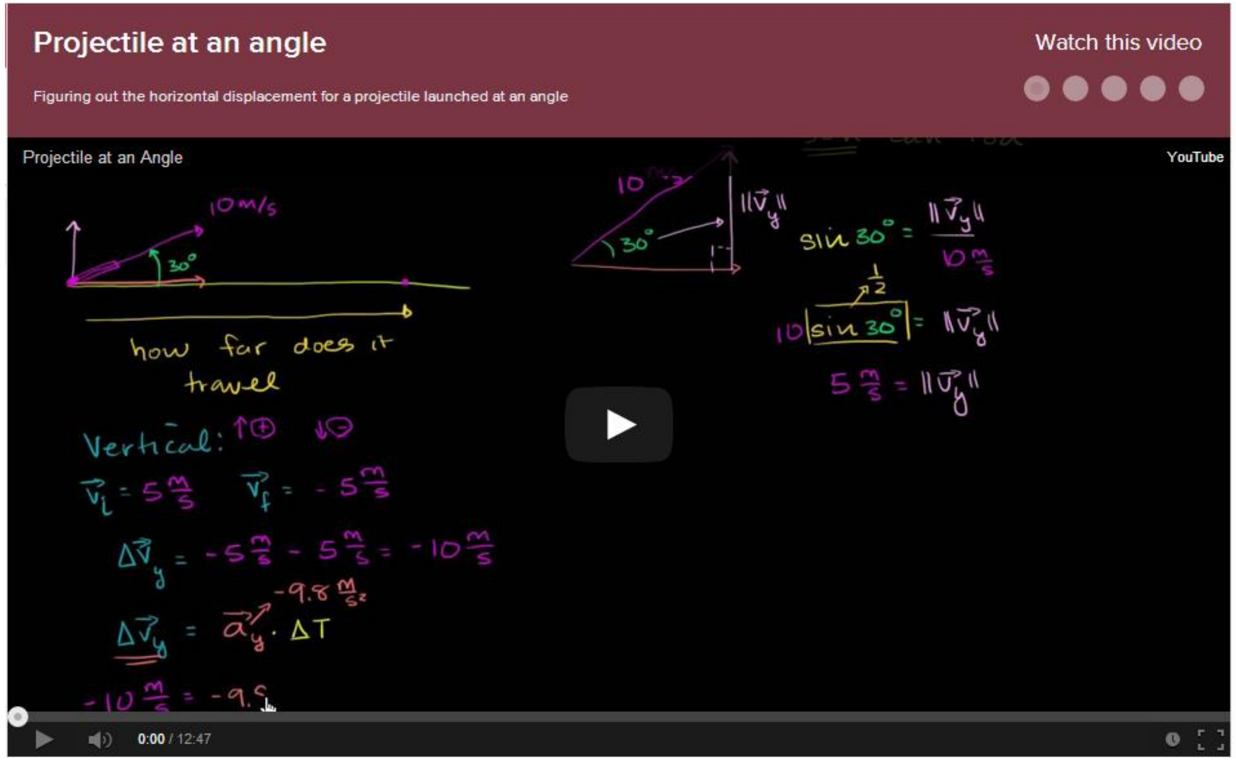
$$v_y = v_{0y} + a_y t \qquad \Delta y = \left(\frac{v_{0y}+v_y}{2}\right) t$$

$$\Delta y = v_{0y} t + \tfrac{1}{2} a_y t^2 \qquad v_y^2 = v_{0y}^2 + 2a\Delta y$$

Range = $(V_o{}^2 * \sin 2\theta) / g$

Projectile at an angle

https://www.khanacademy.org/science/physics/two-dimensional-motion/two-dimensional-projectile-mot/v/projectile-at-an-angle



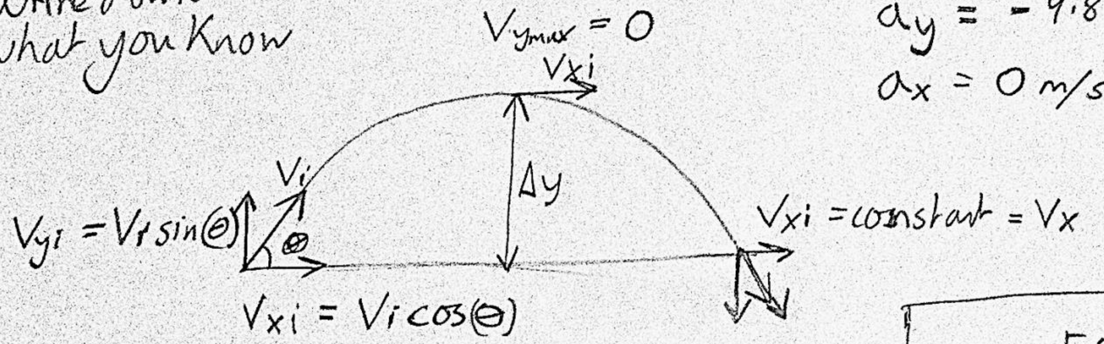
Projetiles

① Write down what you know

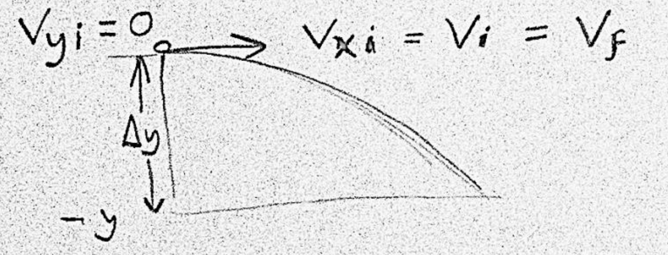
$a_y = -9.8\ m/s^2$

$a_x = 0\ m/s^2$

DEAL WITH HORIZONTAL AND VERTICAL SEPARATLY



If launched horizontaly



② Write down what you want to find

③ Choose the correct equation.

④ Rearange and solve carefully

EQUATIONS

Horizontal (x)

$$\Delta x = v \Delta t$$

Vertical (y)

$$V_f = V_i + a \Delta t$$

$$\Delta y = V_i \Delta t + \frac{1}{2} a \Delta t^2$$

$$V_f^2 = V_i^2 + 2 a \Delta y$$

8. • A physics book slides off a horizontal tabletop with a speed of 1.10 m/s. It strikes the floor in 0.350 s. Ignore air resistance. Find (a) the height of the tabletop above the floor, (b) the horizontal distance from the edge of the table to the point where the book strikes the floor, and (c) the horizontal and vertical components of the book's velocity, and the magnitude and direction of its velocity, just before the book reaches the floor.

$$v_x = v_{0x} = \frac{\Delta x}{t}$$

$$v_y = v_{0y} + a_y t \qquad \Delta y = \left(\frac{v_{0y}+v_y}{2}\right) t$$

$$\Delta y = v_{0y} t + \tfrac{1}{2} a_y t^2 \qquad v_y^2 = v_{0y}^2 + 2a\Delta y$$

a) 0.60 m b) 0.385 m c) 3.60 m/s angle 288 deg

9. • A tennis ball rolls off the edge of a tabletop 0.750 m above the floor and strikes the floor at a point 1.40 m horizontally from the edge of the table. (a) Find the time of flight of the ball. (b) Find the magnitude of the initial velocity of the ball. (c) Find the magnitude and direction of the velocity of the ball just before it strikes the floor.

$$v_x = v_{0x} = \frac{\Delta x}{t}$$

$$v_y = v_{0y} + a_y t \qquad \Delta y = \left(\frac{v_{0y}+v_y}{2}\right) t$$

$$\Delta y = v_{0y} t + \tfrac{1}{2} a_y t^2 \qquad v_y^2 = v_{0y}^2 + 2a\Delta y$$

a) 0.39 s b) 3.58 m/s c) 5.24 m/s angle 313 deg

A

3.8. Set Up: Take $+y$ downward, so $a_x = 0$, $a_y = +9.80\ \text{m/s}^2$, $v_{0x} = 1.10\ \text{m/s}$ and $v_{0y} = 0$.

Solve: (a) $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2 = \frac{1}{2}(9.80\ \text{m/s}^2)(0.350\ \text{s})^2 = 0.600\ \text{m}$. This is the height of the tabletop.

(b) $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 = (1.10\ \text{m/s})(0.350\ \text{s}) = 0.385\ \text{m}$

(c) $v_x = v_{0x} = 1.10\ \text{m/s}$, $v_y = v_{0y} + a_y t = (9.80\ \text{m/s}^2)(0.350\ \text{s}) = 3.43\ \text{m/s}$. $v = \sqrt{v_x^2 + v_y^2} = 3.60\ \text{m/s}$.

$$\tan\theta = \frac{|v_y|}{|v_x|} = \frac{3.43\ \text{m/s}}{1.10\ \text{m/s}}$$

and $\theta = 72.2°$. The velocity of the book just before it hits the floor has magnitude 3.60 m/s and is directed at $72.2°$ below the horizontal.

***3.9. Set Up:** Take $+y$ downward, so $a_x = 0, a_y = +9.80\ \text{m/s}^2$ and $v_{0y} = 0$. When the ball reaches the floor, $y - y_0 = 0.750\ \text{m}$.

Solve: (a) $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $t = \sqrt{\frac{2(y - y_0)}{a_y}} = \sqrt{\frac{2(0.750\ \text{m})}{9.80\ \text{m/s}^2}} = 0.391\ \text{s}$.

(b) $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ gives $v_{0x} = \frac{x - x_0}{t} = \frac{1.40\ \text{m}}{0.391\ \text{s}} = 3.58\ \text{m/s}$. Since $v_{0y} = 0$, $v_0 = v_{0x} = 3.58\ \text{m/s}$.

(c) $v_x = v_{0x} = 3.58\ \text{m/s}$. $v_y = v_{0y} + a_y t = (9.80\ \text{m/s}^2)(0.391\ \text{s}) = 3.83\ \text{m/s}$. $v = \sqrt{v_x^2 + v_y^2} = 5.24\ \text{m/s}$.

$$\tan\theta = \frac{|v_y|}{|v_x|} = \frac{3.83\ \text{m/s}}{3.58\ \text{m/s}}$$

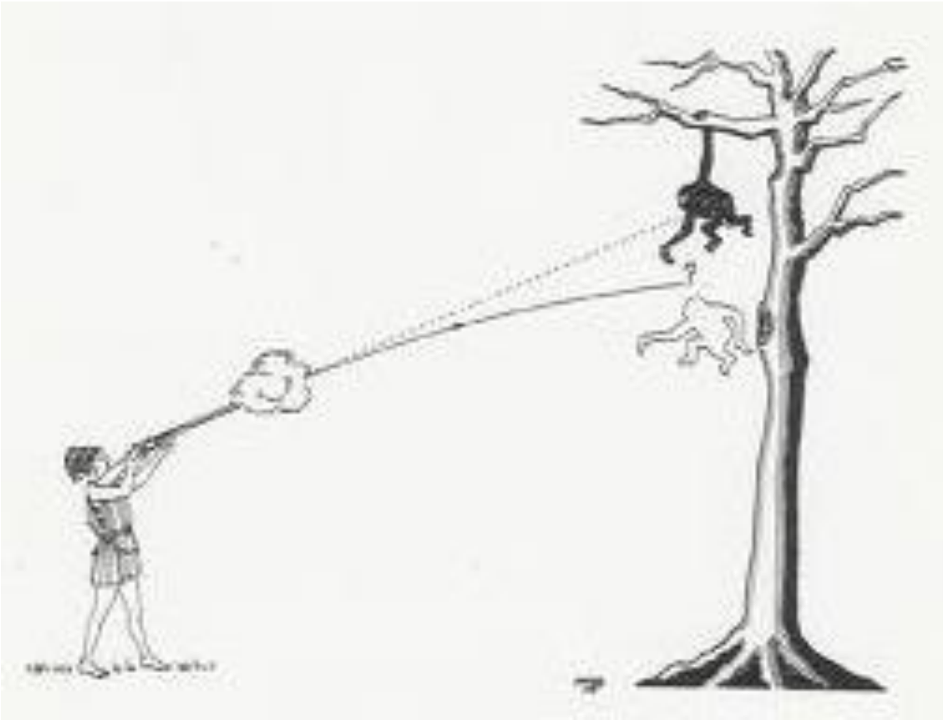
and $\theta = 46.9°$. The final velocity of the ball has magnitude 5.24 m/s and is directed at 46.9° below the horizontal.

Reflect: The time for the ball to reach the floor is the same as if it had been dropped from a height of 0.750 m; the horizontal component of velocity has no effect on the vertical motion.

VTD Range of gun at two firing angles

Monkey and gun

A shooter hiding under a bush in a forest sees a monkey on a tree 50m high and immediately points his gun at the monkey and shoots him. The monkey notices the shooter and jumps from the tree at the same time when the bullet is released from the gun.
Where should he aim the gun?



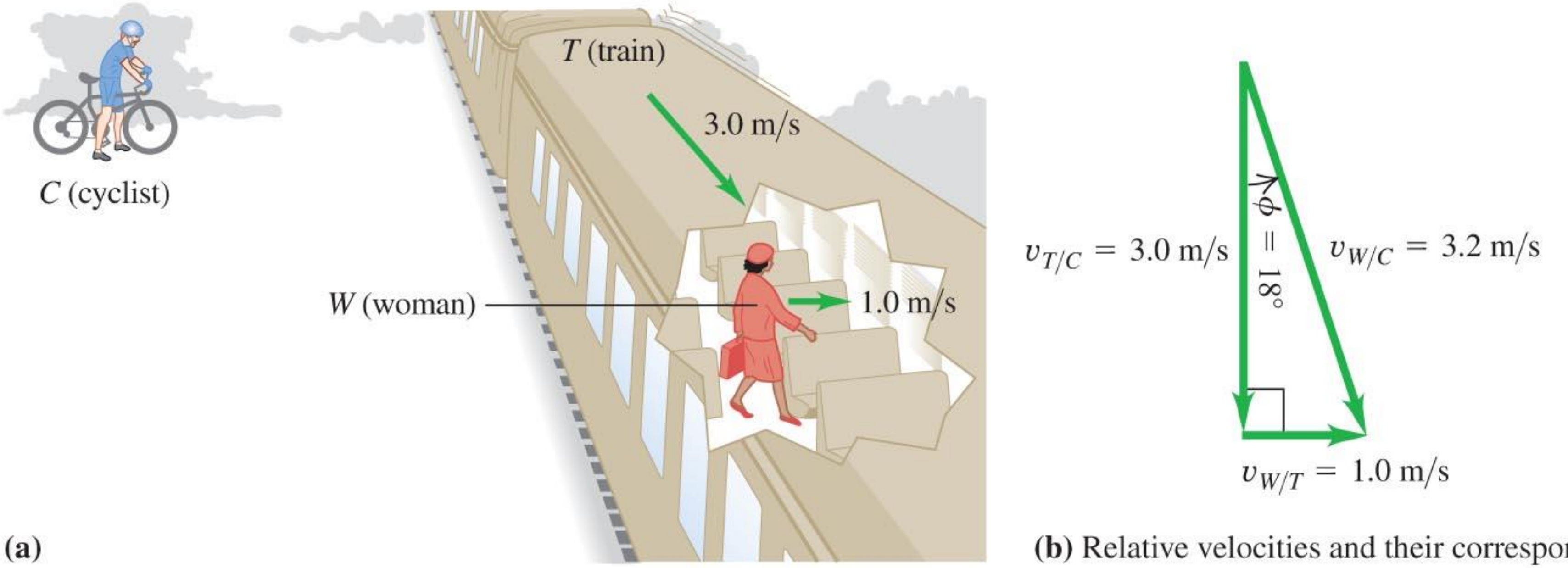
http://www.youtube.com/watch?v=cxvsHNRXLjw&feature=kp

Like VTS Ex 3.7

Relative Velocity

- Velocities can carry multiple values depending on the position and motion of the object and the observer.

$$\vec{v}_{W/C} = \vec{v}_{W/T} + \vec{v}_{T/C}$$



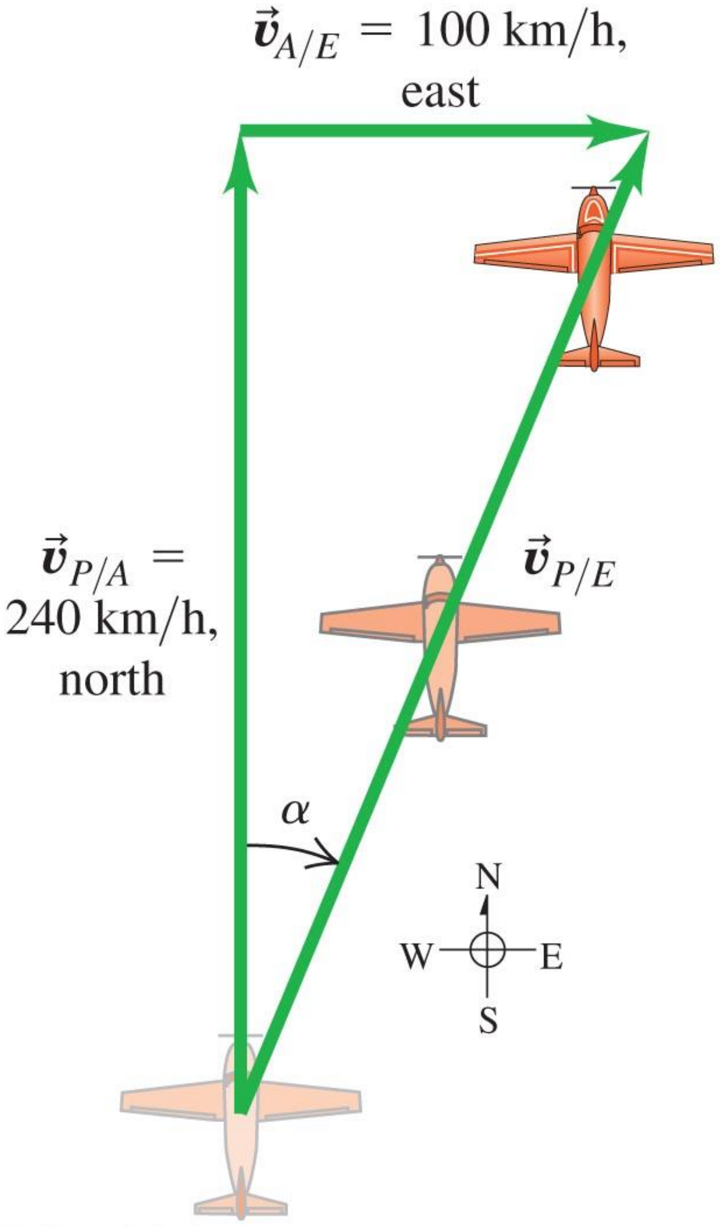
(b) Relative velocities and their corresponding magnitudes as seen from above

An Airplane in a Crosswind

- A solved application of relative motion.
- This is just velocity vector addition.

$$\vec{v}_{P/E} = \vec{v}_{P/A} + \vec{v}_{A/G}$$

VTS Ex 3.10

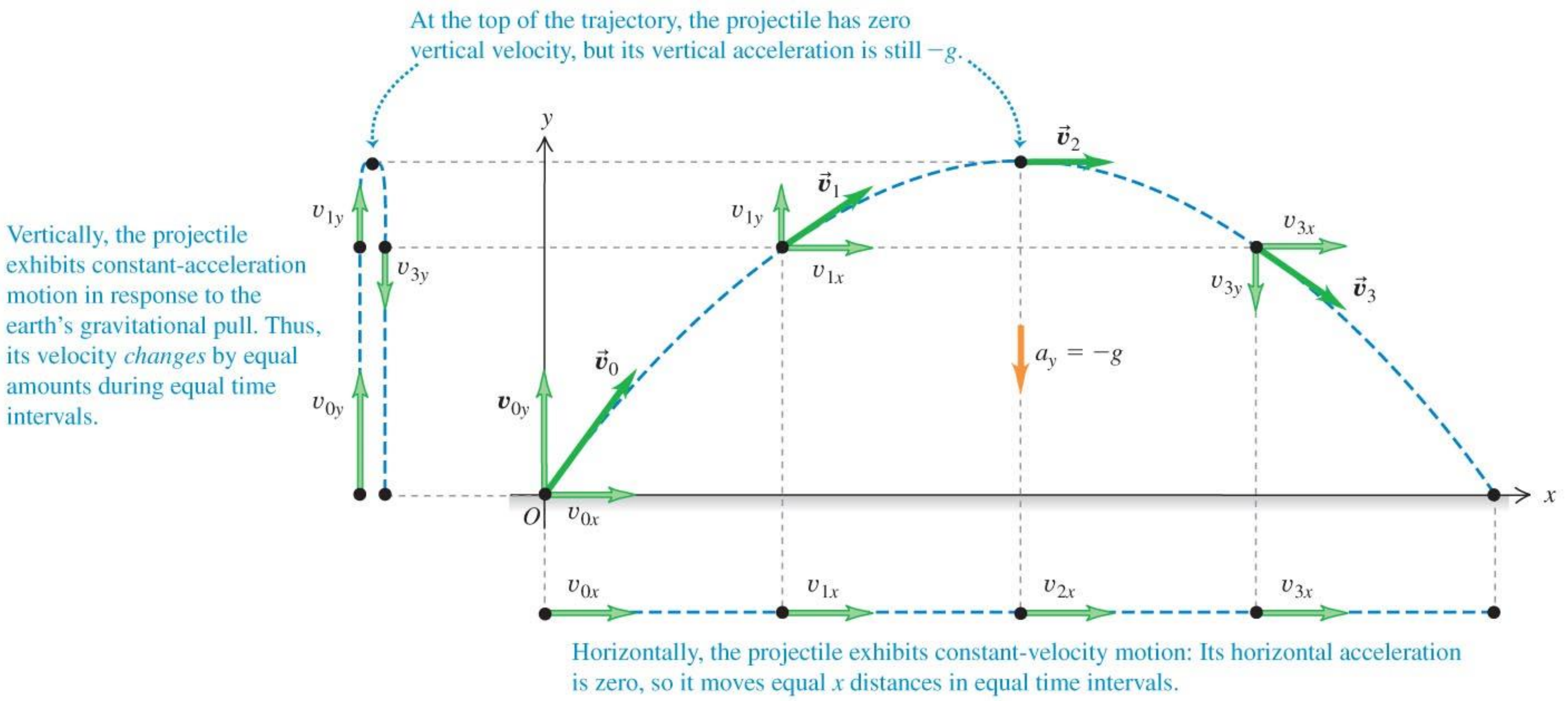


Summary

Motion (CLO3, Chapters 2 & 3)

- Δy and Δx can be interchanged.
- For free fall (in the y direction) $a = -g = -9.8 m/s^2$
- For free fall the y component of velocity at the highest point is zero.
- Deal with x (horizontal) and y (vertical) separately
- Time is the common value for x and y directions.
- I take the launch point as zero time and zero x and y coordinates.

Summary



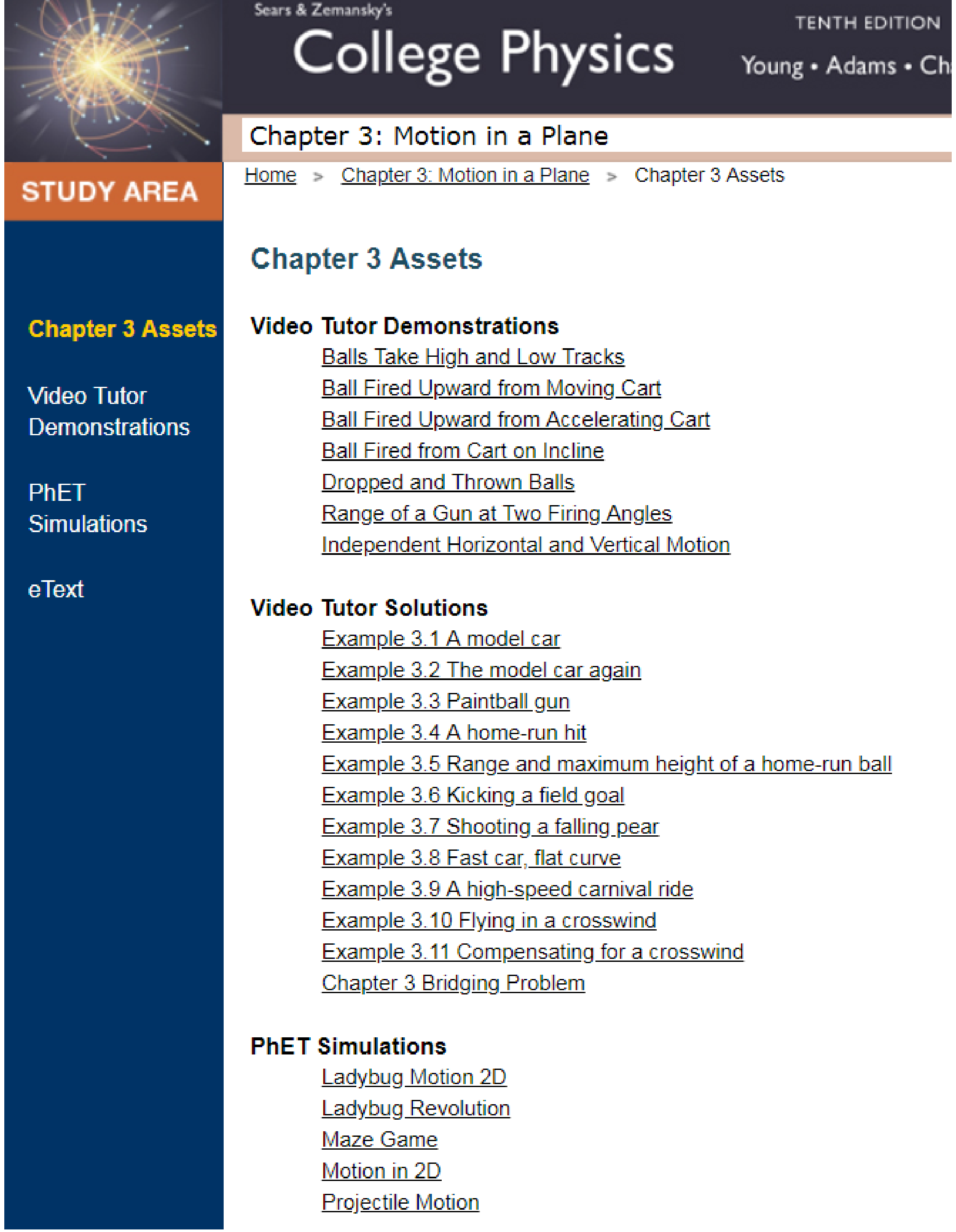
At the top of the trajectory, the projectile has zero vertical velocity, but its vertical acceleration is still $-g$.
Vertically, the projectile exhibits constant-acceleration motion in response to the earth's gravitational pull. Thus, its velocity *changes* by equal amounts during equal time intervals.
y
x
O
$\vec{v}_0$
$\vec{v}_1$
$\vec{v}_2$
$\vec{v}_3$
v_{0x}
v_{0y}
v_{1x}
v_{1y}
v_{2x}
v_{3x}
v_{3y}
$a_y = -g$
Horizontally, the projectile exhibits constant-velocity motion: Its horizontal acceleration is zero, so it moves equal x distances in equal time intervals.

Use the textbook for more details

VTD Balls take high and low tracks

VTS Ex 3.1

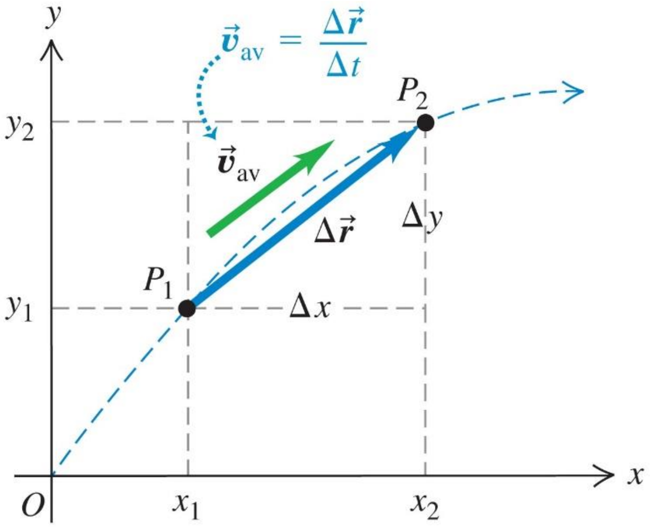
PhET Projectile motion



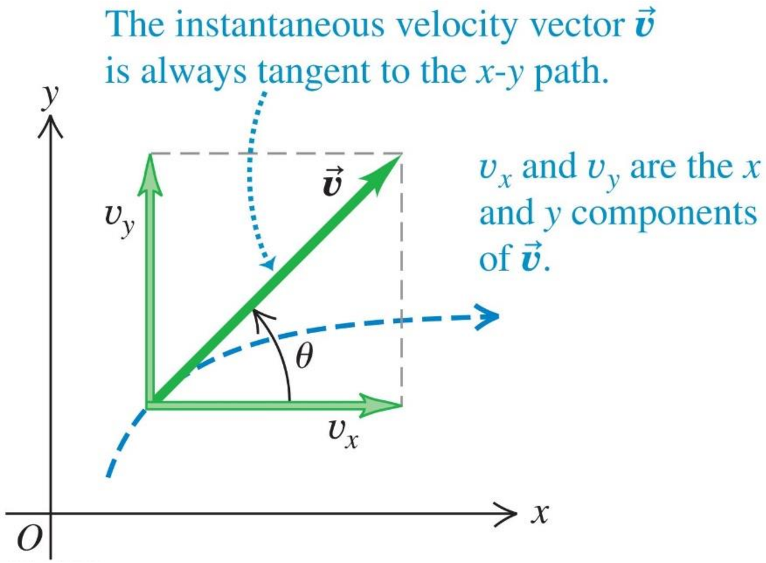
Appendix
Extra information and questions

Velocity in a Plane

- From the graphs, we see both average and instantaneous velocity vectors.



(b)



Average velocity of a particle over displacement $\Delta\vec{r}$

$$\vec{v}_{av} = \frac{\Delta \vec{r}}{\Delta t}$$

Instantaneous velocity of particle at point $\vec{r}$

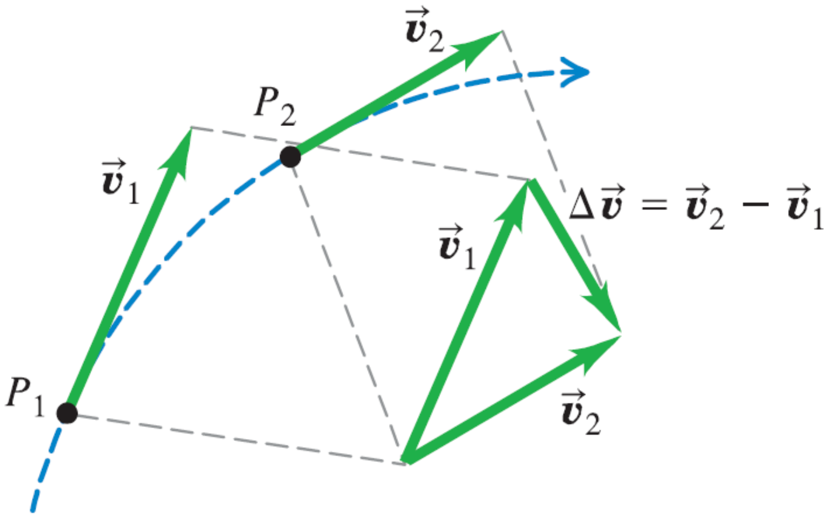
$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t}$$

Average Acceleration in a Plane

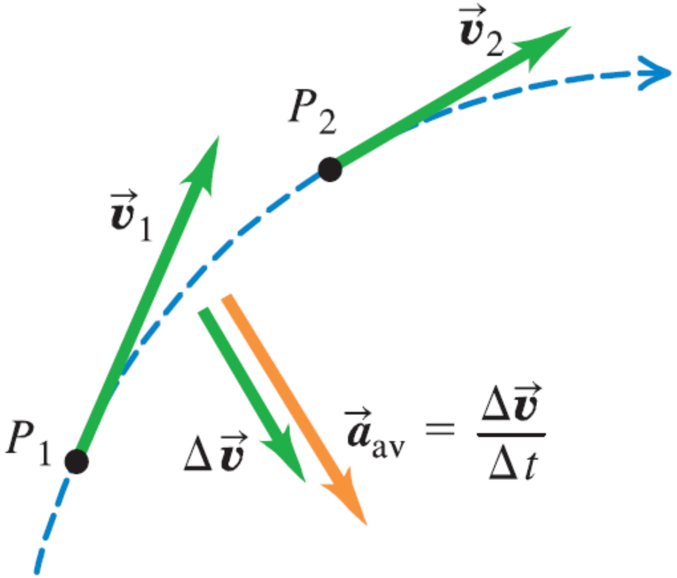
Definition of average acceleration $\vec{a}_{av}$

As an object undergoes a displacement during a time interval Δt, its average acceleration is its change in velocity, $\Delta\vec{v}$, divided by Δt:

$$\text{Average acceleration} = \vec{a}_{av} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} = \frac{\Delta\vec{v}}{\Delta t}. \qquad (3.4)$$



To find the car's average acceleration between P_1 and P_2, we first find the change in velocity $\Delta\vec{v}$ by subtracting $\vec{v}_1$ from $\vec{v}_2$. (Notice that $\vec{v}_1 + \Delta\vec{v} = \vec{v}_2$.)



The average acceleration has the same direction as the change in velocity, $\Delta\vec{v}$.

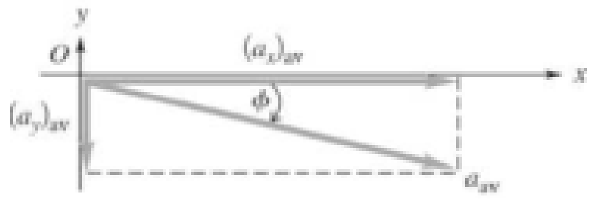
• At an air show, a jet plane has velocity components v_x = 625 km/h and v_y = 415 km/h at time 3.85 s and v_x = 838 km/h and v_y = 365 km/h at time 6.52 s. For this time interval, find (a) the x and y components of the plane's average acceleration and (b) the magnitude and direction of its average acceleration.

3.2. Set Up: Since the velocity is expressed in units of km/h and the time interval is in seconds, it is convenient to express the acceleration in mixed units of km/h · s.

Solve: (a)

$$(a_x)_{av} = \frac{\Delta v_x}{\Delta t} = \frac{838 \text{ km/h} - 625 \text{ km/h}}{6.52 \text{ s} - 3.85 \text{ s}} = 79.8 \text{ km/h} \cdot \text{s}; \; (a_y)_{av} = \frac{\Delta v_y}{\Delta t} = \frac{365 \text{ km/h} - 415 \text{ km/h}}{6.52 \text{ s} - 3.85 \text{ s}} = -18.7 \text{ km/h} \cdot \text{s}$$

(b) $\vec{a}_{av}$ and its components are shown in the figure below.



$$a_{av} = \sqrt{(a_x)_{av}^2 + (a_y)_{av}^2} = 820 \text{ km/h} \cdot \text{s}; \; \tan\phi = \frac{(v_y)_{av}}{(v_x)_{av}} \text{ and } \phi = 13.2°, \text{ below the } +x\text{-axis.}$$