Momentum

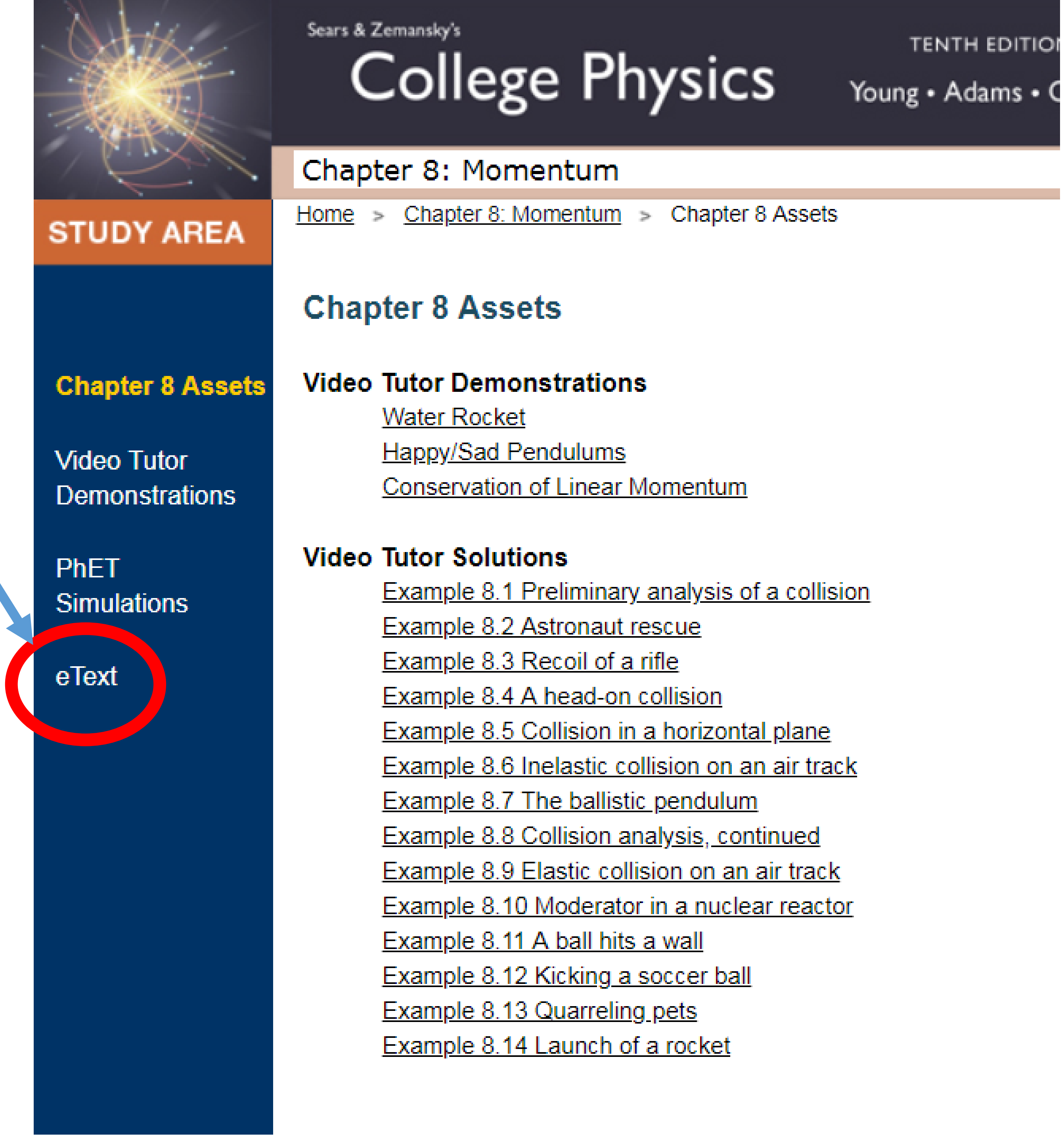
ch8

10/06/2022

Use textbook for greater detail

VTD Water rocket

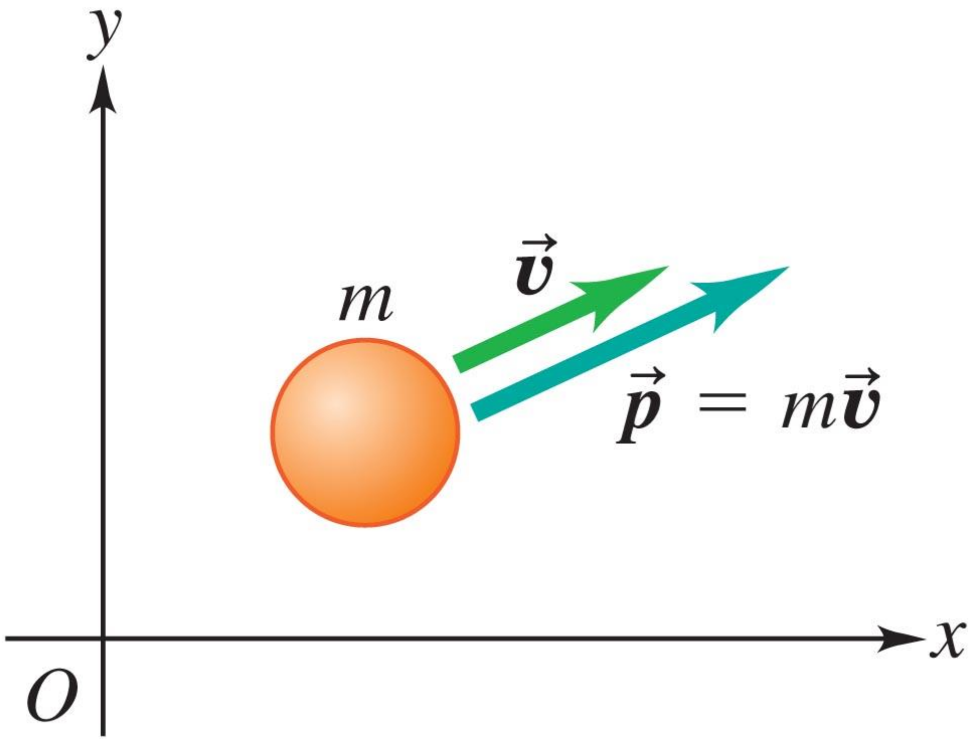
VTS Ex 8.1



Goals for Chapter 8

- Study momentum.
- Understand conservation of momentum.
- Study momentum changes during collisions.
- Add time and study impulse.
- Understand center of mass and how forces act on the c.o.m.
- Apply momentum to rocket propulsion.

Momentum $\vec{p}$ is a vector quantity; a particle's momentum has the same direction as its velocity $\vec{v}$.



Linear momentum and Newton's 2nd law

Assuming a resultant force acts on a body to accelerate it

$$p = mv$$

Momentum is a vector
In the same direction as acceleration
Units of kg m/s = N s

- $F = ma$
- $F = m\frac{\Delta v}{t}$
- $F = m\frac{(v_f - v_i)}{t}$
- $F = \frac{mv_f - mv_i}{t}$
- $F = \frac{p_f - p_i}{t}$
- $F = \frac{\Delta p}{t}$

Example 1

- Two objects on a head-on collision course have masses of 200 g and 300 g and speeds of 4.0 m/s and 2.5 m/s, respectively.

(a) Calculate the momentum of each object.

(b) Calculate the total momentum of the system made up of these two objects.

Force and Linear Momentum

- The net force acting on a particle is equal to the rate of change in its linear momentum.

  - $F = \frac{mv_f - mv_i}{t}$
  - $F = \frac{p_f - p_i}{t}$
  - $F = \frac{\Delta p}{t}$

Momentum and Newton's 3rd Law

$$F_1 = -F_2$$

$$\frac{\Delta p_1}{t} = -\frac{\Delta p_2}{t}$$

$$\Delta p_1 = -\Delta p_2$$

$$p_{1f} - p_{1i} = -(p_{2f} - p_{2i})$$

$$p_{1i} + p_{2i} = p_{1f} + p_{2f}$$

$$p_{ti} = p_{tf}$$

$$\sum p_i = \sum p_f$$

Newton's third law is: For every action, there is an equal and opposite reaction.

So if 2 objects collide the force on one is equal and opposite to the other when they are in contact with each other.

Conservation of linear momentum

$$\sum p_i = \sum p_f$$

The ***total*** momentum of an isolated system remains constant during any interaction

Two gliders move toward each other on a linear air track, which we assume is frictionless. Glider A has a mass of 0.50 kg, and glider B has a mass of 0.30 kg; both gliders move with an initial speed of 2.0 m/ s. After they collide, glider B moves away with a final velocity whose x component is +2.0 m/s. What is the final velocity of A?

Hint: Velocity is a vector so has direction

VTS Ex 8.4

- A Beretta 92 FS pistol has a mass of 970 g. It can shoot a 147-g bullet with a muzzle velocity of 318 m/s.
- What is the ‘recoil’ velocity of the gun?

Like VTS Ex 8.3

Solution for Beretta 92 Pistol….

Momentum Initial = Momentum Final

Before the Pistol is Fired the Bullet is inside the gun, therefore the **Momentum Initial = 0**

This must mean the **Final Momentum** is also **= 0**

Initial Momentum = (0.970 x 0) + (0.147 x 0) = 0

Final Momentum = (0.970 x v_p) + (0.147 x 318) = 0

Final Momentum = 0.970 x v_p + 46.746 = 0

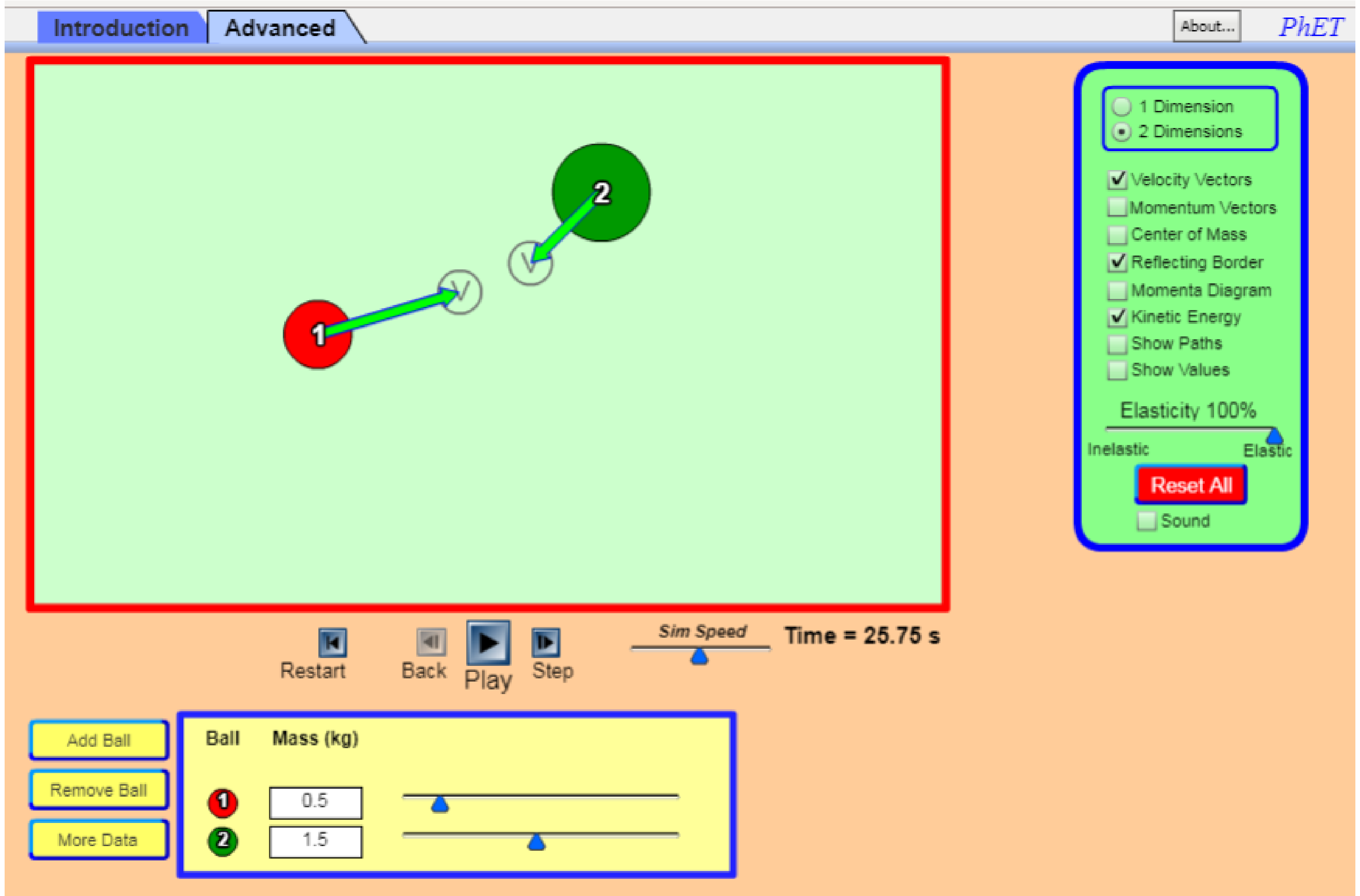
Final Momentum = 0.970 x v_p = - 46.746

v_p = - 46.746 / 0.970 = - 48.19 m/s

The **–ve** velocity means the gun travels in the **opposite** direction to the bullet and 48.19 m/s is ~173.5 km/h !!


© Getty Images

VTD Water rocket



https://phet.colorado.edu/en/simulation/collision-lab

This problem now has 2 dimensions!

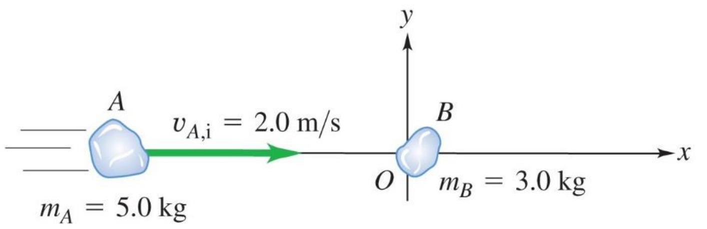
Like VTS Ex 8.1

- Two chunks of ice collide on the surface of a frictionless frozen pond. Chunk A, with mass $m_A = 5.0$ kg, moves with initial velocity, $V_{A,i} = 2.0$ m/s parallel to the x axis. It collides with chunk B, which has mass $m_B = 3.0$ kg and is initially at rest.
- After the collision, the velocity of Chunk A is found to be 1.0 m/s in a direction at an angle $\alpha = 30°$ with the initial direction.
- What is the final velocity of B (magnitude and direction)?

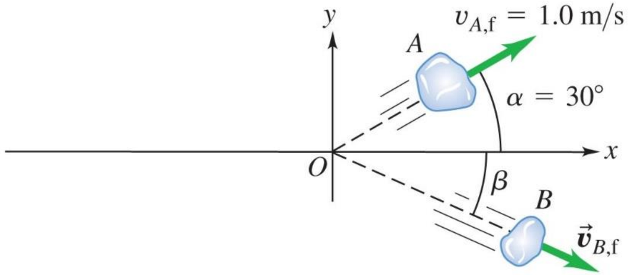
Let Us See How This Problem is Solved……

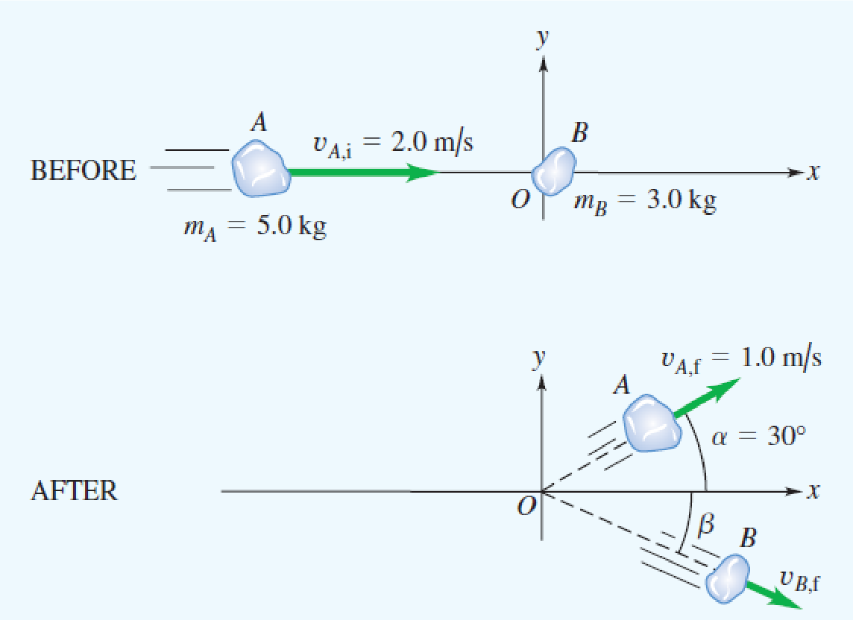
See Handbook Page 239 for this problem and solution…..

BEFORE



AFTER





X - Component

SOLVE We start by writing expressions for the total x component of momentum before and after the collision. For the x components, we have

$$m_A(v_{A,i,x}) + m_B(v_{B,i,x}) = (5.0\text{ kg})(2.0\text{ m/s}) + (3.0\text{ kg})(0), \quad \text{(before)}$$

$$m_A(v_{A,f,x}) + m_B(v_{B,f,x}) = (5.0\text{ kg})(1.0\text{ m/s})(\cos 30°) + (3.0\text{ kg})(v_{B,f,x}). \quad \text{(after)}$$

Equating these two expressions and solving for $v_{B,f,x}$, we find that

$$v_{B,f,x} = 1.89\text{ m/s}. \quad (x\text{ component of final velocity of } B)$$

Y - Component

Conservation of the y component of total momentum gives

$$m_A(v_{A,i,y}) + m_B(v_{B,i,y}) = (5.0\text{ kg})(0) + (3.0\text{ kg})(0), \quad \text{(before)}$$

$$m_A(v_{A,f,y}) + m_B(v_{B,f,y}) = (5.0\text{ kg})(1.0\text{ m/s})(\sin 30°) + (3.0\text{ kg})(v_{B,f,y}). \quad \text{(after)}$$

Equating these two expressions and solving for $v_{B,f,y}$, we obtain

$$v_{B,f,y} = -0.83\text{ m/s}. \quad (y\text{ component of final velocity of } B)$$

Equate Both Directions for Magnitude

We now have the x and y components of the final velocity $\vec{v}_{B,f}$ of chunk B. The magnitude of $\vec{v}_{B,f}$ is

$$|\vec{v}_{B,f}| = \sqrt{(1.89\text{ m/s})^2 + (-0.83\text{ m/s})^2} = 2.1\text{ m/s}, \quad \text{(final } \textit{speed} \text{ of } B)$$

And the Angle……

and the angle β of its direction from the positive x axis is

$$\beta = \tan^{-1}\frac{-0.83\text{ m/s}}{1.89\text{ m/s}} = -24°.$$

Now Try This One……

Practice Problem: If chunk B has an initial velocity of magnitude 2.0 m/s in the $+y$ direction instead of being initially at rest, find its final velocity (magnitude and direction). *Answer:* 2.2 m/s, 32°.

Collisions

Elastic

Total kinetic energy is conserved.

$$K_{t,i} = K_{t,f}$$

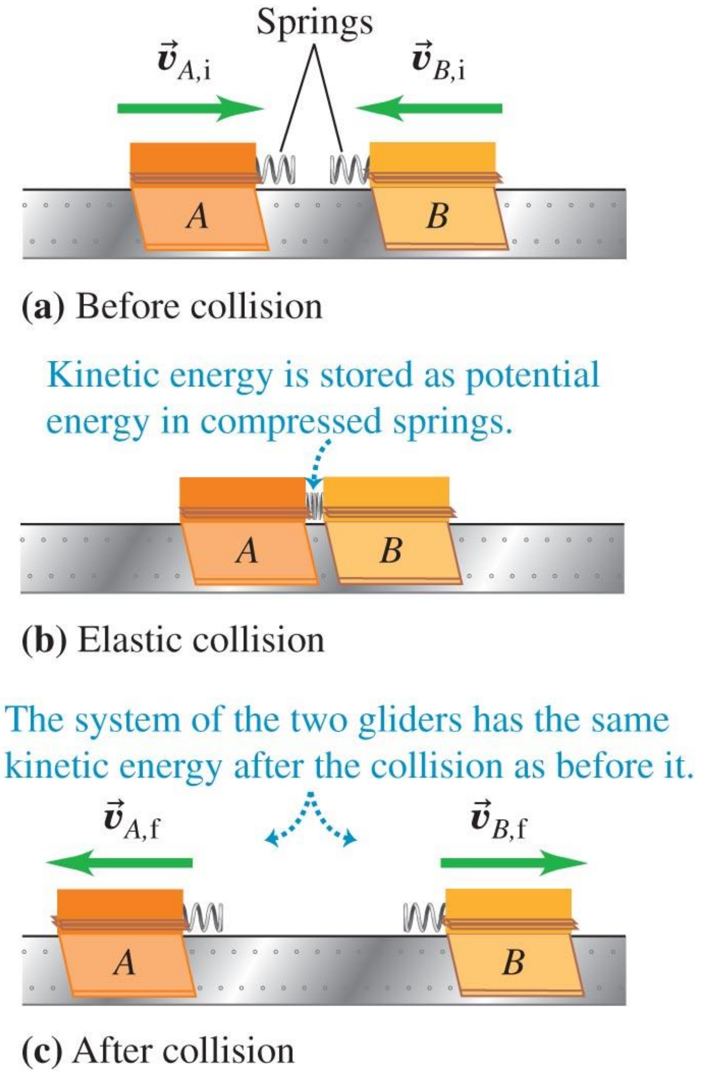
Inelastic

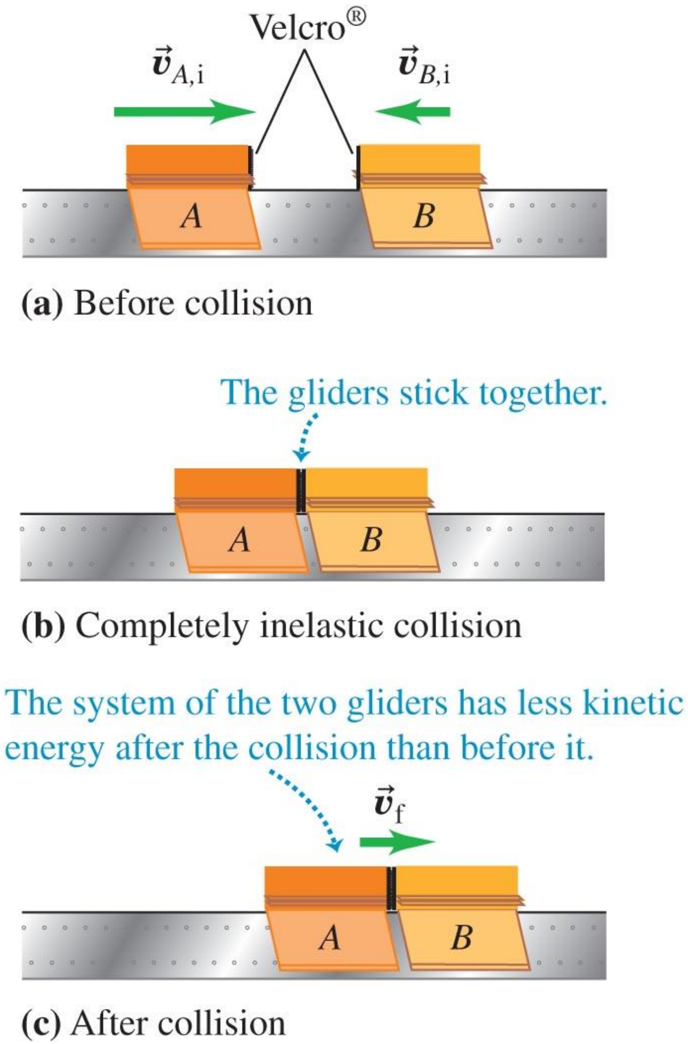
Total kinetic energy is NOT conserved.

$$K_{t,i} \neq K_{t,f}$$

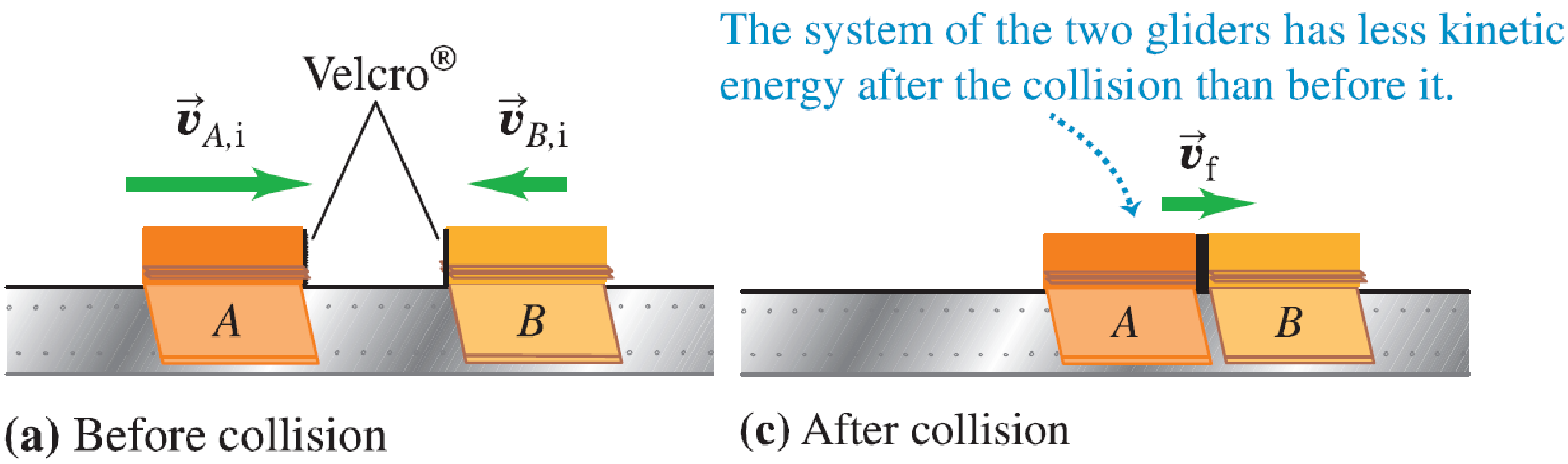
Elastic

Inelastic





Inelastic collisions



- Momentum is conserved since the system is isolated.
- KE is not conserved since the collision is not elastic.

Inelastic collisions: example

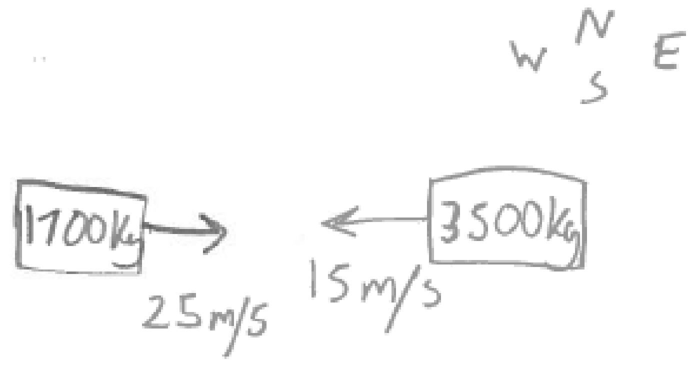
- A truck of mass 3500 kg moving at 15 m/s westward collides with a 1700 kg car moving at 25 m/s eastward. The two vehicles are enmeshed and move together.

(a) Find their common speed and direction.

(b) Determine whether the collision is elastic.

Solution….

- A truck of mass 3500 kg moving at 15 m/s westward collides with a 1700 kg car moving at 25 m/s eastward. The two vehicles are enmeshed and move together.



a) First, we check the initial momentum……

I've selected East as my **+Ve** x-axis…..

So I have to calculate initial momentum of the system:

(1700 x 25) + (3500 x – 15) = 42,500 – 52,500 = – 10,000 kgm/s

So here we see the momentum is in the –x-direction (or West).

Solution….

If the truck and car then enmesh our final momentum will be:

(Mass of Truck + Mass of Car) x (Velocity)

Since we know **momentum** ***MUST*** be conserved, we have:

(3500 + 1700) x (Velocity) = -10,000

5200 x Velocity = -10,000

Velocity = -1.923 m/s

So when the truck and car crash and stick together, the overall velocity of both is **1.923 m/s** to the Westward direction.

Solution….

b) To check for an elastic collision we must calculate the initial and final kinetic energies – we know that an Elastic Collision must conserve both Energy & Momentum.

Initial K.E.= ($\frac{1}{2}$ x 1700 x 25^2) + ($\frac{1}{2}$ x 3500 x $(-15)^2$) = 925,000 Joules

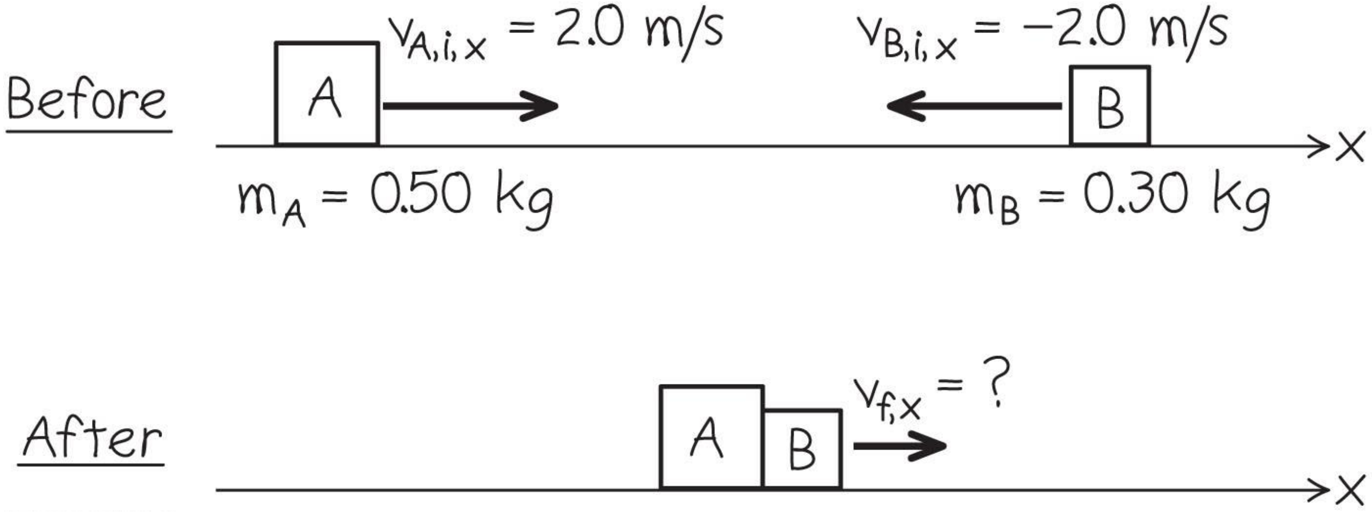
Final K.E. = $\frac{1}{2}$ x 5200 x $(-1.923)^2$ = 9,614.62 Joules

So K.E. is not conserved, this amount is only ~1.04% of the Original K.E. But Momentum is conserved, so the collision is *Inelastic* !!

Another similar example

VTS Ex 8.6

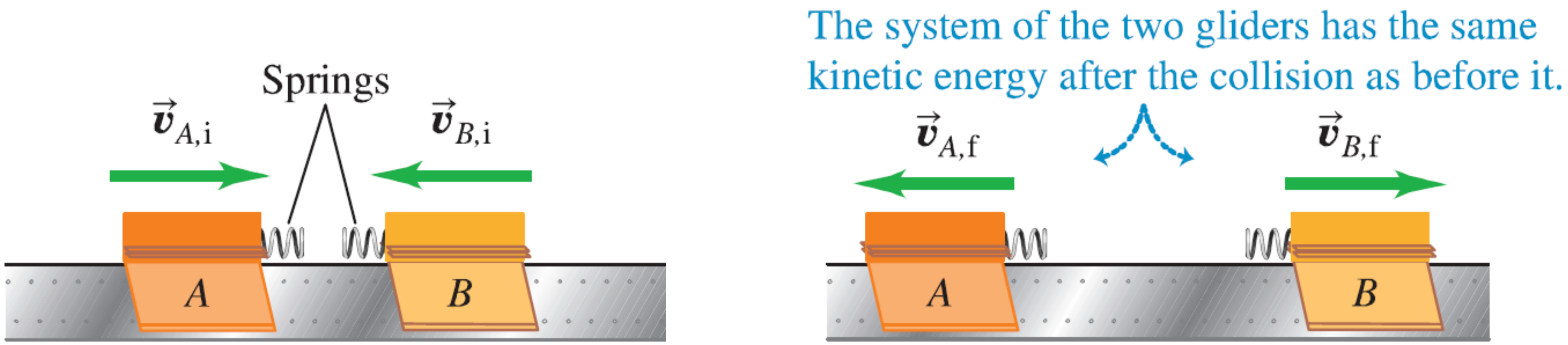
- Find final velocity and compare initial and final KEs



ANS v = 0.50 m/s.

Elastic collisions

- Total kinetic energy *is* conserved during collision.
- Momentum is conserved since the system is isolated.



(a) Before collision

(c) After collision

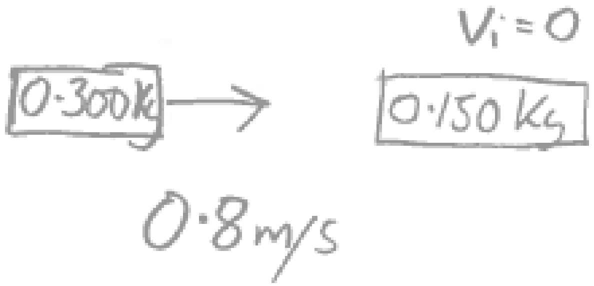
Elastic collisions (Pages 244-245)

- Since *both* total momentum and total kinetic energy are conserved, it can be shown that:

the initial relative velocity and the final relative velocity have equal magnitudes but opposite signs.

$$\vec{v}_{B,f} - \vec{v}_{A,f} = -(\vec{v}_{B,i} - \vec{v}_{A,i})$$

Elastic collisions:
For you to try !!



Like VTS Ex 8.9

A 0.300 kg glider is moving to the right on a frictionless, horizontal air track with a speed of 0.80 m/s when it makes a head-on collision with a stationary 0.150 kg glider.

(a) Find the initial K.E. of each glider.

(b) Find the magnitude and direction of the final velocity of each glider.

(c) Find the final kinetic energy of each glider.

(d) Can you prove the collision is Elastic ?

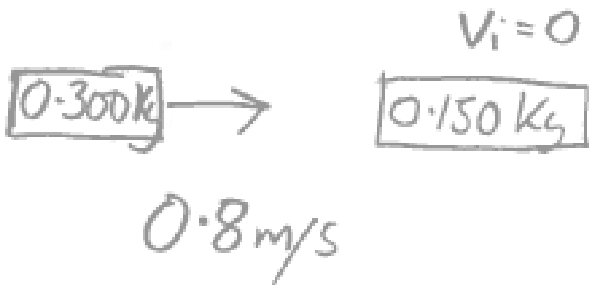
Also use for revision

ANS b) va_{final} = 0.267 m/s, thus vb_{final} = 1.07 m/s

A 0.300 kg glider is moving to the right on a frictionless, horizontal air track with a speed of 0.80 m/s when it makes a head-on collision with a stationary 0.150 kg glider.

ANS b) va_{final} = 0.267 m/s, thus vb_{final} = 1.07 m/s

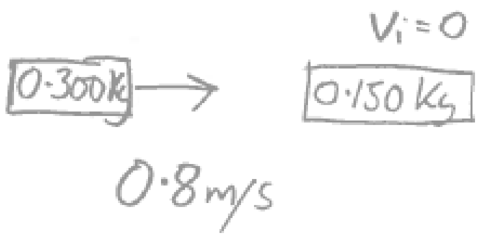
Solution (2)….



a) Here we calculate the K.E. of each glider, hence Total KE.

Eq1. $(\frac{1}{2} \times 0.3 \times 0.8^2) + (\frac{1}{2} \times 0.150 \times 0^2) = 0.096$ Joules

b) Now the final magnitude and direction of the gliders from conservation of momentum…



$$(0.3 \times 0.8) + (0.150 \times 0) = 0.24 = (0.3\ va_{final}) + (0.150\ vb_{final})$$

$$\text{We know: } vb_{final} - va_{final} = -(vb_{initial} - va_{initial})$$

$$= -(0 - 0.8) = +0.8 \text{ m/s}$$

$$\text{So } vb_{final} = 0.8 + va_{final}$$

$$0.24 = (0.3 \times va_{final}) + (0.150 \times (0.8 + va_{final}))$$

$$0.24 = 0.3\ va_{final} + 0.12 + 0.150\ va_{final}$$

$$+0.12 \text{ m/s} = 0.450\ va_{final}$$

$$va_{final} = 0.2666 \text{ m/s, thus } vb_{final} = 1.0666 \text{ m/s}$$

Solution

c) The final K.E. of each glider:

$(½ \times 0.3 \times 0.2666^2) + (½ \times 0.150 \times 1.0666^2) = 0.096$ Joules (2dp)

d) **As The *KE Before = KE After*, thus this is an *elastic* collision as the *KE has been conserved* as well as the Momentum !!**

Impulse $\vec{J}$

Definition

$$\vec{J} = \vec{F}\Delta t$$

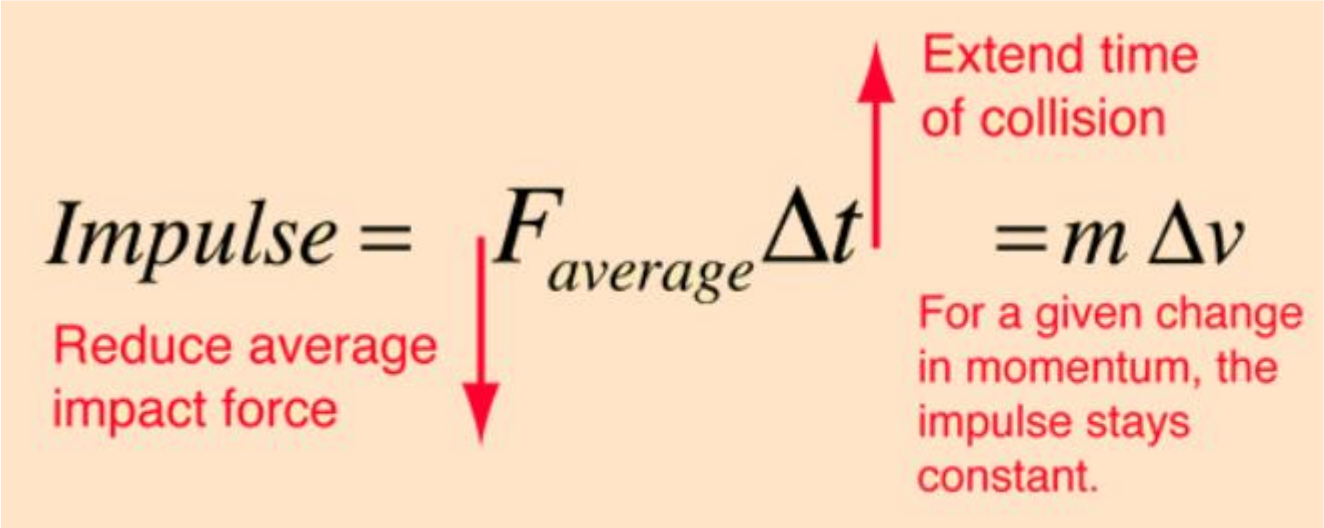
Use definition to calculate impulse if $\vec{F}$ is constant

Which means…

$$\vec{J} = \Delta\vec{p} = \vec{F}\Delta t$$

Impulse is change in momentum.

This can be applied to safety features on cars.



- If you jump to the ground from any height, you bend your knees upon impact, extending the time of collision and lessening the impact force.
- A boxer moves away from a punch, extending the time of impact and lessening the force.
- Automobiles are made to collapse upon impact, extending the time of collision and lessening the impact force.

Impulsive driver

- A driver suddenly steps on the accelerator of his 2000-kg vehicle, causing a constant force of 2000N to be applied over in 10 seconds.

$$\vec{J} = \Delta\vec{p} = \vec{F}\Delta t$$

a) Calculate the impulse during this time.

b) By how much did the speed increase (assuming it's travelling in a straight line)

c) Is it possible to determine its final speed using the given information?

Solution

Firstly, we must use the **Impulse** equation.

a) Therefore: $J = F * \Delta t = 2000 * 10 = 20,000$ Ns

b) Well, $J = \Delta p$, so the ***Change in Momentum*** = 20,000 Ns

$$\text{So, } \Delta p = m \times v_{final} - m \times v_{initial}$$

$$20,000 = 2000 * v_{final} - 2000 * v_{initial}$$

Dividing through by 2000, we get….

$$10 = v_{final} - v_{initial}$$

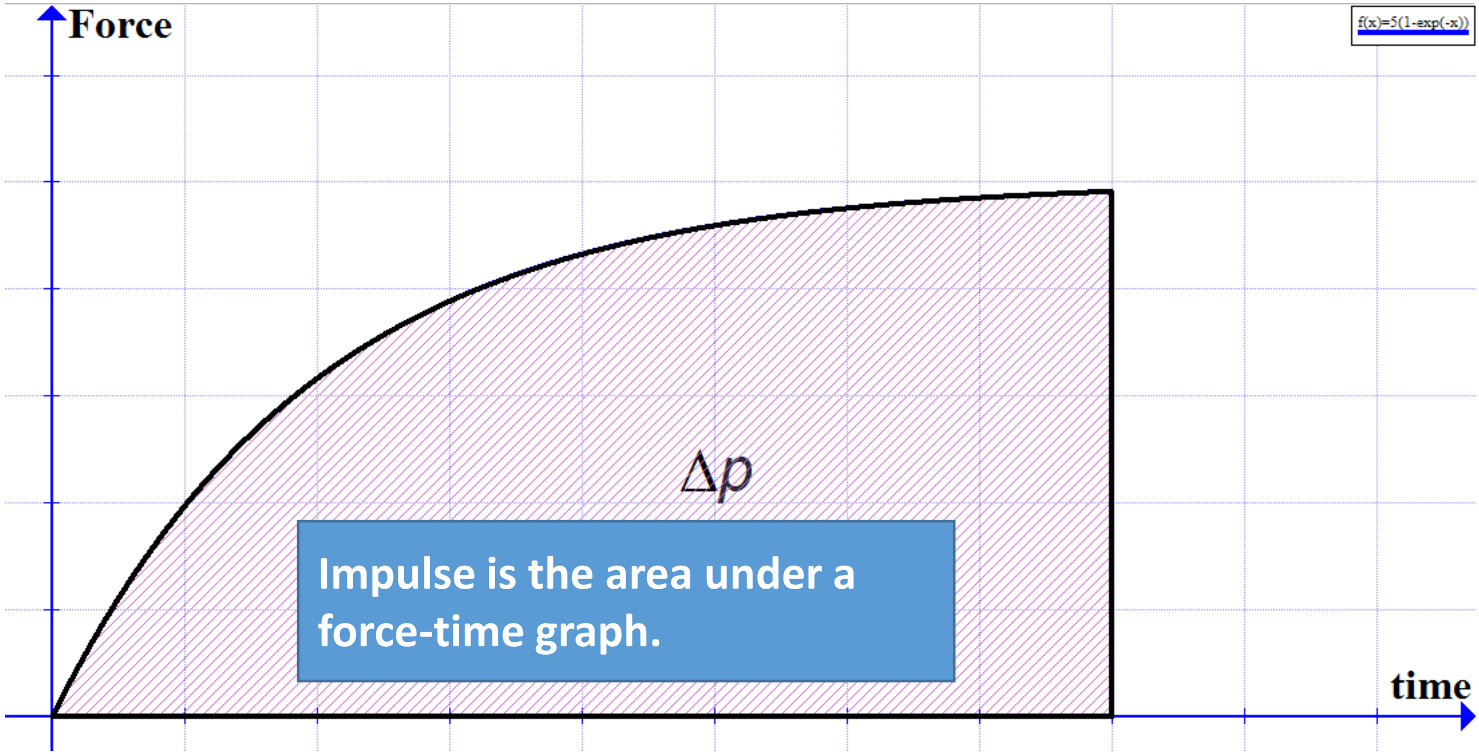
So the *change in velocity* is +10 m/s.

c) We cannot calculate the ***final velocity*** as we did not know what the ***initial velocity*** was.

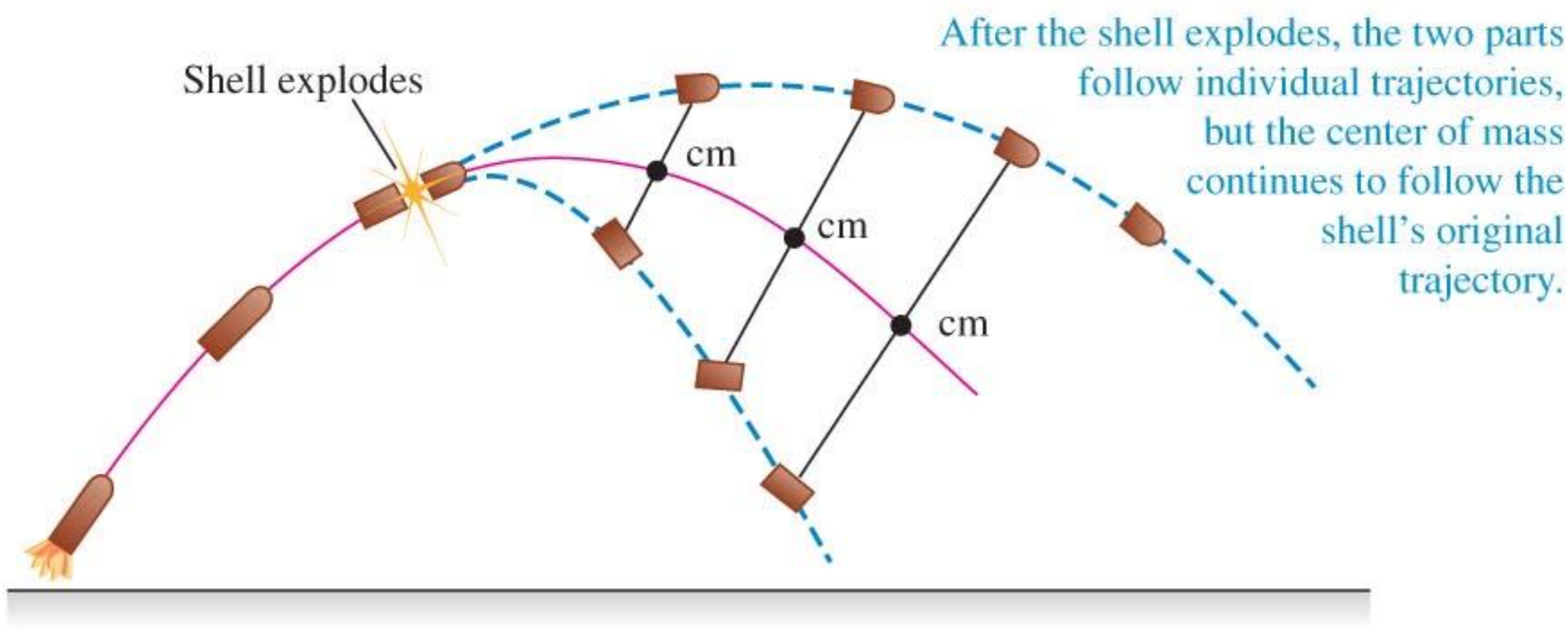
We only know that the ***change in velocity*** is **+10 m/s**.

If F is not constant we take the area under the graph

If $\vec{F}$ is NOT constant



- Momentum changes themselves are altered as the projectile uses fuel or explodes.



(a)

(b)

Linear momentum $\vec{p}$

- The linear momentum $\vec{p}$ of an object is the product of its mass m and velocity $\vec{v}$.

$$\vec{p} = m\vec{v}$$

• Vector
• Same direction as $\vec{v}$

kg m/s = N s

- Collisions
  - Elastic
    - Total kinetic energy is conserved.
    - $K_{t,i} = K_{t,f}$
  - Inelastic
    - Total kinetic energy is NOT conserved.
    - $K_{t,i} \neq K_{t,f}$

Principle of conservation of linear momentum

The *total* momentum of an isolated system remains constant during any interaction

$$\sum p_i = \sum p_f$$

$$\vec{J} = \Delta\vec{p} = \vec{F}\Delta t$$

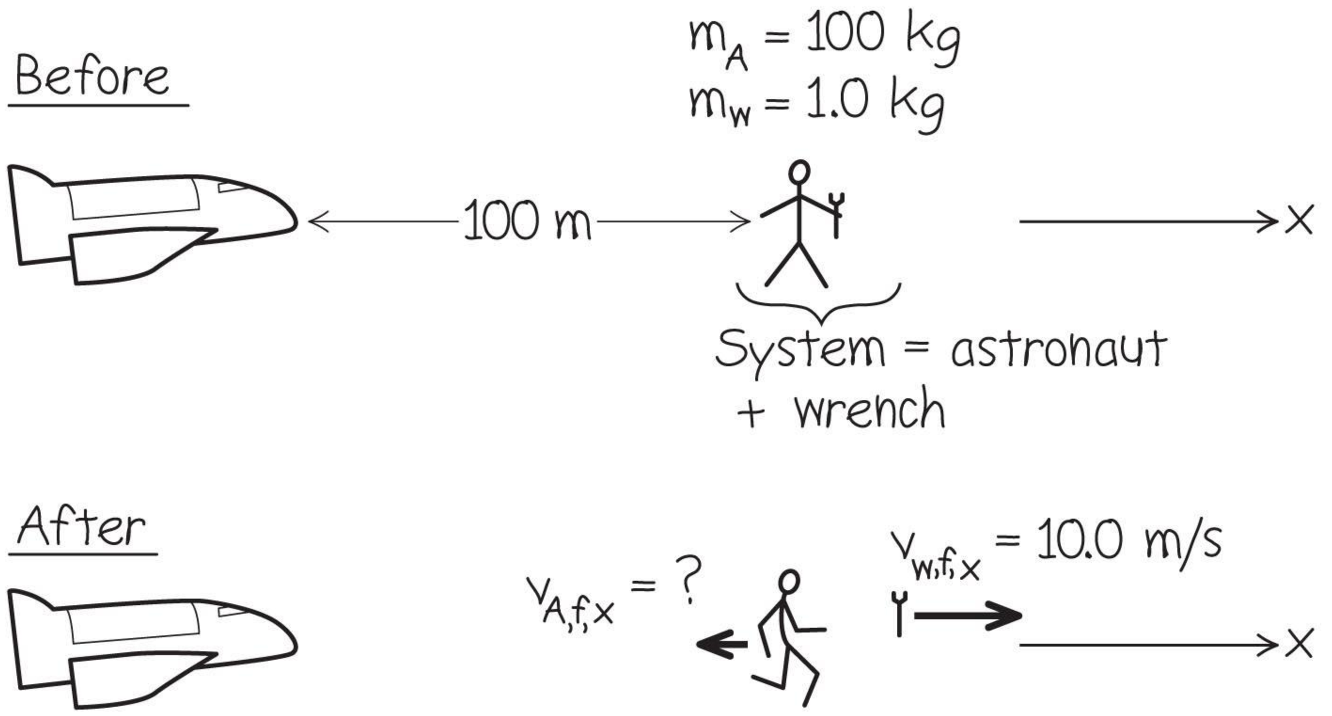
Extra information and examples to try yourself, or do in class, if we have time

Momentum
At the time Newton published the Principia, impetus was the quality of an object that was moving independent of an observed force. Impetus comes from the Latin in- + petere to go to, seek -- from Greek petesthai to fly, piptein to fall, pteron wing. Also, push and pull derive from the Latin pellere.

The symbol for displacement is Δs where the s stands for Latin spatium ‘space’.

An Astronaut Rescue

VTS Ex 8.2



How long does it take the astronaut to get back to the spacecraft?

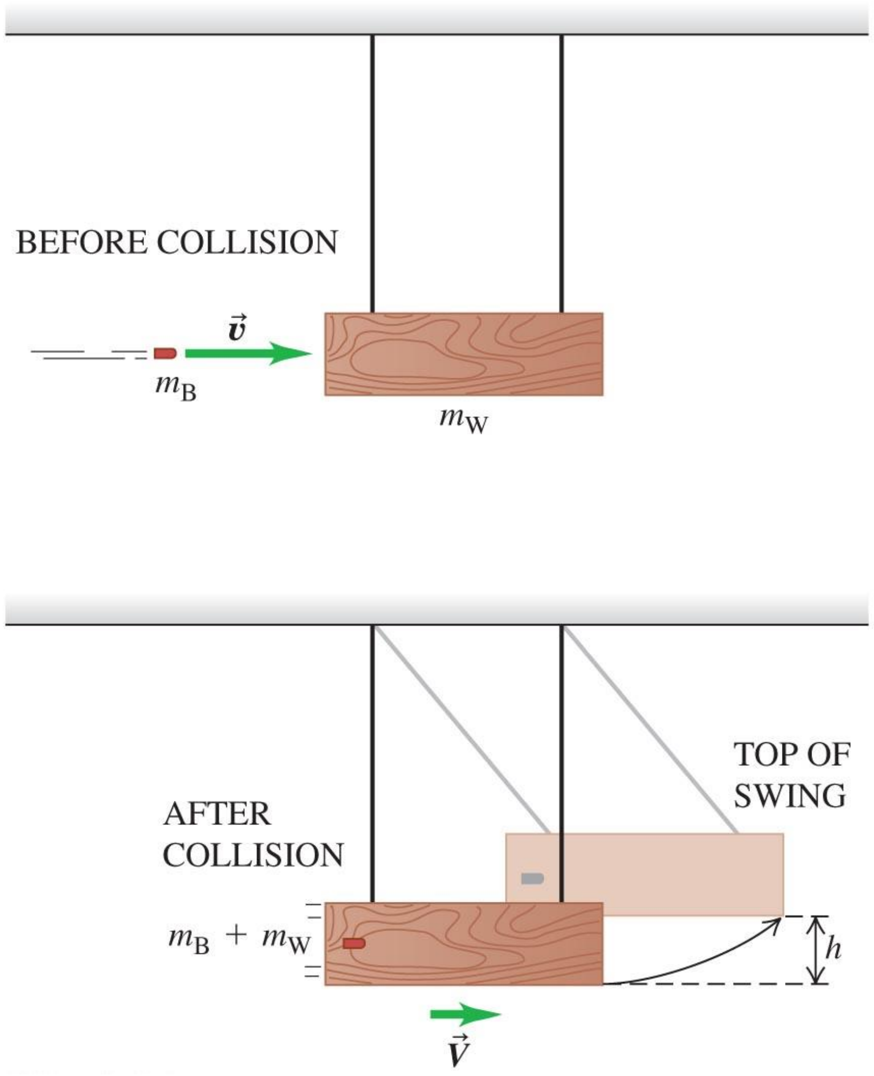
ANS 16 mins 40 s

The Ballistic Pendulum – Example 8.7

- Often done with a .22 in lab before firearms were forbidden on campus.

Video Tutor Solutions

Example 8.1 Preliminary analysis of a collision
Example 8.2 Astronaut rescue
Example 8.3 Recoil of a rifle
Example 8.4 A head-on collision
Example 8.5 Collision in a horizontal plane
Example 8.6 Inelastic collision on an air track
Example 8.7 The ballistic pendulum
Example 8.8 Collision analysis, continued
Example 8.9 Elastic collision on an air track
Example 8.10 Moderator in a nuclear reactor
Example 8.11 A ball hits a wall
Example 8.12 Kicking a soccer ball
Example 8.13 Quarreling pets
Example 8.14 Launch of a rocket



Do this from Mastering physics

VTS Ex 8.7

Covers
- Momentum
- Conservation of momentum
- Rearranging equations
- KE
- PE
- Conservation of energy
- Elastic and inelastic collisions

Momentum and Kinetic Energy

Prove (show, derive). $p = \sqrt{2\, m\, KE}$

Start with the equations you know.
Notice that what you want to prove contains momentum and kinetic energy

Notice that the equation we want does not have v in it. So I will need to substitute and arrange.

$$p = m\, v \qquad KE = \frac{1}{2}\, m\, v^2$$

$$v = \frac{p}{m} \qquad KE = \frac{1}{2}\, m\, \frac{p^2}{m^2}$$

$$KE = \frac{1}{2}\, \frac{p^2}{m}$$

$$p^2 = 2\, m\, KE$$

$$p = \sqrt{2\, m\, KE}$$

Video Tutor Demonstrations

- Water Rocket
- Happy/Sad Pendulums
- Conservation of Linear Momentum

Everything Acts on the Center of Mass

VTS Ex 8.13