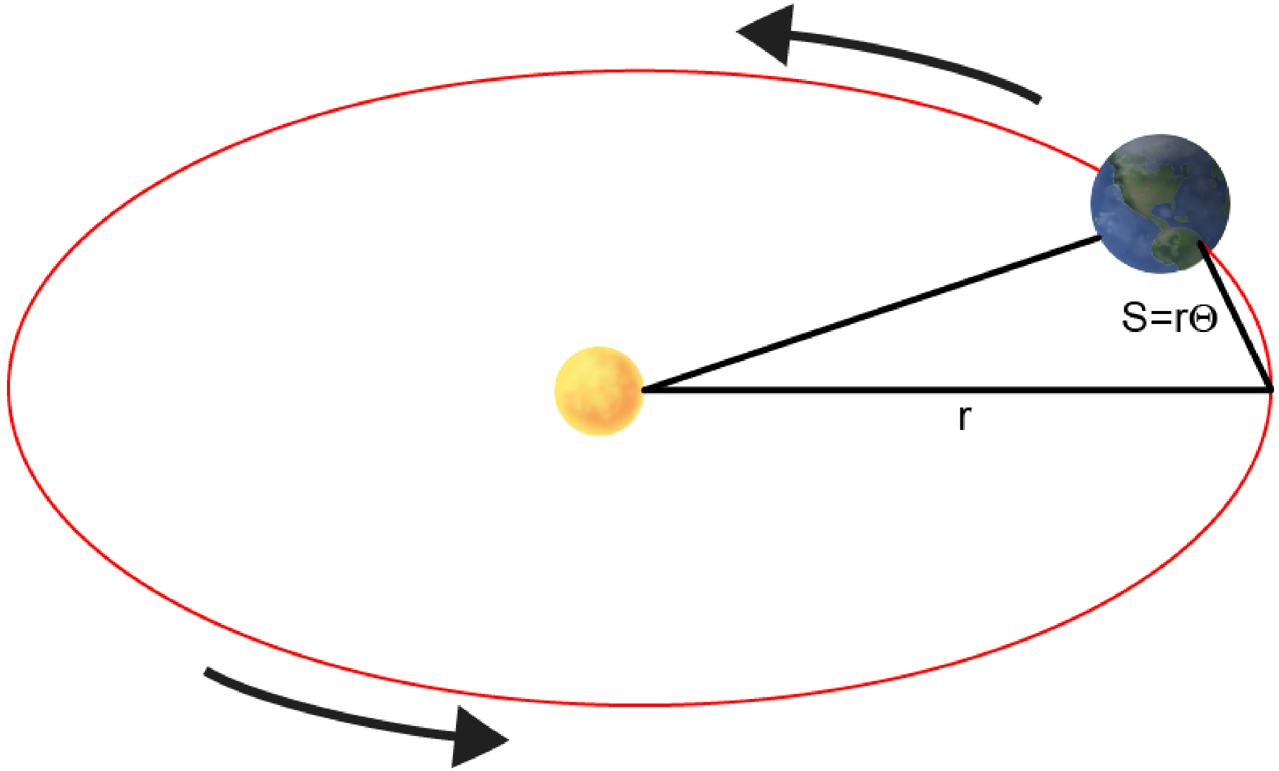
Rotational Motion

Chapter 9
Going around in circles



10 June 2022

Sears & Zemansky's

College Physics

TENTH

Young • Ad

Chapter 9: Rotational Motion

STUDY AREA

Chapter 9 Assets

Video Tutor Demonstrations

PhET Simulations

eText

Chapter 9 Assets

Video Tutor Demonstrations

- Canned Food Race

Video Tutor Solutions

- Example 9.1 Rotation of a compact disc
- Example 9.2 Rotation of a bicycle wheel
- Example 9.3 Throwing a discus
- Example 9.4 Designing a propeller
- Example 9.5 Bicycle gears
- Example 9.6 An abstract sculpture
- Example 9.7 A cable unwinding from a winch
- Example 9.8 Conservation of energy in a well
- Example 9.9 A primitive yo-yo
- Example 9.10 Race of the rolling objects
- Chapter 9 Bridging Problem

VTS Ex 9.1

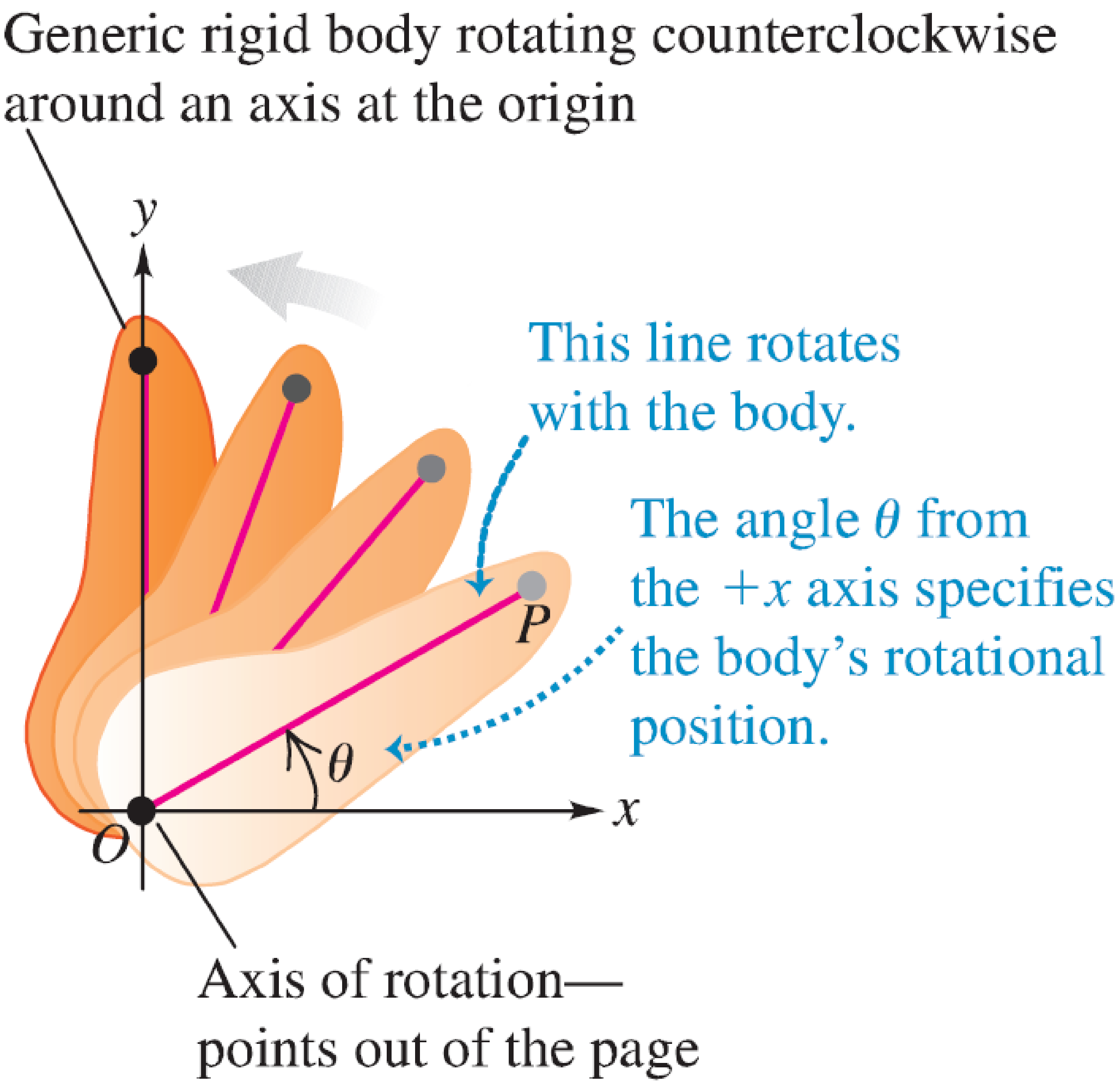
Goals for Chapter 9

- To study angular velocity and angular acceleration.
- To examine rotation with constant angular acceleration.
- To understand the relationship between linear and angular quantities.
- To determine the kinetic energy of rotation and the moment of inertia.
- To study rotation about a moving axis.

Quantities in Rotational Kinematics

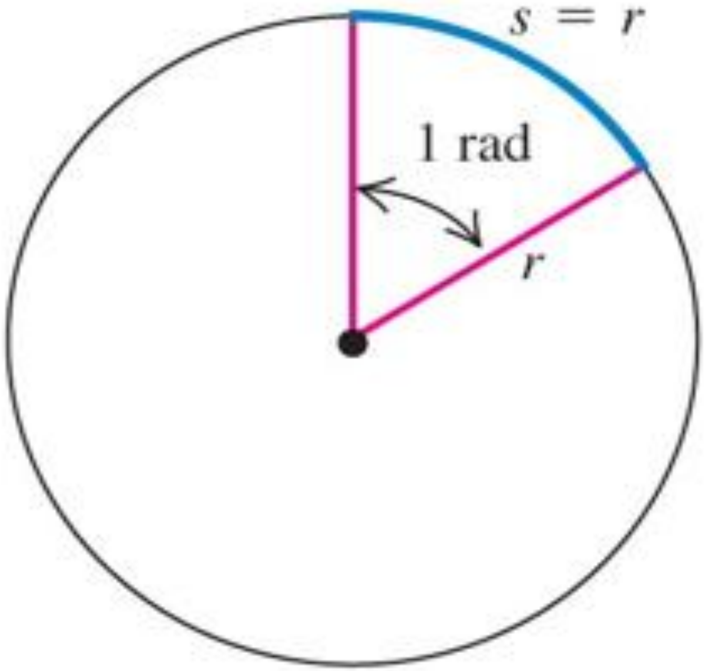
- Angle of rotation θ
- Angular displacement $\Delta\theta$
- Angular velocity ω
- Angular acceleration α

Rotation of rigid bodies



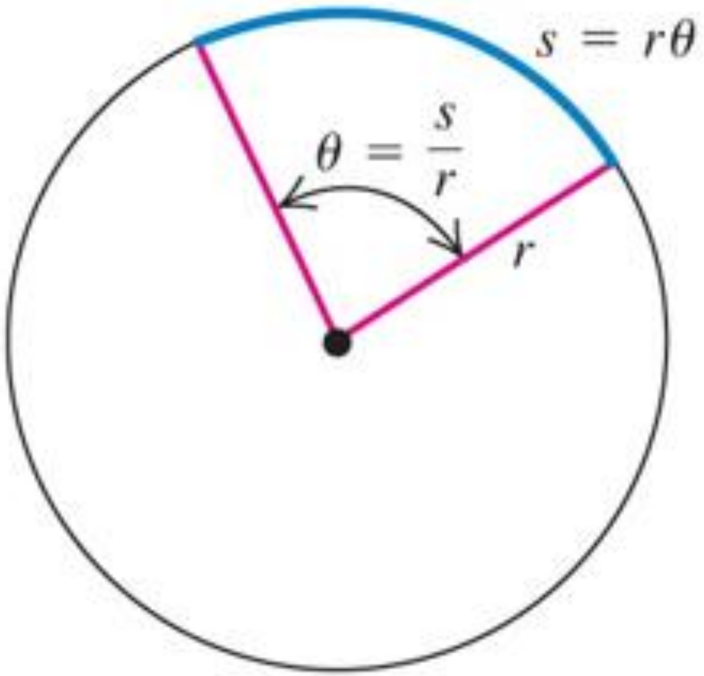
Radians

One radian is the angle at which the arc s has the same length as the radius r.



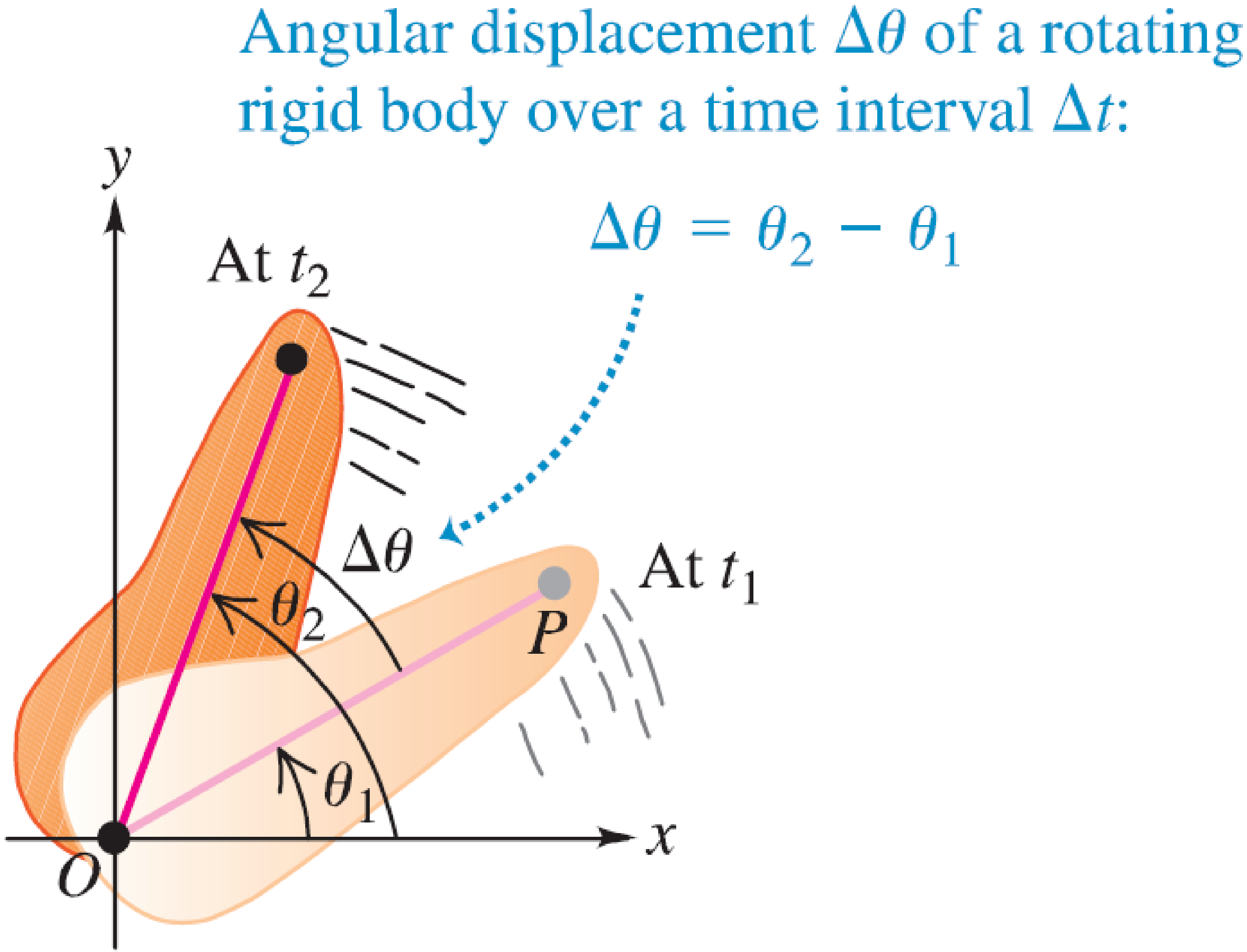
(a)

An angle θ in radians is the ratio of the arc length s to the radius r.



(b)

Angular Displacement



Angular Velocity

- Average Angular Velocity
  - $\omega_{av} = \frac{\Delta\theta}{\Delta t} = \frac{\theta_2 - \theta_1}{t_2 - t_1}$
- Instantaneous Angular Velocity
- $\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t}$
- Note: instantaneous velocity is the slope of $\Delta\theta$ vs. Δt graph.

Angular Velocity

- Useful conversions

$$1\ \text{rev/s} = 2\pi\ \text{rad/s}$$

$$1\ \text{rev/min} = 1\ \text{rpm} = \frac{2\pi}{60}\ \text{rad/s}$$

Early records
about 78 rpm

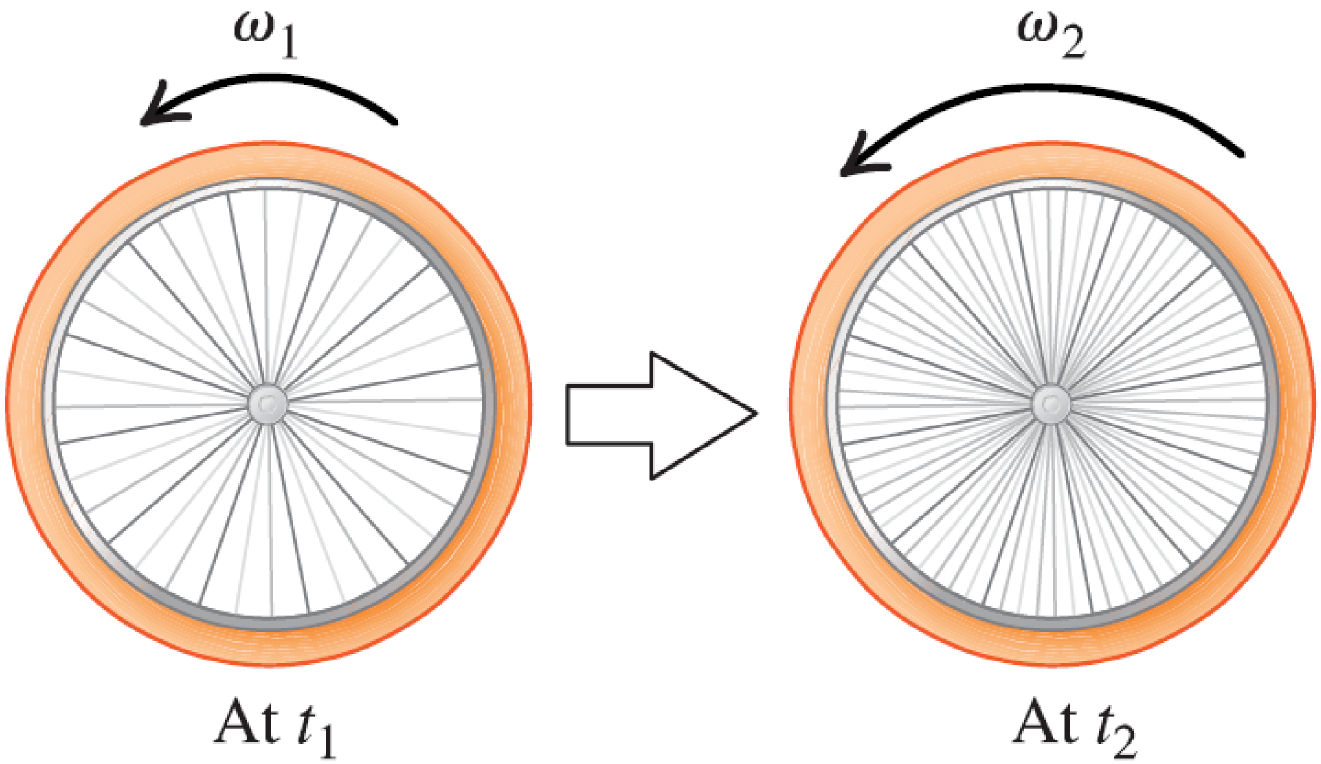
LP 33 1/3 rpm
Single 45 rpm

CD 7200 rpm

Angular Acceleration

The average angular acceleration is the change in angular velocity divided by the time interval:

$$\alpha_{av} = \frac{\omega_2 - \omega_1}{t_2 - t_1} = \frac{\Delta\omega}{\Delta t}$$



Angular Acceleration

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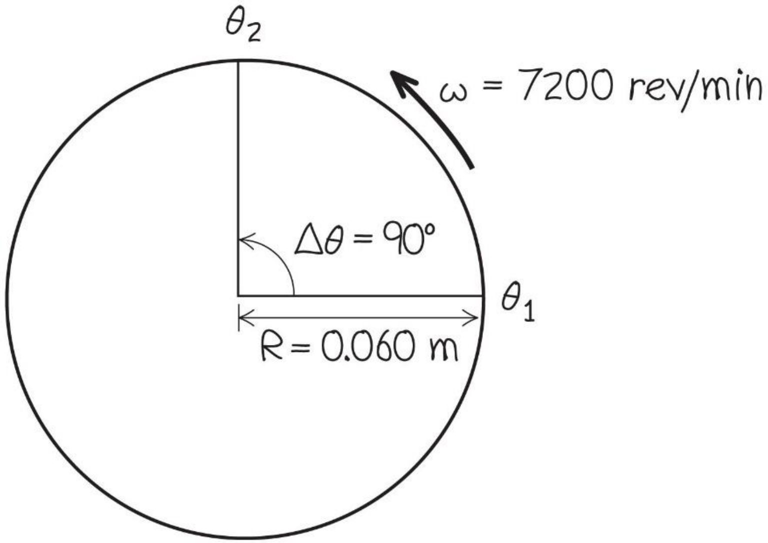
- Instantaneous Angular Acceleration
- $\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t}$
- Note: instantaneous acceleration is the slope of ω vs. Δt graph.

Example 1

- A compact disk (CD) rotates at high speed while a laser reads data encoded in a spiral pattern. The disk has radius R = 6.0 cm; when data are being read, it spins at 7200 rev/min.

a) What is the CD's angular velocity in radians per second?



b) How much time is required for it to rotate through 90°?

c) If it starts from rest and reaches full speed in 4.0 s, what is its average angular acceleration?

VTS Ex 9.1

Solution……

$$1 \text{ rev/s} = 2\pi \text{ rad/s}$$

$$1 \text{ rev/min} = 1 \text{ rpm} = \frac{2\pi}{60} \text{ rad/s}$$

- R = 6.0 cm; when data are being read, it spins at 7200 rev/min.

(a) What is the CD's angular velocity in radians per second?

Need to convert to **Radians per second**…..

$$7200 \text{ rev/min} = 7200 \,*\, \frac{2\pi}{60} = 753.98 \text{ rad/s}$$

Solution cont…

(b) How much time is required for it to rotate through 90°?

Convert degrees to radians….

$90^{\circ} = \frac{2\pi}{4} = 1.57 \text{ rad}$

Time taken = Angle (rad) / Angular Velocity (rad/s)

1.57 rad / 753.98 rad/s = 2.08 x 10^{-3} seconds or 2.08ms

Solution cont…

(c) If it starts from rest and reaches full speed in 4.0 s, what is its average angular acceleration?

Starting Speed = 0 rads^{-1} Full speed = 753.98 rads^{-1}

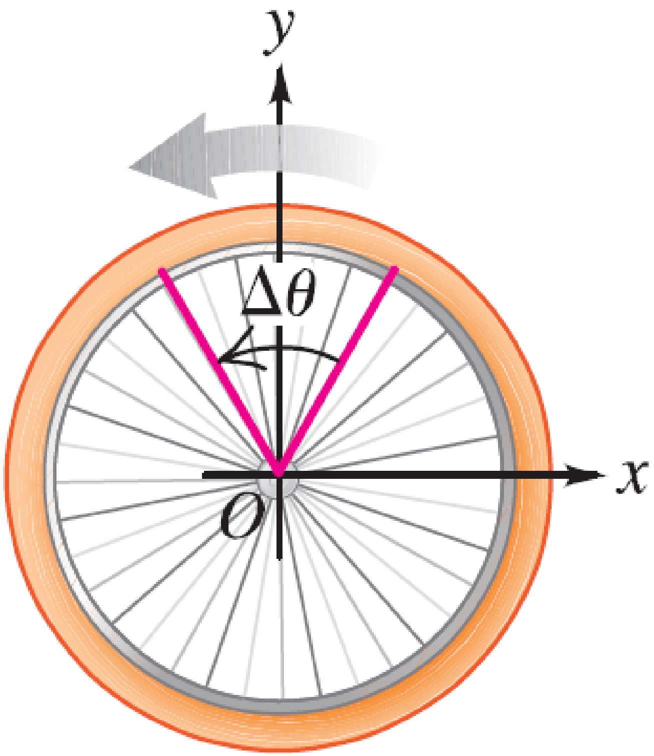
Angular Acceleration (ave) =

Change in Angular Velocity / Change in time

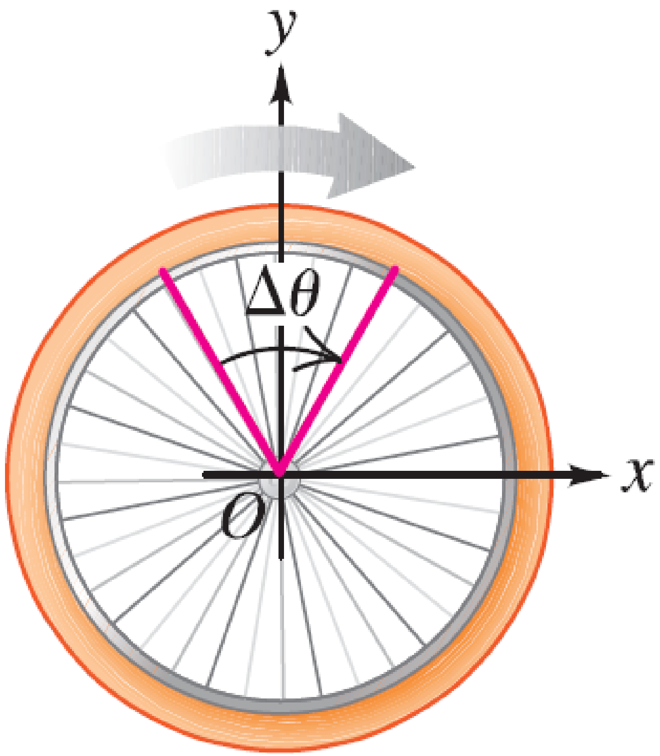
Ang. Accel.=(753.98 rads^{-1} – 0)/(4 – 0) = 188.50 rads^{-2}

Sign Convention

Counterclockwise rotation positive:
$\Delta\theta > 0$, so
$\omega = \Delta\theta/\Delta t > 0$



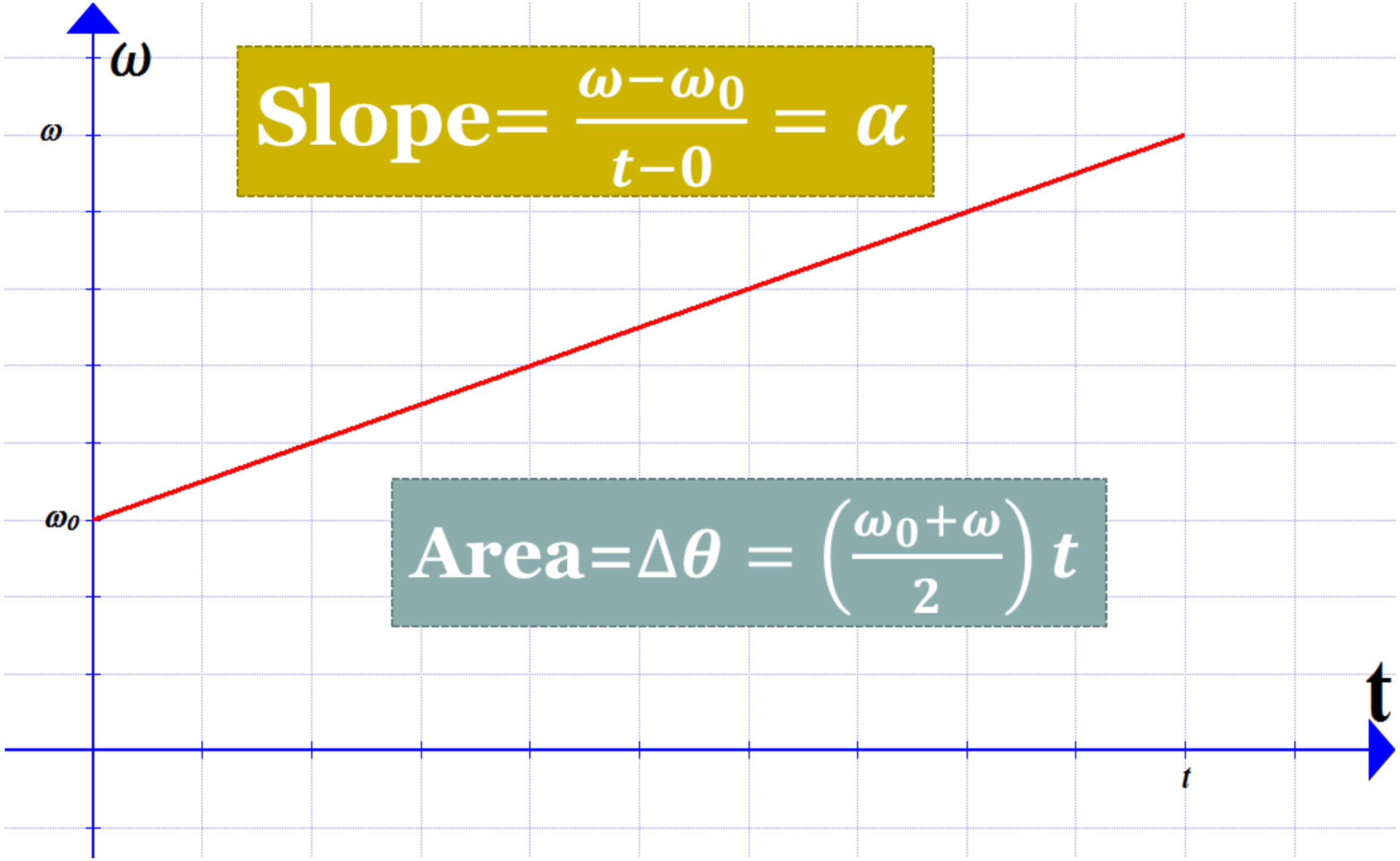
Clockwise rotation negative:
$\Delta\theta < 0$, so
$\omega = \Delta\theta/\Delta t < 0$



Rotation with constant angular acceleration

Angular acceleration α is the rate of change in angular velocity ω.

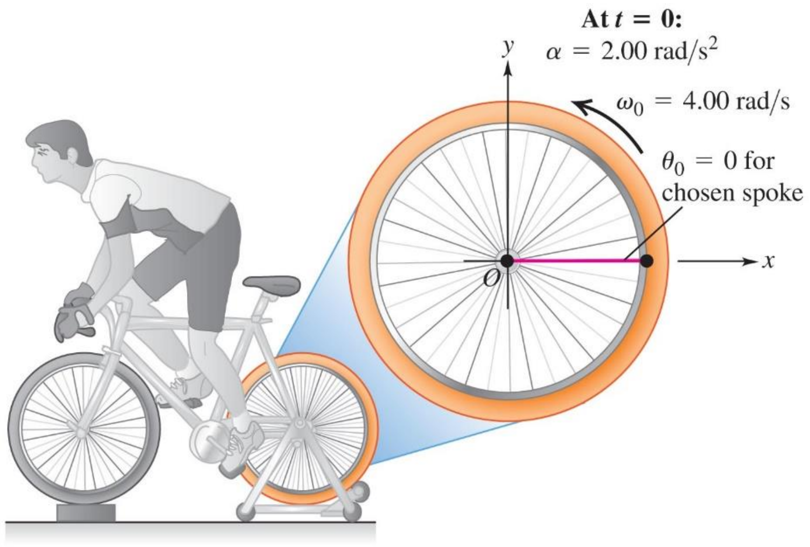
$$\alpha = \frac{\Delta\omega}{\Delta t}$$



Rotation with constant angular acceleration

- *Angular Equations of Motion*
- $\boldsymbol{\alpha = \frac{\Delta\omega}{\Delta t} = \frac{\omega - \omega_0}{t} \Rightarrow \omega = \omega_0 + \alpha t}$
- $\boldsymbol{\Delta\theta = \left(\frac{\omega_0 + \omega}{2}\right) t}$
- $\boldsymbol{\Delta\theta = \omega_0 t + \frac{1}{2}\alpha t^2}$
- $\boldsymbol{\Delta\theta = \omega t - \frac{1}{2}\alpha t^2}$
- $\boldsymbol{\omega^2 = {\omega_0}^2 + 2\alpha\Delta\theta}$
- Note the Similarity to the original Linear Equations !

The angular velocity of the rear wheel of a stationary exercise bike is 4.00 rad/s at time t = 0, and its angular acceleration is constant and equal to 2.00 rad/s^2.



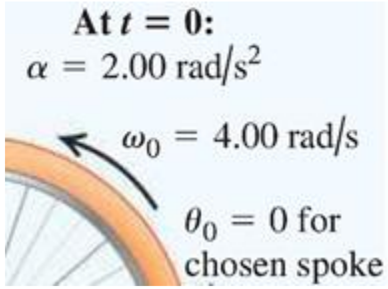
A particular spoke coincides with the +x axis at time t = 0.

(a) What angle does this spoke make with the +x axis at time t = 3.00 s?

(b) What is the wheel's angular velocity at this time?

VTS Ex 9.2

Solution…



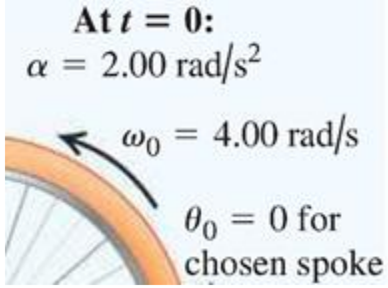
Firstly we write down the information we are given and if necessary draw a diagram of the situation….

- Initial angular velocity = 4.0 rad/s
- Angular Acceleration = 2.0 rad/s^2
- Angle at t=0 is 0 rad.

Now we select the desired equation to solve this, I am going to use: $\Delta\theta = \omega_0 t + \frac{1}{2}\alpha t^2$ for the first part.

So, $\Delta\theta = (4 * 3) + \left(\frac{1}{2} * 2 * 3^2\right) = 12 + 9 = 21.0\ rad.$

Now 21 rad is more than one revolution, so we need to look at the numbers of revs to calculate the correct angle.



Solution continued…

21.0 rad * / 2π rad = 3.34 revolutions, so we have 0.34 revolutions extra.

Thus, $\Delta\theta = 0.34 * 2\pi = 2.136$ **rads** or **122.4°**

For the new Angular Velocity we again select the desired equation…

I've chosen: $\omega = \omega_0 + \alpha t$

$\omega = 4 + (2 * 3) = 10\ rad/s.$

For YOU to do.

Let us now double the angular acceleration – Now you need to calculate the new results……

PTO for question

For YOU to do.

Let us now double the angular acceleration – Now you need to calculate the new results……

The angular velocity of the rear wheel of a stationary exercise bike is 4.00 rad/s at time t = 0, and its angular acceleration is constant and equal to **4.00 rad/s²**.
A particular spoke coincides with the +x axis at time t = 0.

(a) What angle does this spoke make with the +x axis at time t = 3.00 s?

(b) What is the wheel's angular velocity at this time?

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{\omega - \omega_0}{t} \Rightarrow \omega = \omega_0 + \alpha t$$

$$\Delta\theta = \left(\frac{\omega_0 + \omega}{2}\right) t$$

$$\Delta\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\Delta\theta = \omega t - \frac{1}{2}\alpha t^2$$

$$\omega^2 = \omega_0{}^2 + 2\alpha\Delta\theta$$

Your Solution...?

We need the same equations this time....but the value of acceleration is now 4 rad/s².

$$\Delta\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\Delta\theta = (4 * 3) + \left(\frac{1}{2} * 4 * 3^2\right) = 30.0\, rad$$

So we have 4.775 revolutions, so the desired angle will be 0.775 * 2π = 4.87 rad or 279°

The final Angular Velocity is:

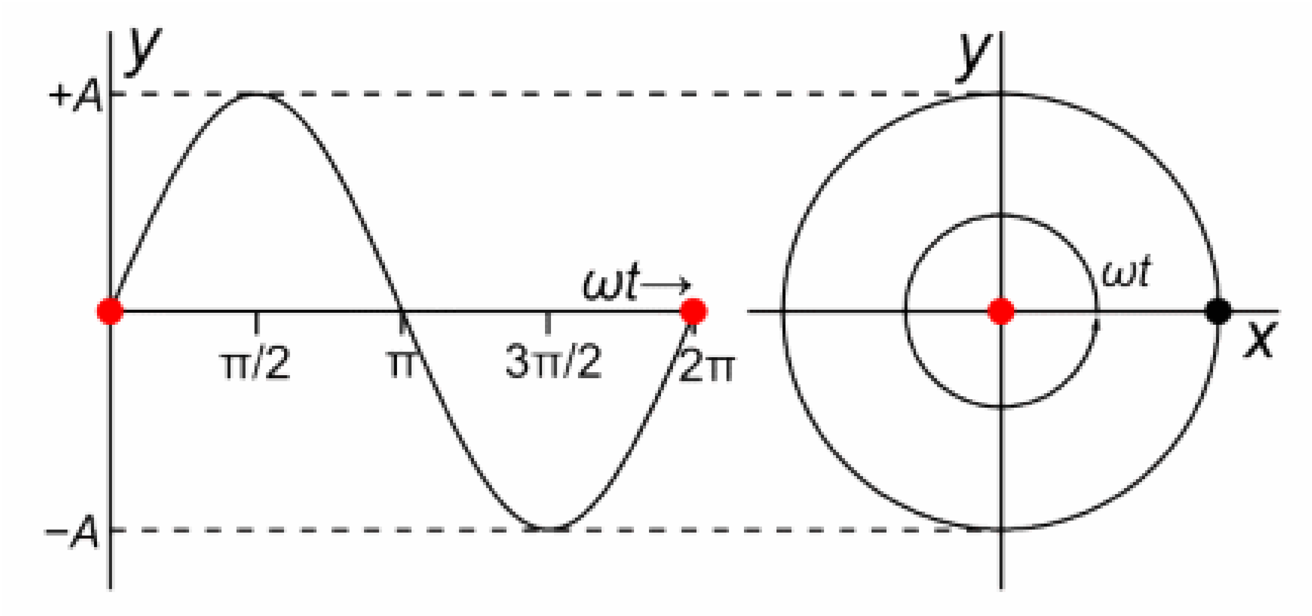
$$\omega = \omega_0 + \alpha t = 4 + (4 * 3) = 16\, rad/s$$

Period

The time taken to go around a complete circle is known as the period (T).

The period is also equal to 1/frequency.

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi}{T} = 2\pi f$$



9. An airplane propeller is rotating at 1900 rpm. (a) Compute the propeller's angular velocity in rad/s. (b) How many seconds does it take for the propeller to turn through 35°? (c) If the propeller were turning at 18 rad/s, at how many rpm would it be turning? (d) What is the period (in seconds) of this propeller? For part d take the airplane propeller to be at 1900 rpm.

9. • An airplane propeller is rotating at 1900 rpm. (a) Compute the propeller's angular velocity in rad/s. (b) How many seconds does it take for the propeller to turn through 35°? (c) If the propeller were turning at 18 rad/s, at how many rpm would it be turning? (d) What is the period (in seconds) of this propeller?

9.9. Set Up: 1 rpm = $(2\pi/60)$ rad/s. Period $T = \dfrac{2\pi}{\omega}$. $\theta - \theta_0 = \omega t$.

Solve: **(a)** $\omega = (1900)(2\pi \text{ rad}/60 \text{ s}) = 199 \text{ rad/s}$

Radians

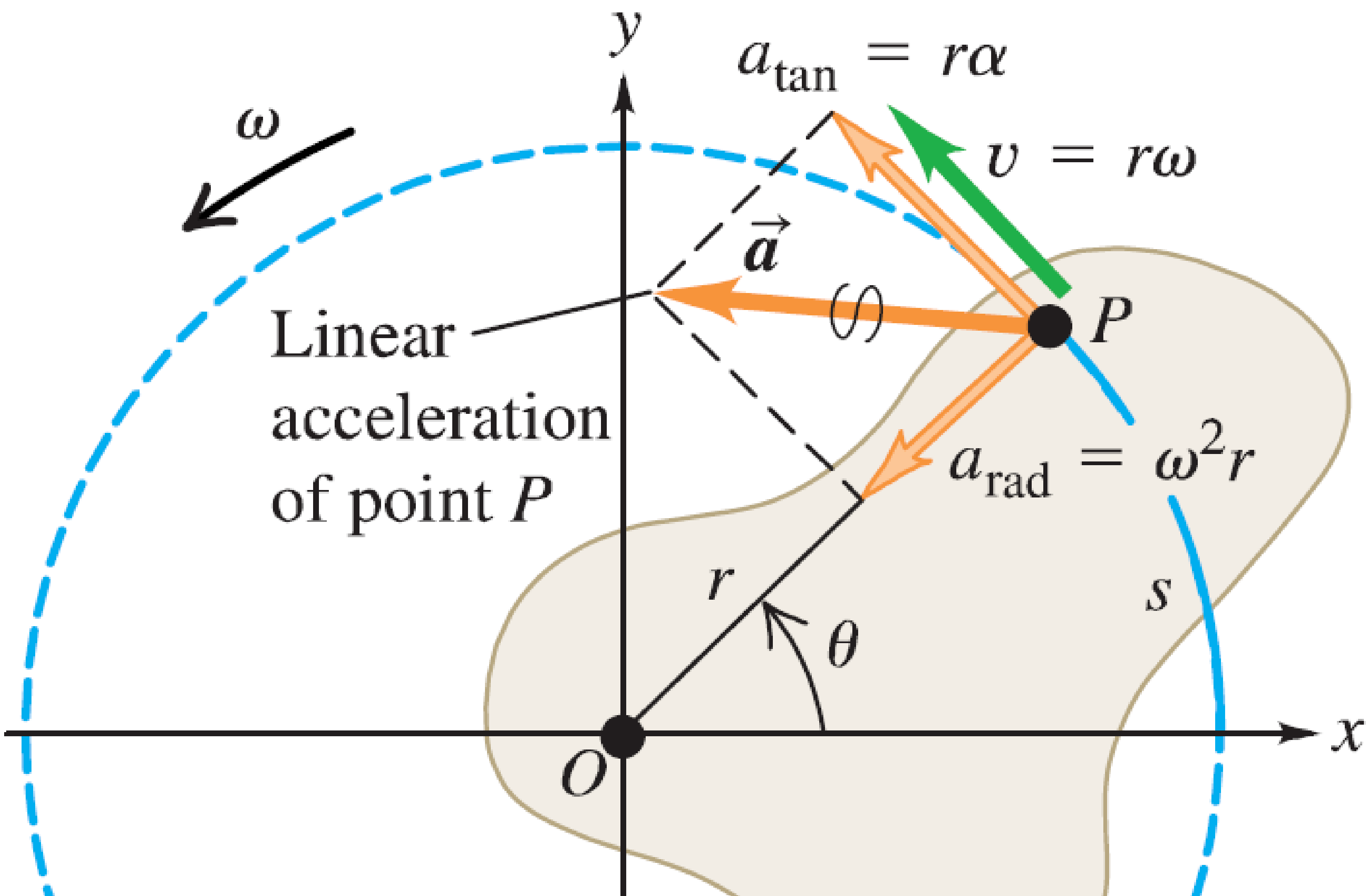
(b) $35° = (35°)(\pi/180°) = 0.611 \text{ rad}$. $t = \dfrac{\theta - \theta_0}{\omega} = \dfrac{0.611 \text{ rad}}{199 \text{ rad/s}} = 3.1 \times 10^{-3} \text{ s}$

(c) $(18 \text{ rad/s})\left(\dfrac{60 \text{ s}}{1 \text{ min}}\right)\left(\dfrac{1 \text{ rev}}{2\pi \text{ rad}}\right) = 172 \text{ rpm}$

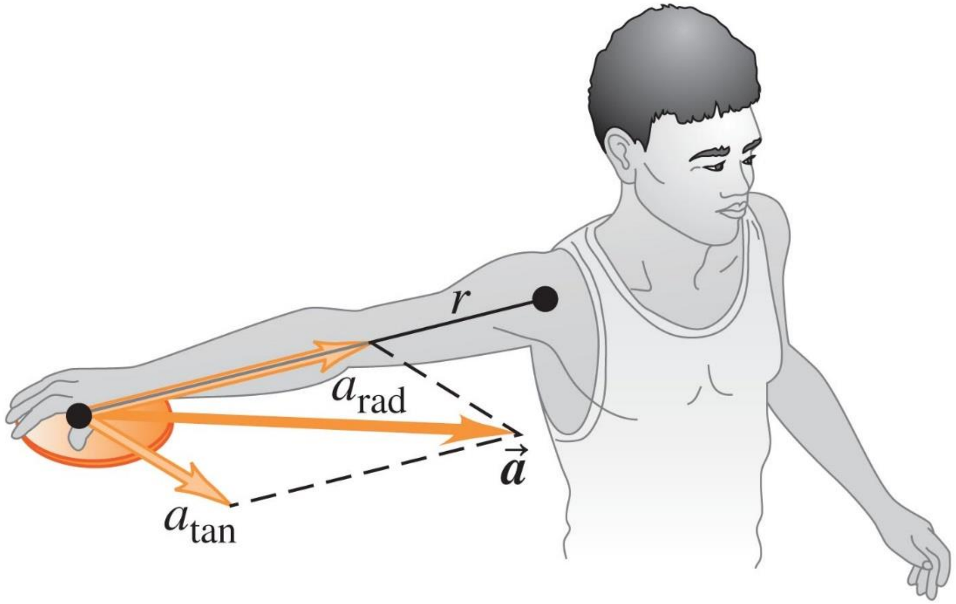
(d) For the propeller rotating at 1900 rpm, $T = \dfrac{2\pi \text{ rad}}{\omega} = \dfrac{2\pi \text{ rad}}{199 \text{ rad/s}} = 0.032 \text{ s}$.

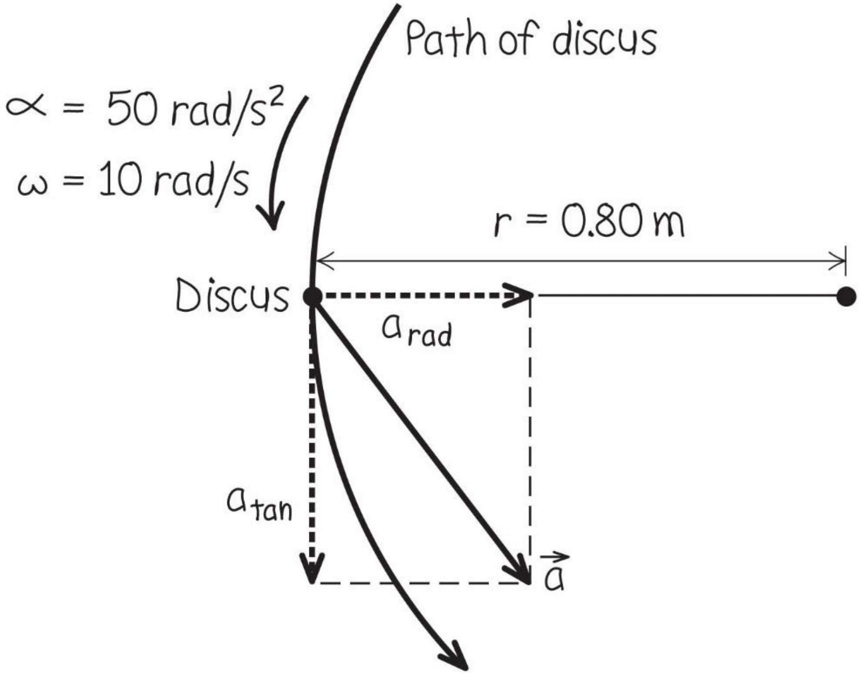
Reflect: The period is inversely proportional to the angular velocity.

Relationship between linear and Angular Quantities



A discus thrower turns with angular acceleration α = 50 rad/s^2, moving the discus in a circle of radius 0.80 m. Find the radial and tangential components of acceleration of the discus (modeled as a point) and the magnitude of its acceleration at the instant when the angular velocity is 10 rad/s.



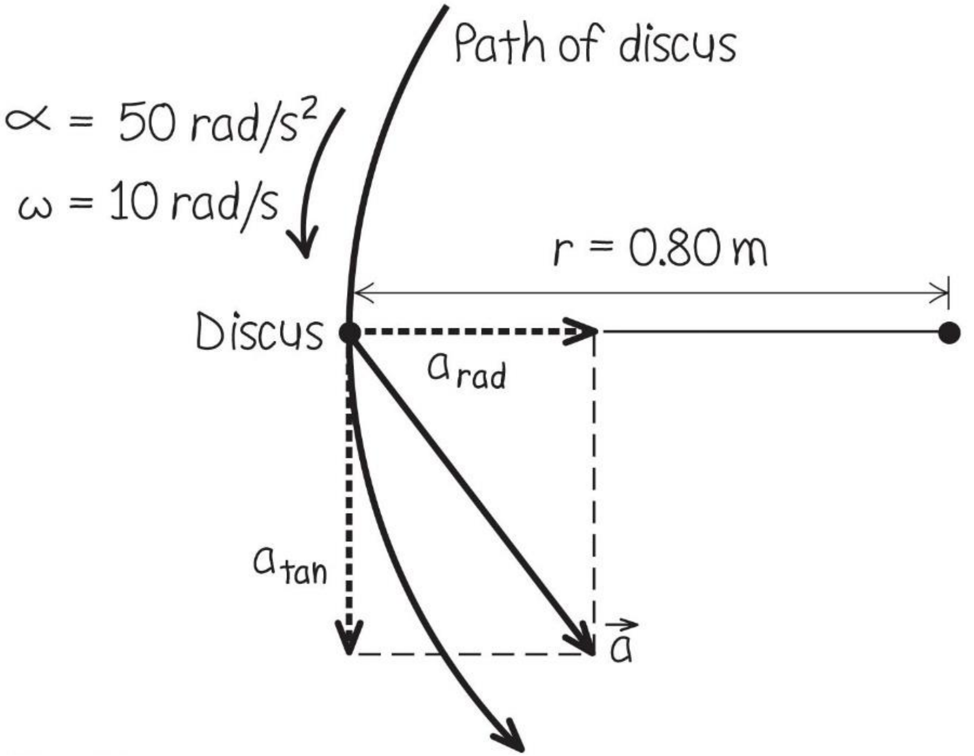


$$a_{tan} = r\alpha$$

VTS Ex 9.3

$$a_{rad} = \omega^2 r$$

$$a_{tan} = r\alpha \qquad\qquad a_{rad} = \omega^2 r$$



Solution……

We first look at the information we are given:

$\alpha = 50\ \text{rad/s}^2$ radius = $\mathbf{r} = 0.80$ m

angular velocity = $\boldsymbol{\omega} = 10$ rad/s

Now we look at the equations we must use……

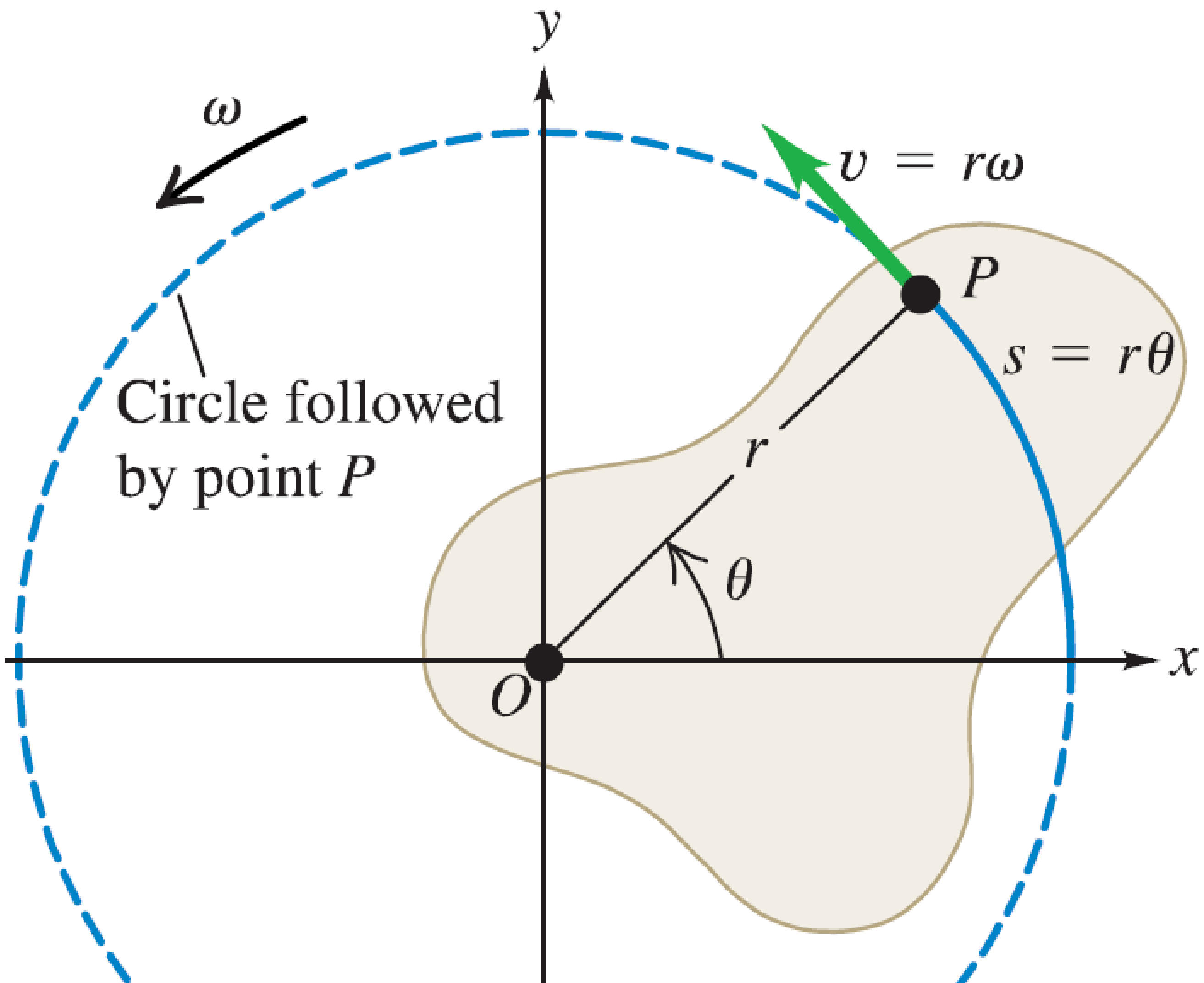
$$\mathbf{a_{tangential} = r\,\alpha = 50 \times 0.80 = 40\ m/s^2}$$

$$\mathbf{a_{radial} = \omega^2 r = 10^2 \times 0.80 = 80\ m/s^2}$$

And now we resolve as usual…..

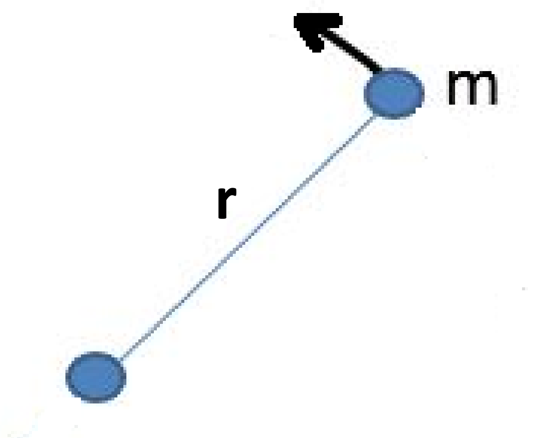
$$\textbf{Magnitude} = \sqrt{a_{rad}{}^2 + a_{tan}{}^2} = \mathbf{89.44\ m/s^2}$$

Kinetic Energy and Moment of Inertia



Moment of Inertia of a particle

For a particle of mass m, rotating at a radius r from the center of rotation, the moment of inertia is



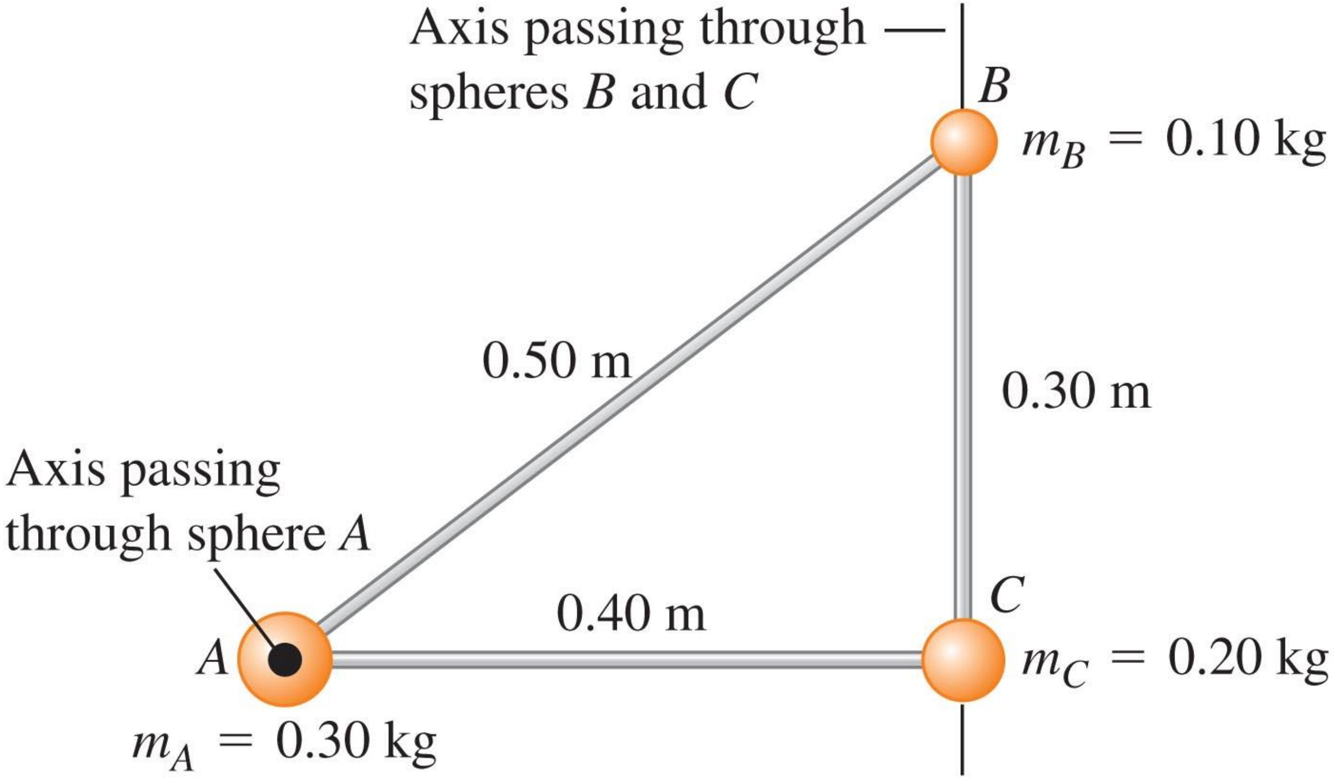
$$I = mr^2$$

and the KE is

$$KE = \frac{1}{2}mv^2 = \frac{1}{2}m(r\omega)^2 = \frac{1}{2}mr^2\omega^2 = \frac{1}{2}I\omega^2$$

Kinetic Energy of Rotation

- An artist is designing a part for a kinetic sculpture. The part consists of three massive disks connected by light supporting rods that can be considered massless. Find the moment of inertia about an axis passing through disk A and is perpendicular to the plane of that disk. If the object rotates about the axis through disk A with angular velocity 4.0 rad/s, find its kinetic energy.



$$I = mr^2$$

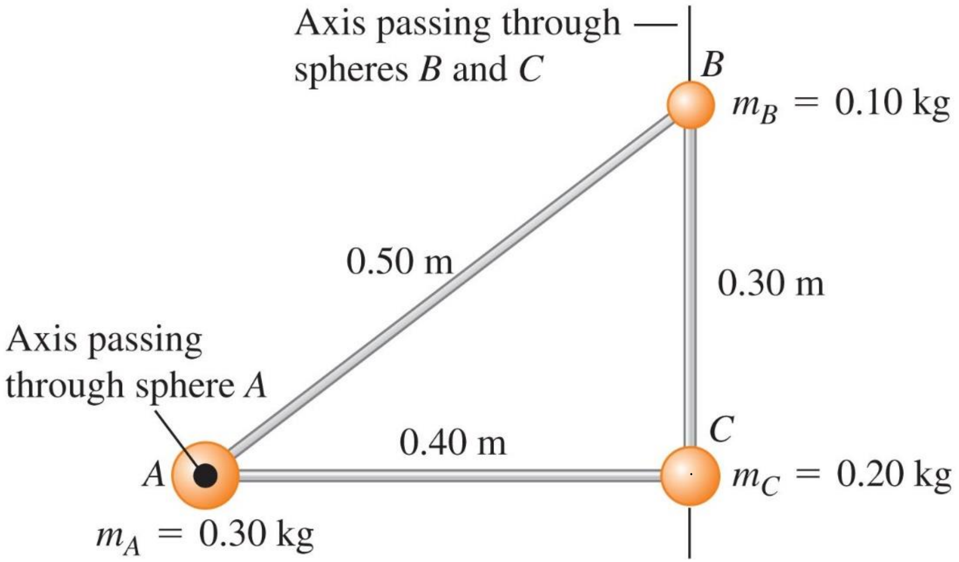
$$KE = \frac{1}{2} I\omega^2$$

VTS Ex 9.6

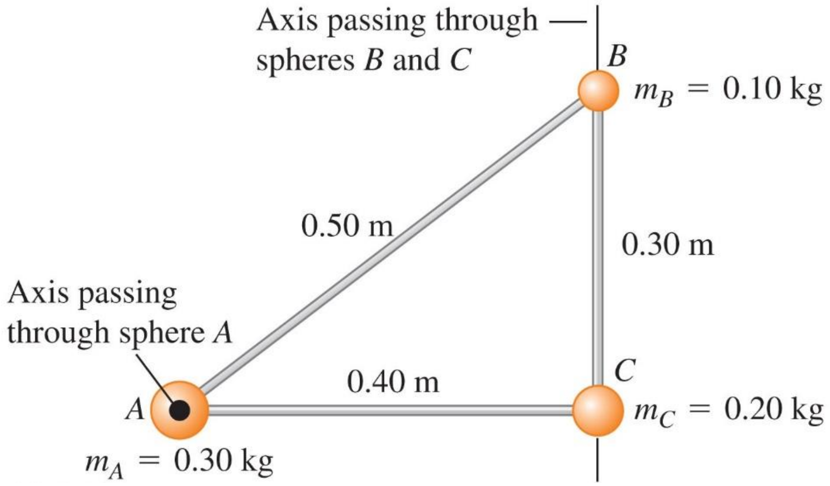
$$I = mr^2$$

$$KE = \frac{1}{2}\mathrm{I}\omega^2$$

ω=4.0 rad/s



Solution.....



Our moment of inertia is for the whole system so we must include all of the masses. But as the system rotates about axis A, then A has zero inertia as it has zero radius.

So we have:

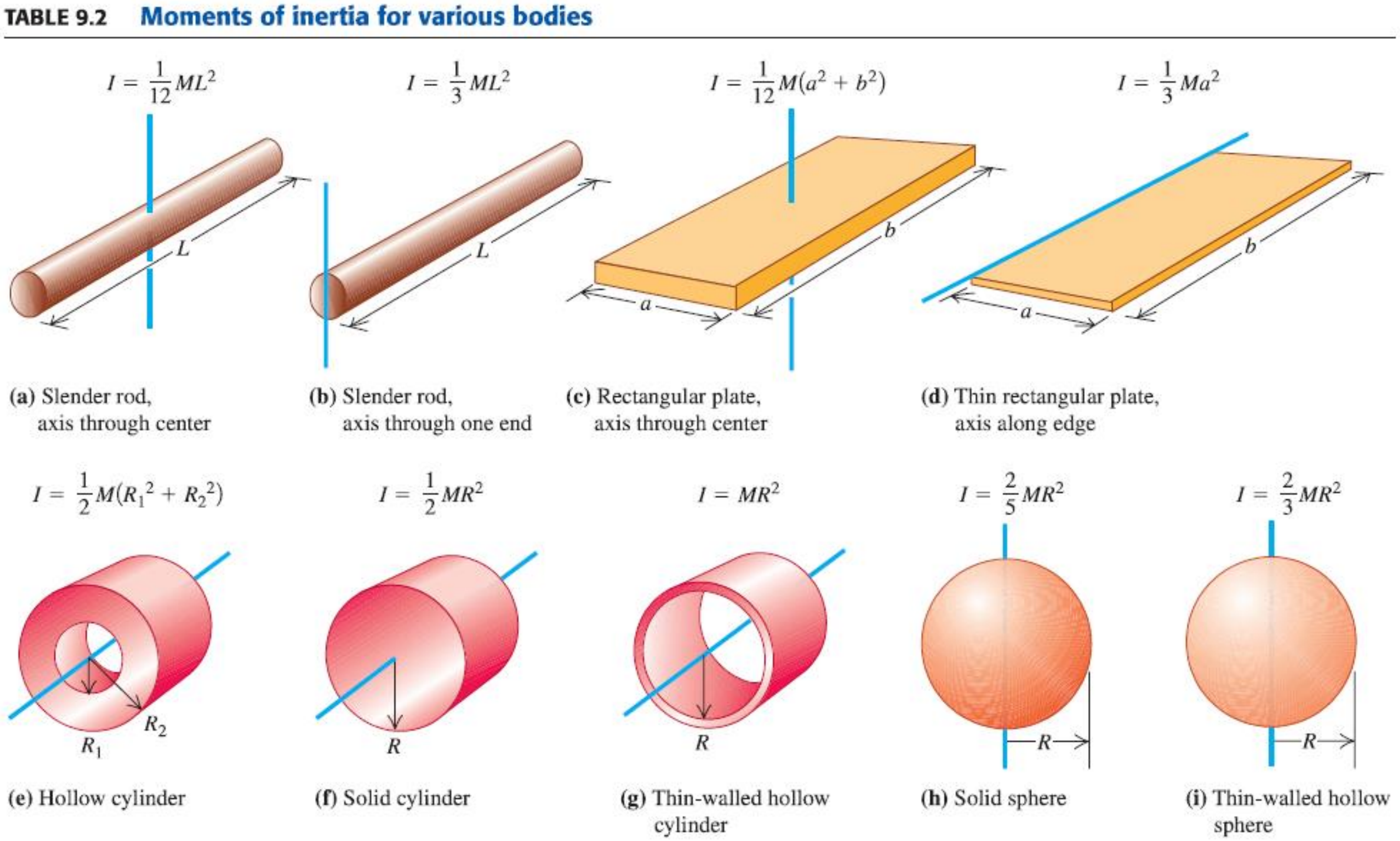
$$I_A = m_b r_b^2 + m_c r_c^2 = (0.10 * 0.50^2) + (0.20 * 0.40^2)$$

$$I_A = 0.057 \text{ kgm}^2$$

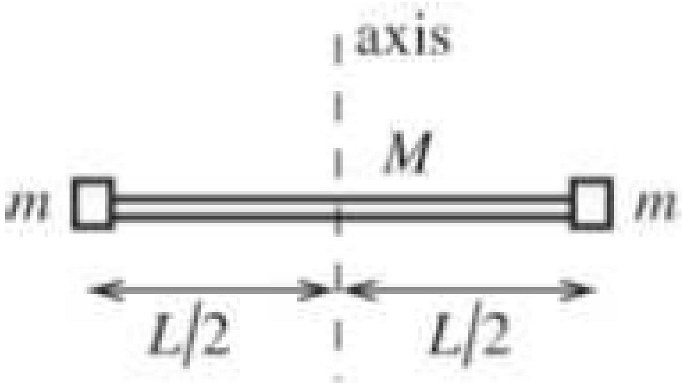
K.E. = ½ * I * ω^2 = ½ * 0.057 * 4^2 = 0.46 Joules

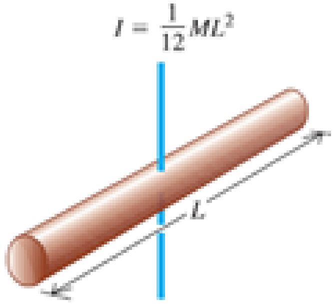
You try for the M.o.I and K.E. through the B-Axis......

Moment of Inertia of rigid bodies



30. • A twirler's baton is made of a slender metal cylinder of mass M and length L. Each end has a rubber cap of mass m, and you can accurately treat each cap as a particle in this problem. Find the total moment of inertia of the baton about the usual twirling axis (perpendicular to the baton through its center).





(a) Slender rod, axis through center

For the end caps we used $I = mr^2$, since they are treated as point particles.

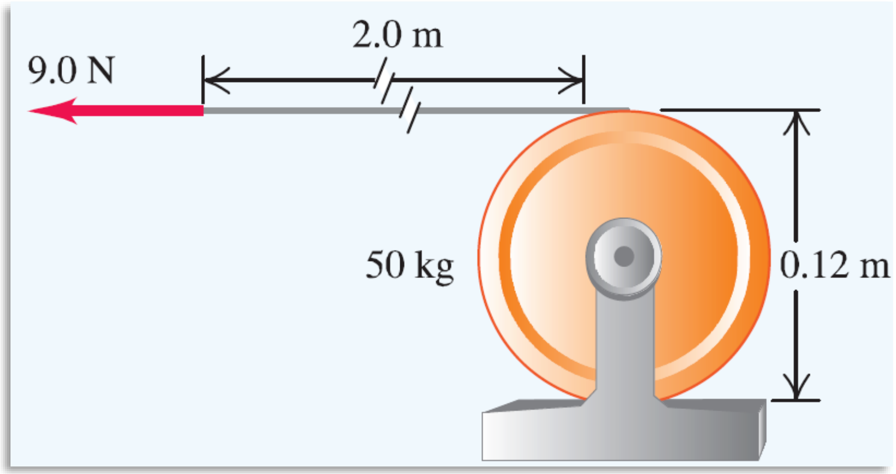
We have $I = I_{\text{rod}} + 2I_{\text{cap}}$. Thus, $I = \frac{1}{12}ML^2 + 2(m)(L/2)^2 = \left(\frac{1}{12}M + \frac{1}{2}m\right)L^2$

Moment of Inertia of rigid bodies

VTS Ex 9.7

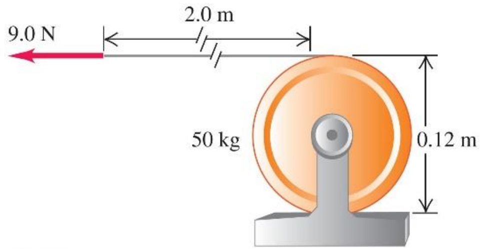
A light, flexible, non-stretching cable is wrapped several times around a winch drum - a solid cylinder with mass 50 kg and diameter 0.12 m that rotates about a stationary horizontal axis that turns in frictionless bearings. The free end of the cable is pulled with a constant force of magnitude 9.0 N for a distance of 2.0 m. It unwinds without slipping, turning the cylinder as it does so. If the cylinder is initially at rest, find its final angular velocity ω and the final speed v of the cable.

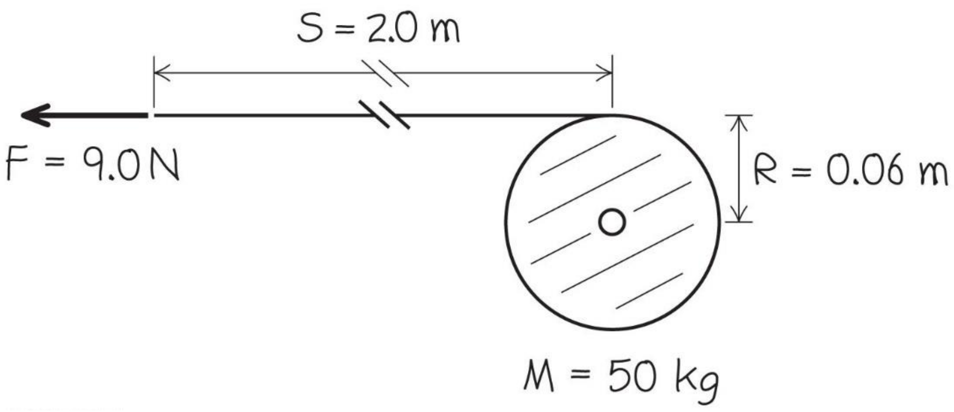
(work = Fs)

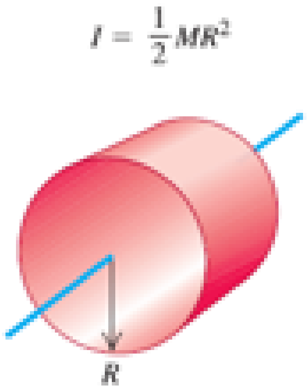


Starts from rest
Find ω and v

hint
Work done > energy transferred
$Fs = KE_{rot}$







(f) Solid cylinder

Ans ω=20 rad/s
v= 1.2 m/s

Solution…

As friction is negligible, no energy is lost. Thus,

Final K.E. Cylinder = Work Done by Force on Cylinder

$$\frac{1}{2} I * \omega^2 = F * s$$

So the: **Work Done = F * s = 9 * 2 = 18 Joules**

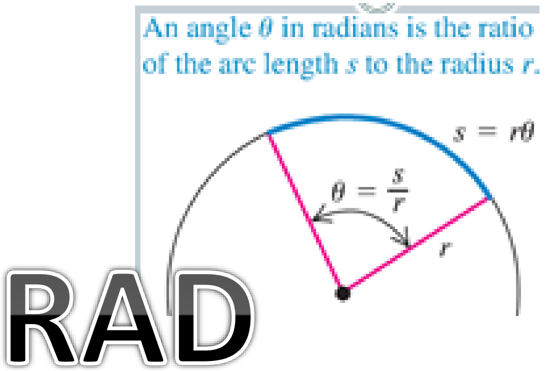
And: Inertia= $\frac{1}{2} MR^2 = \frac{1}{2} * 50 * 0.06^2 = 0.090\ kgm^2$

These values must equate from: Work Done = ΔK.E.

$$18 = \frac{1}{2}(0.090) * \omega^2$$

$$\omega = \sqrt{\frac{18}{0.045}} = 20\ rad/s$$

Final linear velocity is: $v = \omega r = 20 * 0.06 = 1.2\ ms^{-1}$



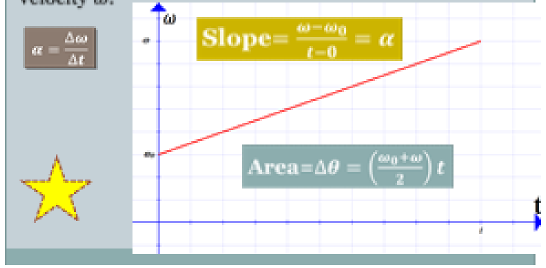
- Average Angular Velocity
  - $\omega_{av} = \frac{\Delta\theta}{\Delta t} = \frac{\theta_2-\theta_1}{t_2-t_1}$
- Instantaneous Angular Velocity
- $\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t}$
- Note: instantaneous velocity is the slope of $\Delta\theta$ vs. Δt graph.

$$1 \text{ rev/s} = 2\pi \text{ rad/s}$$

$$1 \text{ rev/min} = 1 \text{ rpm} = \frac{2\pi}{60} \text{ rad/s}$$

Rotation with constant angular acceleration

Angular acceleration α is the rate of change in angular velocity ω.



For a particle of mass m and rotating at a radius r from the center of rotation, the moment of inertia is

$$I = mr^2$$

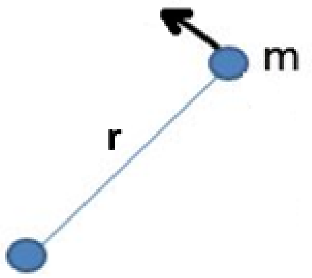
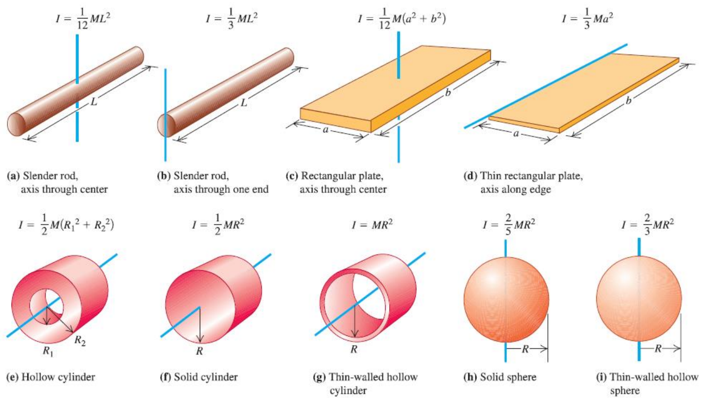
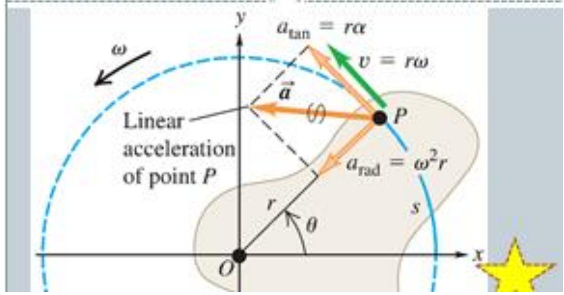


TABLE 9.2 Moments of inertia for various bodies



K.E. = ½ * I * ω²

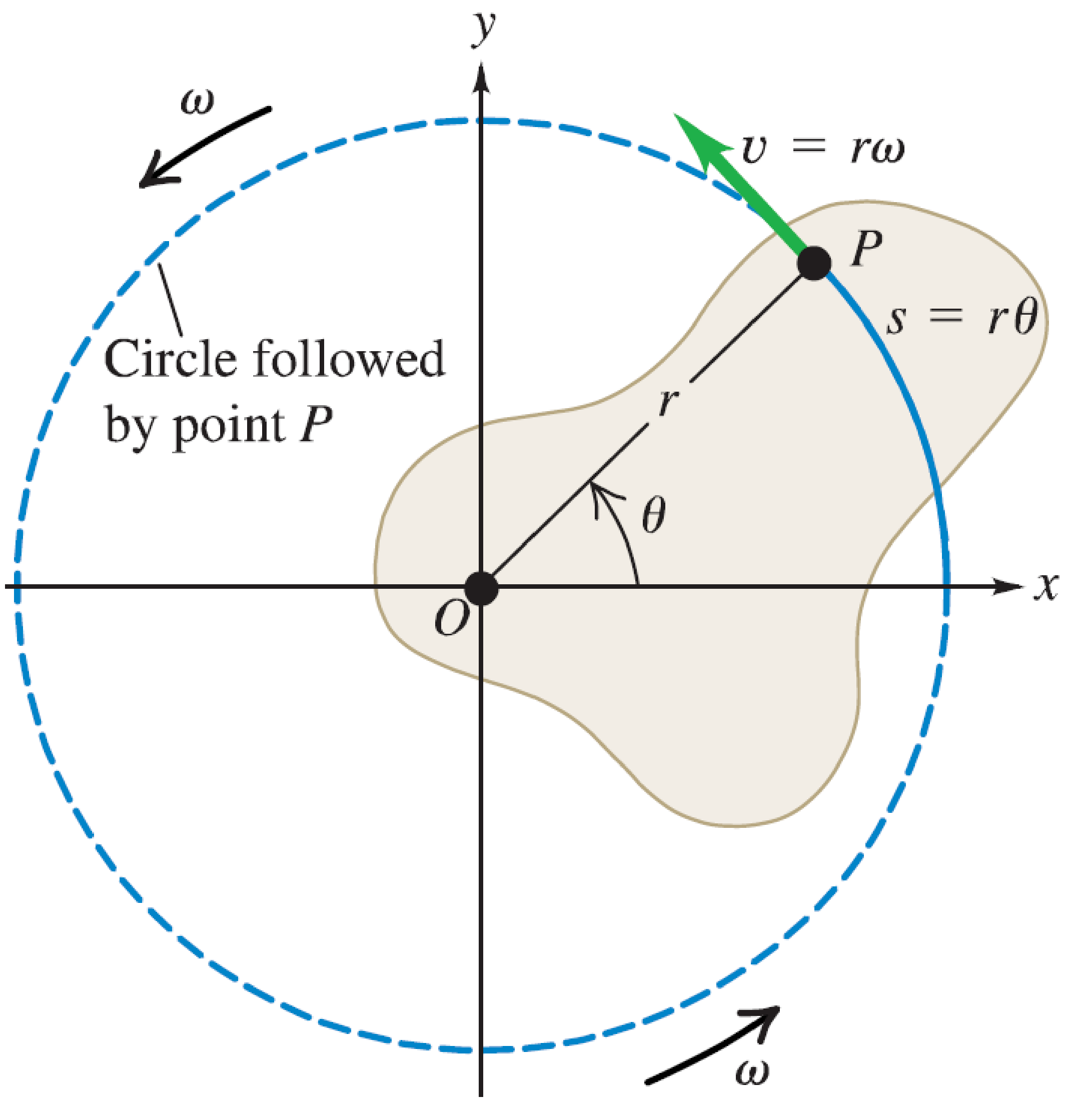
Relationship between linear and Angular Quantities



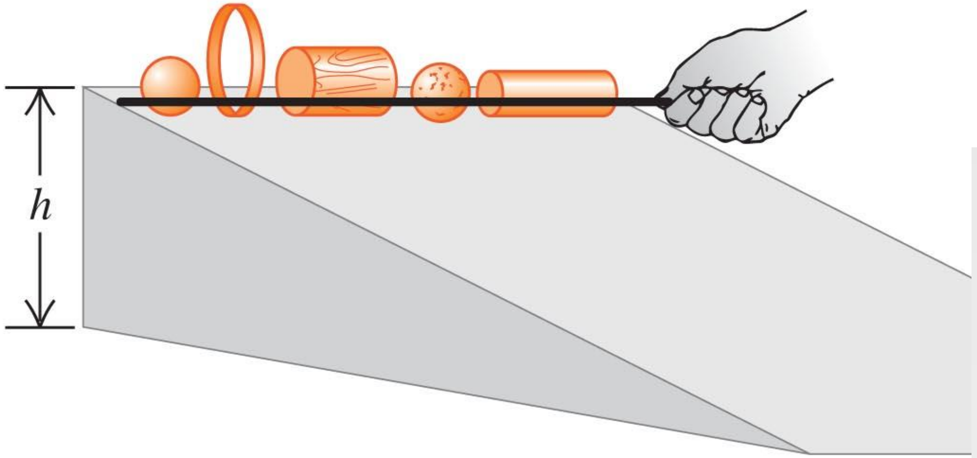
- *Angular Equations of Motion*
- $\alpha = \frac{\Delta\omega}{\Delta t} = \frac{\omega-\omega_0}{t} \Rightarrow \omega = \omega_0 + \alpha t$
- $\Delta\theta = \left(\frac{\omega_0+\omega}{2}\right)t$
- $\Delta\theta = \omega_0 t + \frac{1}{2}\alpha t^2$
- $\Delta\theta = \omega t - \frac{1}{2}\alpha t^2$
- $\omega^2 = \omega_0{}^2 + 2\alpha\Delta\theta$

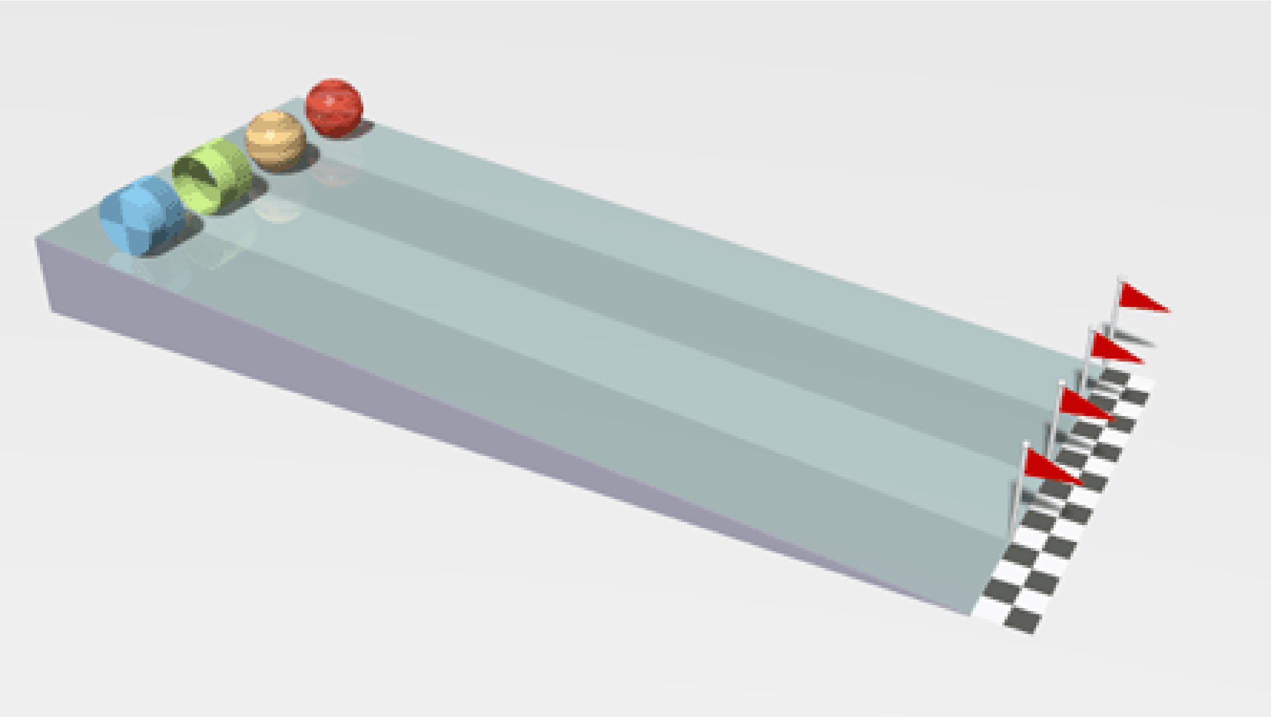
Extra information and examples to try yourself, or do in class, if we have time

Relationship between linear and Angular Quantities



Race of the Rolling Objects – Example 9.10





Prove the solid cylinder will beat the cylindrical shell

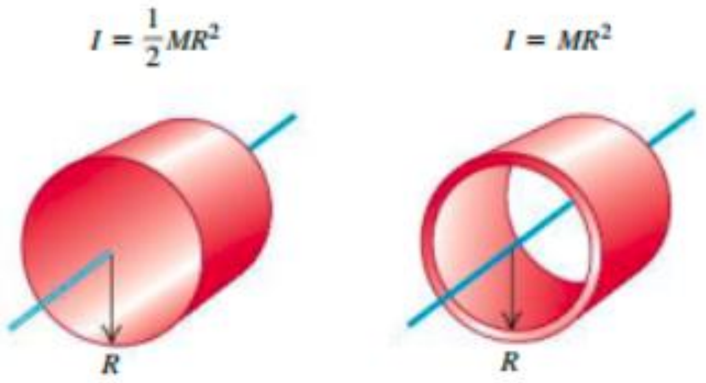
SOLVE For the cylindrical shell, Table 9.2g gives $I_{\text{shell}} = MR^2$. Conservation of energy then results in

$$\begin{aligned} 0 + Mgh &= \tfrac{1}{2}Mv_{\text{cm}}^{\;2} + \tfrac{1}{2}I_{\text{shell}}\omega^2 \\ &= \tfrac{1}{2}Mv_{\text{cm}}^{\;2} + \tfrac{1}{2}(MR^2)(v_{\text{cm}}/R)^2 \\ &= \tfrac{1}{2}Mv_{\text{cm}}^{\;2} + \tfrac{1}{2}Mv_{\text{cm}}^{\;2} = Mv_{\text{cm}}^{\;2}, \\ v_{\text{cm}} &= \sqrt{gh}. \end{aligned}$$

cm is center of mass

For the solid cylinder, Table 9.2f gives $I_{\text{solid}} = \frac{1}{2}MR^2$, and the corresponding equations are

$$\begin{aligned} 0 + Mgh &= \tfrac{1}{2}Mv_{\text{cm}}^{\;2} + \tfrac{1}{2}I_{\text{solid}}\omega^2 \\ &= \tfrac{1}{2}Mv_{\text{cm}}^{\;2} + \tfrac{1}{2}(\tfrac{1}{2}MR^2)(v_{\text{cm}}/R)^2 \\ &= \tfrac{1}{2}Mv_{\text{cm}}^{\;2} + \tfrac{1}{4}Mv_{\text{cm}}^{\;2} = \tfrac{3}{4}Mv_{\text{cm}}^{\;2}, \\ v_{\text{cm}} &= \sqrt{\tfrac{4}{3}gh}. \end{aligned}$$



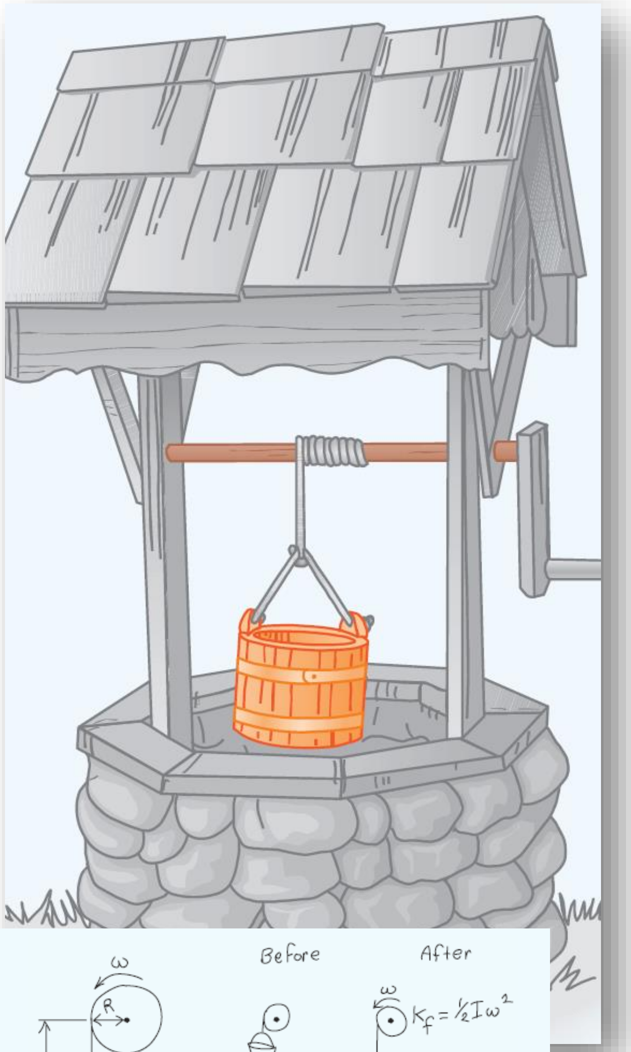
(f) Solid cylinder

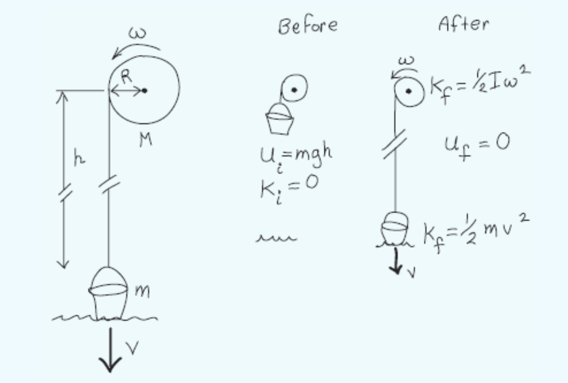
(g) Thin-walled hollow cylinder

VTS Ex 9.10

Energy method

VTS Ex 9.8





▲ **FIGURE 9.20** Our sketches for this problem.

In an old-fashioned well, a bucket is suspended over the well shaft by a winch and rope. The winch includes a solid cylinder with mass M and radius R and rotates without friction about a horizontal axis. The bucket (mass m) must descend a height h to reach the water; it is suspended by a rope of negligible mass that wraps around the winch. If the winch handle falls off, releasing the bucket to fall to the water, rotating the cylinder as it falls, find the speed v of the bucket and the angular velocity ω of the cylinder just before the bucket hits the water.

Just before the bucket hits the water, both it and the cylinder have kinetic energy. The total kinetic energy K_f at that time is

$$K_f = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2.$$

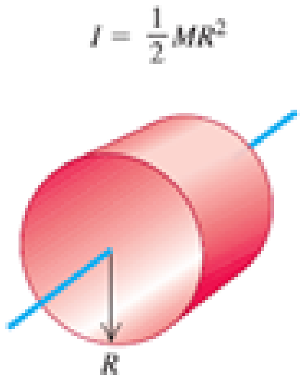
From Table 9.2, we find that the moment of inertia of the cylinder is $I = \tfrac{1}{2}MR^2$. Also, v and ω are related by $v = R\omega$, because the speed of mass m must equal the tangential speed of the outer surface of the cylinder. Using these relations and equating the initial and final total energies, we find that

$$K_i + U_i = K_f + U_f,$$

$$0 + mgh = \tfrac{1}{2}mv^2 + \tfrac{1}{2}\left(\tfrac{1}{2}MR^2\right)\left(\frac{v}{R}\right)^2 = \tfrac{1}{2}\left(m + \tfrac{1}{2}M\right)v^2,$$

$$v = \sqrt{\frac{2gh}{1 + M/2m}}.$$

The final angular velocity ω is obtained from $\omega = v/R$.



(f) Solid cylinder