Chapter 2

Motion in a straight line

Learning Outcomes Ch 2

- I can calculate the average velocity.
- I can find the instantaneous velocity at a particular point.

- I can calculate the average acceleration.
- I can find the instantaneous acceleration at a particular point.
- I can understand and interpret distance-time graphs and velocity-time graphs.

- **I can use the equations of motions to solve motion problems**

- I understand what free fall is and what assumptions have been made.

Motion

- Motion is divided into two areas of study:
  - Kinematics
    - This will be our focus in Chapter 2 (1-dimension). [2-D in Chapter 3]
    - Kinematics describes the movement of the object.
  - Dynamics
    - Will come in Chapter 4 and after
    - Dynamics answers the "Why is this object moving?" question.

2.2 Displacement and Average Velocity: "Are we there yet?"

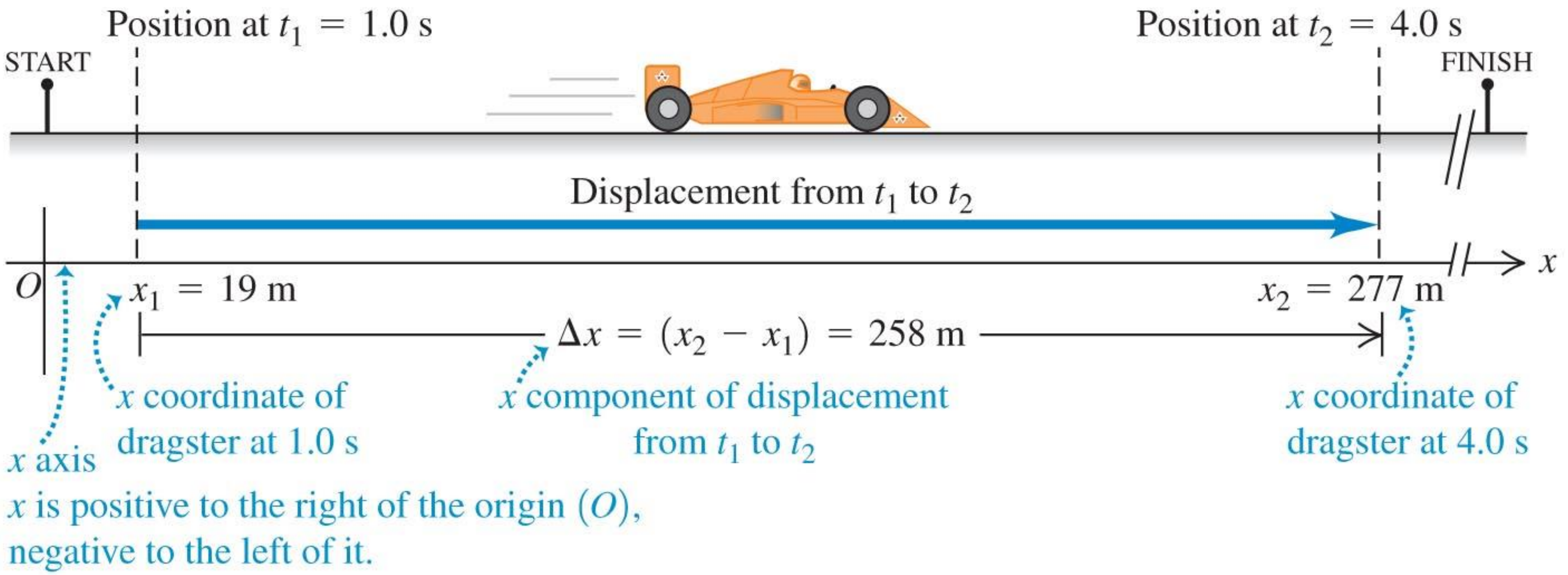
- Displacement, the distance from *here* to *there* → *a vector quantity*

$$\Delta x = x_2 - x_1$$

- Units
  - SI: Meters (m), CGS: Centimeters (cm), US Cust: Feet (ft)
- Average velocity → *a vector quantity*
  - Stop, speed up, slow down
  - Focus on total time and total displacement

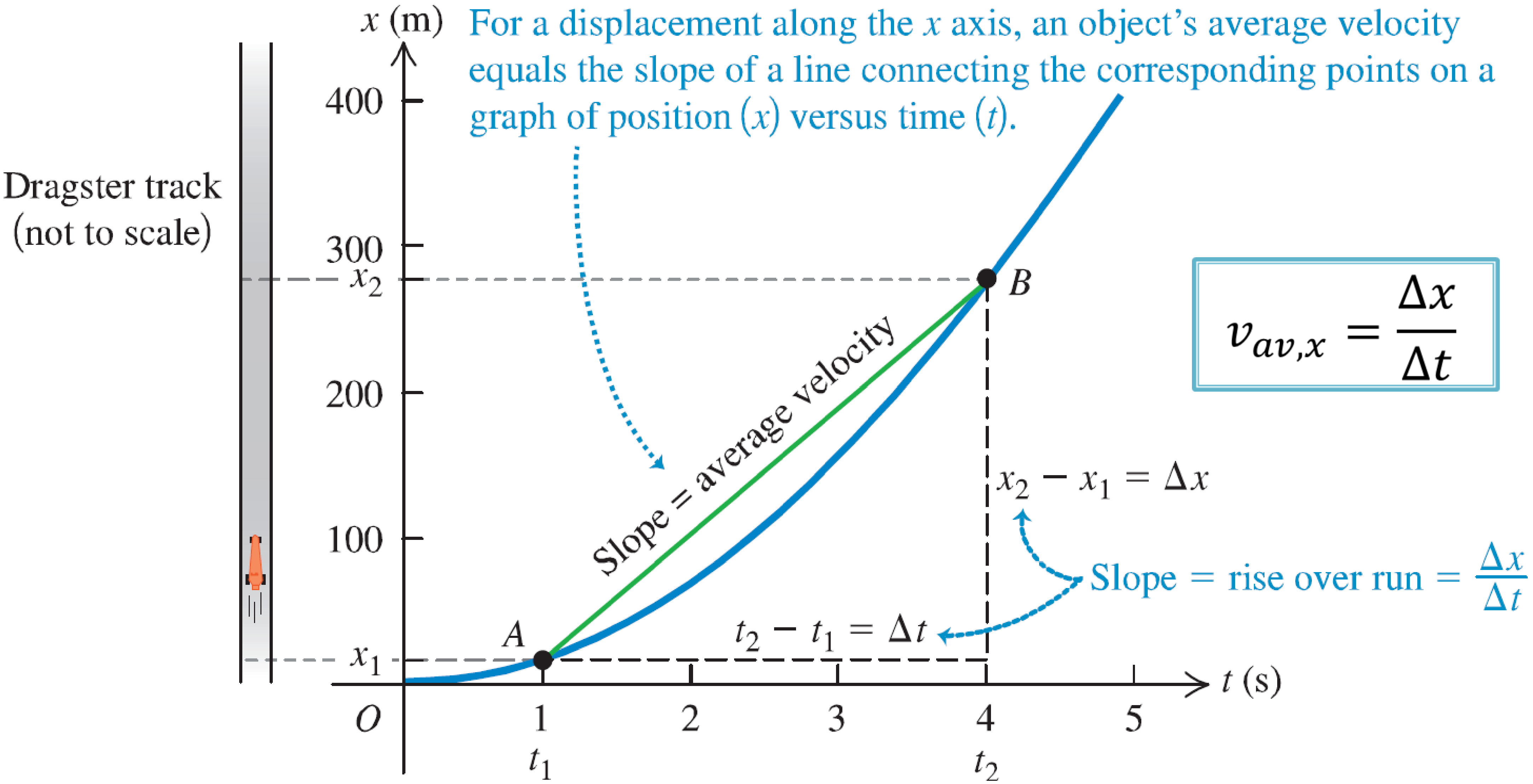
$$v_{av,x} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t} \qquad \text{Units: } m/s$$

Average Velocity



$$v_{av,x} = \frac{277\text{ m} - 19\text{ m}}{4.0\text{ s} - 1.0\text{ s}} = 86\,\text{m/s}$$

Average Velocity



Example (ans 23.7 m/s)

- A vehicle travels from Abu Dhabi to Dubai. The vehicle leaves Abu Dhabi at 9.45am and arrives in Dubai at 11.20am. The distance from Abu Dhabi to Dubai is said to be 135km. Calculate the average velocity of the car during this journey. (in m/s)

- We know that:

$$V_{average} = \text{Change in Distance / Change in Time} = \frac{\Delta x}{\Delta t}$$

Firstly, we need both the time and distance in SI units.....

Ans = 23.7 m/s

Example Solution

Firstly, we need both the time and distance in SI units.....
Time is 9.45am until 11.20am, which is 1 hour and 35 mins.
Well, 1 hour 35 mins = 95 mins. As there are 60 seconds in a minute we multiply this out.

95 * 60 = **5,700 seconds.**

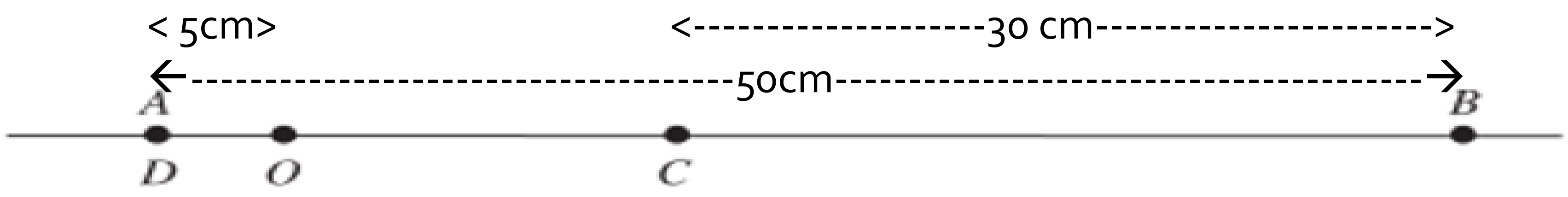
The distance is 135km. Well there are 1000m in 1km, so there must be 135 * 1000 metres in 135 km, which gives the distance as **135,000 metres.**

We know Ave. Velocity $= \frac{\Delta x}{\Delta t}$

So, Ave, Velocity = $\frac{\mathbf{135000}}{\mathbf{5700}} = 23.68\ ms^{-1}$

2.1 Displacement and Average Velocity

1. • An ant is crawling along a straight wire, which we shall call the *x* axis, from *A* to *B* to *C* to *D* (which overlaps *A*), as shown in Figure 2.39. *O* is the origin. Suppose you take measurements and find that *AB* is 50 cm, *BC* is 30 cm, and *AO* is 5 cm. (a) What is the ant's position at points *A*, *B*, *C*, and *D*? (b) Find the displacement of the ant and the distance it has moved over each of the following intervals: (i) from *A* to *B*, (ii) from *B* to *C*, (iii) from *C* to *D*, and (iv) from *A* to *D*.



Position

A

B

C

D

Displacement

A-B

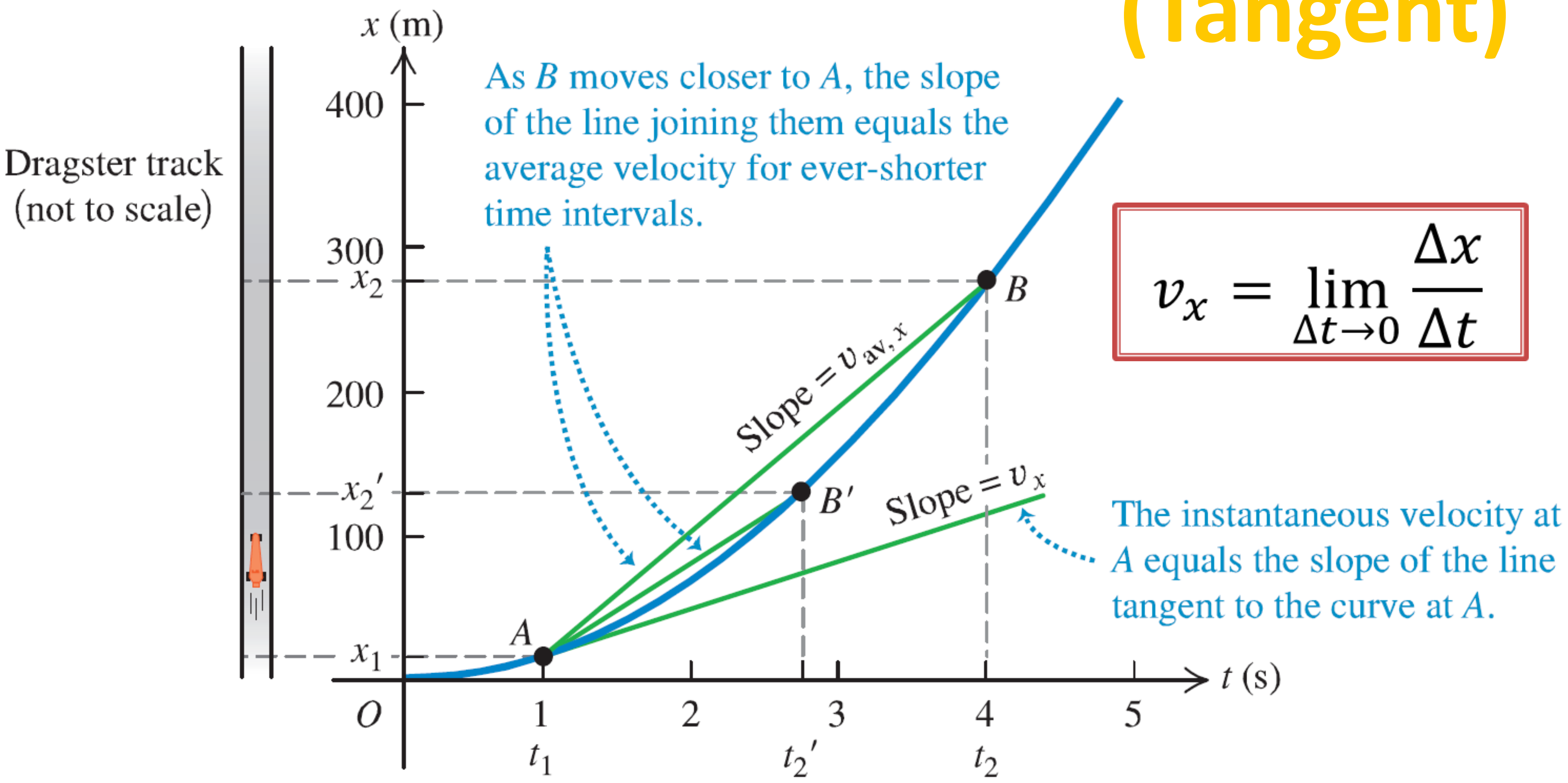
B-C

C-D

A-D

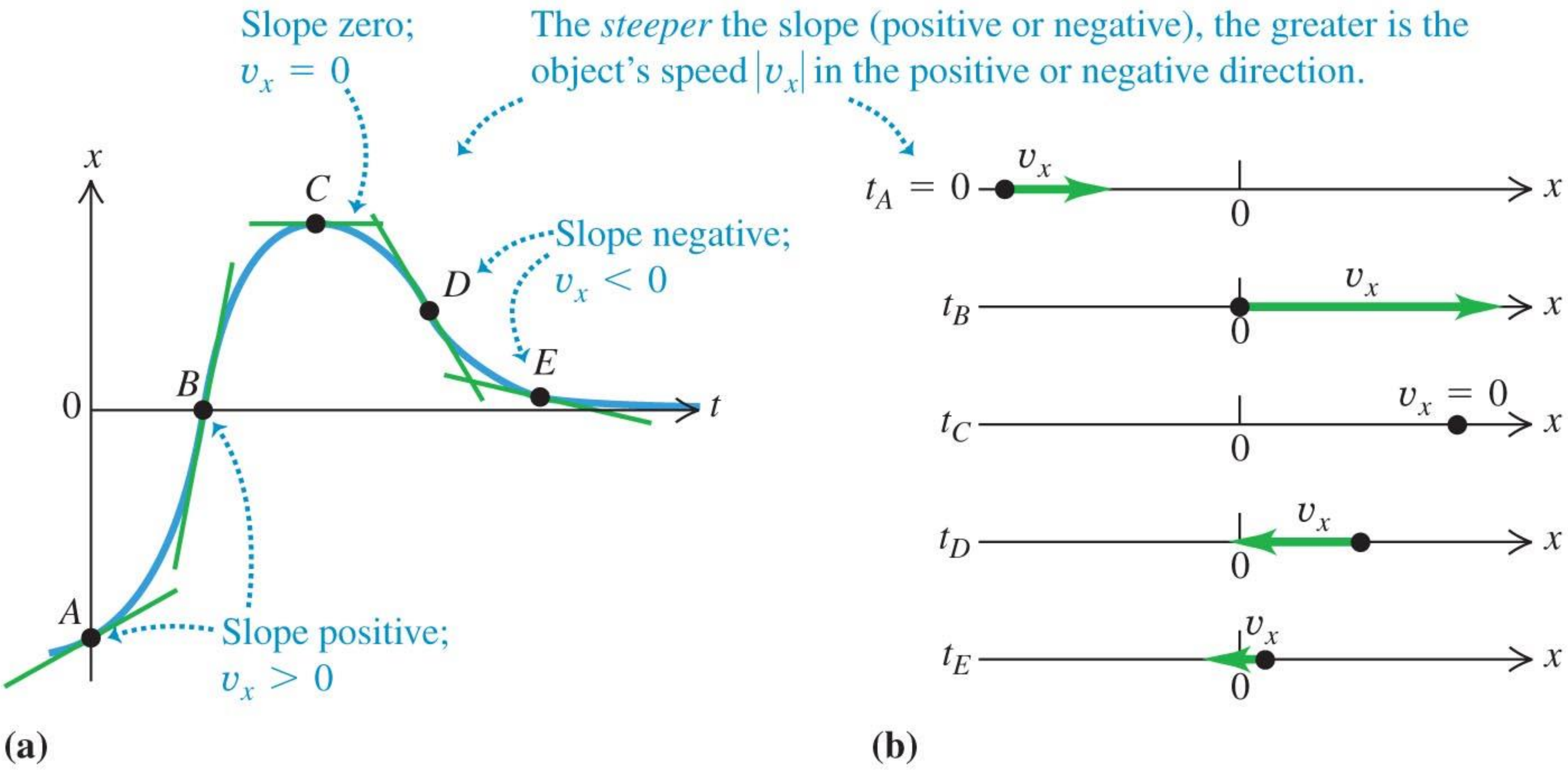
2.2 Instantaneous Velocity

(Tangent)

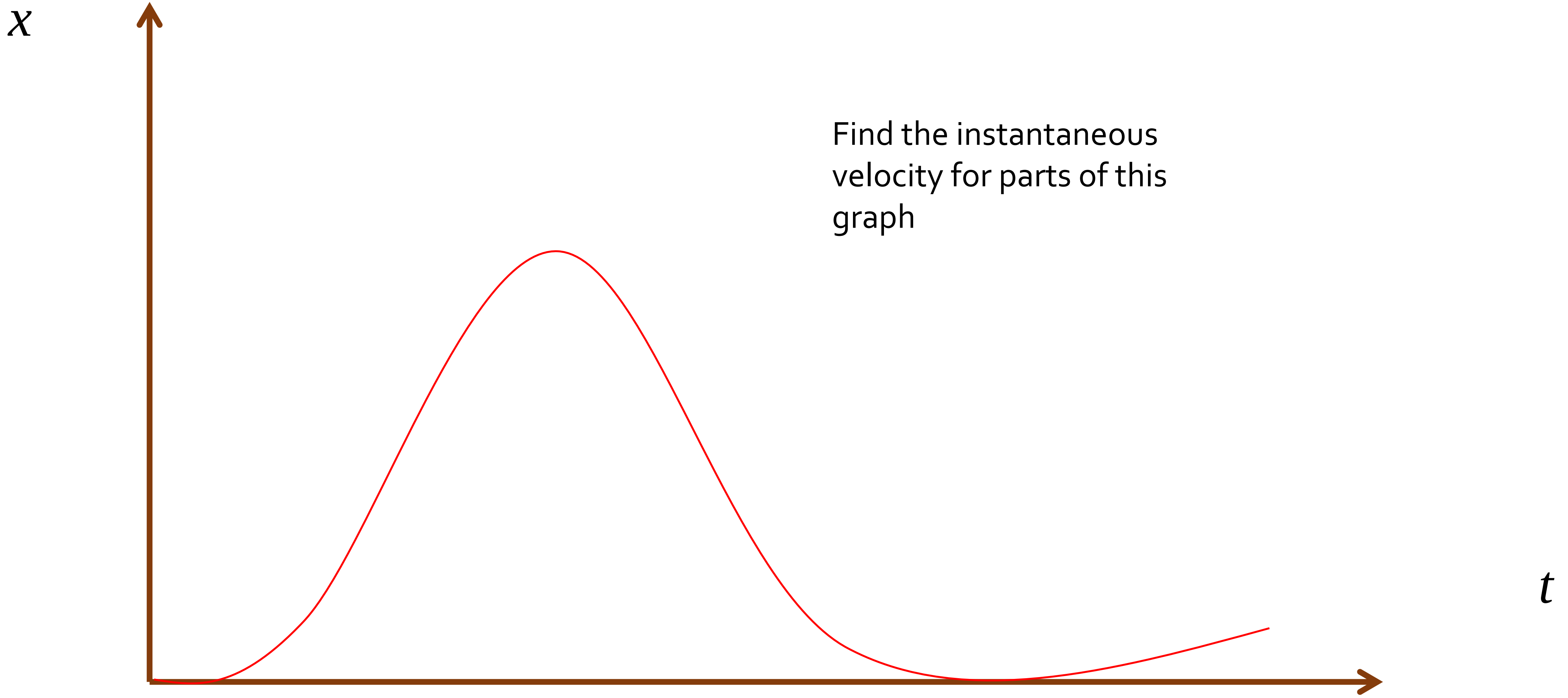


$$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

Interpretation of Motion via Graphing



Instantaneous Velocity

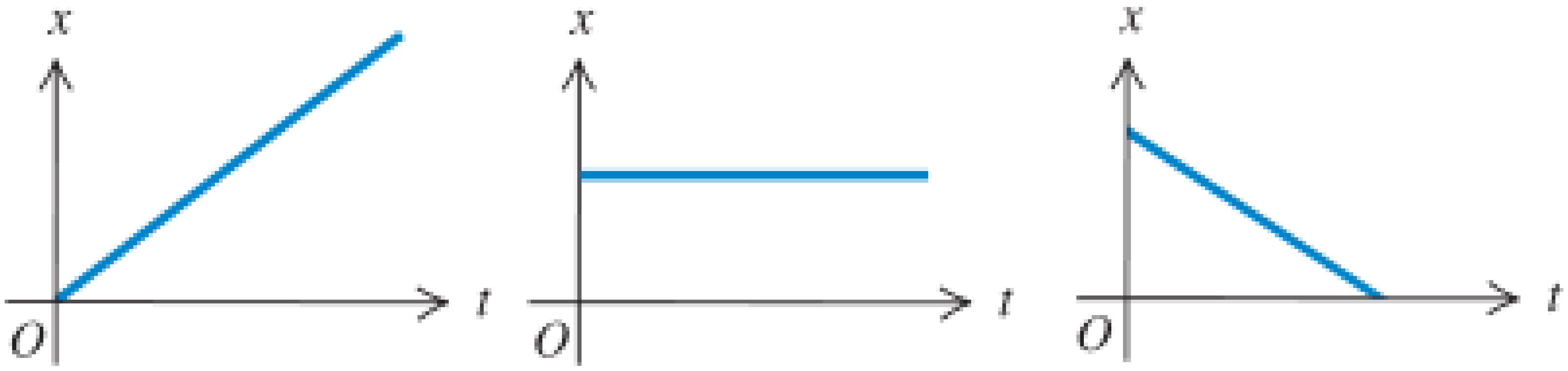


Find the instantaneous velocity for parts of this graph

?

8. •• Each graph in Figure 2.45 shows the position of a running cat, called Mousie, as a function of time. In each case, sketch a clear *qualitative* (no numbers) graph of Mousie's velocity as a function of time.

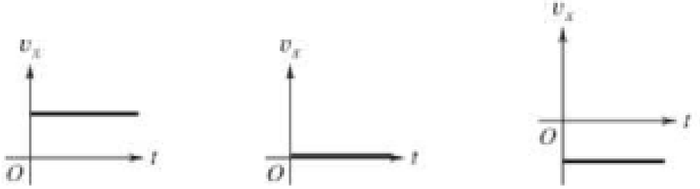
$$v_{av,x} = \frac{\Delta x}{\Delta t}$$



(a) (b) (c)

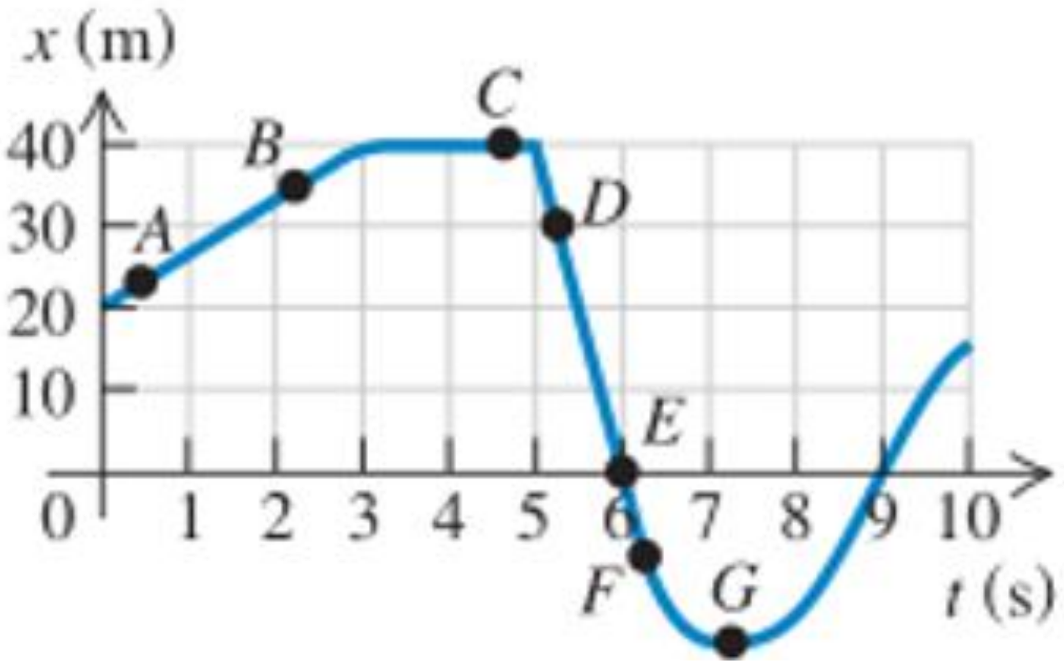
2.8. Set Up: $v_x(t)$ is the slope of the x versus t graph. In each case this slope is constant, so v_x is constant.

Solve: The graphs of v_x versus t are sketched in the figure below.



v_x constant and positive (a) v_x zero (b) v_x constant and negative (c)

20. ●● A test car travels in a straight line along the x axis. The graph in Figure 2.47 shows the car's position x as a function of time. Find its instantaneous velocity at points A through G.



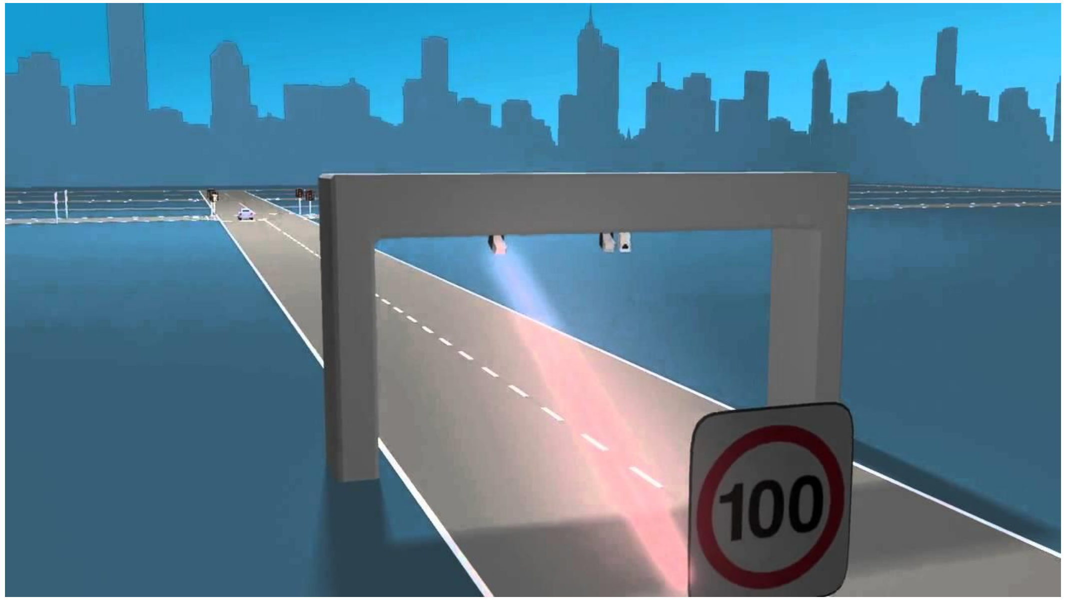
A

2.20. Set Up: The instantaneous velocity at any point is the slope of the x versus t graph at that point. Estimate the slope from the graph.

Solve: A: $v_x = 6.7$ m/s; B: $v_x = 6.7$ m/s; C: $v_x = 0$; D: $v_x = -40.0$ m/s; E: $v_x = -40.0$ m/s; F: $v_x = -40.0$ m/s; G: $v_x = 0$.

Reflect: The sign of v_x shows the direction the car is moving. v_x is constant when x versus t is a straight line.

Instantaneous speed camera



average speed camera

2.3 Acceleration

- **Acceleration is the rate of change in velocity.**
- Average Acceleration

$$a_{av,x} = \frac{\Delta v_x}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1}$$

Slope between two points

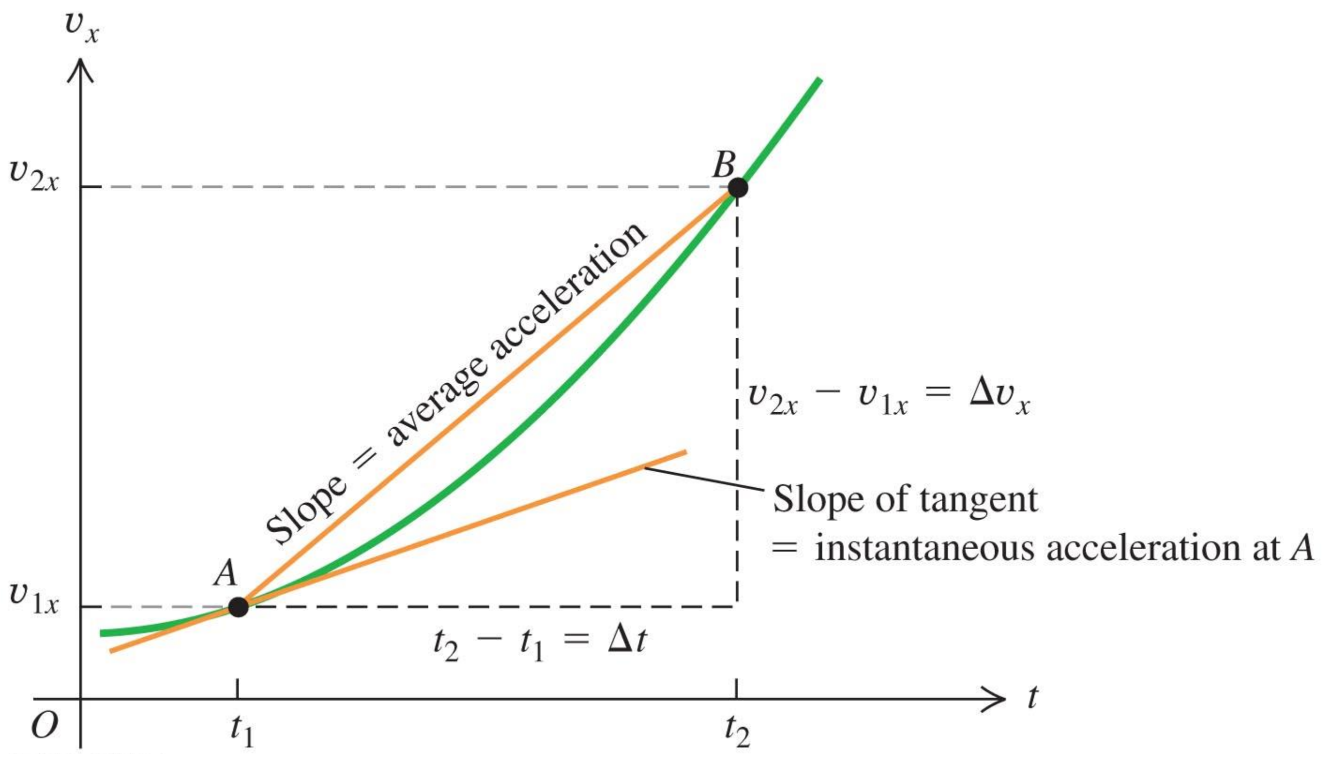
- Instantaneous acceleration

$$a_x = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$$

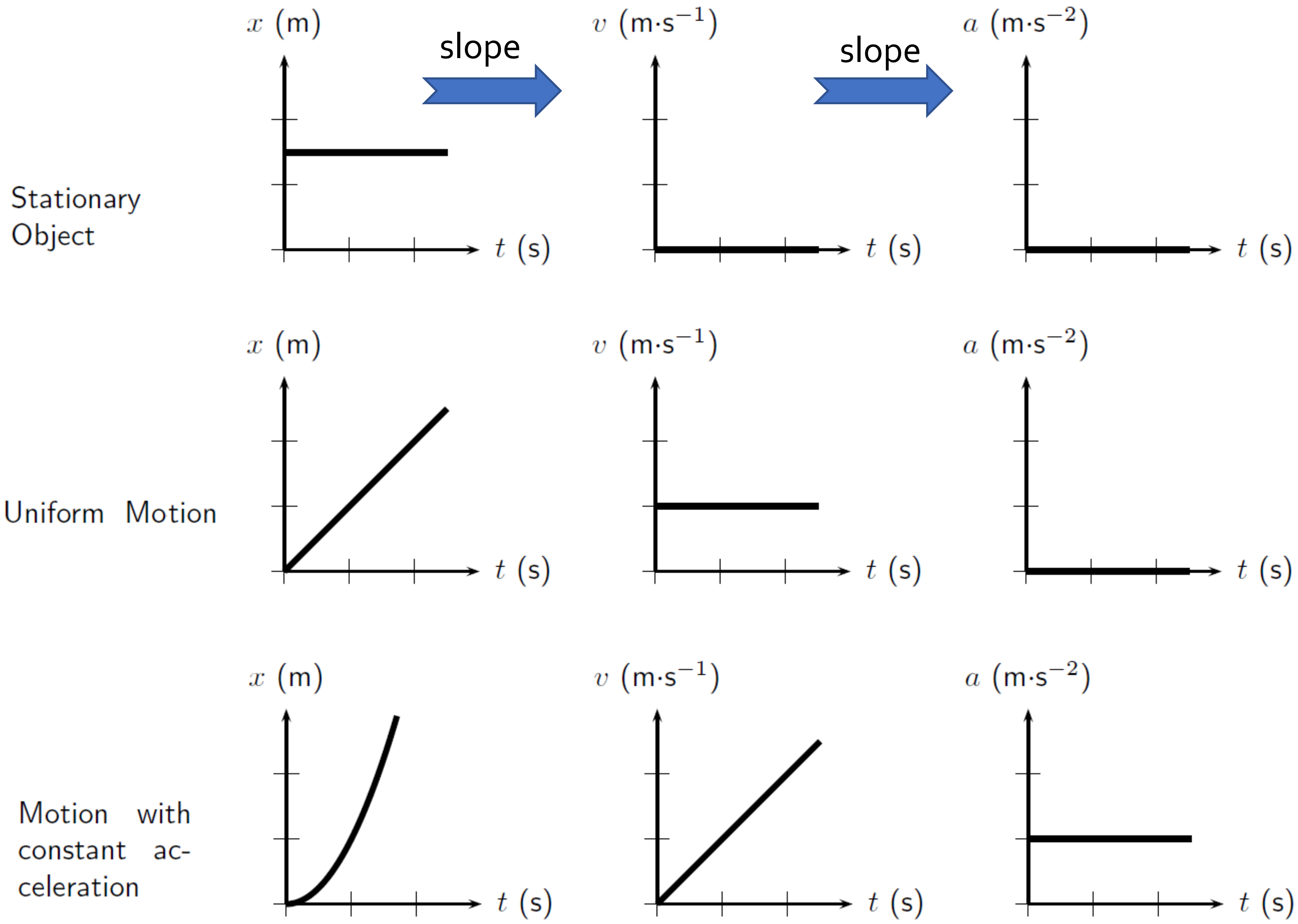
Slope at a point

- *Acceleration is the slope of v-t graph*

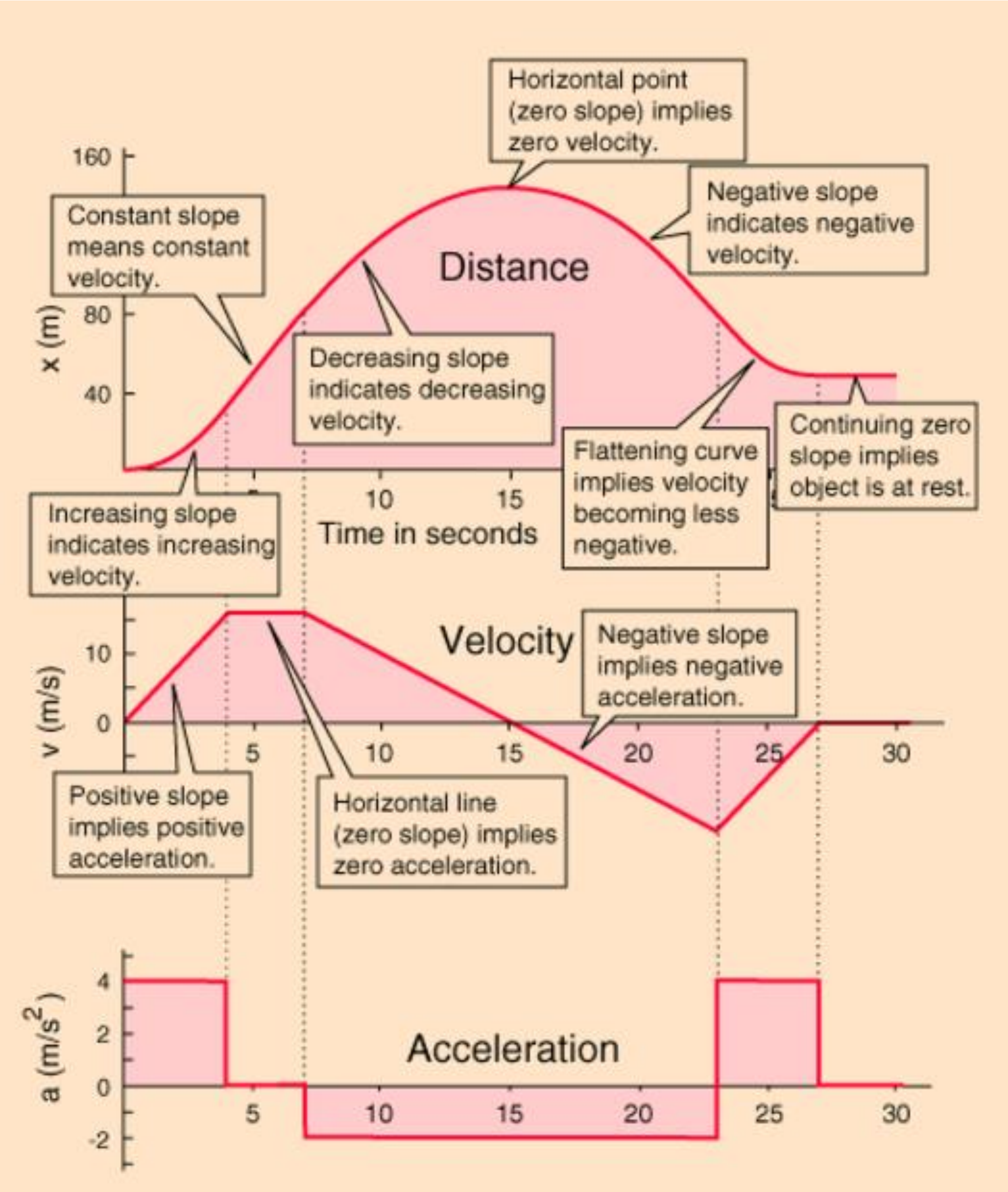
Acceleration from a Velocity vs Time Plot



- Average acceleration → slope between two points
- Instantaneous acceleration → line tangent at point


slope
slope
Stationary Object
Uniform Motion
Motion with constant acceleration
x (m)
v (m·s⁻¹)
a (m·s⁻²)
t (s)

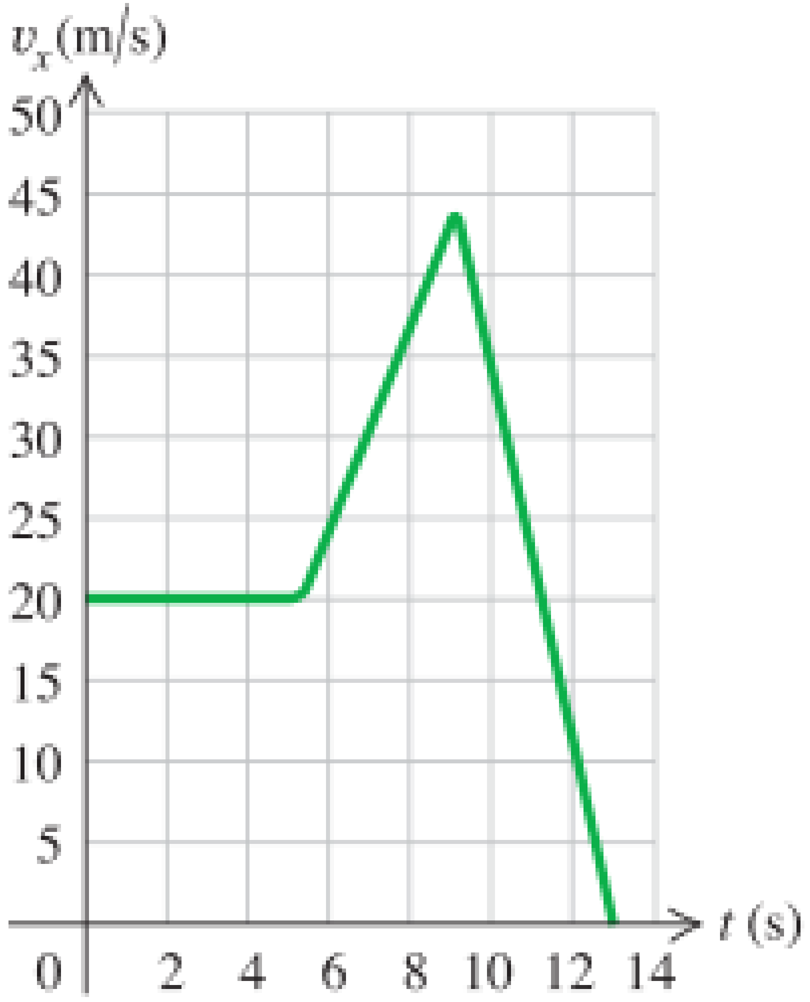
From Hyperphysics site



?

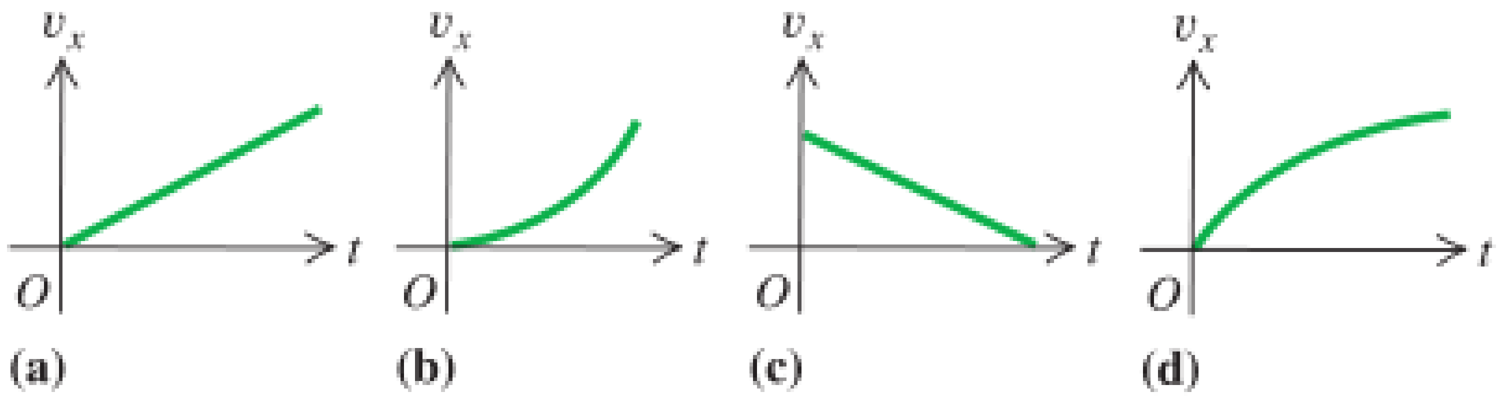
22. ●● The graph in Figure 2.49 shows the velocity of a motorcycle police officer plotted as a function of time. Find the instantaneous acceleration at times $t = 3\text{ s}$, at $t = 7\text{ s}$, and at $t = 11\text{ s}$.

Acceleration is the slope of a v-t graph



▲ **FIGURE 2.49** Problem 22.

25. ●● For each graph of velocity as a function of time in Figure 2.50, sketch a qualitative graph of the acceleration as a function of time.



▲ **FIGURE 2.50** Problem 25.

A

2.22. Set Up: The instantaneous acceleration is the slope of the v_x versus t graph.

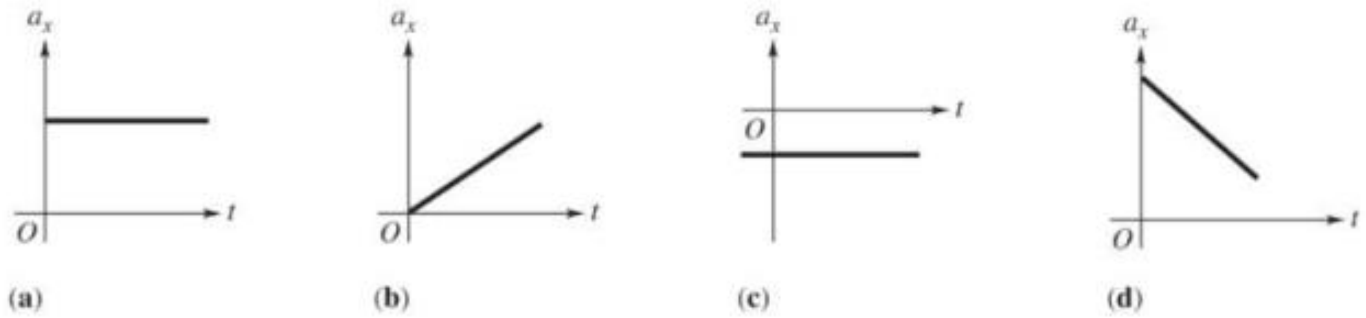
Solve: $t = 3$ s: The graph is horizontal, so $a_x = 0$.

$t = 7$ s: The graph is a straight line with slope $\dfrac{44\text{ m/s} - 20\text{ m/s}}{4\text{ s}} = 6.0\text{ m/s}^2$; $a_x = 6.0\text{ m/s}^2$.

$t = 11$ s: The graph is a straight line with slope $\dfrac{0 - 44\text{ m/s}}{4\text{ s}} = -11\text{ m/s}^2$; $a_x = -11\text{ m/s}^2$.

***2.25. Set Up:** The acceleration a_x equals the slope of the v_x versus t curve.

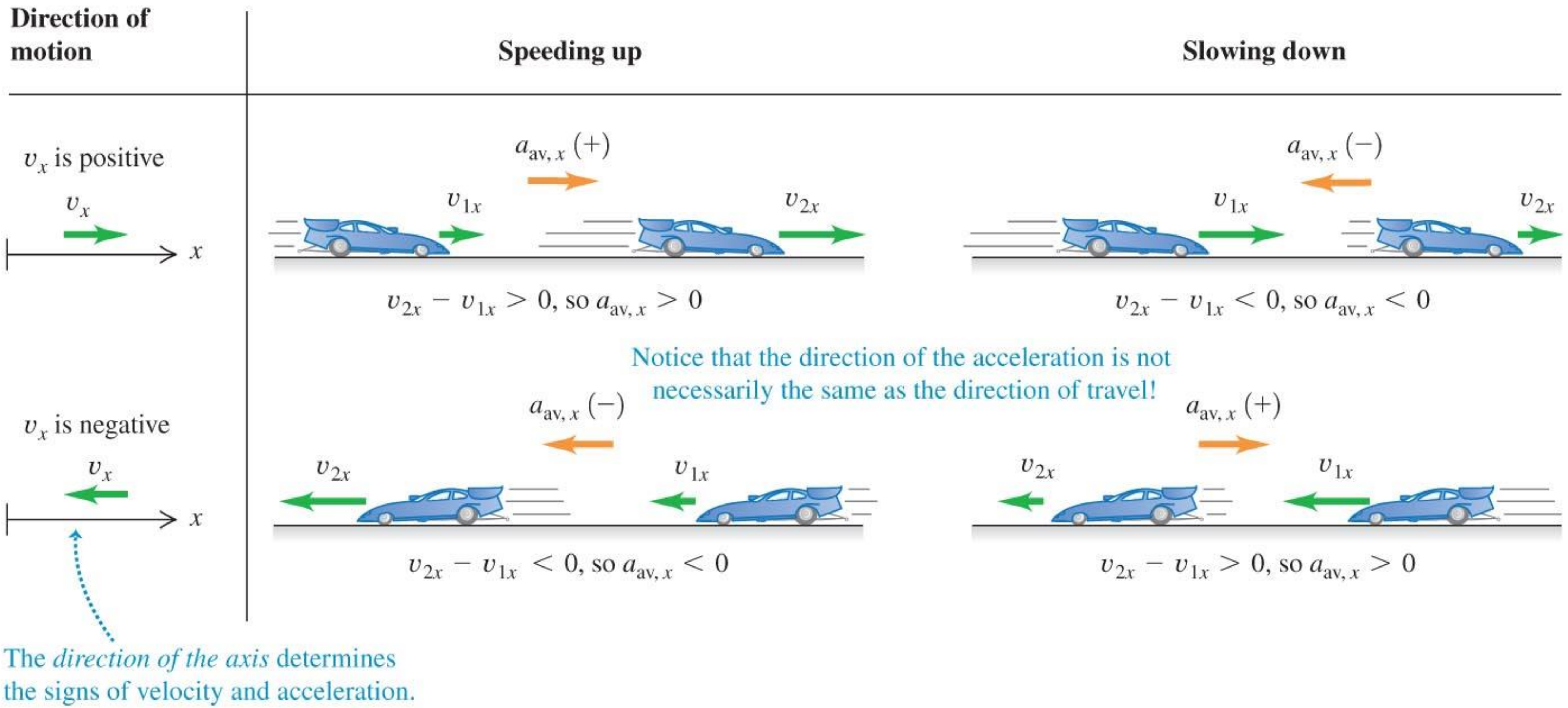
Solve: The qualitative graphs of acceleration as a function of time are given in the figure below.



The acceleration can be described as follows: (**a**) positive and constant, (**b**) positive and increasing, (**c**) negative and constant, (**d**) positive and decreasing.

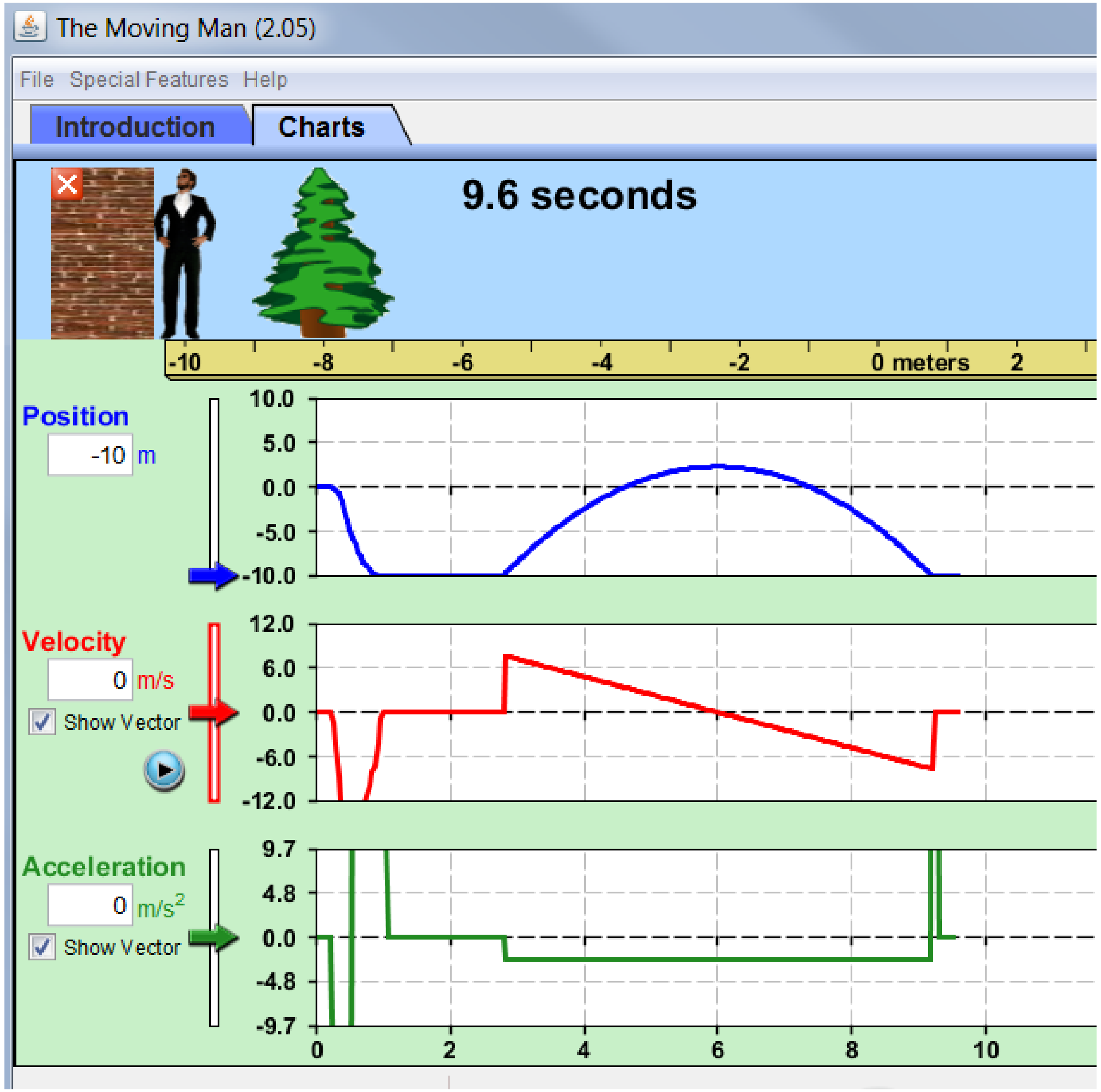
Reflect: When $\boldsymbol{v}_x$ and a_x have the same sign then the speed is increasing. In (**c**) the velocity and acceleration have opposite signs and the speed is decreasing.

Motion in Pictures and Graph



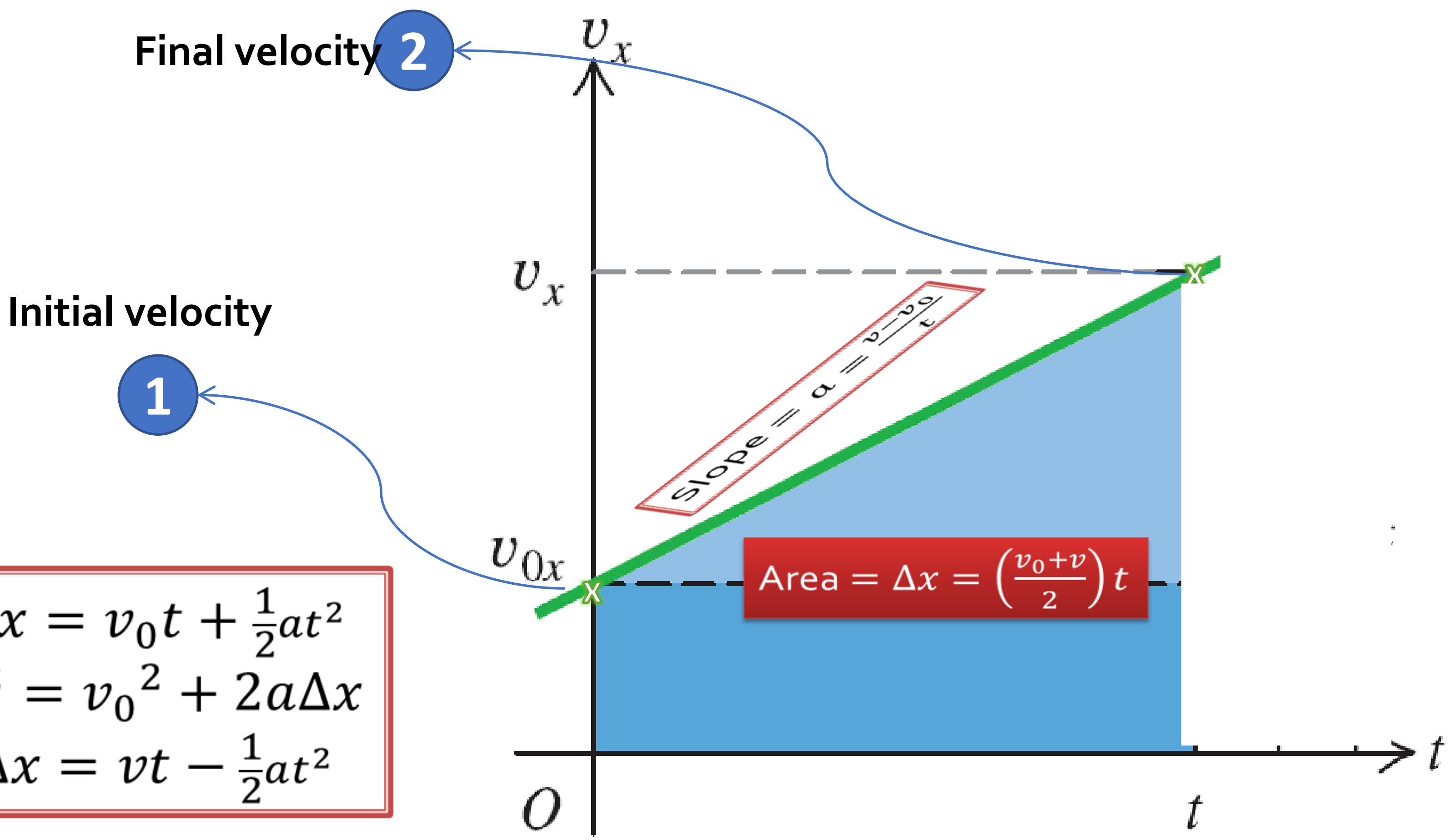
PHet

http://phet.colorado.edu/en/simulation/moving-man



PhET The moving man

2.4 Motion with constant acceleration



$$\Delta x = v_0 t + \frac{1}{2}at^2$$
$$v^2 = {v_0}^2 + 2a\Delta x$$
$$\Delta x = vt - \frac{1}{2}at^2$$

Equations of motion for constant acceleration

1 $$v = v_0 + at$$

2 $$\Delta x = \left(\frac{v_0 + v}{2}\right) t$$

3 $$\Delta x = v_0 t + \tfrac{1}{2} a t^2$$

4 $$v^2 = {v_0}^2 + 2a\Delta x$$

5 $$\Delta x = vt - \tfrac{1}{2} a t^2$$

v = velocity.
v_o = velocity at t = 0
a = acceleration
t = time
Δ = 'change in'

(equation 5 is 3 + 1)

Example 1

VTS Ex 2.5

A car initially traveling along a straight stretch of highway at 15 m/s accelerates with a constant acceleration of 2.0 m/s^2 in order to overtake a slower truck.
(a) What is the velocity of the car after 5.0 s?

1 $v = v_0 + at$

2 $\Delta x = \left(\frac{v_0 + v}{2}\right)t$

3 $\Delta x = v_0 t + \frac{1}{2}at^2$

4 $v^2 = {v_0}^2 + 2a\Delta x$

5 $\Delta x = vt - \frac{1}{2}at^2$

Example 1

(b) Sketch a velocity-time graph for these five seconds.

Example 1

(c) Sketch an acceleration time graph for the same time interval.

Example 1

VTS Ex 2.6

(d) How far did the car travel during this interval?

1 $$v = v_0 + at$$

2 $$\Delta x = \left(\frac{v_0 + v}{2}\right)t$$

3 $$\Delta x = v_0 t + \frac{1}{2}at^2$$

4 $$v^2 = {v_0}^2 + 2a\Delta x$$

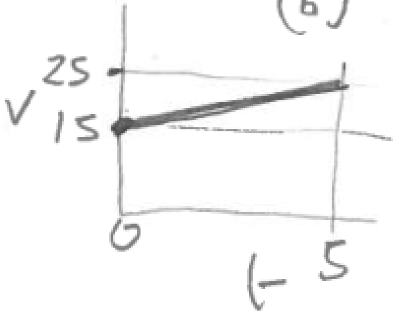
5 $$\Delta x = vt - \frac{1}{2}at^2$$

① $v_0 = 15 \text{ m/s}$

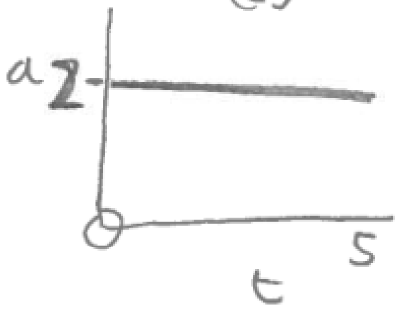
$a = 2.0 \text{ m/s}^2$

(a) $v = v_0 + at = 15 + 2.0(5) = 25 \text{ m/s}$

(b)



(c)



d

$$\Delta x = \left(\frac{v_0 + v}{2}\right)t$$

$$= \left(\frac{15+25}{2}\right)5$$

$$= 100 \text{ m}$$

Example 2

A car approaches a traffic signal at 20 m/s. The signal turns red and the driver applies the brakes causing the car to stop just before the signal line in 5.0 s.

(a) Calculate the acceleration of the car.

(b) How far was the car from the signal line when the driver started applying the brakes?

(c) Sketch a v-t and an a-t graph.

$$v = v_0 + at \qquad \Delta x = \left(\frac{v_0+v}{2}\right)t$$

$$\Delta x = v_0 t + \frac{1}{2}at^2 \qquad v^2 = {v_0}^2 + 2a\Delta x$$

$$\Delta x = vt - \frac{1}{2}at^2$$

A car approaches a traffic signal at 20 m/s. The signal turns red and the driver applies the brakes causing the car to stop just before the signal line in 5.0 s.

(a) Calculate the acceleration of the car.

(b) How far was the car from the signal line when the driver started applying the brakes?

(c) Sketch a v-t and an a-t graph.

$$v = v_0 + at \qquad \Delta x = \left(\frac{v_0+v}{2}\right)t$$

$$\Delta x = v_0 t + \frac{1}{2}at^2 \qquad v^2 = {v_0}^2 + 2a\Delta x$$

$$\Delta x = vt - \frac{1}{2}at^2$$

(2)

$V_0 = 20\ m/s$

$V = 0\ m/s$

$t = 5.0\ s$

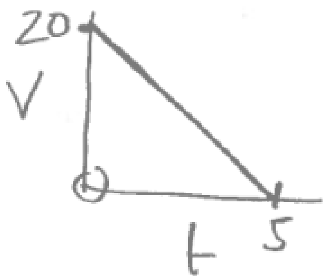
(a) $V = V_0 + at$

$$a = \frac{V - V_0}{t} = \frac{0 - 20}{5} = -4\ m/s^2$$

(b)

$$\Delta x = \left(\frac{V_0 + V}{2}\right)t = \left(\frac{20 + 0}{2}\right)5 = 50m$$

(c)



(d) Acceleration is constant

Example 3

A ball thrown horizontally hits a vertical wall at 4.8 m/s and rebounds at 3.6 m/s. It remains in contact with the wall for 280 ms.

(a) Sketch the ball's v-t graph.

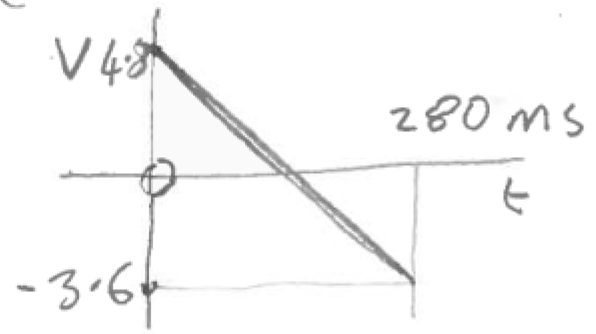
(b) From your graph, find the acceleration of the ball during contact with the wall.

(3) $v_0 = 4.8\ m/s$ → +ve

$v = -3.6\ m/s$

$t = 280 \times 10^{-3}\ s$

(a)



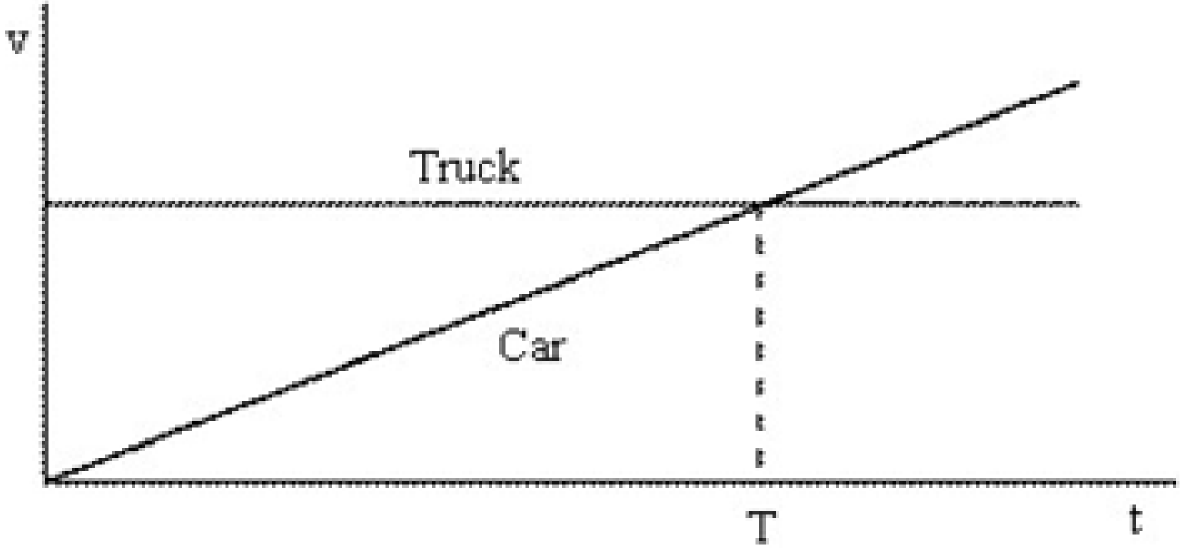
(b)

$$a = \frac{-3.6-4.8}{280 \times 10^{-3}}$$

$$= -30\ m/s^2$$

?

A1



At time T, what is true of the distances travelled by the vehicles since time t = 0?

A) They will have travelled the same distance.
B) The truck will not have moved.
C) The car will have travelled further than the truck.
D) The truck will have travelled further than the car.

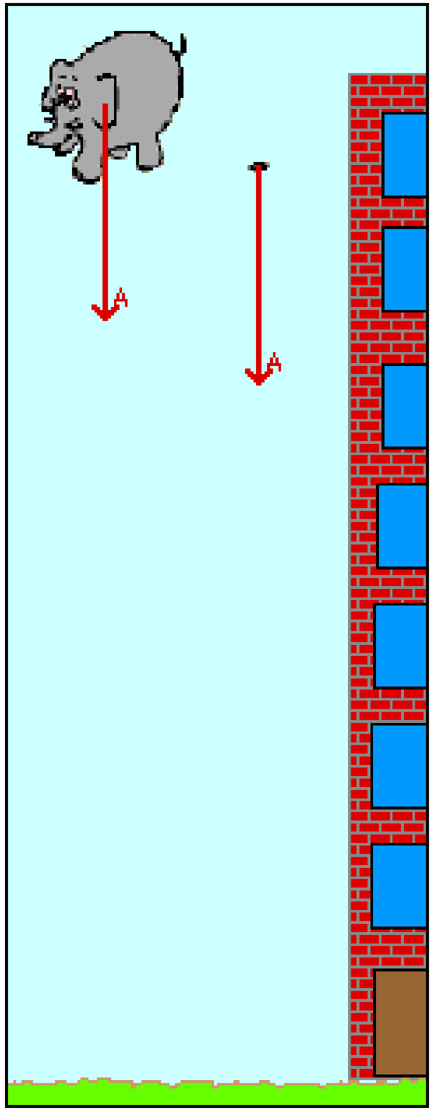
Hint: Distance is the area under a velocity - time graph.

A

A1 The answer is D . The distance traveled is the area under a v-t curve

2.6 Free Fall: Does weight matter?

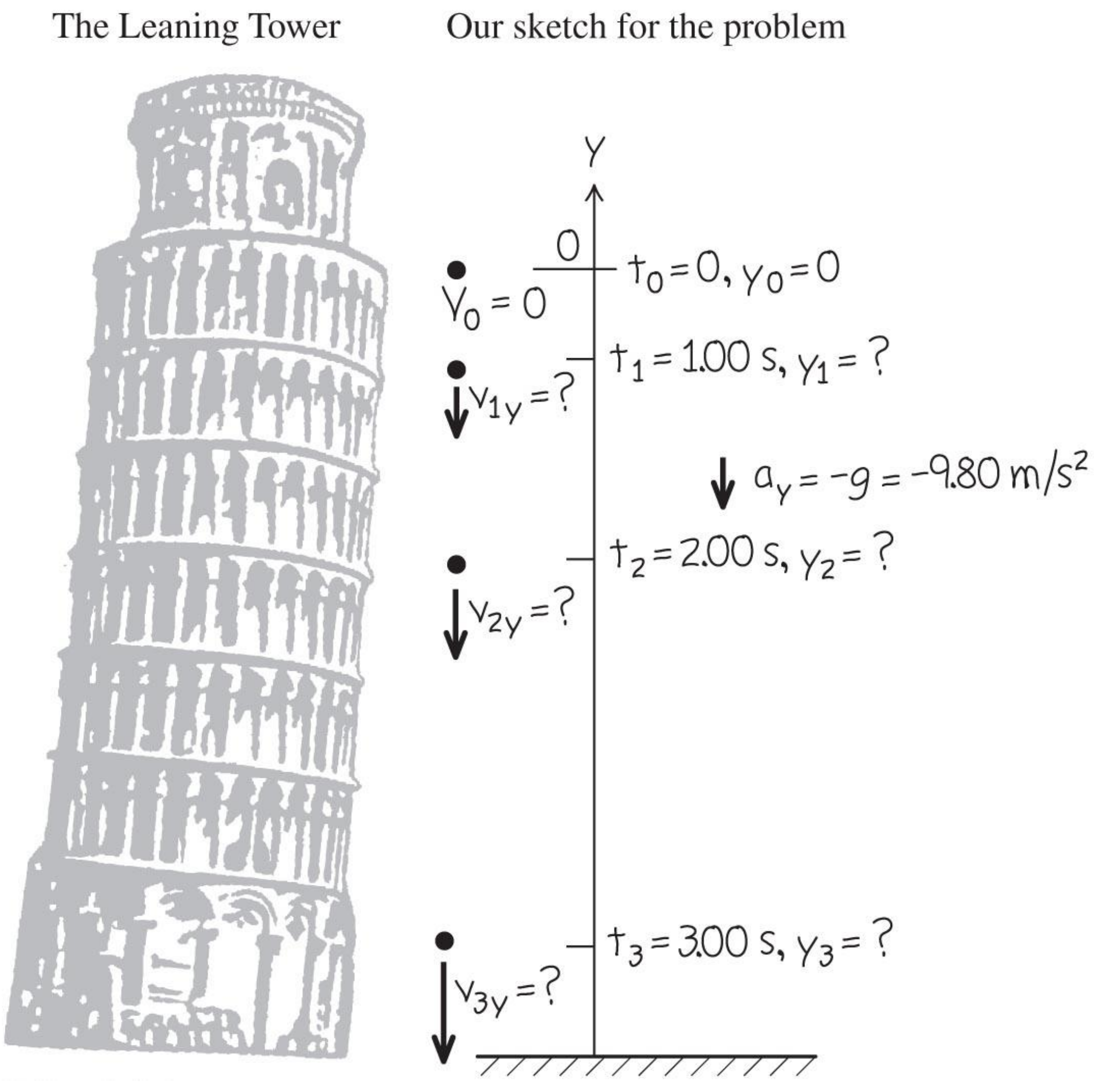
Does a falling elephant or a falling feather reach the ground first?



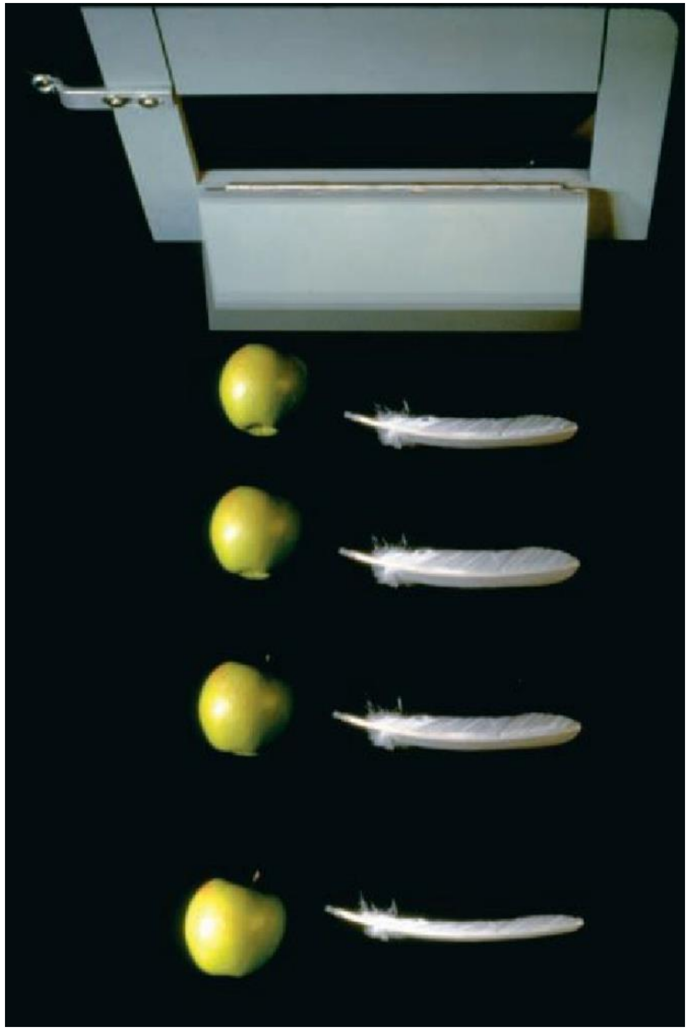
"Thank You Galileo"

VTS Ex 2.10

- As the story has it, he dropped objects from the Leaning Tower of Pisa; one heavy, one light. They hit simultaneously, disproving Aristotle's assertion that heavier objects fall faster.
- A feather and a hammer falling on the moon during the Apollo 15 mission by astronaut Dave Scott.
- The key is that Galileo is right for motion *in the absence of air resistance*.



Free Fall: Does weight matter?



Hammer and feather (Apollo 15)

http://www.youtube.com/watch?v=KDp1tiUsZw8

Free Fall: a special case of constant acceleration

Assumptions

1. No air resistance
   a) $\equiv$ air resistance is negligible
   b) $\equiv$ the objects moves only under the effect of gravity
2. the gravitational field strength is constant
   a) $\equiv g \cong 9.8\ m\ s^{-2}$
   b) $\equiv$ the object is close to the surface of the earth

Free Fall: Illustration

A tennis serving machine is directed vertically upwards and shoots a ball at 30 m/s.

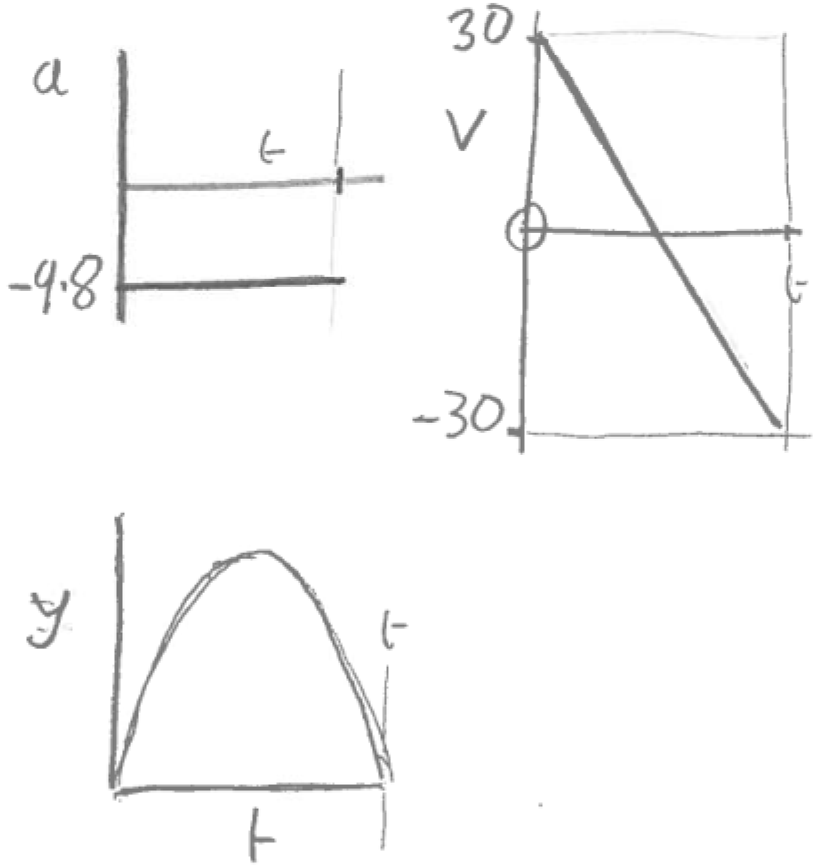
For the time interval taken by the ball to return to the same initial height, sketch its

(a) acceleration-time graph,

(b) velocity-time graph, and

(c) position-time graph

Free fall



Free Fall: important pointers

- The acceleration at **every position** is $\boldsymbol{g \cong 9.8\, m\, s^{-2}\, downwards}$
- The velocity at the highest position is *zero*.
- If the object is initially going upwards:
  - The time taken to reach maximum height is the same as the time taken to return to launch position.
  - The velocity has the same magnitude at the same height when going up and down.

You throw a ball vertically upward from the flat roof of a tall building. The ball leaves your hand at a point even with the roof railing, with an upward velocity of 15.0 m/s. On its way back down, it just misses the railing. Find

(a) the position and velocity of the ball 1.00 s and 4.00 s after it leaves your hand;
(b) the velocity of the ball when it is 5.00 m above the railing; and
(c) the maximum height reached and the time at which it is reached. Ignore the effects of the air.

VTS Ex 2.11

$$v = v_0 + at \qquad \Delta x = \left(\frac{v_0+v}{2}\right)t$$

$$\Delta x = v_0 t + \frac{1}{2}at^2 \qquad v^2 = {v_0}^2 + 2a\Delta x$$

$$\Delta x = vt - \frac{1}{2}at^2$$

You throw a ball vertically upward from the flat roof of a tall building. The ball leaves your hand at a point even with the roof railing, with an upward velocity of 15.0 m/s. On its way back down, it just misses the railing. Find

(a) the position and velocity of the ball 1.00 s and 4.00 s after it leaves your hand;
(b) the velocity of the ball when it is 5.00 m above the railing; and
(c) the maximum height reached and the time at which it is reached. Ignore the effects of the air.

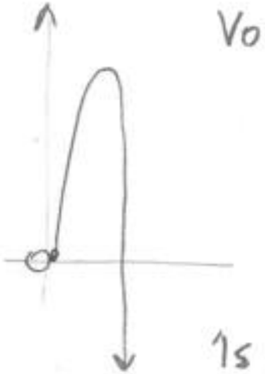
VTS Ex 2.11

$v = v_0 + at$ $\qquad \Delta x = \left(\frac{v_0 + v}{2}\right)t$

$\Delta x = v_0 t + \frac{1}{2}at^2$ $\qquad v^2 = {v_0}^2 + 2a\Delta x$

$\Delta x = vt - \frac{1}{2}at^2$

2.11
p53



$a = -9.8\ m/s^2$
$V_0 = 15.0\ m/s$

(a) find y + V_y
at t = 1.00 s + 4.00 s

$v = V_0 + at$

1s → $v = 15.0 + (-9.8)1$
$= 5.20\ m/s$

4s $v = 15.0 + (-9.8)4$
$= -24.2\ m/s$

$\Delta y = \left(\frac{V_0 + V}{2}\right)t$

1s $\Delta y = \left(\frac{15.0 + 5.20}{2}\right)1$
$= 10.1\ m$

4s $\Delta y = \left(\frac{15.0 + (-24.2)}{2}\right)4$
$= -18.4\ m$

(b) Velocity at 5m

$v^2 = V_0^2 + 2a\Delta y$
$v^2 = 15.0^2 + 2(-9.8)5$
$v = 11.3\ m/s$

+11.3 m/s going up
-11.3 m/s going down

(c) Max height + time
at Max height v = 0

$v^2 = V_0^2 + 2a\Delta y$
$0 = 15.0^2 + 2(-9.8)\Delta y$
$\Delta y = 11.5\ m$

max time $v = V_0 + at$
$0 = 15.0 + (-9.8)t$

$t = 1.53\ s$

?

48. • (a) If a flea can jump straight up to a height of 22.0 cm, what is its initial speed (in m/s) as it leaves the ground, neglecting air resistance? (b) How long is it in the air? (c) What are the magnitude and direction of its acceleration while it is (i) moving upward? (ii) moving downward? (iii) at the highest point?

$v = v_0 + at$ $\qquad \Delta x = \left(\frac{v_0+v}{2}\right)t$

$\Delta x = v_0 t + \frac{1}{2}at^2$ $\qquad v^2 = {v_0}^2 + 2a\Delta x$

$\Delta x = vt - \frac{1}{2}at^2$

49. • A brick is released with no initial speed from the roof of a building and strikes the ground in 2.50 s, encountering no appreciable air drag. (a) How tall, in meters, is the building? (b) How fast is the brick moving just before it reaches the ground? (c) Sketch graphs of this falling brick's acceleration, velocity, and vertical position as functions of time.

$v = v_0 + at$ $\quad\quad$ $\Delta x = \left(\frac{v_0 + v}{2}\right)t$

$\Delta x = v_0 t + \frac{1}{2}at^2$ $\quad\quad$ $v^2 = {v_0}^2 + 2a\Delta x$

$\Delta x = vt - \frac{1}{2}at^2$

49. • A brick is released with no initial speed from the roof of a building and strikes the ground in 2.50 s, encountering no appreciable air drag. (a) How tall, in meters, is the building? (b) How fast is the brick moving just before it reaches the ground? (c) Sketch graphs of this falling brick's acceleration, velocity, and vertical position as functions of time.

$v = v_0 + at$ $\Delta x = \left(\frac{v_0 + v}{2}\right)t$

$\Delta x = v_0 t + \frac{1}{2}at^2$ $v^2 = {v_0}^2 + 2a\Delta x$

$\Delta x = vt - \frac{1}{2}at^2$

Answer

2.48. Set Up: Let $+y$ be upward. $a_y = -9.80\ \text{m/s}^2$. $v_y = 0$ at the maximum height.

Solve: (a) $y - y_0 = 0.220\ \text{m}$, $a_y = -9.80\ \text{m/s}^2$, $v_y = 0$. $v_y{}^2 = v_{0y}{}^2 + 2a_y(y - y_0)$ gives

$$v_{0y} = \sqrt{-2a_y(y - y_0)} = \sqrt{-2(-9.80\ \text{m/s}^2)(0.220\ \text{m})} = 2.08\ \text{m/s}.$$

(b) When the flea returns to ground, $v_y = -v_{0y}$. $v_y = v_{0y} + a_y t$ gives

$$t = \frac{v_y - v_{0y}}{a_y} = \frac{-2.08\ \text{m/s} - 2.08\ \text{m/s}}{-9.80\ \text{m/s}^2} = 0.424\ \text{s}$$

(c) $a = 9.80\ \text{m/s}^2$, downward, at all points in the motion.

NOTE: I take the launch point as zero so I get different answers

2.49. Set Up: Let $+y$ be downward. $a_y = 9.80\ \text{m/s}^2$

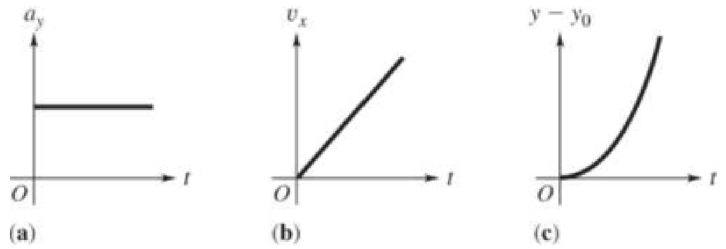
Solve: (a) $v_{0y} = 0$, $t = 2.50\ \text{s}$, $a_y = 9.80\ \text{m/s}^2$.

$$y - y_0 = v_{0y}t + \tfrac{1}{2}a_y t^2 = \tfrac{1}{2}(9.80\ \text{m/s}^2)(2.50\ \text{s})^2 = 30.6\ \text{m}.$$

The building is 30.6 m tall.

(b) $v_y = v_{0y} + a_y t = 0 + (9.80\ \text{m/s}^2)(2.50\ \text{s}) = 24.5\ \text{m/s}$

(c) The graphs of a_y, v_y, and y versus t are given in the figure below. Take $y = 0$ at the ground.



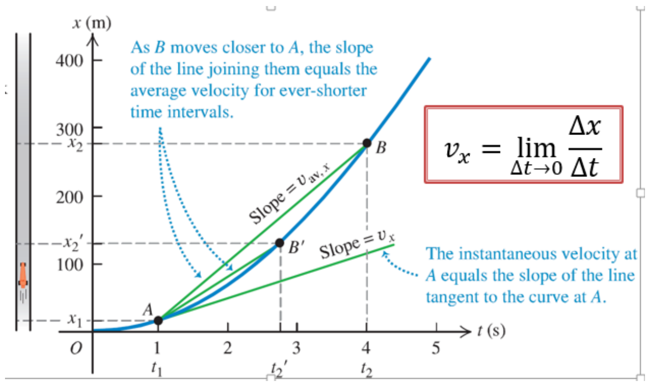
Summary

Motion (CLO3, Chapters 2)

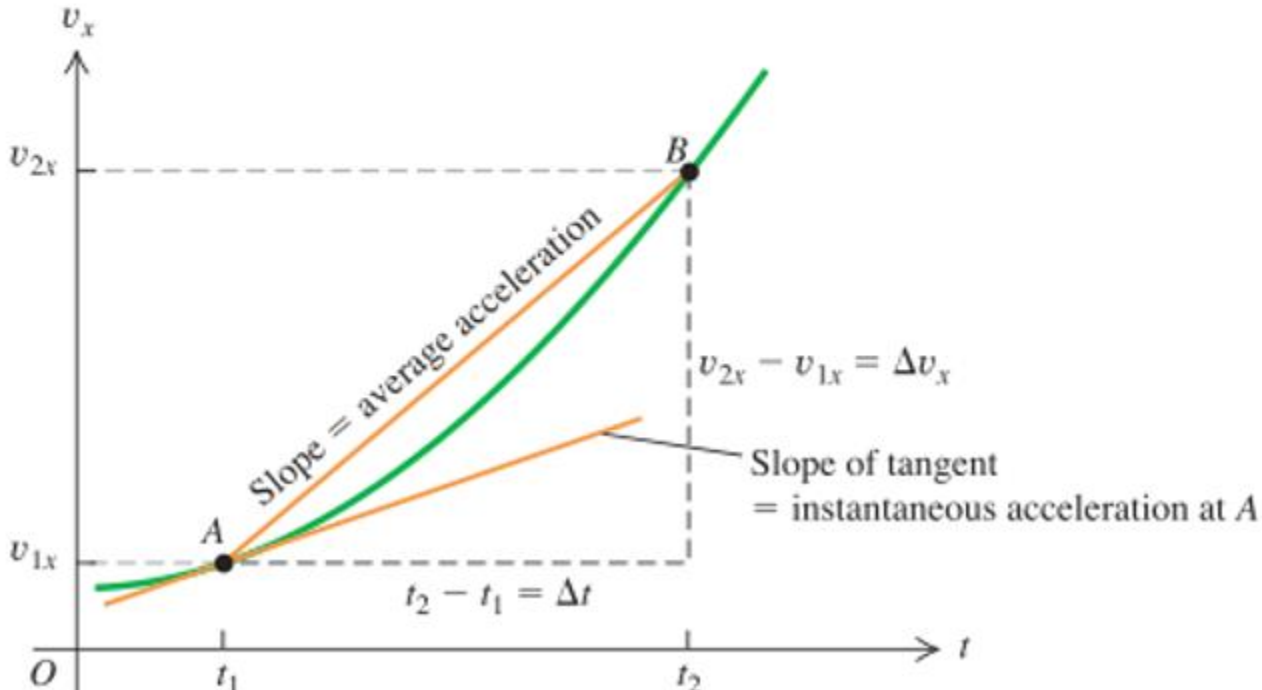
- Δy and Δx can be interchanged.
- For free fall (in the y direction) $a = -g = -9.8 m/s^2$
- For free fall the y component of velocity at the highest point is zero.
- I take the launch point as zero time and zero x and y coordinates.

P1 Ch 2

$$v_{av,x} = \frac{\Delta x}{\Delta t}$$



$$v = v_0 + at$$
$$\Delta x = \left(\frac{v_0 + v}{2}\right) t$$
$$\Delta x = v_0 t + \tfrac{1}{2} a t^2$$
$$v^2 = {v_0}^2 + 2a\Delta x$$
$$\Delta x = vt - \tfrac{1}{2} a t^2$$



Take launch point as zero

FREE FALL

$a = -9.8\ m/s^2$

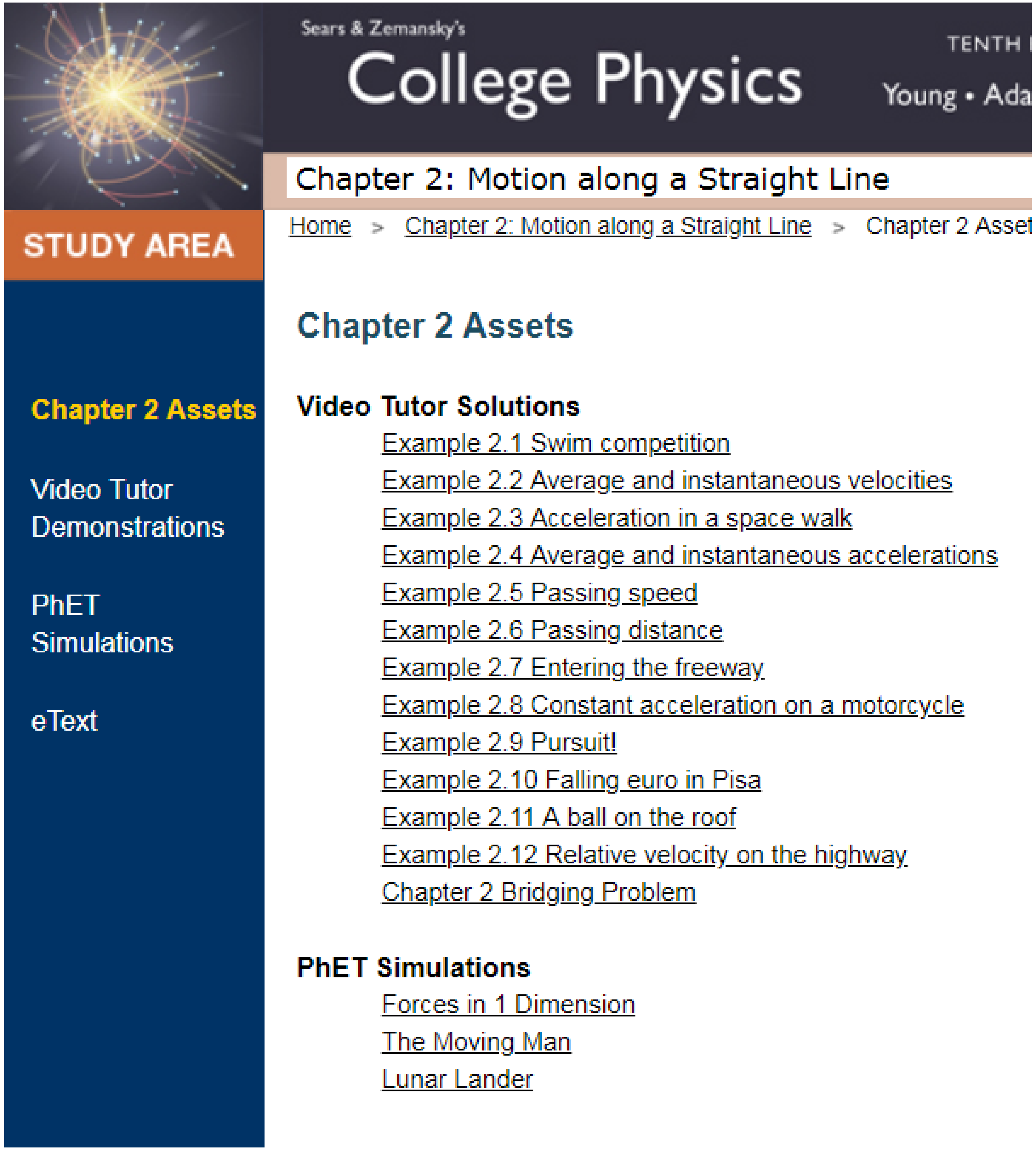
$v_{ymax} = 0$

Use the study area as we go through the slides

VTS Ex 2.1

Use the textbook for more details

PhET The moving man



Appendix
Extra information and questions

Using a Spreadsheet

Create an Excel sheet that

(a) Solves example 2.10 in your textbook

(b) Plots a velocity-time graph and a position-time graph for the same problem.

$$v = v_0 + at$$

$$\Delta x = \left(\frac{v_0+v}{2}\right)t$$

$$\Delta x = v_0 t + \tfrac{1}{2}at^2$$

$$v^2 = {v_0}^2 + 2a\Delta x$$

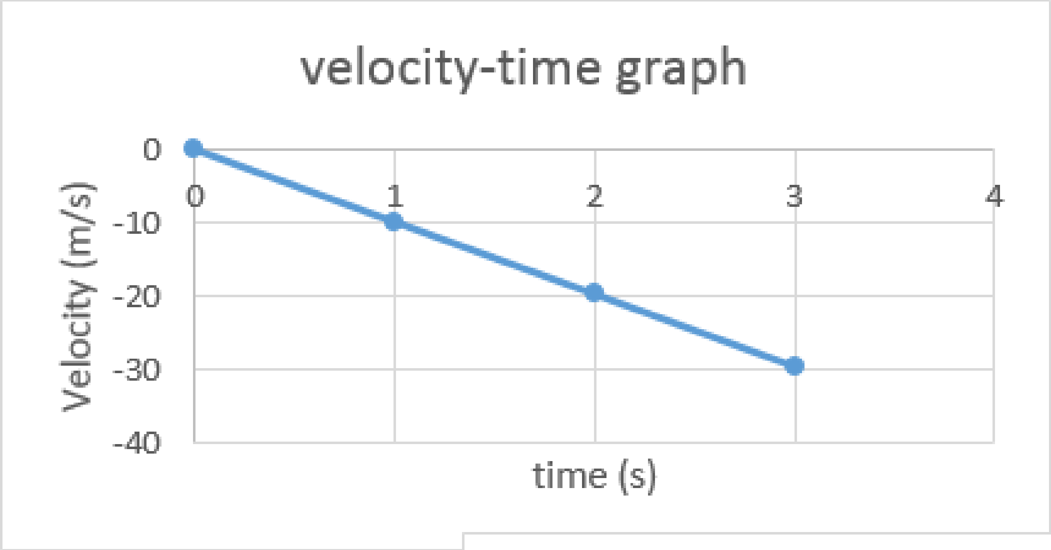
$$\Delta x = vt - \tfrac{1}{2}at^2$$

$$v = 0 - 9.8t$$

$$y = \frac{vt}{2}$$

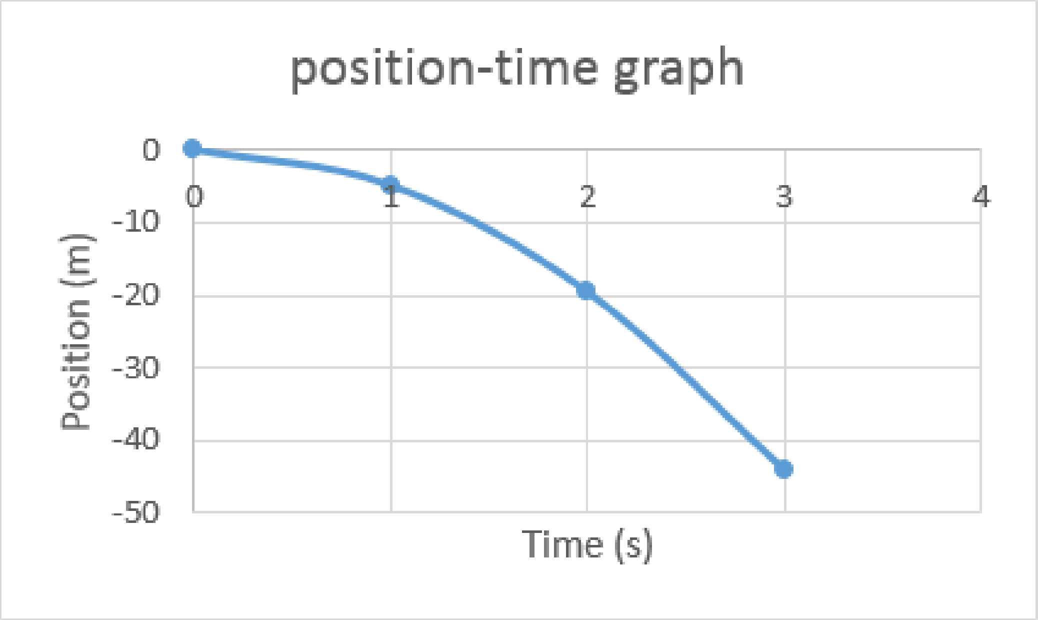
time	velocity
0	0
1	-9.8
2	-19.6
3	-29.4

time	position
0	0
1	-4.9
2	-19.6
3	-44.1



$$v = 0 - 9.8t$$

$$y = \frac{vt}{2}$$



Reading Assignment

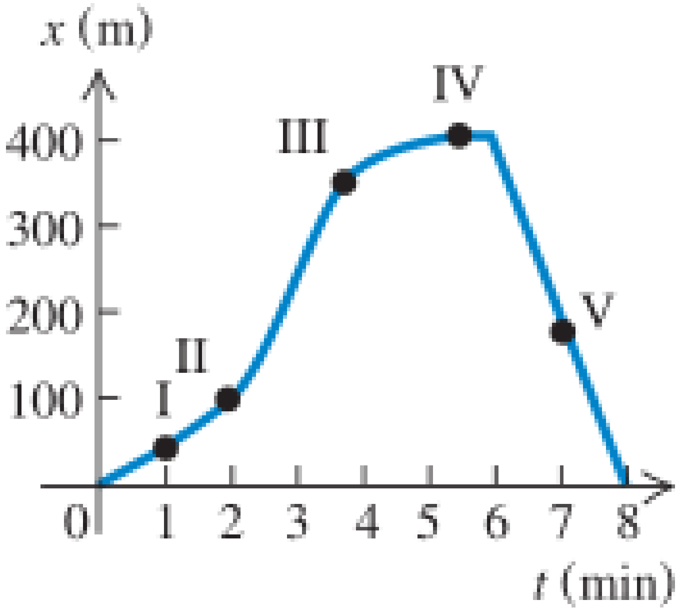
- Please read Section 2.7: Relative velocity along a straight line (don't worry about the *Theory of Relativity* portion at the end of the section)

VTS Ex 2.12

FOR INTEREST ONLY

?

19. ●● A physics professor leaves her house and walks along the sidewalk toward campus. After 5 min, she realizes that it is raining and returns home. The distance from her house as a function of time is shown in Figure 2.46. At which of the labeled points is her velocity (a) zero? (b) constant and positive? (c) constant and negative? (d) increasing in magnitude? and (e) decreasing in magnitude?



▲ **FIGURE 2.46** Problem 19.

***2.19. Set Up:** The instantaneous velocity is the slope of the tangent to the *x* versus *t* graph.
Solve: (a) The velocity is zero where the graph is horizontal; point IV.
(b) The velocity is constant and positive where the graph is a straight line with positive slope; point I.
(c) The velocity is constant and negative where the graph is a straight line with negative slope; point V.
(d) The slope is positive and increasing at point II.
(e) The slope is positive and decreasing at point III.