Mathematics I (Maths 101)

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Course Topics

1. **Basic algebraic operations**. Transposition. Rounding errors. Significant figures. Measurements. Scientific and engineering notation. Calculations. Conversion of numbers.

1. **Geometry**. Geometrical shapes and solids.

1. **Dimensional analysis**. Unit conversions, unit modifiers, and metric measurements such as conversion between metric and the U.S. customary system.

1. **Logarithms**. Laws of indices. Solving for the exponent. Solving for any variable, base e, and base 10. Exponential growth and decay.

1. **Quadratic equations**. Methods of solutions.

1. **Graphical solution of linear equations**.

Assessments

Weekly quizzes, weekly problem sets, midterm examination, and final examination.

Grading policy

Grading policy:

-	Weekly Quizzes	(20%)
-	Problem Sets	(20%)
-	Midterm Exam	(25%)
-	Final Exam	(35%)
Total		(100%)

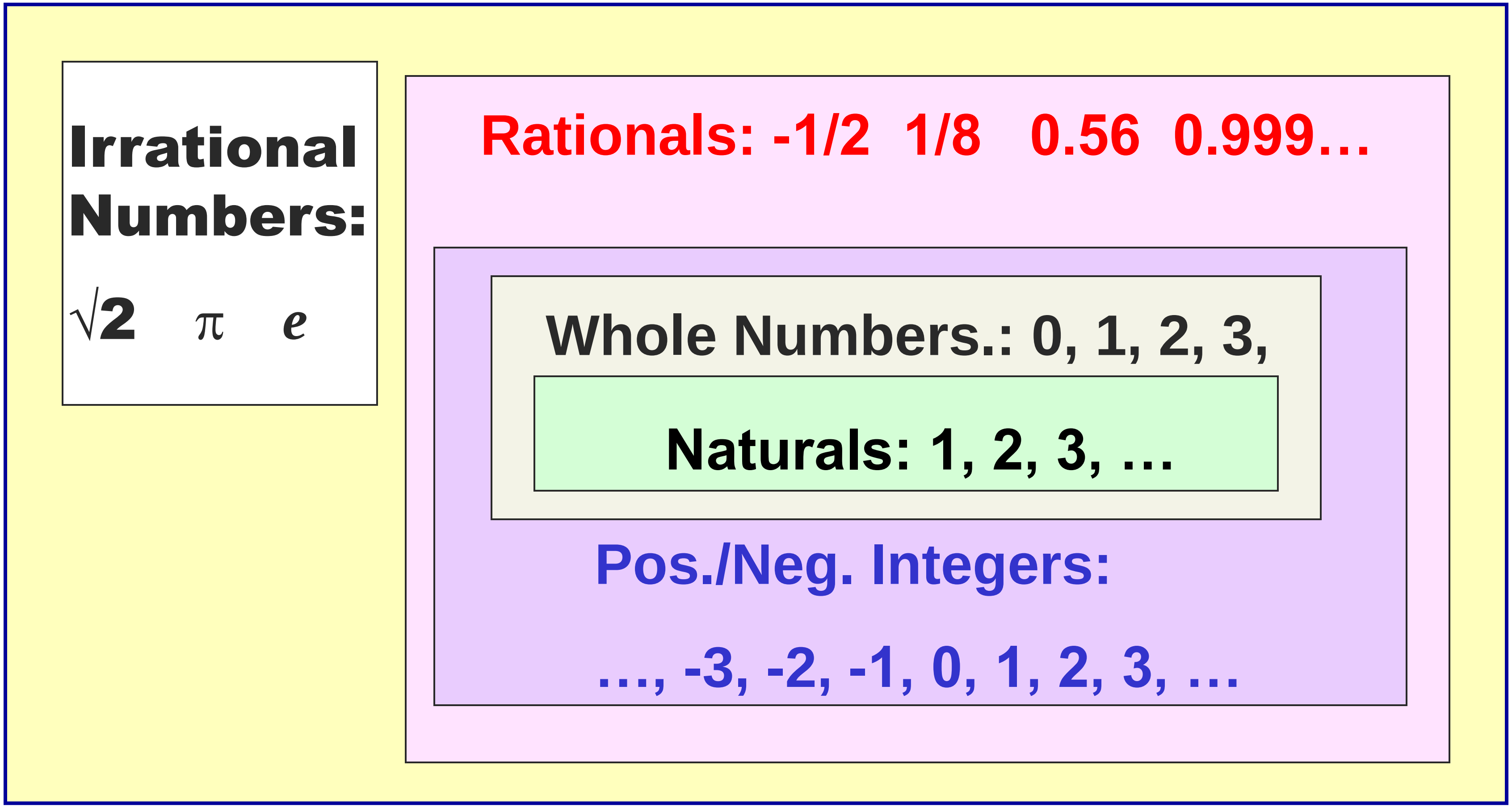
At the end of Week 1

- I know what the Real Number System is.
- I know commutative, associated and distributed laws are.
- I can give examples of exact and approximate numbers.
- I can quote values to a number of significant figures.
- I can solve problems with exponents.
- I can solve problems with roots and radicals
- I can define the terminology in algebraic expressions.

Ch. 1.1: Numbers

- The Real Number System includes:
  - Natural numbers,
  - Whole numbers,
  - Positive & negative integers,
  - Rational & irrational numbers.
- We also work with complex numbers.

The Real Number System



The Number Line

- Real numbers can be represented as points on a line.

Absolute Value

- The absolute value of a ***positive*** number is the number itself.
- The absolute value of a ***negative*** number is the corresponding positive number.
- It represents the ***distance*** from the number and zero on a number line.
- ***Example***: **|-3| = 3**

Signs of Inequality

- ***Greater than***:
- ***Example***: **5 > -1**

Signs of Inequality *(continued)*

- ***Less than***:
- ***Example***: **-3 < -1**

Example

Basic Technical maths(9)
Ex 1.1 q5-16

5. 3: integer, rational $\left(\frac{3}{1}\right)$, real

$\sqrt{-4}$: imaginary

$-\frac{\pi}{6}$: irrational, real

9. $6 < 8$

Answers

5. 3: integer, rational $\left(\frac{3}{1}\right)$, real

$\sqrt{-4}$: imaginary

$-\frac{\pi}{6}$: irrational, real

6. $-\sqrt{-6}$: imaginary

$-2.33 = -\frac{233}{100}$: rational, real

$\frac{\sqrt{7}}{3}$: irrational

7. $|3| = 3$, $|-4| = 4$, $\left|-\frac{\pi}{2}\right| = \frac{\pi}{2}$

8. $|-0.857| = 0.857$, $\left|\sqrt{2}\right| = \sqrt{2}$, $\left|-\frac{19}{4}\right| = \frac{19}{4}$

9. $6 < 8$

10. $7 > 5$

11. $\pi > -3.2$, $\pi = 3.14$ is greater than -3.2.

12. $-4 < 0$

13. $-|-3| = -3 \Rightarrow -4 < -3 = -|-3|$

14. $-\sqrt{2} > -1.42$

15. $-\frac{1}{3} > -\frac{1}{2}$

16. $-0.6 < 0.2$

Reciprocals

- Every number, except zero, has a reciprocal.
- The reciprocal of a number is **1** divided by the number.
- ***Example***:
- The reciprocal of **12** is $\frac{1}{12}$

a unit fraction

Denominate Numbers

- Numbers which represent a measurement and are written with units.
- ***Example***:
- A temperature of **25** degrees Celsius

25°C

Literal Numbers

- We use literal numbers to represent the wording of numbers.
- ***Example***:
- Five times a ***number***, where ***x*** is that number

$$5x$$

Example

Ex 1.1 q 17 and 18

17. The reciprocal of $3 = \frac{1}{3}$. The reciprocal of

$-\frac{4}{\sqrt{3}}$ is $\frac{1}{-\frac{4}{\sqrt{3}}} = -\frac{\sqrt{3}}{4}$.

The reciprocal of $\frac{y}{b}$ is $\frac{1}{\frac{y}{b}} = \frac{b}{y}$.

18. The reciprocal of $-\frac{1}{3}$ is $\frac{1}{-\frac{1}{3}} = -3$.

The reciprocal of 0.25 is $\frac{1}{0.25} = 4$.

The reciprocal of x is $\frac{1}{x}$.

Ch. 1.2: Fundamental Operations of Algebra

- The commutative law of addition:
  - ***Definition***:
  - $a + b = b + a$

- The associative law of addition:
  - ***Definition***:
  - $(a + b) + c = a + (b + c)$

Fundamental Operations of Algebra *(continued)*

- The commutative law of multiplication:
  - ***Definition***:
  - $a \times b = b \times a$

- The associative law of multiplication:
  - ***Definition***:
  - $(a \times b) \times c = a \times (b \times c)$

Fundamental Operations of Algebra *(continued)*

- The distributive law of multiplication over addition:
  - ***Definition***:
  - $a\,(b + c) = ab + ac$

Operations on Positive & Negative Numbers

- Addition of 2 numbers of the same sign:
  - Add their absolute values & assign the sum their common sign.
- Addition of 2 numbers of different signs:
  - Subtract the number of smaller absolute value from the number of larger absolute value & assign to the result the sign of the number of larger absolute value.

Operations on Positive & Negative Numbers *(continued)*

- Subtraction of 1 number from another:
  - Change the sign of the number being subtracted & change the subtraction to addition.
  - Perform the addition.

Operations on Positive & Negative Numbers *(continued)*

- Multiplication & division of 2 numbers:
  - The product (or quotient) of 2 numbers of the same sign is ***positive***.
  - The product (or quotient) of 2 numbers of different signs is ***negative***.

Order of Operations

1. Operations within specific groupings are done first.
2. Perform multiplications and divisions (from left to right).
3. Then perform additions and subtractions (from left to right).

Operations with Zero

- Working with addition & subtraction:
  - $a + 0 = a$
  - $a - 0 = a$; $0 - a = -a$
- Working with multiplication & division:
  - $a \times 0 = 0$
  - $0 \div a = 0$; (if $a \neq 0$)

Division by zero is undefined.

Example

Ex 1.2 q 5-10 and 33 to 36

5. $8+(-4)=8-4=4$

33.
$$\begin{aligned}-7-\frac{|-14|}{2(2-3)}-3|6-8| &= -7-\frac{14}{2(-1)}-3|-2| \\ &= -7-\frac{14}{-2}-3(2) \\ &= -7-(-7)-6 \\ &= 0-6 \\ &= -6\end{aligned}$$

Answers

5. $8+(-4)=8-4=4$

6. $-4+(-7)=-11$

7. $-3+9=-(3-9)=-(-6)=6$

8. $18-21=-3$

9. $-19-(-16)=-19+16=-3$

10. $8-(-4)=8+4=12$

33. $$\begin{aligned}-7-\frac{|-14|}{2(2-3)}-3|6-8| &= -7-\frac{14}{2(-1)}-3|-2|\\ &= -7-\frac{14}{-2}-3(2)\\ &= -7-(-7)-6\\ &= 0-6\\ &= -6\end{aligned}$$

34. $$\begin{aligned}-7(-3)+\frac{6}{-3}-(-9) &= 21+(-2)+9\\ &= 19+9=28\end{aligned}$$

35. $$\begin{aligned}\frac{3(-9)-2(-3)}{3-10} &= \frac{-(3\times 9)+(2\times 3)}{-7}\\ &= \frac{-27+6}{-7}\\ &= +\left(\frac{27-6}{7}\right)\\ &= \frac{21}{7}=3\end{aligned}$$

36. $$\begin{aligned}\frac{20(-12)-40(-15)}{98-|-98|} &= \frac{-240-(-600)}{98-98}\\ &= \frac{360}{0} \text{ is undefined}\end{aligned}$$

Ch. 1.3: Calculators & Approximate Numbers

- You will want to use one of two general types of calculators:
  - A scientific calculator
  - A graphing calculator
- Be sure to keep the manual for reference.
- Otherwise, check for one on the internet.

Approximate & Exact Numbers

- The final result of a calculation should not be written with any more accuracy than is proper.
- We use:
  - ***Approximate numbers***
  - ***Exact numbers***

Approximate Numbers

- ***Approximate numbers*** are determined by some measurement.
  - ***Example***: a shaft is approximately **1.75 *m*** in diameter.
- Many fractions are approximate.
  - ***Example***: **2/3 = 0.6667**
- Irrational numbers are approximate.
  - ***Example***: π = **3.1415927…**

Exact Numbers

- ***Exact numbers*** are determined by definition or by counting.
  - ***Example***: There are **24** hours in a day, no more no less.
  - ***Example***: A car has exactly **4** wheels.
- On the other hand, a certain town has population of approximately **3500** people***.***

Significant Digits

- Zeros are used in approximate numbers to properly locate the decimal point.
- Except for these zeros, all other digits are called ***significant digits***.

Significant Digits

- For example, the following numbers have **4** significant digits.

497.3

39.05

8003

2.008

Notice how the zeros are surrounded by natural numbers.

Approximate & Exact Numbers

- When adding or subtracting approximate numbers, keep as many decimal places in your answer as contained in the number having the fewest decimal places.
- When multiplying 2 or more approximate numbers, round the result to as many digits as are in the factor having the fewest significant digits.

Accuracy & Precision

- Accuracy:
  - the number of significant digits a number has.
- Precision:
  - the decimal position of the last significant digit.

Example

- A measurement of **1.125** is more precise than a measurement of **1.13**.
- It (**1.125**) is also considered more precise since it is more accurate to **3** significant digits and **1.13** has **2** significant digits.

Accuracy:

the number of significant digits a number has.

Precision:

the decimal position of the last significant digit.

When **adding or subtracting approximate** numbers, keep as many decimal places in your answer as contained in the number having the **fewest decimal places.**

When **multiplying** 2 or more approximate numbers, round the result to as many digits as are in the factor having the **fewest significant digits.**

THIS IS FOR **APPROXIMATE** NUMBERS.

Eg

2041.2 has 5 significant figures and 1 decimal place

0.005 has 1 significant figure an 3 decimal places

So to add them

2041.2 + 0.005 = 2041.205 BUT the fewest decimal places is 1 (2041.2) so our answer is quoted to 1 decimal place = **2041.2**

Multiply them

2041.2 * 0.005 = 10.206 BUT 0.005 has only **1** significant digit so the answer is = **10**

(Highest = 2041.244 * 0.00544 = 11.104

Lowest = 2041.150 * 0.00450 = 9.185)

Rounding

- To round off a number to a specified number of significant digits, discard all digits to the right of the last significant digit (replace them with zeros if needed to properly place the decimal point).
  - If the first digit discarded is 5 or more, increase the last significant digit by 1 (***round up***).
  - If the first digit discarded is less than 5, do not change the last significant digit (***round down***).

Operations with Approximate Numbers

1. When approximate number are added or subtracted, the result is expressed with the precision of the least precise number.
2. When approximate numbers are multiplied or divided, the result is express with the accuracy of the least accurate number.

Operations with Approximate Numbers

3. When the root of an approximate number is found, the result is expressed with the accuracy of the number.
4. When approximate numbers and exact numbers are involved, the accuracy of the result is limited only by the approximate numbers.

Example

Ex 1.3 q 9-20

9. 107 has 3 significant digits. 3004 has 4 significant digits.

9. 107 has 3 significant digits. 3004 has 4 significant digits.

10. 3600 has 2 significant digits
730 has 2 significant digits

11. 6.80 has 3 significant digits; the zero indicates precision.
6.08 has 3 significant digits; the zero is not used for decimal location, and is not a place-holder only.

12. 0.8735 has 4 significant digits
0.0075 has 2 significant digits

13. 3000 has 1 significant digit. 3000.1 has 5 significant digits.

14. 1.00 has 3 significant digits
0.01 has 1 significant digit

15. (a) 0.01 is more precise (more decimal places).
(b) 30.8 is more accurate (more significant digits).

16. (a) 0.041 and 7.673 have the same precision
(b) 7.673 is more accurate than 0.041

17. (a) Both numbers have the same precision with digits in the tenths place.
(b) 78.0 with 3 significant digits is more accuate than 0.1 with 1 significant digit.

18. (a) 0.004 is more precise than 7040
(b) 7040 is more accurate than 0.004

19. (a) 0.004 is more precise (more decimal places).
(b) Both have the same accuracy

20. (a) 50.060 and 8.914 have the same precision
(b) 50.060 is more accurate than 8.914

Ch. 1.4: Exponents

- We use exponents to demonstrate when a number is multiplied by itself $\boldsymbol{n}$ times.

base $\longrightarrow$ $\boldsymbol{a^n}$ $\longleftarrow$ exponent

- Only exponents of the same base may be combined.

Laws of Exponents

- Product Law: $a^m \times a^n = a^{m+n}$

- Quotient Law: $\dfrac{a^m}{a^n} = a^{m-n}$ $\quad$ $\boldsymbol{m > n,\ a \neq 0}$

$$\frac{a^m}{a^n} = \frac{1}{a^{n-m}} \qquad \boldsymbol{m < n,\ a \neq 0}$$

Laws of Exponents (*continued*)

- Power Law:

$$\boxed{(a^m)^n = a^{mn}}$$

$$\boxed{(ab)^n = a^n b^n}$$

$$\boxed{\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}} \quad \boldsymbol{b \neq 0}$$

Zero & Negative Exponents

- Any number or variable raised to a zero exponent **0** is equal to **1**.
  - That is: $a^0 = 1$, $a \neq 0$.
- A negative exponent is defined by:

$$\boxed{a^{-n} = \frac{1}{a^n}}$$

Order of Operations

1. Operations within specific groupings
2. Powers
3. Multiplications and divisions (from left to right)
4. Additions and divisions (from left to right)

Evaluating Algebraic Expressions

- An algebraic expression is evaluated by substituting given values of the literal numbers in the expression and calculating the result.

Example

- Evaluate the following algebraic expression when $x = -1$.

$$5x^3 + 7x^2 - 2x + 1$$

$$= 5(-1)^3 + 7(-1)^2 - 2(-1) + 1$$

$$= -5 + 7 + 2 + 1$$

$$= 5$$

Example

Ex 1.4 q 5 -12

5. $x^3 \cdot x^4 = x^{3+4} = x^7$

5. $x^3 \cdot x^4 = x^{3+4} = x^7$

6. $y^2 y^7 = y^{2+7} = y^9$

7. $2b^4 b^2 = 2b^{4+2} = 2b^6$

8. $3k^5(k) = 3k^{5+1} = 3k^6$

9. $\dfrac{m^5}{m^3} = m^{5-3} = m^2$

10. $\dfrac{x^6}{x} = 2x^{6-1} = 2x^5$

11. $\dfrac{n^5}{7n^9} = \dfrac{n^{5-9}}{7} = \dfrac{n^{-4}}{7} = \dfrac{1}{7n^4}$

12. $\dfrac{3s}{s^4} = 3s^{1-4} = 3s^{-3} = \dfrac{3}{s^3}$

Ch. 1.5: Scientific Notation

- How is a very large or a very small number expressed?
- We use express the number in ***scientific notation***.
- We use $P \times 10^k$
- The exponent of **10** tells us how many decimal places are in the number.

Examples

1. $6.873 \times 10^{11} = 687\ 300\ 000\ 000$
2. $5.67 \times 10^{-6} = .000\ 005\ 67$

- Check to see how you can use your calculator to express numbers in scientific notation and perform multiplication & division with numbers that are expressed in scientific notation.

Example

Ex 1.5 q 29 to 36

29. $1280(865{,}000)(43.8) = 4.85 \times 10^{10}$

29. $1280(865{,}000)(43.8) = 4.85\times10^{10}$

30. $0.0000569(3{,}190{,}000) = 1.82\times10^{2}$

31. $$\frac{0.0732(6710)}{0.00134(0.0231)} = \frac{7.32\times10^{-2}\times6.71\times10^{3}}{1.34\times10^{-3}\times2.31\times10^{-2}}$$

$$= 1.59\times10^{7}$$

32. $$\frac{0.00452}{2430(97{,}100)} = 1.92\times10^{-11}$$

33. $(3.642\times10^{-8})(2.736\times10^{5}) = 9.965\times10^{-3}$

34. $$\frac{(7.309\times10^{-1})^{2}}{5.9843(2.5036\times10^{-20})} = 3.566\times10^{18}$$

35. $$\frac{(3.69\times10^{-7})(4.61\times10^{21})}{0.0504} = 3.40\times10^{16}$$

36. $$\frac{(9.907\times10^{7})(1.08\times10^{12})^{2}}{(3.603\times10^{-5})(2054)} = 1.56\times10^{33}$$

Ch. 1.6: Roots & Radicals

- The **square** root of a number x is one of **two** equal factors whose product is x.

$$\sqrt{144} = \sqrt{\mathbf{2} \times \mathbf{2} \times \mathbf{2} \times \mathbf{2} \times \mathbf{3} \times \mathbf{3}}$$

$$= \mathbf{2} \times \mathbf{2} \times \mathbf{3} = \mathbf{12}$$

Roots & Radicals

- The **cube** root of a number x is one of **three** equal factors whose product is x.

$$\sqrt[3]{216} = \sqrt[3]{\mathbf{2} \times \mathbf{2} \times \mathbf{2} \times \mathbf{3} \times \mathbf{3} \times \mathbf{3}}$$

$$= \mathbf{2} \times \mathbf{3} = \mathbf{6}$$

We usually use prime numbers to give us our simplest answer.

A **prime number** (or a prime) is a natural number greater than 1 that has no positive divisors other than 1 and itself.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

n Roots

- The n^{th} root of a number a is one of n equal factors whose product is a. This is denoted by:

$$\sqrt[n]{a}$$

radicand

index

radical sign

Properties of Roots & Radicals

1. We define the principal n^{th} root of a to be positive if a is positive & to be negative if a is negative and n is odd.
2. The square root of a product of positive numbers is the product of their square roots.

$$\boxed{\sqrt{ab} = \sqrt{a}\sqrt{b}}$$

Example

Ex 1.6 q 17 to 24

17. $\left(\sqrt{5}\right)^2 = 5$

16. $\sqrt[5]{-32} = -2$

17. $\left(\sqrt{5}\right)^2 = 5$

18. $\left(\sqrt[3]{31}\right)^3 = \sqrt[3]{31}\sqrt[3]{31}\sqrt[3]{31} = 31$

19. $\left(-\sqrt[3]{-47}\right)^3 = (-1)^3\left(\sqrt[3]{-47}\right)^3 = -1(-47) = 47$

20. $\left(\sqrt[5]{-23}\right)^5 = -23$

21. $\left(-\sqrt[4]{53}\right)^4 = 53$

22. $-\sqrt{32} = -\sqrt{16\cdot 2} = -4\sqrt{2}$

23. $\sqrt{1200} = \sqrt{400(3)} = 20\sqrt{3}$

24. $\sqrt{50} = \sqrt{25\cdot 2} = 5\sqrt{2}$

Ch. 1.7: Addition & Subtraction of Algebraic Expressions

- We can work with algebraic expressions as we would with any real numbers.
- Be sure to follow the order of operations.
- Collect like terms watching for any negative terms.

Terminology used with Algebraic Expressions

- Monomial:
  - an algebraic expression containing only one term.
  - ***Examples***: $3\boldsymbol{x}^5$, 7, $-15\boldsymbol{x}^2$
- Binomial:
  - an algebraic expression containing two terms.
  - ***Examples*** : $3\boldsymbol{x}^5 - 4\boldsymbol{x}$, $2\boldsymbol{x} + 7$, $-15\boldsymbol{x}^2 - 20$

Terminology used with Algebraic Expressions (*continued*)

- Trinomial:
  - an algebraic expression containing three terms.
  - ***Examples***: $3\boldsymbol{x}^5 - 4\boldsymbol{x} + 1$, $\boldsymbol{x}^3 - 2\boldsymbol{x} + 7$
- Multinomial:
  - Any expression containing two or more terms.
  - ***Examples*** : $3\boldsymbol{x}^5 - 8\boldsymbol{x}^2 + 4\boldsymbol{x} + 1$

In mathematics, a **coefficient** is a multiplicative factor in some term of a polynomial, a series or any expression; it is usually a number, but in any case does not involve anyvariables of the expression. For instance in

$$7x^2 - 3xy + 1.5 + y$$

the first two terms respectively have the coefficients 7 and −3. The third term 1.5 is a constant. The final term does not have any explicitly written coefficient, but is considered to have coefficient 1,

Terminology used with Algebraic Expressions (*continued*)

- Coefficient:
  - The numbers & literal symbols multiplying any given factor in an algebraic expression.
- Numerical coefficient:
  - The product of all the numbers in explicit form.
- Similar or like terms:
  - All terms that differ at most in their numerical coefficients.

Addition & Subtraction of Algebraic Expressions

- In adding and subtracting algebraic expressions, we combine similar (or like) terms into a single term.
- The final simplified expression will contain only terms that are not similar.

Example

- Simplify by collecting like terms:

$$(-6x^4 + 3x^2 + 6) - (2x^4 + 5x^3 - 5x^2 + 7)$$

$$= -6x^4 + 3x^2 + 6 - 2x^4 - 5x^3 + 5x^2 - 7$$

$$= (-6x^4 - 2x^4) - 5x^3 + (3x^2 + 5x^2) + (6 - 7)$$

$$= -8x^4 - 5x^3 + 8x^2 - 1$$

Notice how
each term in
algebraic ex
rever

Example

Ex 1.7 q 5 to 12

5. $5x + 7x - 4x = 12x - 4x = 8x$

5. $5x+7x-4x=12x-4x=8x$

6. $6t-3t-4t=-t$

7. $2y-y+4x=y(2-1)+4x=y+4x$

8. $4C+L-6C=-2C+L$

9. $2F-2T-2+3F-T=5F-3T-2$

10. $x-2y+3x-y+z=4x-3y+z$

11. $a^2b-a^2b^2-2a^2b=a^2b-2a^2b-a^2b^2=-a^2b-a^2b^2$

12. $xy^2-3x^2y^2+2xy^2=3xy^2-3x^2y^2$

Ch. 1.8: Multiplication of Algebraic Expressions

- To find the products of two or more monomials, we use the laws of exponents & the laws for multiplying signed numbers.
  1. Multiply the numerical coefficients.
  2. Multiply the literal numbers.
  3. Combine any exponents when bases are the same.

Example

- Multiply:

$$5x^3(9x^2 - 7x)$$

1. Multiply the numerical coefficients.

2. Multiply the literal numbers.

$$= 45x^5 - 35x^4$$

3. Combine any exponents when bases are the same.

Multiplication of Algebraic Expressions

- When working with multinomials, be sure to use the distributive property over each term.
- ***Example***:

$$(x-3)^2 = (x-3)(x-3)$$

$$= x^2 - 3x - 3x + 9$$

$$\boxed{= x^2 - 6x + 9}$$

Ex 1.8 q 19-22, 55 and 56

19. $$\begin{aligned} ax\left(cx^2\right)\left(x+y^3\right) &= ax\left(cx^2\left(x\right)+cx^2\left(y^3\right)\right) \\ &= ax\left(cx^3\right)+ax\left(cx^2y^3\right) \\ &= acx^4+acx^3y^3 \end{aligned}$$

20. $-2\left(-3st^3\right)(3s-4t) = 6st^3(3s-4t) = 18s^2t^3 - 24st^4$

21. $(x-3)(x+5) = x^2 + 5x - 3x - 15 = x^2 + 2x - 15$

22. $(a+7)(a+1) = a^2 + 8a + 7$

55. $$\begin{aligned}(x+y)^3 &= (x+y)(x+y)(x+y) \\ &= \left(x^2 + 2xy + y^2\right)(x+y) \\ &= x^3 + x^2y + 2x^2y + 2xy^2 + xy^2 + y^3 \\ &= x^3 + 3x^2y + 3xy^2 + y^3 \neq x^3 + y^3\end{aligned}$$

56. $$\begin{aligned}&(x+y)\left(x^2 - xy + y^2\right) \\ &= x^3 - x^2y + xy^2 + x^2y - xy^2 + y^3 \\ &= x^3 + y^3\end{aligned}$$

Ch. 1.9: Division of Algebraic Expressions

- When dividing algebraic expressions once again use the laws of exponents and the laws for dividing signed numbers.
- Combine the exponents if the bases are the same.

Division of Algebraic Expressions

- The quotient of a multinomial divided by a monomial is found by dividing each term of the multinomial by the monomial and adding the results.
- That is: $\boxed{\dfrac{a+b}{c} = \dfrac{a}{c} + \dfrac{b}{c}}$

Polynomials in x

- If each term in an algebraic sum is a number or is of the form ax^n, where n is a nonnegative integer.
- ***Degree of the polynomial***:
  - The greatest value of the exponent n that appears.

Division of One Polynomial by Another

This is long division using polynomials

First watch a video on long division

http://www.youtube.com/watch?v=FXgV9ySNusc

Then we will look at a procedure.

Division of One Polynomial by Another

- Arrange the dividend & the divisor in descending powers of the variable.
- We divide similarly as we would with long division.

Example

- We are asked to solve

$$\left(6x^2 - 13x + 7\right) \div \left(x + 1\right)$$

- Note that each polynomial is arranged in descending order of powers.

Solution

$$
\begin{array}{r|l}
 & 6x \quad -19 \\
\hline
x+1 & 6x^2 - 13x + 7 \\
 & \underline{6x^2 + 6x} \\
 & \qquad -19x + 7 \\
 & \qquad \underline{-19x - 19} \\
\end{array}
$$

remainder $\longrightarrow 26$

Final Answer

- Therefore, the quotient for:

$$\left(6x^2 - 13x + 7\right) \div \left(x + 1\right)$$

$$= 6x - 19 + \boxed{\frac{26}{x+1}}$$ remainder

quotient.

divisor.

$$\begin{array}{r|l} \multicolumn{1}{r}{} & \quad 6x - 19 \\ \cline{2-2} x + 1 & 6x^2 - 13x + 7 \\ & \underline{6x^2 + 6x} \\ & \qquad -19x + 7 \\ & \qquad \underline{-19x - 19} \\ & \qquad\qquad 26 \end{array}$$

dividend

remainder

1. Arrange the dividend & the divisor in descending powers of the variable. We divide similarly as we would with long division.
2. Divide the first term of the dividend by the first term of the divisor. This gives us our first term in the quotient.
3. Multiply the entire divisor by the first term of the quotient. Subtract this product from the dividend.
4. Divide the first term of the difference by the first term of the divisor. This gives us the second term.
5. Multiply the entire divisor by this term of the quotient. Subtract this product from the difference.
6. This gives the remainder (repeat if needed)

Ex 1.9 questions 7,8,9 and 32

7. $$\frac{-16r^3t^5}{-4r^5t} = \frac{4t^{5-1}}{r^{5-3}} = \frac{4t^4}{r^2}$$

7. $\dfrac{-16r^3t^5}{-4r^5t} = \dfrac{4t^{5-1}}{r^{5-3}} = \dfrac{4t^4}{r^2}$

8. $\dfrac{51mn^5}{17m^2n^2} = \dfrac{3n^3}{m}$

9. $\dfrac{(15x^2)(4bx)(2y)}{30bxy} = 4x^2$

32.
$$
\begin{array}{r}
2x+1 \\
3x-4\,\overline{)\,6x^2-5x-9} \\
\underline{6x^2-8x} \\
3x-9 \\
\underline{3x-4} \\
-5
\end{array}
$$

$$(6x^2-5x-9)\div(3x-4) = 2x+1+\frac{-5}{3x-4}$$

Ch. 1.10: Solving Equations

- An equation is an algebraic statement that two algebraic expressions are equal.
- Any value of the literal numbers representing the ***unknown*** that produces equality when ***substituted*** in the equation is said to ***satisfy*** the equation.
- An equation valid only for certain values of the unknown is a ***conditional equation***.

Solving Equations

- To solve an equation we find the values of the unknown that satisfy it.
- ***Key Rule*** when solving equations.

Perform the same operation on both sides of the equation.

Procedure for Solving Equations

1. Remove grouping symbols (distributive law).
2. Combine any like terms of each side (also after *step 3*).
3. Perform the same operations on both sides, until ***x = result*** is obtained.
4. Check the solution in the original equation.

Example

- Solve this linear equation:

$$3(x-4)-6(1-3x)=20$$

$$3x-12-6+18x=20$$

$$21x=38$$

$$x=\frac{38}{21}\cong 1.81$$

1. Remove grouping symbols (distributive law).
2. Combine any like terms of each side (also after *step 3*).
3. Perform the same operations on both sides, until ***x = result*** is obtained.

Check

- Solve this linear equation:

$$3\left(\boxed{\frac{38}{21}}-4\right)-6\left(1-3\times\boxed{\frac{38}{21}}\right)\overset{?}{=}20$$

$$20 = 20$$

Ex 1.10 questions 29-32, 50

29. $$\frac{4x-2(x-4)}{3}=8$$

$$4x-2(x-4)=24$$

$$4x-2x+8=24$$

$$2x=16$$

$$x=8$$

29. $\frac{4x-2(x-4)}{3}=8$

$$4x-2(x-4)=24$$
$$4x-2x+8=24$$
$$2x=16$$
$$x=8$$

30. $2x=\frac{3-5(7-3x)}{4}$

$$8x=3-5(7-3x)$$
$$8x=3-35+15x$$
$$-7x=-32$$
$$x=\frac{32}{7}$$

31. $|x|-1=8$

$$|x|=9$$
$$x=-9 \text{ or } x=9$$

32. $2-|x|=4$

$$-|x|=2$$
$$|x|=-2, \text{ no solution}$$

50. $210(3x)=55.3x+38.5(8.25-3x)$

$$630x=55.3x+317.625-115.5x$$
$$690.2x=317.625$$
$$x=0.46 \text{ m}$$

Ch. 1.11: Formulas & Literal Equations

- A ***formula*** is an equation that expresses the relationship between two or more related quantities.
- We can isolate the desired symbol by using algebraic operations on the literal numbers.
- When expected to substitute in a given value into the formula, we should first isolate the given variable.

Example

- In the given formula, isolate for e.

$$C = \frac{2eAk_1k_2}{d(k_1 + k_2)}$$

- ***Solution***:

1. We multiply both sides by $d(k_1 + k_2)$.
2. Divide both sides by $2Ak_1k_2$.

Solution

1. We multiply both sides by $\boldsymbol{d(k_1 + k_2)}$
2. Divide both sides by $\boldsymbol{2Ak_1k_2}$.

$$C = \frac{2eAk_1k_2}{d(k_1 + k_2)} \Rightarrow Cd(k_1 + k_2) = 2eAk_1k_2$$

$$\boxed{\frac{Cd(k_1 + k_2)}{2Ak_1k_2} = e}$$

Ex 1.11 questions 5-8, 35

5. $E = IR$

$$\frac{E}{I} = \frac{IR}{I}$$

$$R = \frac{E}{I}$$

5. $E = IR$

$$\frac{E}{I} = \frac{IR}{I}$$

$$R = \frac{E}{I}$$

6. $PV = nRT$

$$T = \frac{PV}{nR}$$

35. $P = \dfrac{V_1(V_2 - V_1)}{gJ}$

$$gJP = V_1V_2 - V_1^2$$

$$gJP + V_1^2 = V_1V_2$$

$$V_2 = \frac{gJP + V_1^2}{V_1}$$

7. $rL = g_2 - g_1$

$$g_1 = g_2 - rL$$

8. $W = S_dT - Q$

$$Q = S_dT - W$$

Ch. 1.12: Applied Word Problems

- Mathematical questions in science, biology, etc. do not present themselves in neat, tidy equations.
- They are often presented as word problems which must be solved.
- The following is a step-by-step approach that you can use to solve word problems.

Procedure for Solving Word Problems

1. Read the statement of the problem. First, read it quickly for a general overview. Then read it slowly & carefully, listing the information given.
2. Clearly identify the unknown quantities & then assign an appropriate letter to represent one of them, stating this choice clearly.
3. Specify the other unknown quantities in terms of the one in step 2.

Procedure for Solving Word Problems (*continued*)

4. If possible, make a sketch using the known & unknown quantities.
5. Analyze the statement of the problem & write the necessary equation.
6. Solve the equation, clearly stating the solution.
7. Check the solution with the original statement of the problem.

Example

- The sum of 3 electric currents that come together at a point in an integrated circuit is zero. If the second current is double the first & the third current is 9.2 μA more than the first, what are the currents?
- (The sign of a current indicates the direction of flow.)

Solution

1. Read the statement of the problem. First, read it quickly for a general overview. Then read it slowly & carefully, listing the information given.

- ***Given information***:
  - ✓ 3 separate electric currents.
  - ✓ Their sum total is zero.
  - ✓ The currents can be positive or negative.

Solution *(continued)*

2. Clearly identify the unknown quantities & then assign an appropriate letter to represent one of them, stating this choice clearly.

- ***Unknown quantities***:
  - ✓ First current: x
  - ✓ Each current listed is in terms of the first electric current.

Solution *(continued)*

3. Specify the other unknown quantities in terms of the one in step 2.

- ***Unknown quantities***:
  - ✓ First current: x
  - ✓ Second current: $2x$
  - ✓ Third current: $x + 9.2$

Solution *(continued)*

4. If possible, make a sketch using the known & unknown quantities.

Solution *(continued)*

5. Analyze the statement of the problem & write the necessary equation.

- ***The equation***:
  - ✓ $\boldsymbol{x} + 2\boldsymbol{x} + \boldsymbol{(x} + 9.2\boldsymbol{)} = \mathbf{0}$

Solution *(continued)*

6. Solve the equation, clearly stating the solution.

- ***The equation***:

$$x + 2x + (x + 9.2) = 0$$

$$x = -2.3$$

- ***Therefore,***
  - ✓ First current: $x = -2.3$
  - ✓ Second current: $2(-2.3) = -4.6$
  - ✓ Third current: $(-2.3 + 9.2) = 6.9$

Solution *(continued)*

7. Check the solution with the original statement of the problem.

Ex 1.12 questions 5-6, 18

5.

$$x = \text{cost 6 years ago}$$
$$x + 5000 = \text{cost today}$$
$$x + (x + 5000) = 49{,}000$$
$$2x + 5000 = 49{,}000$$
$$2x = 44{,}000$$
$$x = 22{,}000$$
$$x + 5000 = 27{,}000$$

\$22,000 six years ago   \$27,000 is the cost today

6. Let x = flow rate of first stream in ft^3/s;

$x + 1700$ = flow rate of second stream in ft^3/s

$$(x + x + 1700) \cdot 3600 = 1.98 \times 10^7$$

$$x = 1900 \text{ ft}^3/\text{s}$$

$$x + 1700 = 3600 \text{ ft}^3/\text{s}$$

18.

$$G_1 + G_2 = 750 \Rightarrow G_2 = 750 - G_1$$

$$0.65G_1 + 0.75G_2 = 530$$

$$0.65G_1 + 0.75(750 - G_1) = 530$$

$$0.65G_1 + 562.2 - 0.75G_1 = 530$$

$$-0.10G_1 = -32.5$$

$$G_1 = 325 \ MW$$

$$G_2 = 750 - G_1 = 425 \ MW$$

SUMMARY BELOW

Irrational Numbers:

$\sqrt{2}$ π e

Can't be written as one integer divided by another

Rationals: -1/2 1/8 0.56 0.999…

Whole Numbers.: 0, 1, 2, 3,

Naturals: 1, 2, 3, …

Pos./Neg. Integers:

…, -3, -2, -1, 0, 1, 2, 3, …

ORDER of operations

1. Operations within specific groupings
2. Powers
3. Multiplications and divisions (from left to right)
4. Additions and divisions (from left to right)

.

- commutative $a + b = b + a$
- associative $(a + b) + c = a + (b + c)$
- distributive $a\,(b + c) = ab + ac$

$\sqrt[n]{a}$

index

radicand

radical sign

Division by zero is undefined

Mon = 1
Bi = 2
Tri = 3

$$\sqrt{ab} = \sqrt{a}\sqrt{b}$$

Accuracy: number of significant digits
Precision: decimal position of the last significant digit.

Product Law: $a^m \times a^n = a^{m+n}$

Quotient Law: $\frac{a^m}{a^n} = a^{m-n}$ $m > n$,

base $\longrightarrow a^n \longleftarrow$ exponent

Approx = measure
Exact = count

Power Law: $(a^m)^n = a^{mn}$

$(ab)^n = a^n b^n$

Scientific notation $P \times 10^k$

M1 W1Ch 1

- Multiply:

$$5x^3(9x^2 - 7x)$$

1. Multiply the numerical coefficients.

2. Multiply the literal numbers.

$$=45x^5 - 35x^4$$

3. Combine any exponents when bases are the same.

$$\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$$

$$\begin{array}{r|l} & \;6x \quad -19 \\ \hline x+1 & 6x^2 - 13x + 7 \\ & 6x^2 + 6x \\ \hline & \qquad -19x + 7 \\ & \qquad -19x - 19 \\ \hline \text{remainder} \rightarrow & \qquad\qquad 26 \end{array}$$

Perform the same operation on both sides of the equation.

1. Remove grouping symbols (distributive law).
2. Combine any like terms of each side (also after *step 3*).
3. Perform the same operations on both sides, until $x = result$ is obtained.
4. Check the solution in the original equation.

M1 W2

Ch 1

Chapter 2.

Geometry

Learning Outcomes

- At the end of this chapter the student will:
  - Identify the types & properties of lines, angles, triangles, quadrilaterals.
  - Be able to calculate the area & perimeter of 2-dimensional geometric shapes.
  - Be able to calculate the volume and surface area of 3-dimensional geometric shapes.

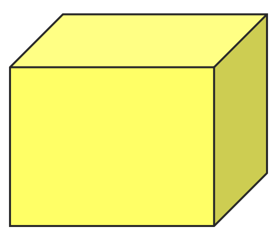
Learning Outcomes

- I can calculate missing angles in lines and triangles.
- I can calculate the area and the perimeter of triangles
- I can apply Hero's formula to triangle problems
- I can apply Pythagoras theorem to problems in right angle triangles
- I can calculate the area of various quadrilaterals.
- I can calculate the area of a circle and know what terms like 'sector' mean.
- I can apply the Trapezoidal Rule to problems.
- I can apply the Simpson's Rule to problems.
- I can find the volume and surface area of geometrical shapes.

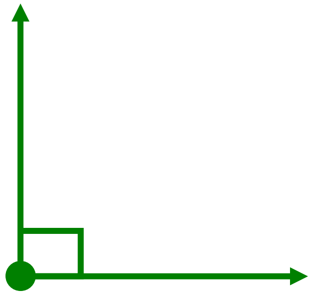
Ch. 2.1: Lines and Angles

- An ***angle*** is generated by rotating a ***ray*** about its fixed endpoint from an initial position to a terminal position.
- The initial position is called the ***initial side*** of the angle, the terminal position is called the ***terminal side***, and the fixed endpoint is the ***vertex***.

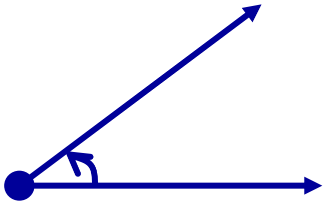
Key Concepts

- Point 
- Line 
- Plane 
- Solid 

Types of Angles

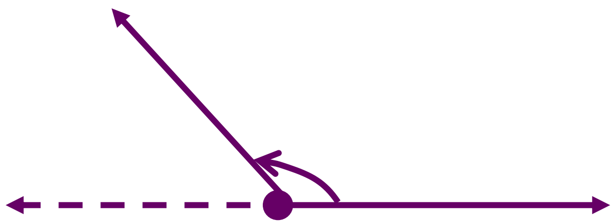


- Right Angle: an angle that **equals 90º**

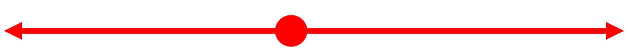


- Acute Angle: an angle that is **less than 90º**

Types of Angles



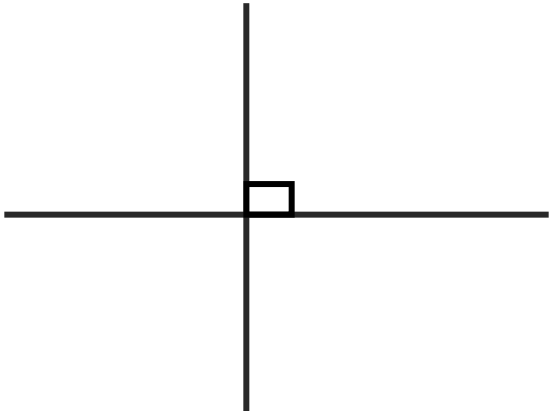
- Obtuse Angle: an angle that is **greater than 90º** & **less than 180º**



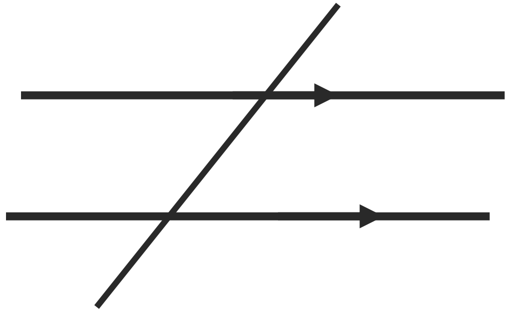
- Straight Angle: an angle that **equals 180º**

Types of Lines

- Perpendicular Lines

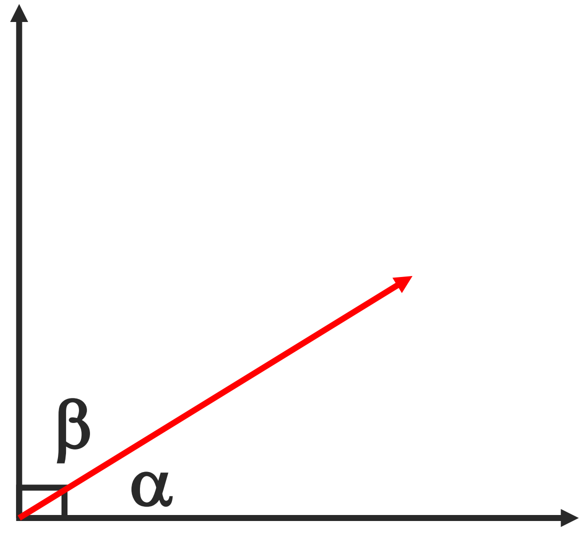


- Parallel Lines



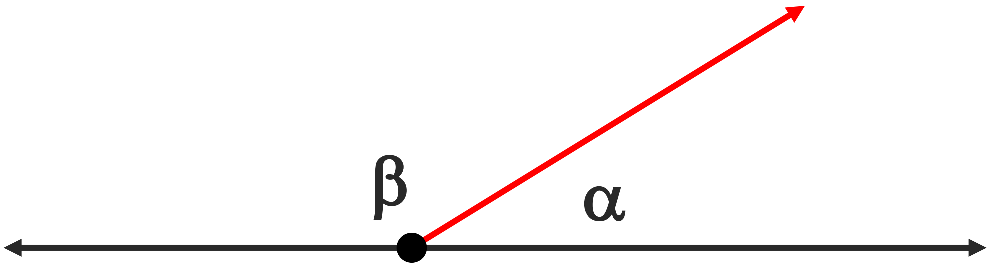
Complementary Angles

- Two angles the sum of whose measures is 90˚
- $\alpha + \beta = 90^\circ$



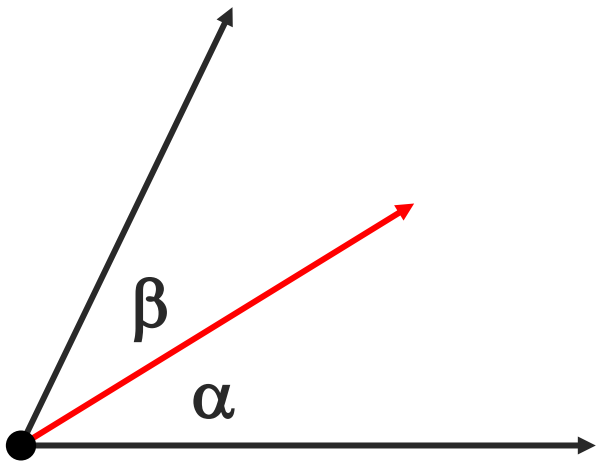
Supplementary Angles

- Two angles the sum of whose measures is equal to 180˚
- $\alpha + \beta = 180^\circ$



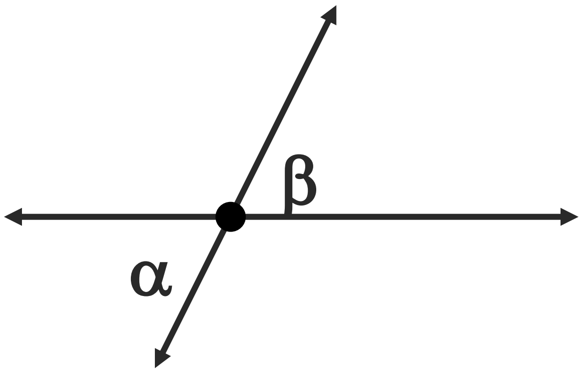
Adjacent Angles

- Two angles that have a common vertex



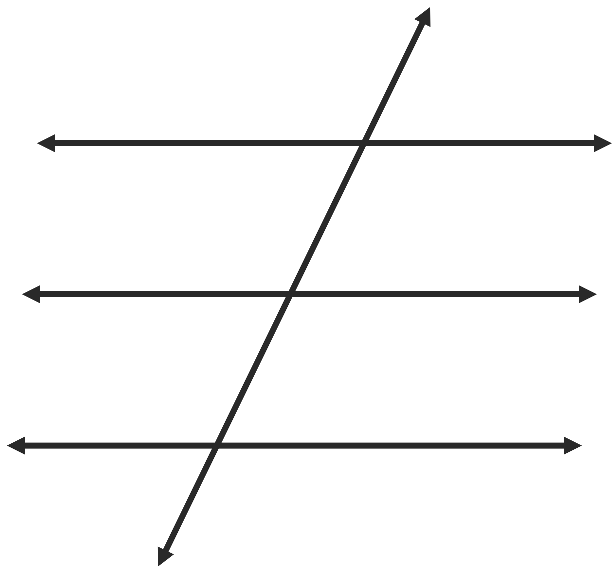
Vertical Angles

- Equal angles formed by two lines which cross on opposite sides of the point of intersection, which is the common vertex
- $\alpha = \beta$



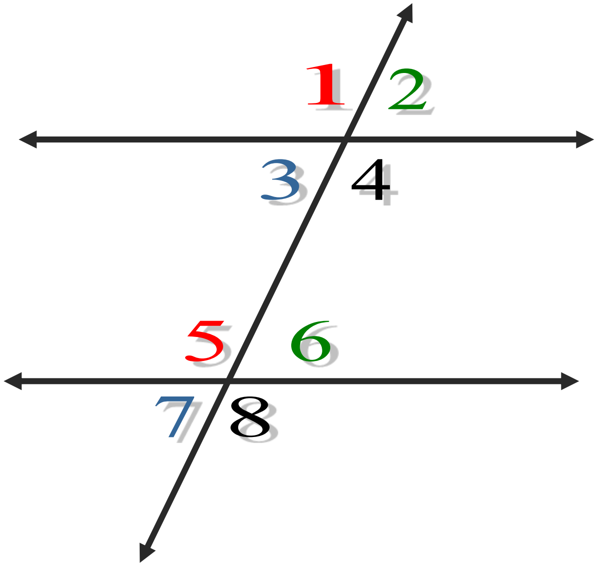
Transversals

- A line which crosses a pair (or more) of parallel or nonparallel lines



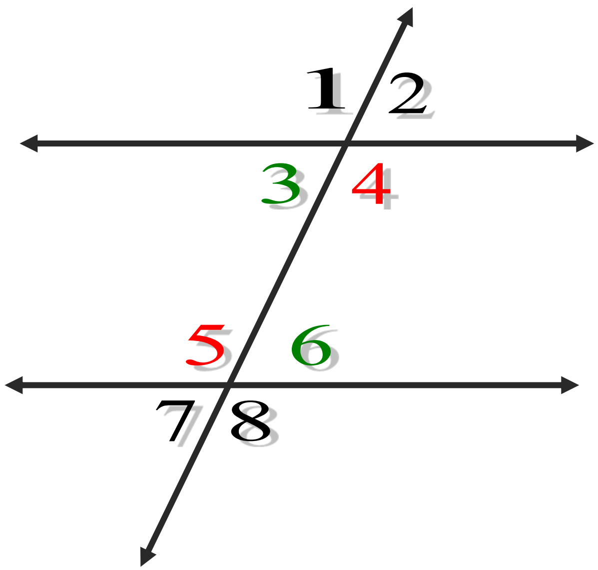
Equal Angles

- Corresponding angles
- 1 = 5
- 2 = 6
- 3 = 7
- 4 = 8



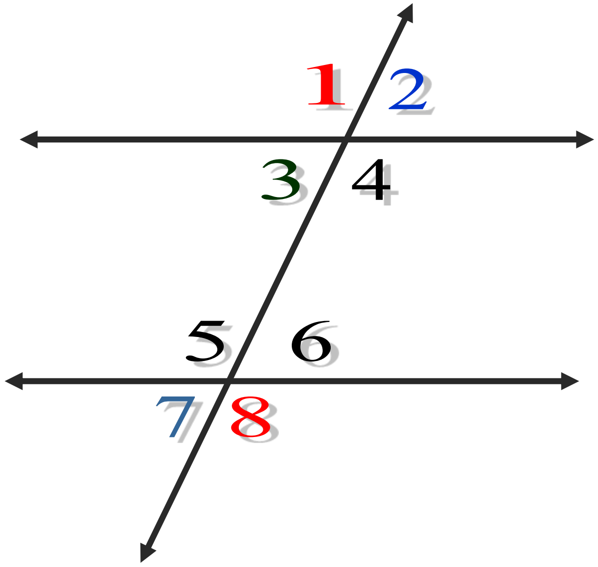
Equal Angles

- Alternate-interior angles
- 4 = 5
- 3 = 6



Equal Angles

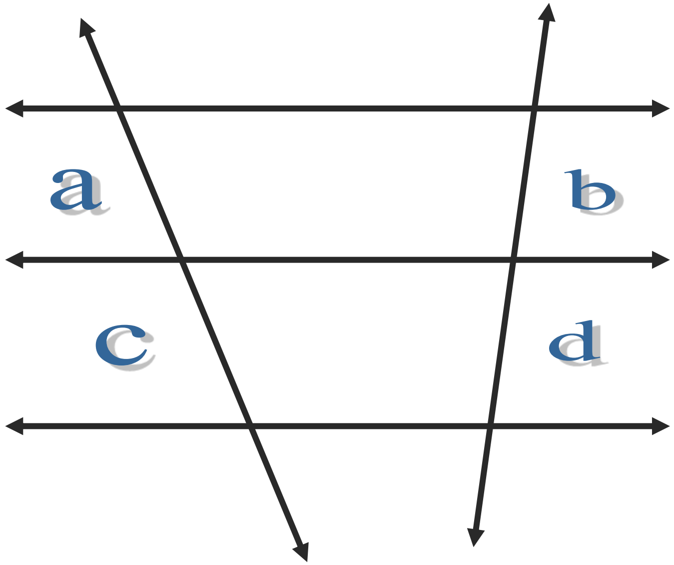
- Alternate-exterior angles
- 1 = 8
- 2 = 7



Corresponding Segments

- The segments of the transversals between the same two parallel lines

$$\frac{a}{b} = \frac{c}{d}$$



?

Ex 2.1 q 5-8, 23-28

In Exercises 5–12, identify the indicated angles and sides in Fig. 2.10. In Exercises 9 and 10, also find the measures of the indicated angles.

5. Two acute angles

6. Two right angles

7. The straight angle

8. The obtuse angle

9. If $\angle CBD = 65^\circ$, find its complement.

10. If $\angle CBD = 65^\circ$, find its supplement.

11. The sides adjacent to $\angle DBC$

12. The acute angle adjacent to $\angle DBC$.

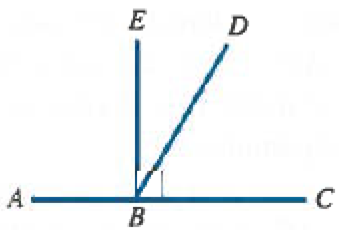


Fig. 2.10

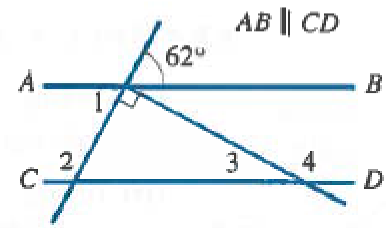


Fig. 2.13

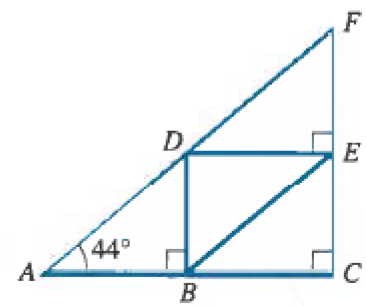


Fig. 2.14

In Exercises 23–28, find the measures of the angles in the truss shown in Fig. 2.14. A truss is a rigid support structure that is used in the construction of buildings and bridges.

23. $\angle BDF$

24. $\angle ABE$

25. $\angle DEB$

26. $\angle DBE$

27. $\angle DFE$

28. $\angle ADE$

A

Angle BDA

23. $\angle BDE = 90^\circ - 44^\circ = 46^\circ$

$\angle BDF = 180^\circ - 46^\circ = 134^\circ$

24. $\angle BDE = 90^\circ - 44^\circ = 46^\circ$

$\angle ABE = 90^\circ + 46^\circ = 136^\circ$

25. $\angle DEB = 44^\circ$

26. $\angle DBE = 46^\circ$

27. $\measuredangle DFE = 90^\circ - \measuredangle FDE$

$= 90^\circ - 44^\circ$

$= 46^\circ$

28. $\measuredangle ADE = \measuredangle ADB + 90^\circ$

$= (90^\circ - 44^\circ) + 90^\circ$

$= 136^\circ$

5. $\angle EBD$ and $\angle DBC$ are acute angles.

6. $\angle ABE$ and $\angle CBE$ are right angles.

7. $\angle ABC$ is a straight angle.

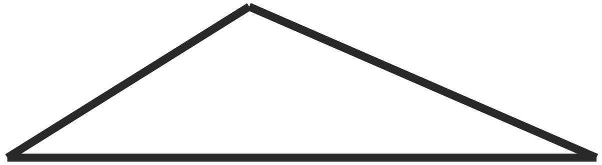
8. $\angle ABD$ is an obtuse angle.

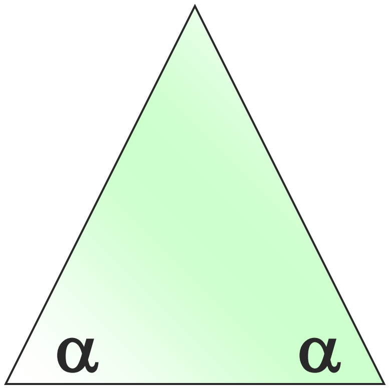
Ch. 2.2: Triangles

- Polygons:
  - a plane figure bounded by 3 or more straight line sides
- Types of Polygons:
  - Triangles
  - Quadrilaterals
  - Pentagons
  - Hexagons

Types & Properties of Triangles

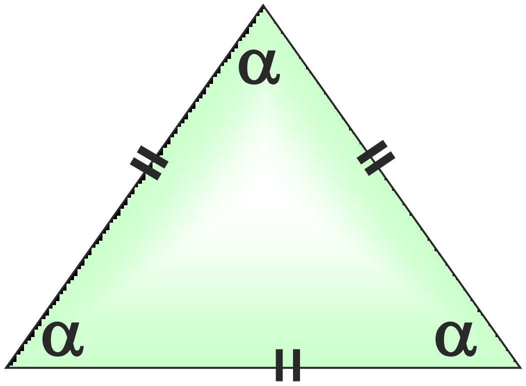
- **Scalene Triangle**:
  - No 2 sides are equal in length



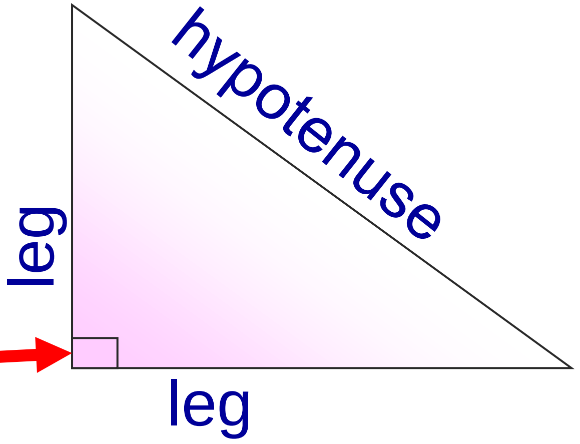


- **Isosceles Triangle**:
  - Two sides are equal in length
  - The sides leading to the base are equal; the base angles of the 2 equal sides are equal.

Types & Properties of Triangles



- **Equilateral Triangle:**
  - All sides & angles are equal.
  - $\alpha = 60°$

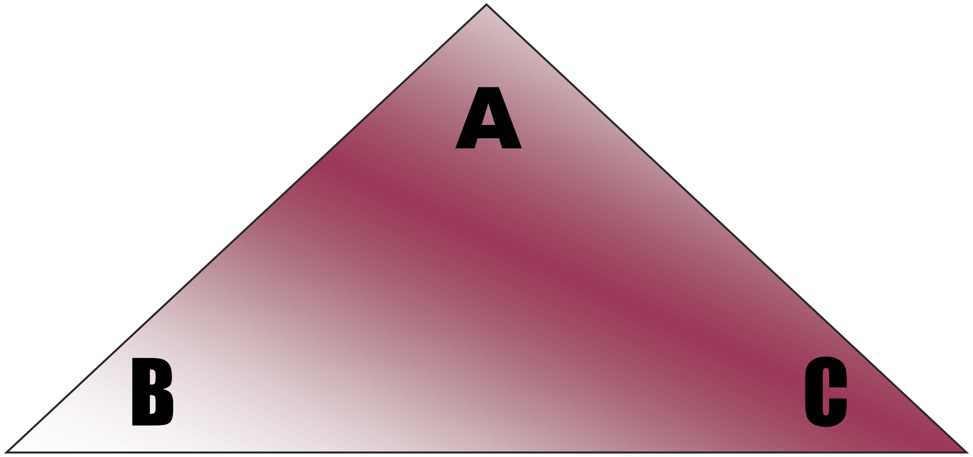


- **Right Angle Triangle:**
  - One of the angles is 90˚

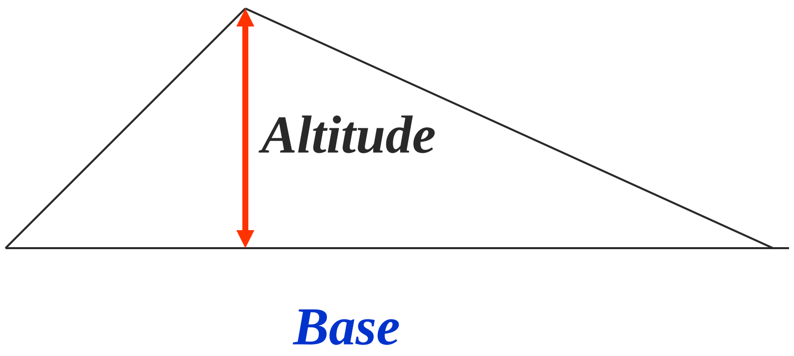
Triangle Sum Theorem

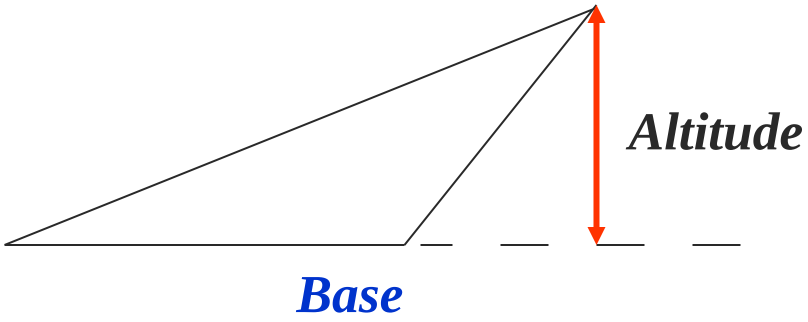
The sum of the three angles of any triangle is **180º**

$$\angle A + \angle B + \angle C = 180^\circ$$



Altitude & Base of a Triangle





- The ***altitude*** of a triangle is the perpendicular distance from an angle (vertex) to the opposite side which is called the ***base.***
- Altitude is also referred to as *'height'.*

Perimeter & Area of a Triangle

- Perimeter:
  - The sum of the lengths of the 3 sides.
- Area: $A = \frac{bh}{2}$

Hero's Formula

- Used in finding area of a triangle when we know the length of all 3 sides of a triangle.

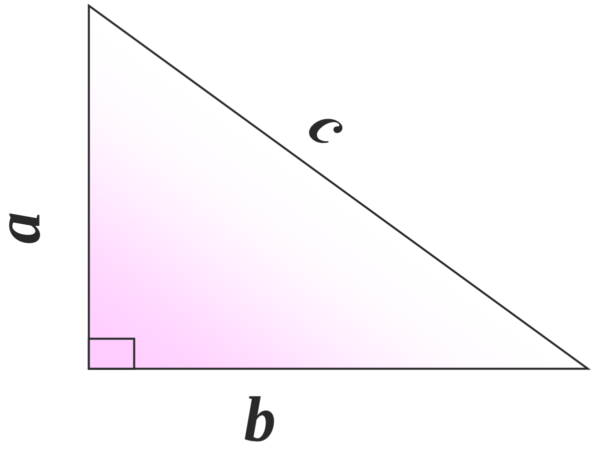
$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

Where, $$s = \frac{a+b+c}{2}$$

And, ***a, b, c*** are the lengths of the sides. This is used when we know the length of all 3 sides of a triangle.

The Pythagorean Theorem

- In a right triangle, the square of the length of the hypotenuse equals the sum of the squares of the lengths of the other two sides.



$$c^2 = a^2 + b^2$$

Similar Triangles

1. The corresponding angles of similar triangles are equal.
2. The corresponding sides of similar triangles are proportional.

Ex 2.2 questions 5-8, 21-24, 41-44

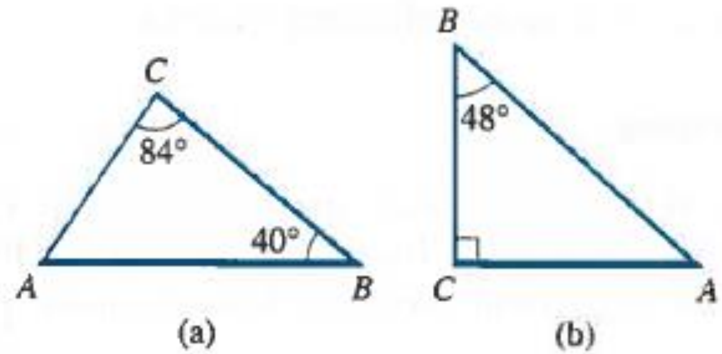
In Exercises 5–8, determine $\angle A$ in the indicated figures.

5. Fig. 2.38(a)

6. Fig. 2.38(b)

7. Fig. 2.38(c)

8. Fig. 2.38(d)



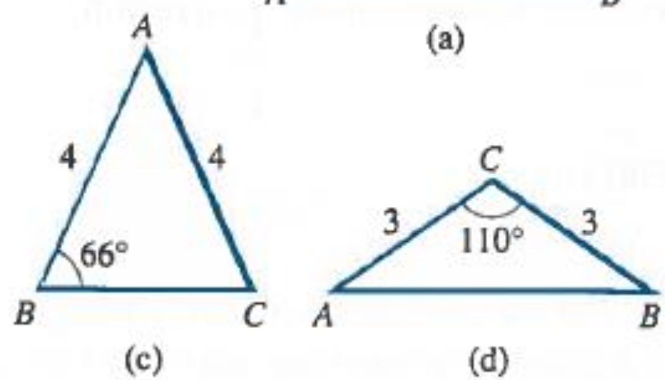


Fig. 2.38

Hero's

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

Where, $s = \dfrac{a+b+c}{2}$

In Exercises 21–24, find the third side of the right triangle shown in Fig. 2.40 for the given values.

21. $a = 13.8$ ft, $b = 22.7$ ft

22. $a = 2.48$ m, $b = 1.45$ m

23. $a = 175$ cm, $c = 551$ cm

24. $b = 0.474$ in., $c = 0.836$ in.

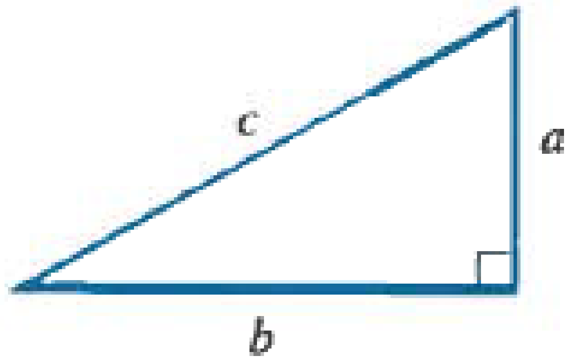


Fig. 2.40

41. A wall pennant is in the shape of an isosceles triangle. If each equal side is 76.6 cm long and the third side is 30.6 cm, what is the area of the pennant?

$A = \sqrt{s(s-a)(s-b)(s-c)}$

Where, $s = \dfrac{a+b+c}{2}$

42. The Bermuda Triangle is sometimes defined as an equilateral triangle 1600 km on a side, with vertices in Bermuda, Puerto Rico, and the Florida coast. Assuming it is flat, what is its approximate area?

43. The sail of a sailboat is in the shape of a right triangle with sides of 8.0 ft, 15 ft, and 17 ft. What is the area of the sail?

44. An observer is 550 m horizontally from the launch pad of a rocket. After the rocket has ascended 750 m, how far is it from the observer?

A

5. $\angle A = 180^\circ - 84^\circ - 40^\circ = 56^\circ$

6. $\angle A = 90^\circ - 48^\circ = 42^\circ$

7. This is an isosceles triangle, so the base angles are equal. $\angle A = 180^\circ - \left(66^\circ + 66^\circ\right) = 48^\circ$

8. $\angle A = \frac{1}{2}\left(180^\circ - 110^\circ\right) = 35^\circ$

21. $c = \sqrt{13.8^2 + 22.7^2} = 26.6 \text{ ft}$

22. $$\begin{aligned} c^2 &= a^2 + b^2 \\ &= 2.48^2 + 1.45^2 \\ c &= 2.87 \text{ m} \end{aligned}$$

23. $b = \sqrt{551^2 - 175^2} = 522 \text{ cm}$

24. $$\begin{aligned} c^2 &= a^2 + b^2 \\ 0.836^2 &= a^2 + 0.474^2 \\ a &= 0.689 \text{ in.} \end{aligned}$$

41. $$\begin{aligned} s &= \frac{2(76.6) + 30.6}{2} = 91.9 \\ A &= \sqrt{91.9(91.9 - 76.6)^2(91.6 - 30.6)} \\ A &= 1150 \text{ cm}^2 \end{aligned}$$

42. $$\begin{aligned} \frac{3(1600)}{2} &= 2400 \\ A &= \sqrt{2400(2400 - 1600)^3} \\ A &= 1{,}100{,}000 \text{ km}^2 \end{aligned}$$

43. $A = \frac{1}{2}bh = \frac{1}{2}(8.0)(15) = 60 \text{ ft}^2$

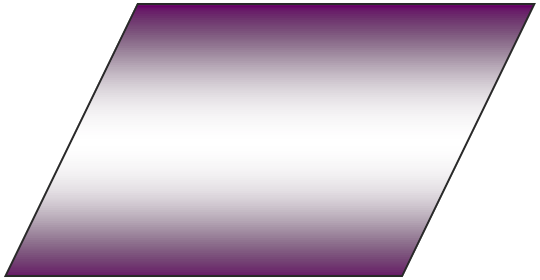
44. $d = \sqrt{750^2 + 550^2} = 930 \text{ m}$

Ch. 2.3: Quadrilaterals

- A quadrilateral is a closed plane figure with four sides with four interior angles.

Types of Quadrilaterals

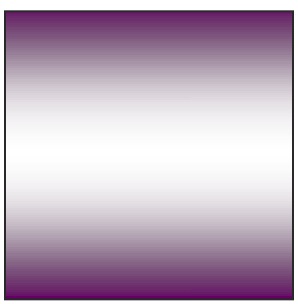
- **Parallelogram**:
  - Opposite sides are parallel



- **Rhombus**:
  - A parallelogram with 4 equal sides

Types of Quadrilaterals

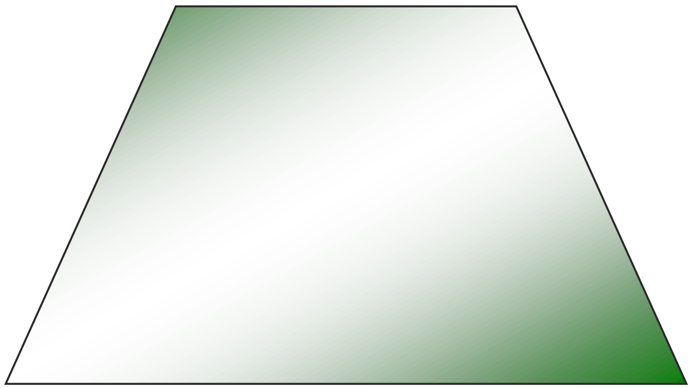
- **Rectangle**:
  - A parallelogram in which intersecting sides are perpendicular



- **Square**:
  - A rectangle with four equal sides

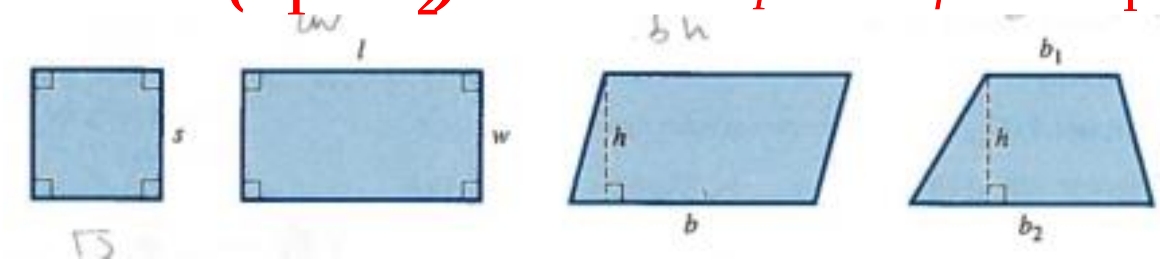
Types of Quadrilaterals

- **Trapezoid**:
  - A quadrilateral in which two sides are parallel
  - The parallel sides are called **bases**.



Perimeter & Area of a Quadrilateral

- Perimeter:
  - The sum of the lengths of the four sides.
- Area:
  - $A = x^2$ Square of side x
  - $A = lw$ *Rectangle of length* **l** *and width* **w**
  - $A = bh$ *Parallelogram of base* **b** *& height* **h**
  - $A = \frac{1}{2}h(b_1 + b_2)$ *Trapezoid of bases* b_1 *&* b_2 *and height*



Ex 2.3 p62 ?

FIND PERIMETER

9. Parallelogram in Fig. 2.59

10. Parallelogram in Fig. 2.60

11. Trapezoid in Fig. 2.61

12. Trapezoid in Fig. 2.62

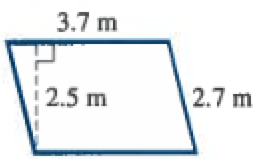


Fig. 2.59

Fig. 2.60

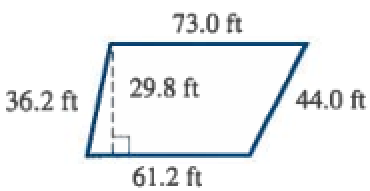


Fig. 2.61

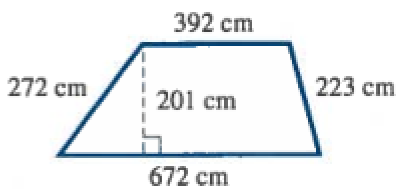


Fig. 2.62

FIND AREA

17. Parallelogram in Fig. 2.59

18. Parallelogram in Fig. 2.60

19. Trapezoid in Fig. 2.61

20. Trapezoid in Fig. 2.62

In Exercises 21–24, set up a formula for the indicated perimeter or area. (Do not include dashed lines.)

21. The perimeter of the figure in Fig. 2.63 (a parallelogram and a square attached)

22. The perimeter of the figure in Fig. 2.64 (two trapezoids attached)

23. Area of figure in Fig. 2.63

24. Area of figure in Fig. 2.64

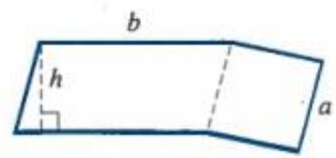


Fig. 2.63

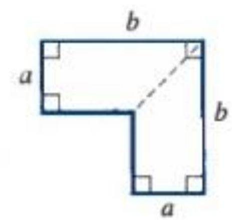


Fig. 2.64

A

9. $p = 2l + 2w = 2(3.7) + 2(2.7) = 12.8 \text{ m}$

10. $p = 2(27.3) + 2(14.2) = 83.0 \text{ in.}$

11. $p = 36.2 + 73.0 + 44.0 + 61.12 = 214.4 \text{ ft}$

12. $p = 272 + 392 + 223 + 672 = 1559 \text{ cm}$

17. $A = bh = 3.7(2.5) = 9.3 \text{ m}^2$

18. $A = 27.3(12.6) = 344 \text{ in.}^2$

19. $A = (1/2)(29.8)(61.2 + 73.0) = 2000 \text{ ft}^2$

20. $A = \frac{1}{2}(392 + 672)(201) = 107{,}000 \text{ cm}^2$

21. $p = 2b + 4a$

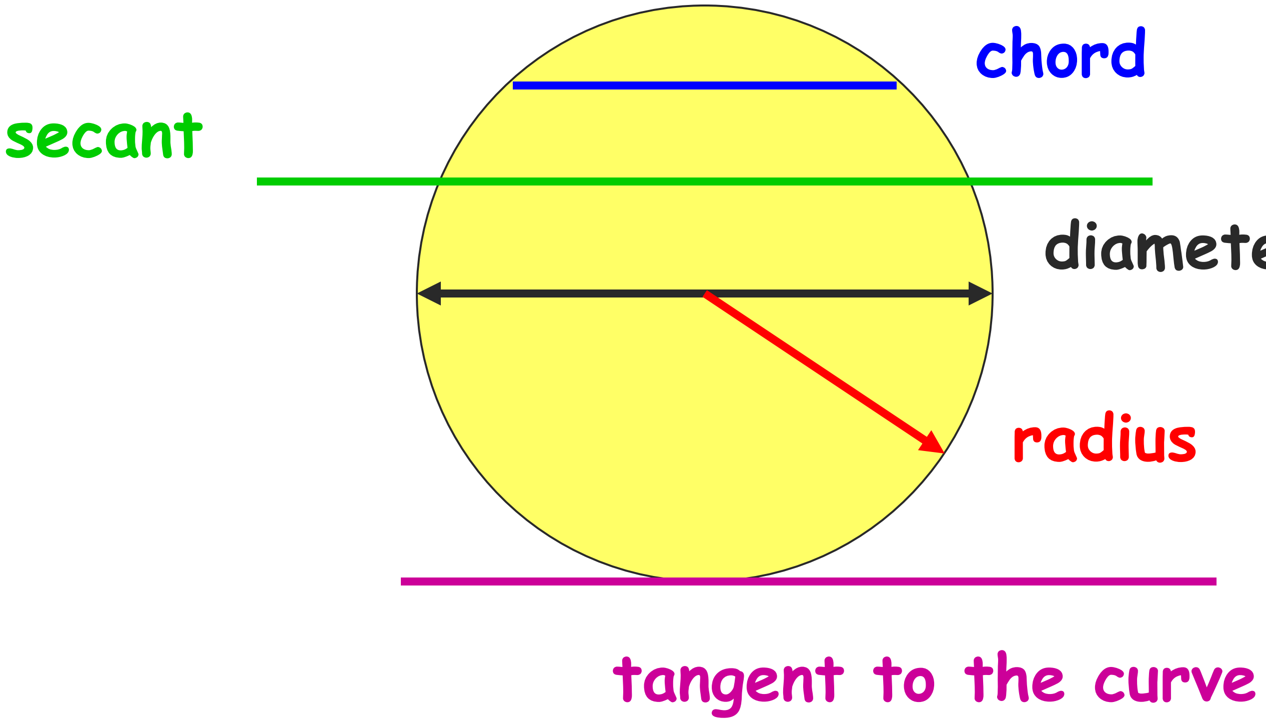
22. $p = a + b + b + a + (b - a) + (b - a) = 4b$

23. $A = b \times h + a^2 = bh + a^2$

24. $A = ab + a(b - a) = 2ab - a^2$

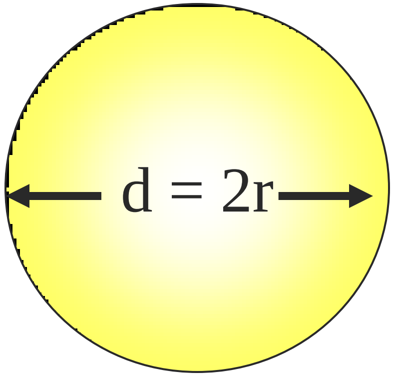
Ch. 2.4: Circles

- Terminology related to the circle.



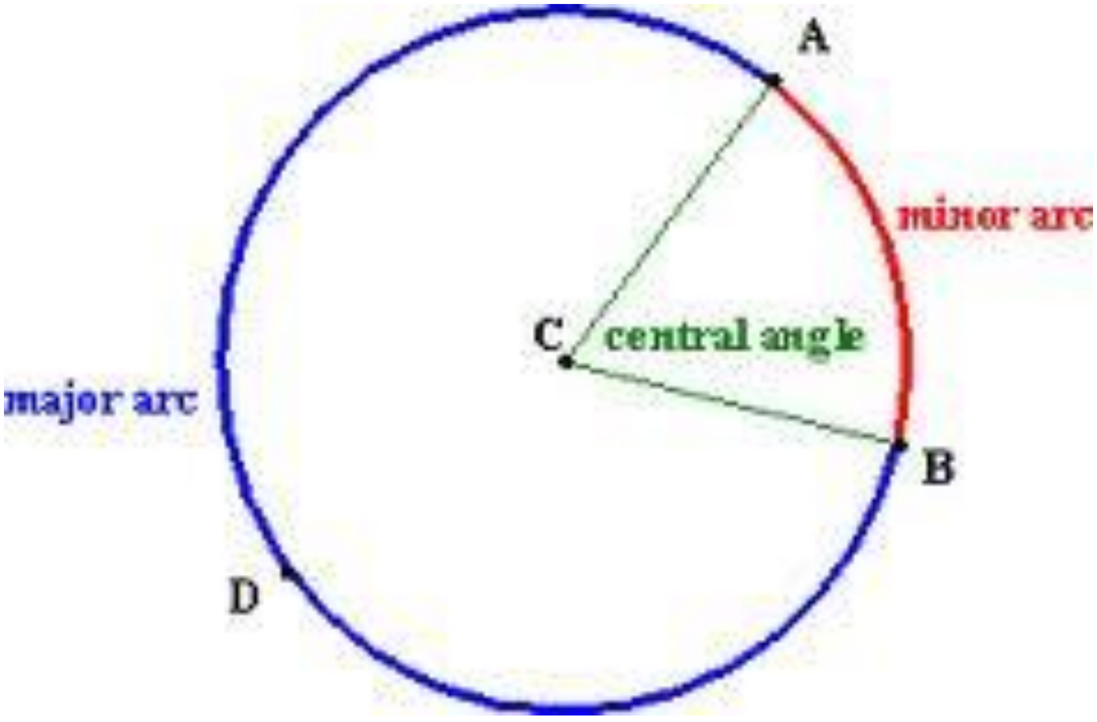
Circumference & Area of a Circle

- Circumference is a perimeter of a circle and is given by $C = 2\pi r$
- $A_{circle} = \pi r^2$



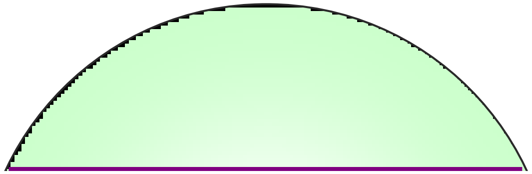
Circular Arcs & Angles

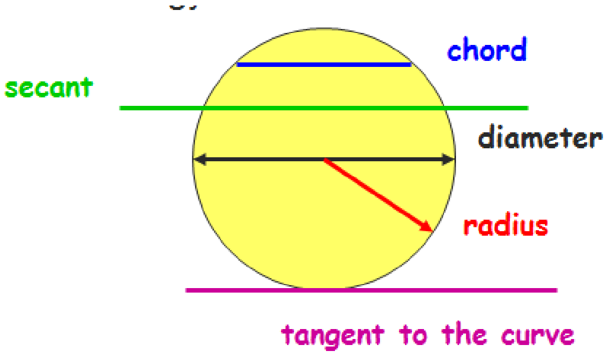
- An ***arc*** is part of a circle.
- A ***central angle*** is an angle formed at the centre by two radii.



Segment of a Circle

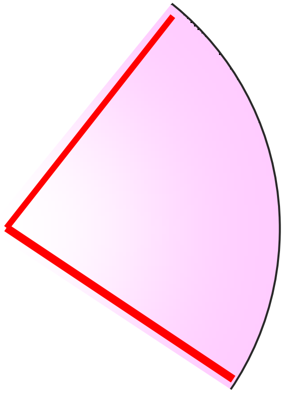
- The region bounded by a chord and its arc.





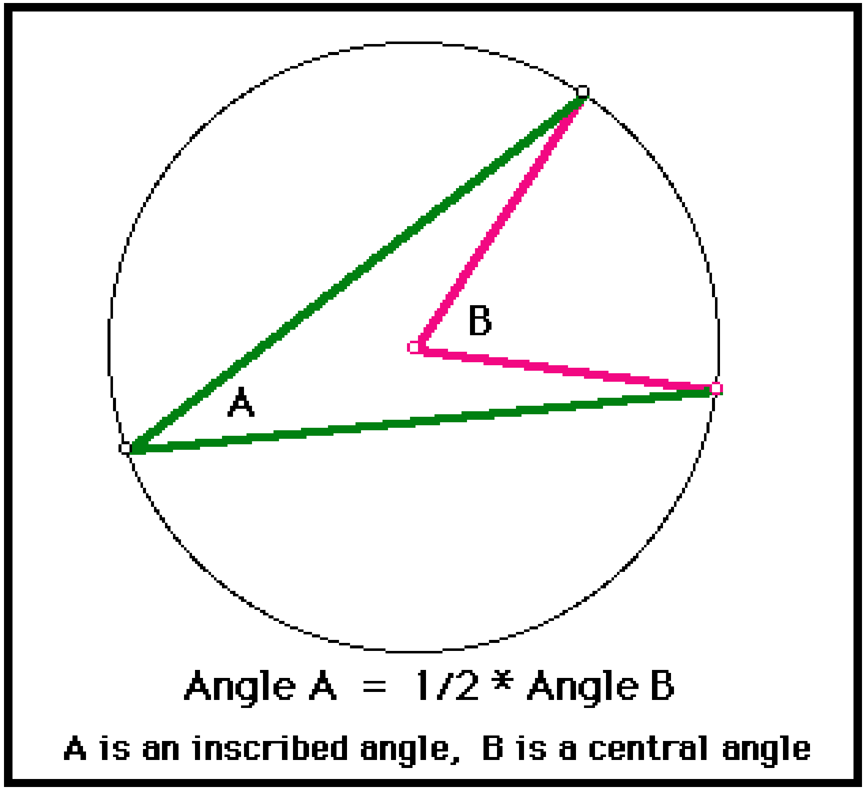
The Sector of a Circle

- The region bounded by 2 radii and the arc they intercept.



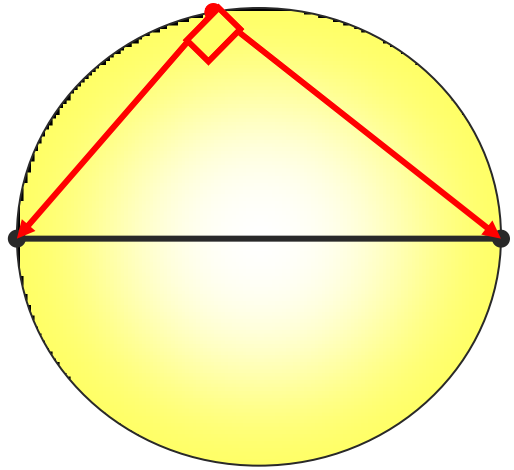
An Inscribed Angle

- An ***inscribed angle*** of an arc is one for which the endpoints of the arc are points on the sides of the angle for which the vertex is a point (not an endpoint) of the arc.



An Inscribed Angle

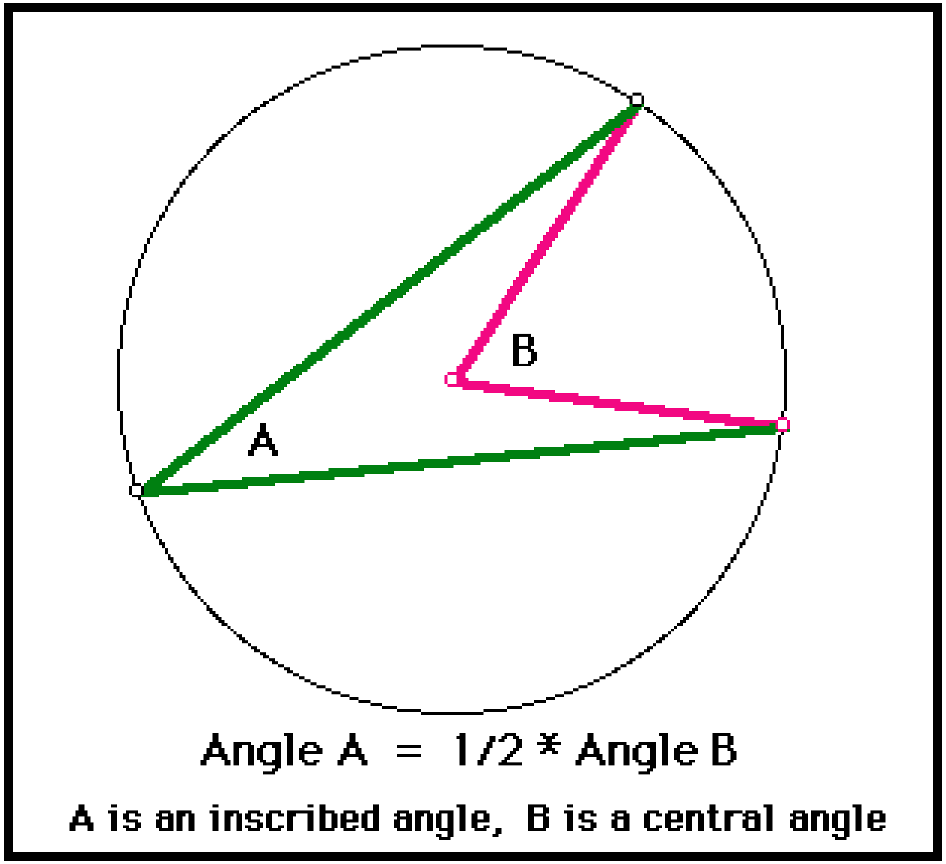
- An angle inscribed in a semi-circle is a right angle.



The measure of an inscribed angle in a circle equals half the measure of its intercepted arc.

<u>Measure of an Inscribed Angle Theorem</u>

If an angle is inscribed in a circle, then its measure is half the measure of its intercepted arc.



- **Inscribed angle:** In a circle, this is an angle formed by two chords with the vertex on the circle.
- **Intercepted arc:** Corresponding to an angle, this is the portion of the circle that lies in the interior of the angle together with the endpoints of the arc.

? Ex 2.4 p66

In Exercises 17–20, refer to Fig. 2.81, where AB is a diameter, TB is a tangent line at B, and $\angle ABC = 65°$. Determine the indicated angles.

17. $\angle CBT$

18. $\angle BCT$

19. $\angle CAB$

20. $\angle BTC$

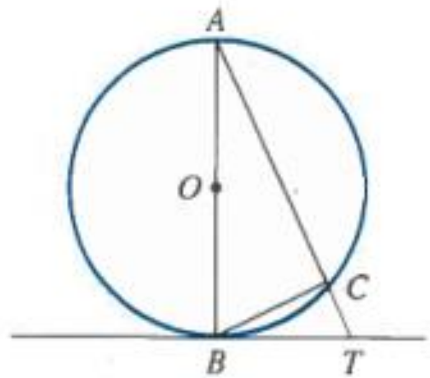


Fig. 2.81

43. A window designed between semicircular regions is shown in Fig. 2.86. Find the area of the window.

- $A_{circle} = \pi r^2$

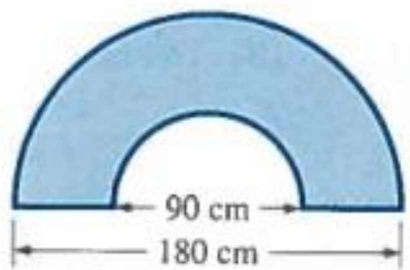


Fig. 2.86

Fig. 2.87

Area of small / Area of large

44. The cross section of a large circular conduit has seven smaller equal circular conduits within it. The conduits are tangent to each other as shown in Fig. 2.87. What fraction of the large conduit is occupied by the seven smaller conduits?

A

17. $\angle CBT = 90^\circ - \angle ABC = 90^\circ - 65^\circ = 25^\circ$

18. $\angle BCT = 90^\circ$, any angle such as $\angle BCA$ inscribed in a semicircle is a right angle and $\angle BCT$ is supplementary to $\angle BCA$.

19. A tangent to a circle is perpendicular to the radius drawn to the point of contact. Therefore,

$\angle ABT = 90^\circ$

$\angle CBT = \angle ABT - \angle ABC = 90^\circ - 65^\circ = 25^\circ$;

$\angle CAB = 25^\circ$

20. $\angle BTC = 65^\circ$; $\angle CBT = 35^\circ$ since it is complementary to $\angle ABC = 65^\circ$.

43. $A = \frac{\pi}{2}\left(90^2 - 45^2\right)$

$A = 9500 \text{ cm}^2$

44. Let D = diameter of large conduit, then

$D = 3d$ where d = diameter of smaller conduit

$$F = \frac{\pi}{4}D^2 = 7 \cdot \frac{\pi}{4} \cdot d^2$$

$$F = \frac{7d^2}{D^2} = \frac{7d^2}{(3d)^2} = \frac{7d^2}{9d^2}$$

$$F = \frac{7}{9}$$

The smaller conduits occupy $\frac{7}{9}$ of the larger conduits.

Ch. 2.5: Measurement of Irregular Areas

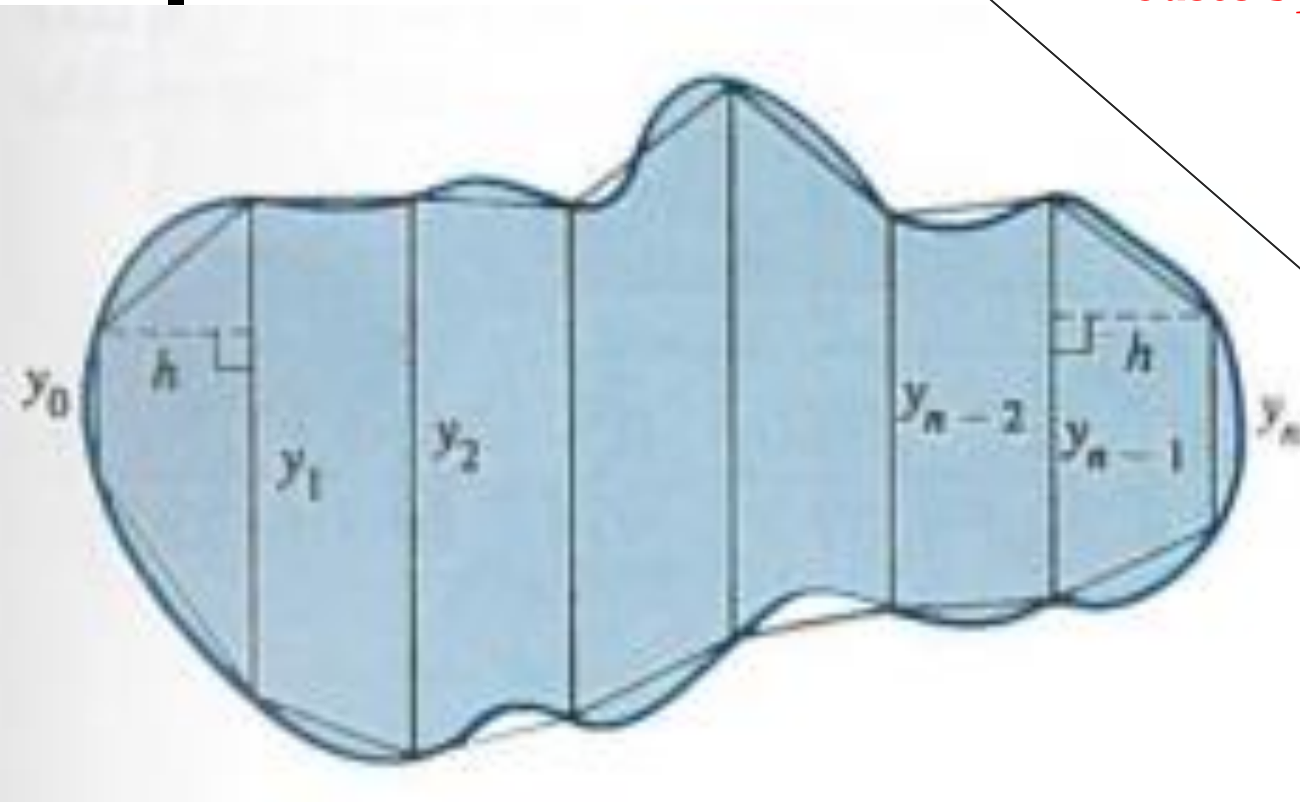
- Some figures are not very well defined, making it difficult to find their area.
- For these figures, the following two methods provide very good approximations of area for irregular figures:
  - The Trapezoidal Rule
  - Simpson's Rule

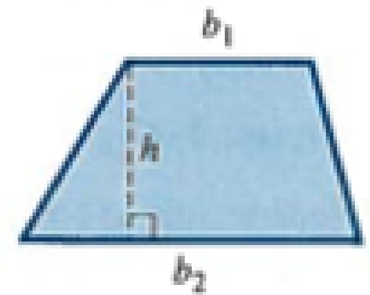
The Trapezoidal Rule

- This method takes advantage of breaking up the area to be found into trapezoids of equal height.
- The formula used is:

$$\boxed{A = \frac{h}{2}\left(y_0 + 2y_1 + 2y_2 + \cdots + 2y_{n-1} + y_n\right)}$$

A* = ½*h*(*b*$_1$ + *b*$_2$)** *Trapezoid of bases* ***b*$_1$** *&* ***b*$_2$** *and height* ***h





first trapezoid	second trapezoid	third trapezoid	next-to-last trapezoid	last trapezoid

$$A = \frac{h}{2}(y_0 + y_1) + \frac{h}{2}(y_1 + y_2) + \frac{h}{2}(y_2 + y_3) + \cdots + \frac{h}{2}(y_{n-2} + y_{n-1}) + \frac{h}{2}(y_{n-1} + y_n)$$

$$= \frac{h}{2}(y_0 + y_1 + y_1 + y_2 + y_2 + y_3 + \cdots + y_{n-2} + y_{n-1} + y_{n-1} + y_n)$$

Simpson's Rule

- This method uses the concepts of parabolas to take into account the curved nature of an irregular shape.
- Once again the segments are of equal height.
- The formula used is:

$$\boxed{A = \frac{h}{3}\left(y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 + \cdots + 4y_{n-1} + y_n\right)}$$

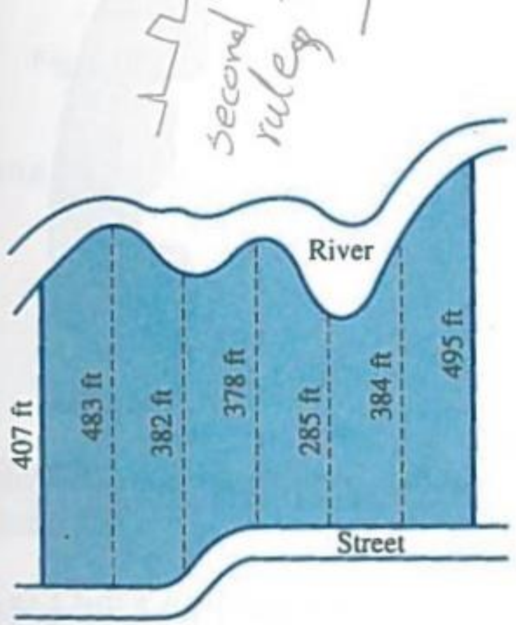


Fig. 2.96

EXAMPLE 3 Application of Simpson's rule

A parking lot is proposed for a riverfront area in a town. The town engineer measured the widths of the area at 100-ft (three sig. digits) intervals, as shown in Fig. 2.96. Find the area available for parking.

First, we see that there are six intervals, which means Eq. (2.13) may be used. With $y_0 = 407$ ft, $y_1 = 483$ ft, . . . , $y_6 = 495$ ft, and $h = 100$ ft, we have

$$A = \frac{100}{3}[407 + 4(483) + 2(382) + 4(378) + 2(285) + 4(384) + 495]$$
$$= 241{,}000 \text{ ft}^2$$

For most areas, Simpson's rule gives a somewhat better approximation than the trapezoidal rule. The accuracy of Simpson's rule is also usually improved by using smaller intervals.

$$\boxed{A = \frac{h}{3}(y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 + \cdots + 4y_{n-1} + y_n)}$$

EXAMPLE 4 Application of Simpson's rule

From an aerial photograph, a cartographer determines the widths of Easter Island (in the Pacific Ocean) at 1.50-km intervals as shown in Fig. 2.97. The widths found are as follows:

Distance from South End (km)	0	1.50	3.00	4.50	6.00	7.50	9.00	10.5	12.0	13.5	15.0
Width (km)	0	4.8	5.7	10.5	15.2	18.5	18.8	17.9	11.3	8.8	3.1

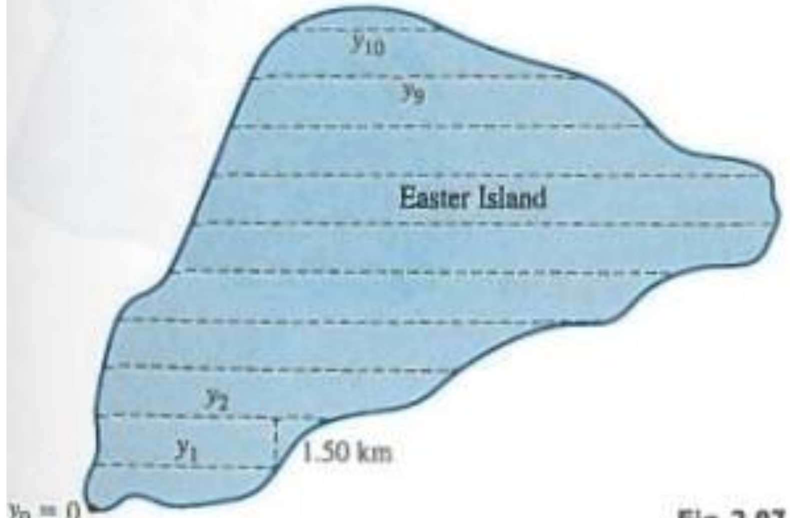


Fig. 2.97

Since there are ten intervals, Simpson's rule may be used. From the table, we have the following values: $y_0 = 0, y_1 = 4.8, y_2 = 5.7, \ldots, y_9 = 8.8, y_{10} = 3.1$, and $h = 1.5$. Using Simpson's rule, the cartographer would approximate the area of Easter Island as follows:

$$A = \frac{1.50}{3}(0 + 4(4.8) + 2(5.7) + 4(10.5) + 2(15.2) + 4(18.5)$$
$$+ 2(18.8) + 4(17.9) + 2(11.3) + 4(8.8) + 3.1) = 174 \text{ km}^2 \blacksquare$$

In Exercises 3 and 4, answer the given questions related to Fig. 2.98.

W **3.** Which should be more accurate for finding the area, the trapezoidal rule or Simpson's rule? Explain.

W **4.** If the trapezoidal rule is used to find the area, will the result probably be too high, about right, or too little? Explain.

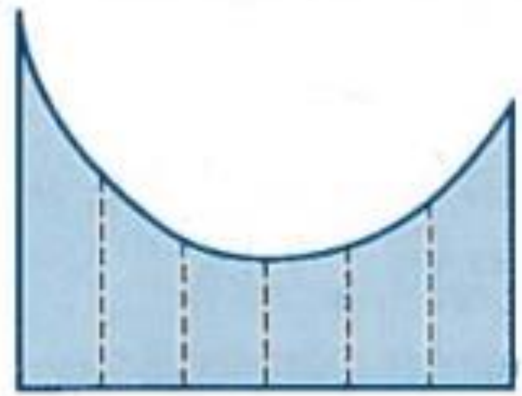


Fig. 2.98

3. Simpson's rule should be more accurate in that it accounts better for the arcs between points on the upper curve.

4. The calculated area would be too high since each trapezoid would include more area than that under the curve.

7. The widths of a cross section of an airplane wing are measured at 1.00-ft intervals, as shown in Fig. 2.100. Calculate the area of the cross section, using Simpson's rule.

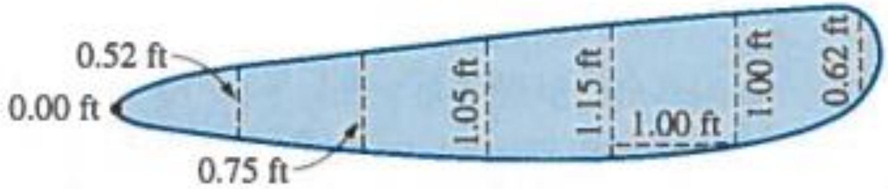


Fig. 2.100

8. Calculate the area of the cross section of the airplane wing in Fig. 2.100, using the trapezoidal rule.

13. The widths of the baseball playing area in Boston's Fenway Park at 45-ft intervals are shown in Fig. 2.102. Find the playing area using the trapezoidal rule.

14. Find the playing area of Fenway Park (see Exercise 13) by Simpson's rule.

Fig. 2.102

Simpson's Rule

$$A=\frac{h}{3}(y_0+4y_1+2y_2+4y_3+2y_4+\cdots+4y_{n-1}+y_n)$$

Trapezoidal Rule

$$A=\frac{h}{2}(y_0+2y_1+2y_2+\cdots+2y_{n-1}+y_n)$$

A

7. $A_{simp} = \dfrac{1.00}{3}(0+4(0.52)+2(0.75)+4(1.05)$
$+2(1.15)+4(1.00)+0.62) = 4.9 \text{ ft}^2$

8. $A_{trap} = \dfrac{1}{2}(0+2(0.52)+2(0.75)+2(1.05)+2(1.15)$
$+2(1.00)+0.62) = 4.8 \text{ ft}^2$

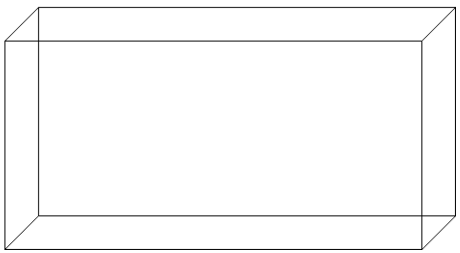
13. $A_{trap} = \dfrac{45}{2}[170+2(360)+2(420)+2(410)+2(390)$
$+2(350)+2(330)+2(290)+230]$
$A_{trap} = 120,000 \text{ ft}^2$

14. $A_{simp} = \dfrac{45}{3}(230+4(290)+2(330)+4(340)$
$+2(390)+4(410)+2(420)+4(360)+170)$
$= 120,000 \text{ ft}^2$

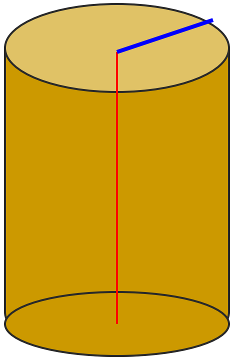
Ch. 2.6: Solid Geometric Figures

- Solid geometric figures are three dimensional figures.
- Their **volume** and **surface area** can be calculated.
- The sides of a solid figure made up of planes are known as **faces**.

Volumes of Various Solids



$V = L\,W\,H$

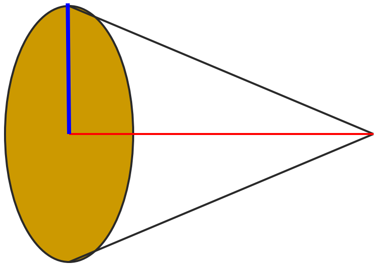


radius

height

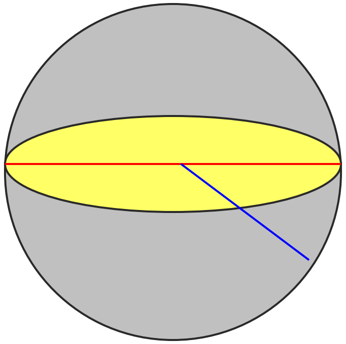
$V = \pi r^2 h$

radius



height

$V = \frac{1}{3}\pi r^2 h$

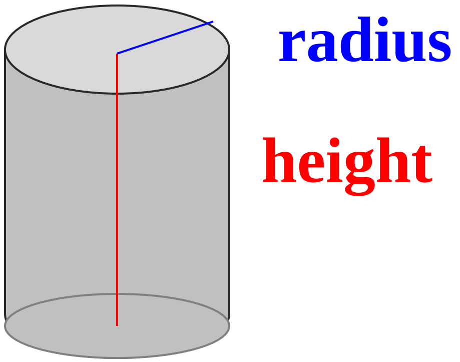


diameter

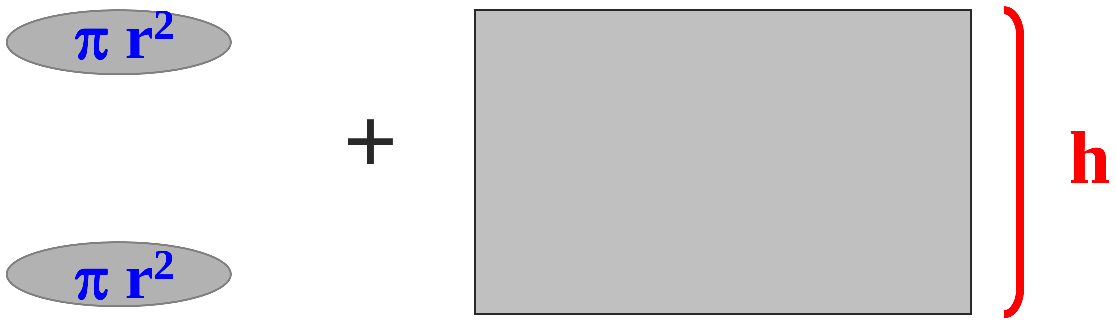
radius

$V = \frac{4}{3}\pi r^3$

Surface Area of a Cylinder



area of circle = $\pi\, r^2$

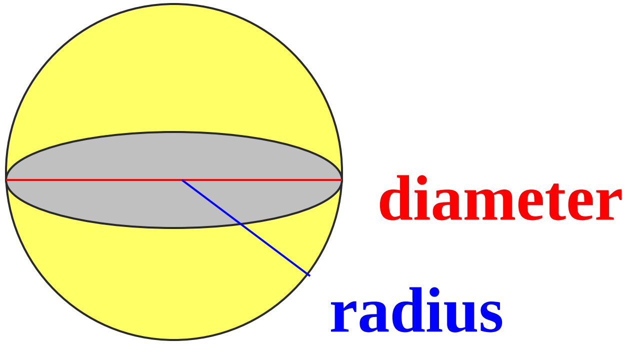


Circumference of the circle top

Surface area for cylinder = $(2)\pi\, r^2 + 2\pi\, r\, h$

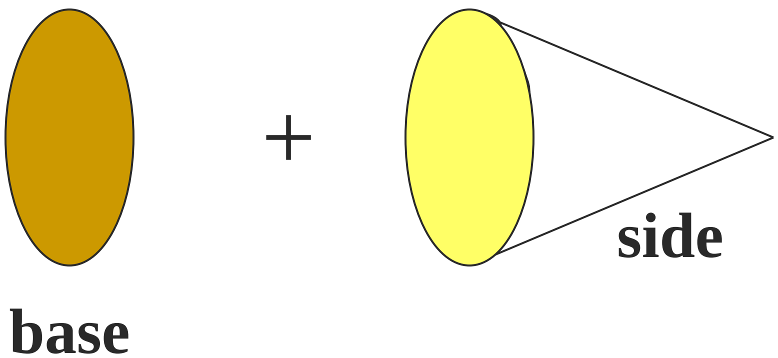
Surface Area & Volume of a Sphere

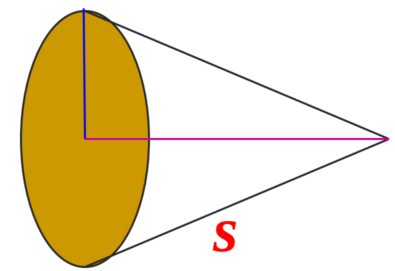
Surface area = $4 \pi r^2$



Volume = $\frac{4}{3} \pi r^3$

Surface Area of a Cone





Surface area $_{\text{cone}} = \pi r^2 + \pi r s$

Summary of Formula Used in Geometry

In the following formulas, V represents the *volume*, A represents the *total surface area*, S represents the *lateral surface area* (bases not included), B represents the *area of the base*, and p represents the *perimeter of the base*.

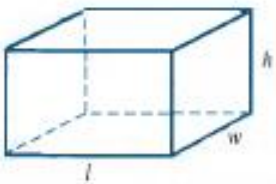


Fig. 2.104

Fig. 2.105

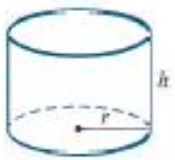


Fig. 2.106

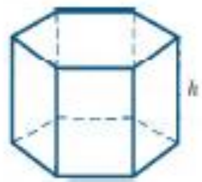


Fig. 2.107

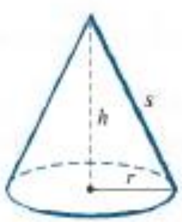


Fig. 2.108

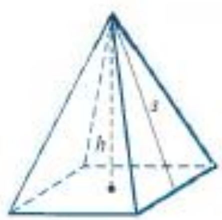


Fig. 2.109

Fig. 2.110

Fig. 2.111

$V = lwh$	Rectangular solid (Fig. 2.104)	(2.14)
$A = 2lw + 2lh + 2wh$		(2.15)
$V = e^3$	Cube (Fig. 2.105)	(2.16)
$A = 6e^2$		(2.17)
$V = \pi r^2 h$	Right circular cylinder (Fig. 2.106)	(2.18)
$A = 2\pi r^2 + 2\pi r h$		(2.19)
$S = 2\pi r h$		(2.20)
$V = Bh$	Right prism (Fig. 2.107)	(2.21)
$S = ph$		(2.22)
$V = \frac{1}{3}\pi r^2 h$	Right circular cone (Fig. 2.108)	(2.23)
$A = \pi r^2 + \pi r s$		(2.24)
$S = \pi r s$		(2.25)
$V = \frac{1}{3}Bh$	Regular pyramid (Fig. 2.109)	(2.26)
$S = \frac{1}{2}ps$		(2.27)
$V = \frac{4}{3}\pi r^3$	Sphere (Fig. 2.110)	(2.28)
$A = 4\pi r^2$		(2.29)

Equation (2.21) is valid for any prism, and Eq. (2.26) is valid for any pyramid. There are other types of cylinders and cones, but we restrict our attention to right circular cylinders and right circular cones, and we will often use "cylinder" or "cone" when referring to them.

The **frustum** of a cone or pyramid is the solid figure that remains after the top is cut off by a plane parallel to the base. Figure 2.111 shows the frustum of a cone.

[? Ex2.6 q16-18, 31-33]

15. Lateral area of regular prism: equilateral triangle base of side 1.092 m, $h = 1.025$ m

16. Lateral area of right circular cylinder: diameter = 250 ft, $h = 347$ ft

17. Volume of hemisphere: diameter = 0.83 yd

18. Volume of regular pyramid: square base of side 22.4 m, $s = 14.2$ m

$V = lwh$	Rectangular solid (Fig. 2.101)	(2.14)
$A = 2lw + 2lh + 2wh$		(2.15)
$V = e^3$	Cube (Fig. 2.102)	(2.16)
$A = 6e^2$		(2.17)
$V = \pi r^2 h$	Right circular cylinder (Fig. 2.103)	(2.18)
$A = 2\pi r^2 + 2\pi rh$		(2.19)
$S = 2\pi rh$		(2.20)
$V = Bh$	Right prism (Fig. 2.104)	(2.21)
$S = ph$		(2.22)
$V = \frac{1}{3}\pi r^2 h$	Right circular cone (Fig. 2.105)	(2.23)
$A = \pi r^2 + \pi rs$		(2.24)
$S = \pi rs$		(2.25)
$V = \frac{1}{3}Bh$	Regular pyramid (Fig. 2.106)	(2.26)
$S = \frac{1}{2}ps$		(2.27)
$V = \frac{4}{3}\pi r^3$	Sphere (Fig. 2.107)	(2.28)
$A = 4\pi r^2$		(2.29)

31. The Great Pyramid of Egypt has a square base approximately 250 yd on a side. The height of the pyramid is about 160 yd. What is its volume? See Fig. 2.117.

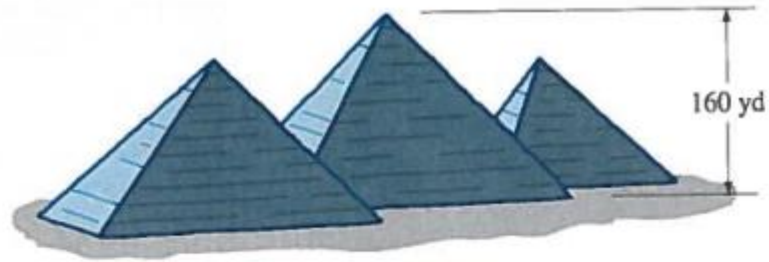


Fig. 2.117

32. A paper cup is in the shape of a cone as shown in Fig. 2.118. What is the surface area of the cup?

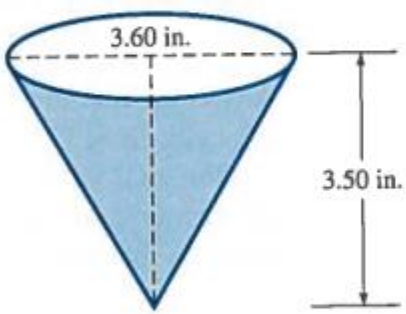


Fig. 2.118

Fig. 2.119

33. *Spaceship Earth* (shown in Fig. 2.119) at Epcot Center in Florida is a sphere of 165 ft in diameter. What is the volume of *Spaceship Earth*?

A

15. $S = \frac{1}{2}ps = \frac{1}{2}(3\times 1.092)(1.025) = 3.358 \text{ m}^2$

16. $S = 2\pi rh = 2\pi(d/2)h = 2\pi(250/2)(347)$
$= 273{,}000 \text{ ft}^2$

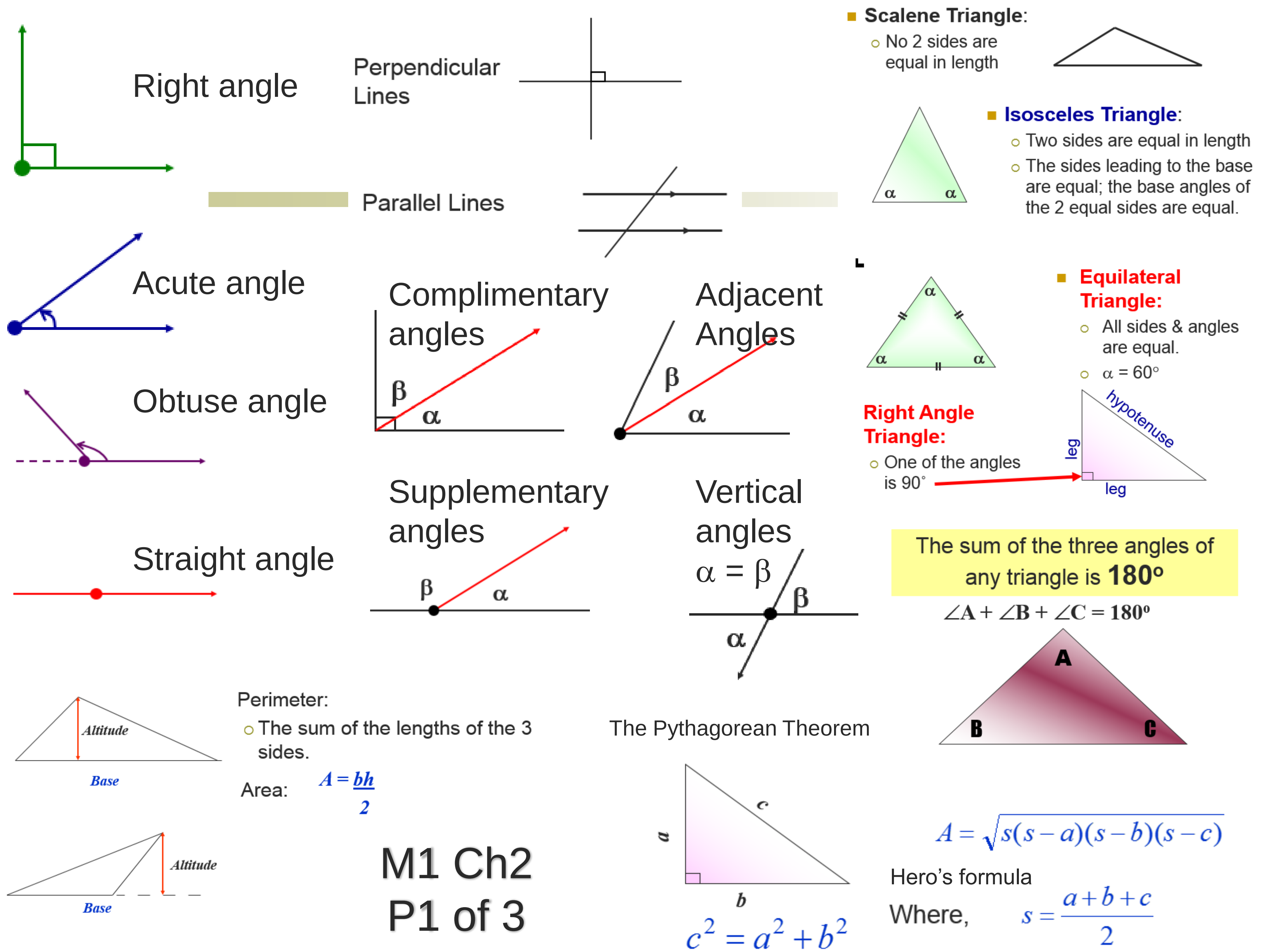
17. $V = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right) = \frac{2}{3}\pi\left(\frac{0.83}{2}\right)^3 = 0.15 \text{ yd}^3$

18. $b = \frac{22.4}{2} = 11.2;\ h = \sqrt{s^2 - b^2} = \sqrt{14.2^2 - 11.2^2}$
$= 8.73 \text{ m}$
$V = \frac{1}{3}Bh = \frac{1}{3}(22.4)^2(8.73) = 1460 \text{ m}^3$

31. $V = \frac{1}{3}BH = \frac{1}{3}(250^2)(160) = 3{,}300{,}000 \text{ yd}^3$

32. $s = \sqrt{h^2 + r^2} = \sqrt{3.50^2 + 1.80^2} = 3.94 \text{ in.}$
$S = \pi rs = \pi(1.80)(3.94) = 22.3 \text{ in.}^2$

33. $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(d/2)^3$
$= \frac{4}{3}\pi(165/2)^3$
$= 2.35\times 10^6 \text{ ft}^3$



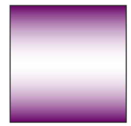
Right angle
Perpendicular Lines
Parallel Lines
Acute angle
Obtuse angle
Straight angle
Complimentary angles
β
α
Adjacent Angles
β
α
Supplementary angles
β
α
Vertical angles
α = β
β
α
Scalene Triangle:
No 2 sides are equal in length
Isosceles Triangle:
Two sides are equal in length
The sides leading to the base are equal; the base angles of the 2 equal sides are equal.
α
α
Equilateral Triangle:
All sides & angles are equal.
α = 60°
α
α
α
Right Angle Triangle:
One of the angles is 90˚
hypotenuse
leg
leg
The sum of the three angles of any triangle is 180º
∠A + ∠B + ∠C = 180º
A
B
C
Altitude
Base
Altitude
Base
Perimeter:
The sum of the lengths of the 3 sides.
Area:
$A = \frac{bh}{2}$
M1 Ch2
P1 of 3
The Pythagorean Theorem
c
a
b
$c^2 = a^2 + b^2$
$A = \sqrt{s(s-a)(s-b)(s-c)}$
Hero’s formula
Where, $s = \frac{a+b+c}{2}$

- **Parallelogram**:
  - Opposite sides are parallel

- **Rectangle**:
  - A parallelogram in which intersecting sides are perpendicular

M1 Ch2
P2 of 3

- **Rhombus**:
  - A parallelogram with 4 equal sides



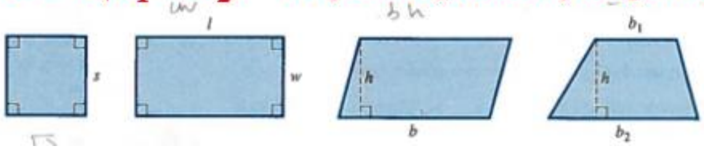
- **Square**:
  - A rectangle with four equal sides

Perimeter:

- The sum of the lengths of the four sides.

Area:

- $A = x^2$ Square of side x
- $A = lw$ *Rectangle of length* l *and width* w
- $A = bh$ *Parallelogram of base* b *&* *height* h
- $A = \frac{1}{2}(b_1 + b_2)$ *Trapezoid of bases* b_1 *&* b_2 *and height* h



$V = lwh$ $A = 2lw + 2lh + 2wh$	Rectangular solid (Fig. 2.101)
$V = e^3$ $A = 6e^2$	Cube (Fig. 2.102)
$V = \pi r^2 h$ $A = 2\pi r^2 + 2\pi rh$ $S = 2\pi rh$	Right circular cylinder (Fig. 2.103)
$V = Bh$ $S = ph$	Right prism (Fig. 2.104)
$V = \frac{1}{3}\pi r^2 h$ $A = \pi r^2 + \pi rs$ $S = \pi rs$	Right circular cone (Fig. 2.105)
$V = \frac{1}{3}Bh$ $S = \frac{1}{2}ps$	Regular pyramid (Fig. 2.106)
$V = \frac{4}{3}\pi r^3$ $A = 4\pi r^2$	Sphere (Fig. 2.107)

Simpson's Rule

$$A = \frac{h}{3}\left(y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 + \cdots + 4y_{n-1} + y_n\right)$$

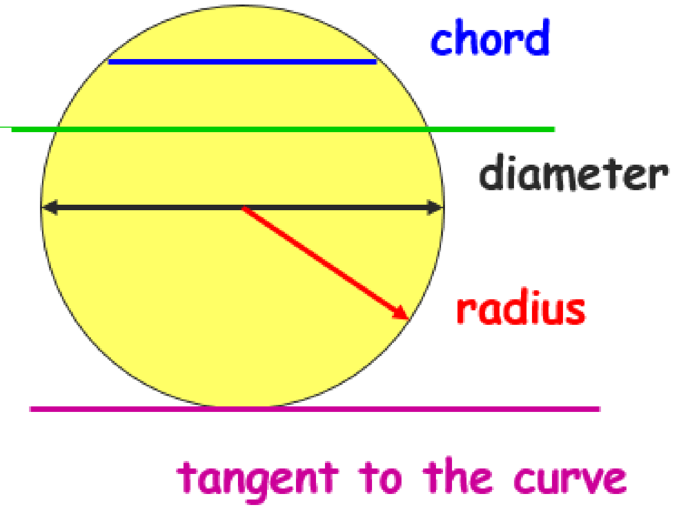
Trapezoidal Rule

$$A = \frac{h}{2}\left(y_0 + 2y_1 + 2y_2 + \cdots + 2y_{n-1} + y_n\right)$$

Circumference is a perimeter of a circle and is given by $C = 2\pi r$

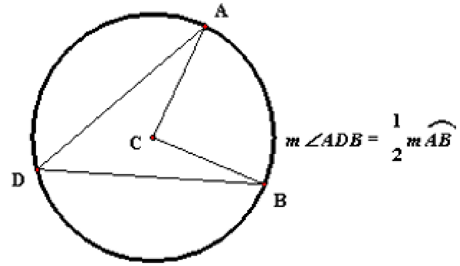
$A_{circle} = \pi r^2$

Terminology related to the circle.



An *arc* is part of a circle.

A *central angle* is an angle formed at the centre by two radii.

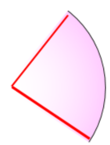


Segment of a Circle

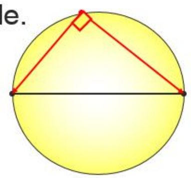
The region bounded by a chord and its arc.

The Sector of a Circle

The region bounded by 2 radii and the arc they intercept.

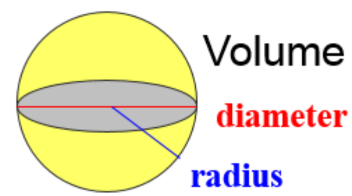


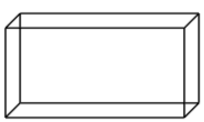
An angle inscribed in a semi-circle is a right angle.



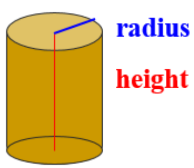
Surface area = $4 \pi r^2$

Volume = $\frac{4}{3} \pi r^3$

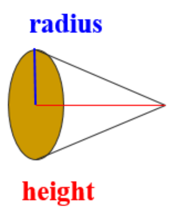




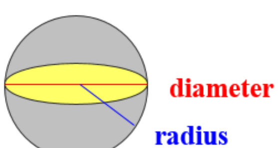
$V = L W H$



$V = \pi r^2 h$

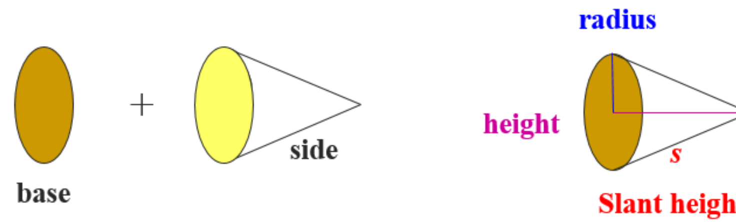


$V = \frac{1}{3} \pi r^2 h$



$V = \frac{4}{3} \pi r^3$

M1 Ch2 P3 of 3



Surface area $_{cone} = \pi r^2 + \pi r s$

Chapter 5

Systems of Linear Equations

Learning Outcomes

- At the end of this chapter the student will:
  - Describe linear equations & their solution graphically and algebraically.
  - Solve a system of linear equations in two and three unknowns graphically, algebraically and using determinants.

Ch. 5.1: Linear Equations

- An equation is termed **linear** in a given set of variables if each term contains only **one** variable to the **first** power or is a constant.

Linear Equations

- A linear equation in one unknown is written in the form:

$$\boxed{ax + b = 0}$$

- The solution to a linear equation is known as the root of the equation.

$$\boxed{x = -\frac{b}{a}}$$

Linear Equations

- A linear equation in two unknowns is written in the form:

$$\boxed{ax + by = 0}$$

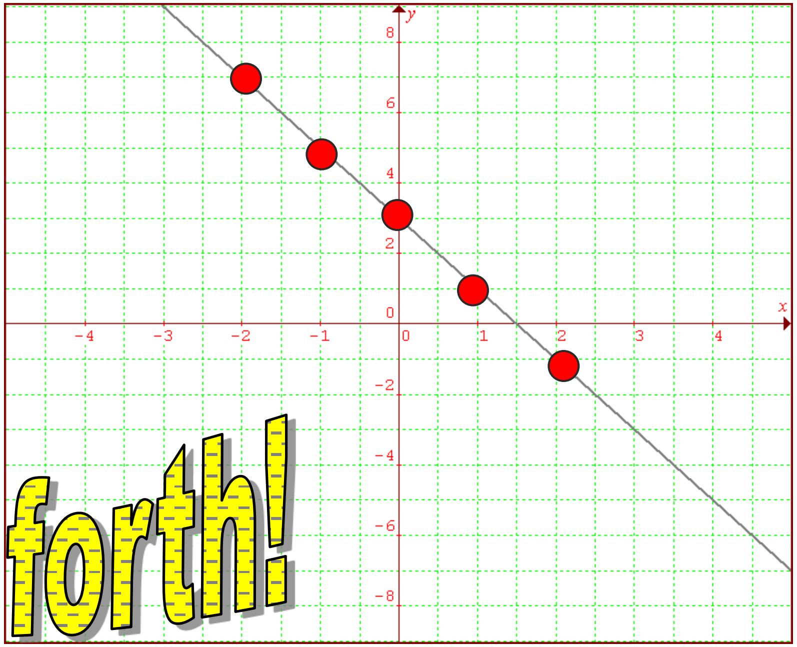
- The solution is any set of numbers, one for each variable, that satisfies the equation.

Linear Equations

- The solution of a linear equation in two unknowns can be graphed as a set of points.
- **Example**:
- What is the solution to $2x + y = 3$?

Solution

- We find a set of points as a part of the solution to $2x + y = 3$:
- (-2, 7)
- (-1, 5)
- (0, 3)
- (1, 1)
- (2, -1)



and so forth!

A System of Simultaneous Linear Equations: ***2 equations in 2 unknowns***

- Two linear equations, each containing the same two unknowns:

$$\boxed{\begin{aligned} a_1 x + b_1 y &= c_1 \\ a_2 x + b_2 y &= c_2 \end{aligned}}$$

- A solution of the system is any pair of values (***x, y***) that satisfies both equations.

■ See the chapter introduction.

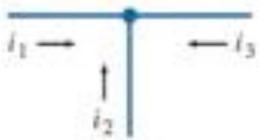


Fig. 5.1

EXAMPLE 3 **Applications of linear equations**

(a) A basic law of direct-current electricity, known as *Kirchhoff's current law*, may be stated as "The algebraic sum of the currents entering any junction in a circuit is zero." If three wires are joined at a junction as in Fig. 5.1, this law leads to the linear equation.

$$i_1 + i_2 + i_3 = 0$$

where i_1, i_2, and i_3 are the currents in each of the wires. (Either one or two of these currents must have a negative sign, showing that it is actually leaving the junction.)

(b) Two forces, F_1 and F_2, acting on a beam might be related by the equation

$$2F_1 + 4F_2 = 200$$

■

An equation that can be written in the form

$$\boxed{ax + by = c} \qquad (5.2)$$

is a **linear equation in two unknowns.** For such equations, in Chapter 3, we found that for each value of x there is a corresponding value of y. Each of these pairs of numbers is a *solution* to the equation. *A* **solution** *is any set of numbers, one for each variable, that satisfies the equation.*

NOTE ▶

? Ex 5.1 p137 q 9-14 and 29

In Exercises 9–14, for each given value of x, determine the value of y that gives a solution to the given linear equations in two unknowns.

9. $3x - 2y = 12; \quad x = 2, x = -3$

10. $-5x + 6y = 60; \quad x = -10, x = 8$

11. $x - 4y = 2; \quad x = 3, x = -0.4$

12. $3x - 2y = 9; \quad x = \frac{2}{3}, x = -3$

13. $24x - 9y = 16; \quad x = 2/3, x = -1/2$

14. $2.4y - 4.5x = -3.0; \quad x = -0.4, x = 2.0$

29. The forces acting on part of a structure are shown in Fig. 5.4. An analysis of the forces leads to the equations

$0.80F_1 + 0.50F_2 = 50$

$0.60F_1 - 0.87F_2 = 12$

Are the forces 45 N and 28 N?

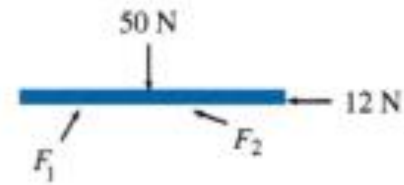


Fig. 5.4

A

9. $3(2) - 2y = 12;\ 2y = 6 - 12 = -6;\ y = -3$

$3(-3) - 2y = 12;\ 2y = -9 - 12 = -21;\ y = -\frac{21}{2}$

A

10. $-5(-10)+6y=60;\ 6y=60-50=10;\ y=\frac{5}{3}$

$-5(8)+6y=60;\ 6y=60+40=100;\ y=\frac{50}{3}$

11. $3-4y=2;\ -4y=-1;\ y=\frac{1}{4}$

$-0.4-4y=2;\ -4y=2.4;\ y=-0.6$

12. $3(2/3)-2y=9;\ 2-2y=9;\ -2y=7;\ y=-7/2$

$3(-3)-2y=9;\ -9-2y=9;\ -2y=18;\ y=-9$

13. $24(2/3)-9y=16;\ 9y=16-16=0;\ y=0$

$24(-1/2)-9y=16;\ 9y=-12-16=-28;$

$y=(-28/9)$

14. $2.4y-4.5(-0.4)=-3.0;\ 2.4y=-4.8;\ y=-2.0$

$2.4y=-4.5(2.0)=-3.0;\ 2.4y=6.0;\ y=2.5$

29. If $F_1=45$ N and $F_2=28$ N, then

$0.80(45)+0.50(28)=50;$

$0.60(45)-0.87(28)=2.64\neq 12$

Since both values do not satisfy both equations, they are not a solution.

Ch. 5.2: Graphs of Linear Equations

- A system of linear equations can be solved graphically.
- Solving graphically will produce at least one line.
- We can determine the steepness of that line by its slope.

Slope of a Line

- Given two points, $A\ (x_1, y_1)$ and $B\ (x_2, y_2)$ existing on a line, the slope, $\boldsymbol{m}$, of this line is defined as:

$$\boxed{m = \frac{y_2 - y_1}{x_2 - x_1}}$$

- Slope is often known as ***rise over run*****.**

Slope of a Line

- Slope is called **positive** as x increases, y increases.
- Slope is considered **negative** when as x increases, y decreases.
- The larger the absolute value of the slope, the steeper is the line.

Slope-Intercept Form of the Equation of a Straight Line

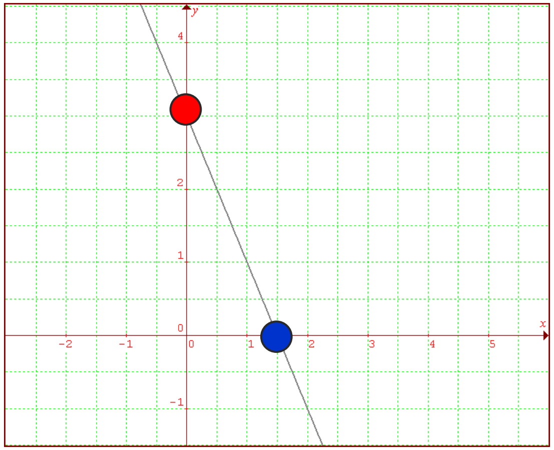
$$y = m x + b$$

Slope

y-intercept

Sketching Lines by Intercepts

- We solve the equation for $x = 0$ (the y-intercept) and $y = 0$ (the x-intercept).
- **Example**:
  - What are the x- and y-intercepts of $2x + y = 3$?
- **Answer:**
  - x-intercept: **(3/2, 0)**
  - y-intercept: **(0, 3)**



EXAMPLE 7 **Sketching line by intercepts**

Sketch the graph of the line $2x - 3y = 6$ by finding its intercepts and one check point. See Fig. 5.13.

First, we let $x = 0$. This gives us $-3y = 6$, or $y = -2$. This gives us the y-intercept, which is the point $(0, -2)$. Next, we let $y = 0$, which gives us $2x = 6$, or $x = 3$. This means the x-intercept is the point $(3, 0)$.

The intercepts are enough to sketch the line as shown in Fig. 5.13. To find a check point, we can use any value for x other than 3 or any value of y other than -2. Choosing $x = 1$, we find that $y = -\frac{4}{3}$. This means that the point $\left(1, -\frac{4}{3}\right)$ should be on the line. In Fig. 5.13, we can see that it is on the line.

Practice Exercise

3. Find the intercepts of the line $3x - 5y - 15 = 0$.

■ Further details and discussion of slope and the graphs of linear equations are found in Chapter 21.

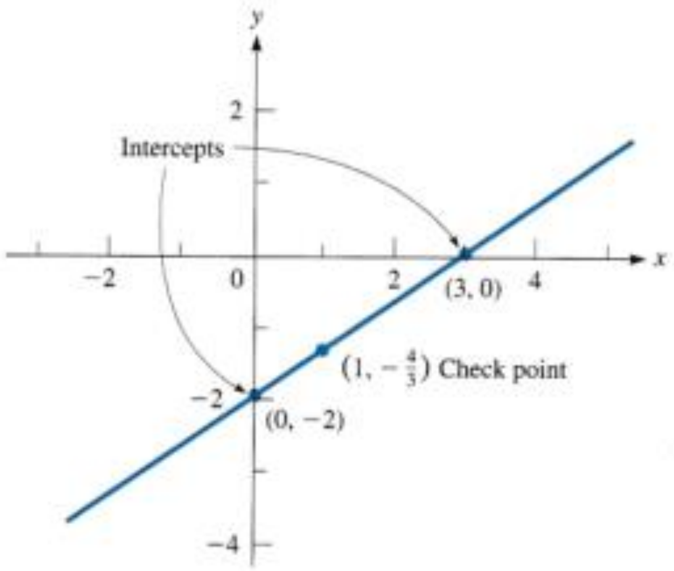


Fig. 5.13

? Ex 5.2 p 141 q 5-8, 21-24

In Exercises 5–12, find the slope of the line that passes through the given points.

5. $(1, 0), (3, 8)$ **6.** $(3, 1), (2, -7)$

7. $(-1, 2), (-4, 17)$ **8.** $(-1, -2), (2, 10)$

A

In Exercises 21–28, find the slope and the y-intercept of the line with the given equation and sketch the graph using the slope and the y-intercept. A graphing calculator can be used to check your graph.

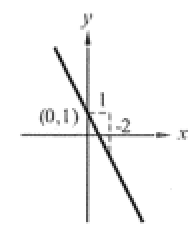
21. $y = -2x + 1$ **22.** $y = -4x$

23. $y = x - 4$ **24.** $y = \frac{4}{5}x + 2$

25. $5x - 2y = 40$ **26.** $-2y = 7$

21. $m = -2x + 1$, $m = -2$, $b = 1$

Plot the y-intercept point $(0, 1)$. Since the slope is $-2/1$, from this point, go over 1 unit and down 2 units, and plot a second point. Sketch the line between the 2 points.



A

5. By taking $(3, 8)$ as (x_2, y_2) and $(1, 0)$ as (x_1, y_1)

$$m = \frac{8-0}{3-1} = \frac{8}{2} = 4$$

6. By taking $(2, -7)$ as (x_2, y_2) and $(3, 1)$ as (x_1, y_1)

$$m = \frac{-7-1}{2-3} = -8$$

7. By taking $(-4, 17)$ as (x_2, y_2) and $(-1, 2)$ as (x_1, y_1)

$$m = \frac{17-2}{-4-(-1)} = \frac{15}{-4+1} = -\frac{15}{3} = -5$$

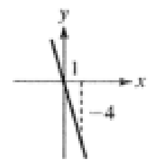
8. By taking $(2, 10)$ as (x_2, y_2) and $(-1, -2)$ as (x_1, y_1)

$$m = \frac{10-(-2)}{2-(-1)} = \frac{10+2}{2+1} = \frac{12}{3} = 4$$

22. $y = -4x$

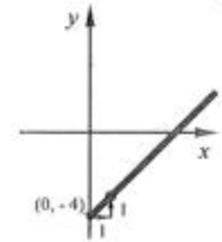
Since the equation is solved for y, the coefficient of x is the slope. Therefore the slope of the line is $-4/1$. Since the b-term is the y-intercept, the y-intercept of this line is $(0,0)$. Plot the y-intercept. From this point go down 4 units and over 1 unit to the right. Draw the line between these points.

$m = -4, b = 0$



23. $y = x - 4$

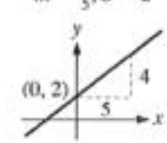
Since the equation is solved for y, the coefficient of x is the slope. Therefore the slope of the line is $1/1$. Since the b-term is the y-intercept, the y-intercept of this line is $(0, -4)$. Plot the y-intercept. From this point go up 1 unit and over 1 unit to the right. Draw the line between these points.



24. $y = \left(\frac{4}{5}\right)x + 2$

Since the equation is solved for y, the coefficient of x is the slope. Therefore the slope of the line is $4/5$. Since the b-term is the y-intercept, the y-intercept of this line is $(0, 2)$. Plot the y-intercept. From this point go up 4 units and over 5 units to the right. Draw the line between these points.

$m = \frac{4}{5}, b = 2$



Ch. 5.3: Solving **Systems** of Linear Equations in Two Unknowns **Graphically**

- Having 2 or more equations and finding their intersection point.
- However, this will not always be possible.

1) The Independent System

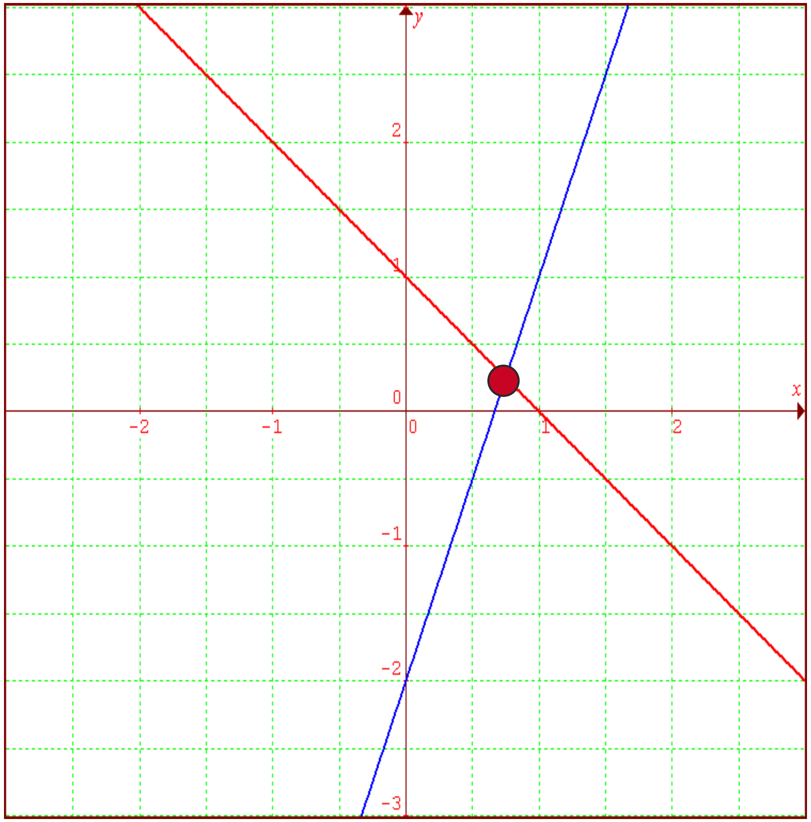
- There will be one point that will satisfy both equations.
- In this way, an intersection point has been found when solving a system of linear equations.

The Independent System

- There exists one unique solution for the two equations.
- Graphically, the two lines intersect at one point (x, y).
- We would obtain some real value for x and y.
  - E.g. $\boldsymbol{x = a}$ and $\boldsymbol{y = b}$.

The Independent System

- Given:
  - $3x - y = 2$
  - $x + y = 1$
- One solution.
- Graphs intersect at **(¾, ¼)** .



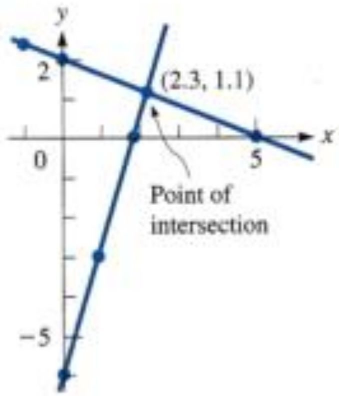


Fig. 5.18

Practice Exercise

1. In Example 2, change the 6 to 3, and then solve. (Can be done using Fig. 5.18.)

EXAMPLE 2 Determine the point of intersection

Solve the system of equations

$$\begin{aligned} 2x + 5y &= 10 \\ 3x - \quad y &= \ \ 6 \end{aligned}$$

We could write each equation in slope-intercept form in order to sketch the lines. Also, we could use the form in which they are written to find the intercepts. Choosing to find the intercepts and draw lines through them, let $y = 0$; then $x = 0$. Therefore, the intercepts of the first line are the points $(5, 0)$ and $(0, 2)$. A third point is $\left(-1, \frac{12}{5}\right)$. The intercepts of the second line are $(2, 0)$ and $(0, -6)$. A third point is $(1, -3)$. Plotting these points and drawing the proper straight lines, we see that the lines cross at about (2.3, 1.1). [The exact values are $\left(\frac{40}{17}, \frac{18}{17}\right)$.] The solution of the system of equations is approximately $x = 2.3$, $y = 1.1$ (see Fig. 5.18).

Checking, we have

$$2(2.3) + 5(1.1) \stackrel{?}{=} 10 \quad \text{and} \quad 3(2.3) - 1.1 \stackrel{?}{=} 6$$
$$10.1 \approx 10 \qquad\qquad 5.8 \approx 6$$

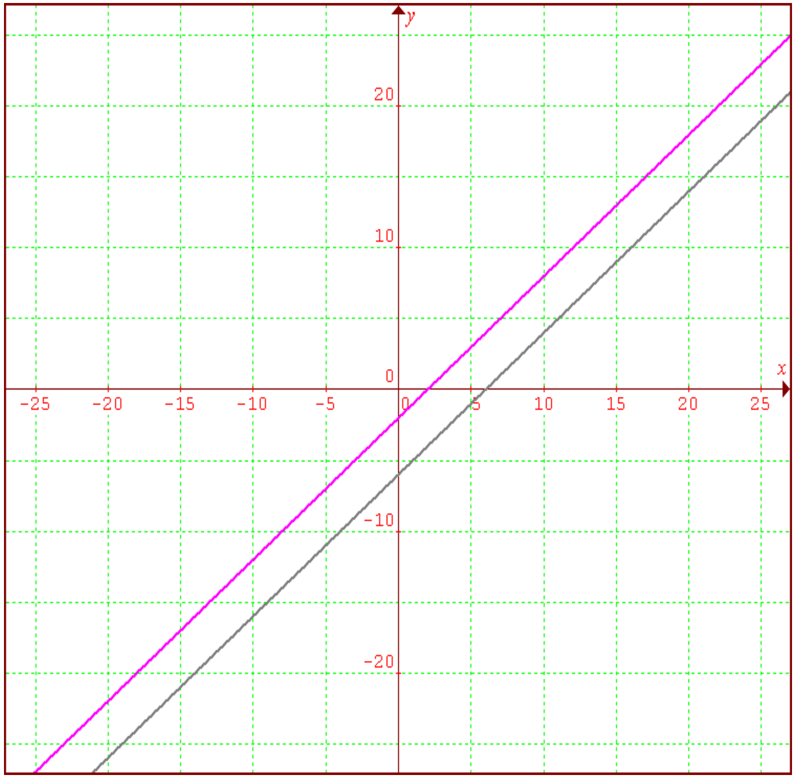
This shows the solution is correct to the accuracy we can get from the graph. ■

2) The Inconsistent System

- There is **no solution** to these types of systems of linear equations.
- Graphically, these **lines never meet** in the plane.
- If we were to work this out algebraically, we would find a false solution.
- That is: $\mathbf{0} = \boldsymbol{a}$

No solution

- Given:
  - $2x - 2y = 4$
  - $x - y = 6$
- Upon graphing this system of linear equations, we find 2 parallel lines.
- These lines do not touch. (No solution)

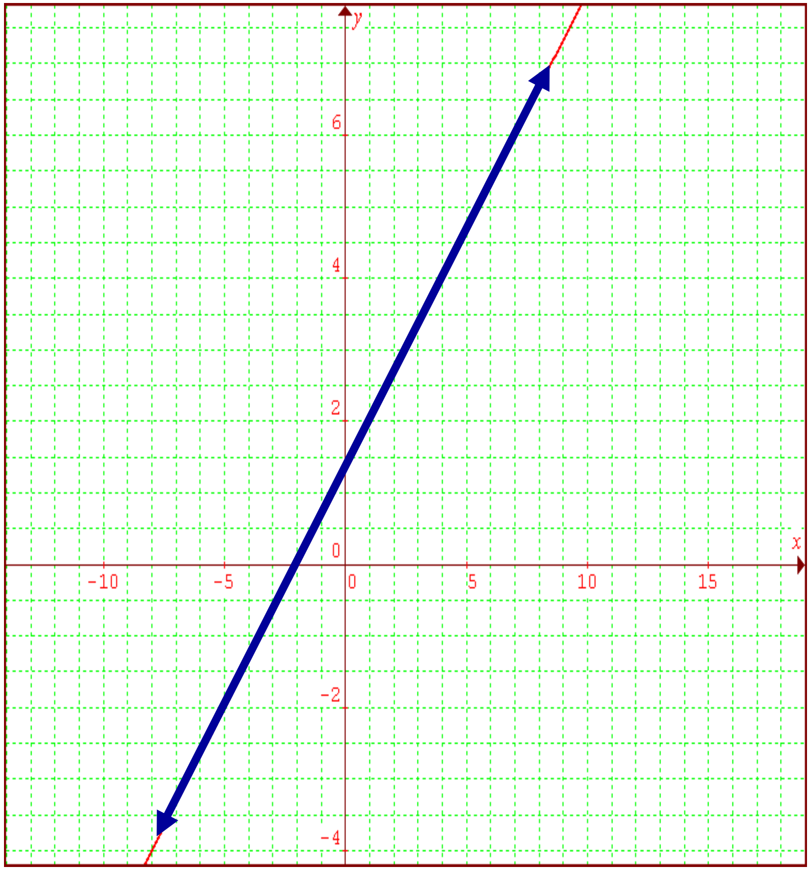


4) Similar lines (Dependent system)

- There are infinitely many solutions to the 2 equations.
- Graphically, the two lines are one and the same line.
- There are an infinite number of solutions (i.e., ordered pairs) to this system.
- When algebraically solving this system, we find a true statement, $\mathbf{0} = \mathbf{0}$.

Infinite solutions

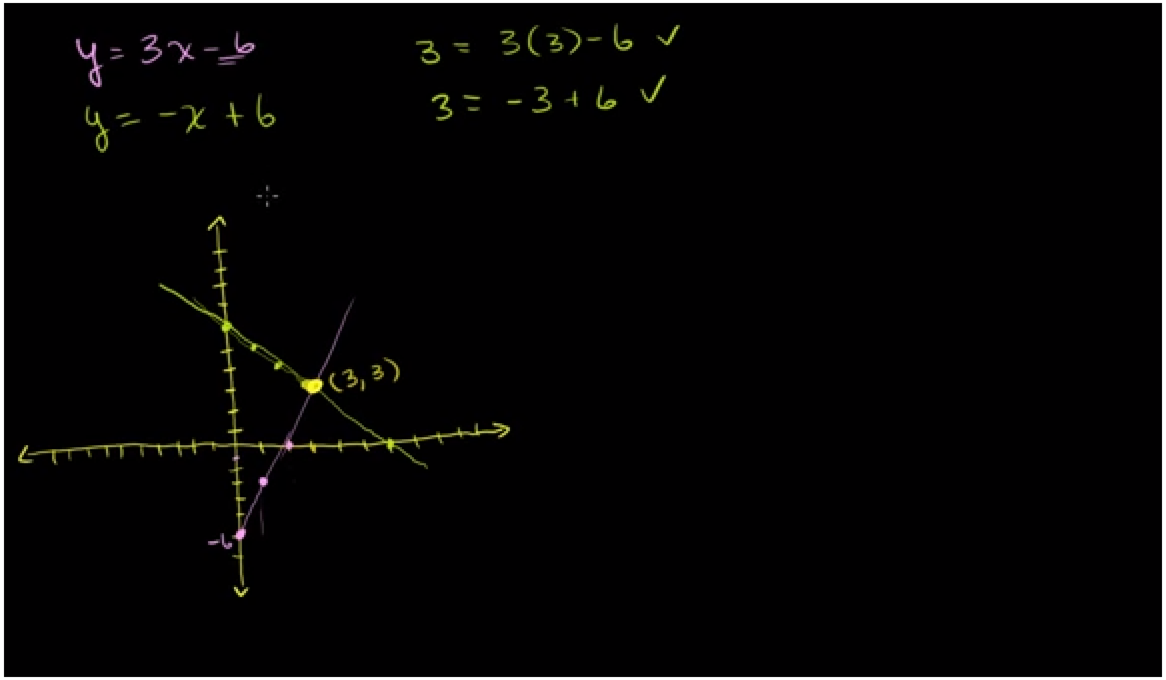
- Given:
  - $-2x + 3y = 4$
  - $4x - 6y = -8$
- By viewing the graph, we note that there is only one line.
- The first equation sits on top of the other one!



Summary

- Graphically, we find the intersection point of the given lines.
- Algebraically, we find the ordered pair that satisfies each equation if the lines are independent.
- There is no solution if the lines are parallel to each other and considered inconsistent.
- There are an infinite number of solutions if the lines lie on top of each other and considered dependent lines.

https://www.khanacademy.org/math/algebra/systems-of-eq-and-ineq/fast-systems-of-equations/v/solving-linear-systems-by-graphing



? Ex5.3 p145 q3-6

In Exercises 3–20, solve each system of equations by sketching the graphs. Use the slope and the y-intercept or both intercepts. Estimate each result to the nearest 0.1 if necessary.

3. $y = -x + 4$
$y = x - 2$

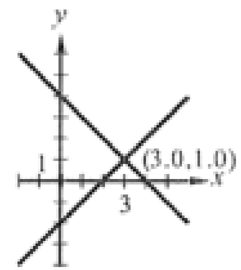
4. $y = \frac{1}{2}x - 1$
$y = -x + 8$

5. $y = 2x - 6$
$y = -\frac{1}{3}x + 1$

6. $y = \frac{1}{2}x - 4$
$y = 2x + 2$

3. $y = -x + 4$ and $y = x - 2$

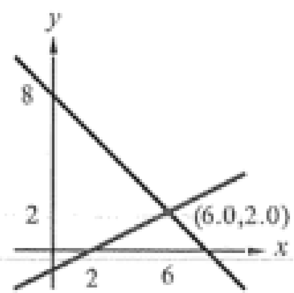
The slope of the first line is -1, and the y-intercept is 4. The slope of the second line is 1 and the y-intercept is -2. From the graph, the point of intersection is $(3.0, 1.0)$. Therefore, the solution of the system of equations is $x = 3.0$, $y = 1.0$.



A

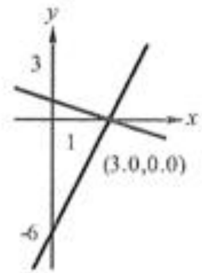
4. $y = \left(\frac{1}{2}\right)x - 1;\ y = -x + 8$

The slope of the first line is $1/2$, and the y-intercept is -1. The slope of the second line is -1, and the y-intercept is 8. From the graph, the point the point of intersection is $(6.0, 2.0)$. Therefore, the solution of the system of equations is $x = 6.0$, $y = 2.0$.



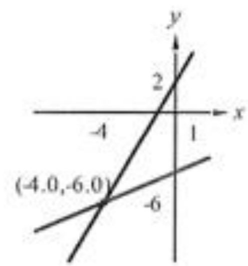
5. $y = 2x - 6;\ y = -\left(\frac{1}{3}\right)x + 1$

The slope of the first line is 2, and the y-intercept is -6. The slope of the second line is $-1/3$ and the y-intercept is 1. From the graph, the point of intersection is $(3.0, 0.0)$. Therefore, the solution of the system of equations is $x = 3.0$, $y = 0.0$.



6. $y = \left(\frac{1}{2}\right)x - 4;\ y = 2x + 2$

The slope of the first line is 1/2, and the y-intercept is -4. The slope of the second line is 2 and the y-intercept is 2. From the graph, the point of intersection is $(-4.0, -6.0)$. Therefore, the solution of the system of equations is $x = -4.0$, $y = -6.0$.



The rest of the chapter is not in maths 1

Summary Ch 5

Straight line

$$y = mx + b$$

Slope

y-intercept

Use intercepts on axis and/or gradient to draw

Solve system of linear equations by drawing a straight line and find the intercept

Chapter 6

Factoring and Fractions

7 8	Quadratic equations	Factoring Solving quadratic equations	6.1 Special Products 6.2 factoring: common factors and difference between two squares 6.3 factoring trinomials 7.1 Quadratic equations: Solution by factoring 7.2 Completing the square 7.3 The quadratic formula 7.4 The graph of the quadratic function

Learning Outcomes

- At the end of this chapter the student will:
  - Solve algebraic products,
  - Factor polynomials,
  - Recognize special products to multiply and factor readily,
  - Solve equations involving fractions.

Ch. 6.1: Special Products

- We apply the distributive law when multiplying algebraic products.
- **We need to become familiar with the *special products* to ease our work with polynomials.**
- Work with literal numbers as you would with numbers.

We could use this equation to expand the brackets

$$(a+b)(c+d)$$

Last

Inner

$$(a+b)(c+d) = ac + ad + bc + bd$$

F O I L

First

Outer

We also have special products to help us do expansiosn

Special Products

Equations to help you

1. $a(x+y) = ax + ay$

2. $(x+y)(x-y) = x^2 - y^2$

3. $(x+y)^2 = x^2 + 2xy + y^2$

Special Products *(continued)*

4 $$(x-y)^2 = x^2 - 2xy + y^2$$

5 $$(x+a)(x+b) = x^2 + (a+b)x + ab$$

6 $$(ax+b)(cx+d) = acx^2 + (ad+bc)x + bd$$

Example 1

- Multiply: $5x\ (3 - 8y)$
- We use: 1. $a(x + y) = ax + ay$

$5x\ (3 - 8y)$

$= 15x - 40xy$

1. Multiply the outside term with the 1st term of the binomial.
2. Multiply the outside term with the 2nd term of the binomial.

- In this way, each term is multiplied together.

Example 2

- Multiply: $(x + 4)^2$
- We use: **3.** $(x + y)^2 = x^2 + 2xy + y^2$
- ***Solution:***

$$(x + 4)^2 = (x + 4)\,(x + 4)$$

$$= x^2 + (2\times 4)x + 4^2$$

$$= x^2 + 8x + 16$$

$$(x + y)^2 \neq x^2 + y^2$$

Example 3

- Multiply: $(x + 7)\ (x - 7)$
- We use: 5. $(x + y)(x - y) = x^2 - y^2$
- ***Solution:***

$$(x + 7)\ (x - 7) = x^2 - 7x + 7x - 7^2$$

$$= x^2 - 49$$

A Difference of Squares

Example 4

- Multiply: $(x - 8)(2x + 1)$
- We use:

  6. $(ax + b)(cx + d) = acx^2 + (ad + bc)x + bd$

- ***Solution:***

$(x - 8)(2x + 1)$

$= (1 \times 2)x^2 + (1 \times 1 - 8 \times 2)x + (8 \times 1)$

$= 2x^2 - 15x + 8$

Each term is multiplied by each other term.

Special Products Involving Cubes

1. $$(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$$

2. $$(x-y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$$

3. $$(x+y)(x^2 - xy + y^2) = x^3 + y^3$$

4. $$(x-y)(x^2 + xy + y^2) = x^3 - y^3$$

Example 5

3 $(x+y)(x^2-xy+y^2)=x^3+y^3$

- Multiply: $(x + 5)(x^2 – 5x + 25)$
- ***Solution:***
- We use **#3 recognizing** the **5** and its square **25**.

$$(x + 5)(x^2 – 5x + 25) = x^3 + 125$$

Summary

- Recognizing the forms that polynomials take will assist in finding their products.
- Being able to identify the products with a difference of squares and perfect squares and cubes will help in simplifying algebraic products.

?6.1 5-11, 18-23

$a(x+y) = ax + ay$

$(x+y)(x-y) = x^2 - y^2$

$(x+y)^2 = x^2 + 2xy + y^2$

$(x-y)^2 = x^2 - 2xy + y^2$

$(x+a)(x+b) = x^2 + (a+b)x + ab$

$(ax+b)(cx+d) = acx^2 + (ad+bc)x + bd$

5. $40(x-y)$

6. $2x(a-3)$

7. $2x^2(x-4)$

8. $3a^2(2a+7)$

9. $(T+6)(T-6)$

10. $(s+2t)(s-2t)$

11. $(3v-2)(3v+2)$

12. $(ab-c)(ab+c)$

17. $(5f+4)^2$

18. $(i_1+3)^2$

19. $(2x+17)^2$

20. $(9a+8b)^2$

21. $(L^2-1)^2$

22. $(b^2-6)^2$

23. $(4a+7xy)^2$

24. $(3A+10z)^2$

SOVLVE USING THE SPECIAL PRODUCTS ABOVE

A

$= x^3 - 12x^2 + 48x - 64$

5. $40(x-y) = 40x - 40y$

6. $2x(a-3) = 2ax - 6x$

7. $2x^2(x-4) = 2x^3 - 8x^2$

8. $3a^2(a+7) = 6a^3 + 21a^2$

9. $(T+6)(T-6) = T^2 - 6^2 = T^2 - 36$

10. $(s+2t)(s-2t) = s^2 - 2st + 2st - 4t^2$
$= s^2 - 4t^2$

11. $(3v-2)(3v+2) = 9v^2 - 6v + 6v - 4$
$= 9v^2 - 4$

18. $(i_i+3)^2 = (i_1)^2 + 2(i_1)(3) + 3^2$
$= i_1^2 + 6i_1 + 9$

19. $(2x+17)^2 = (2x)^2 + 2(2x)(17) + 17^2$
$= 4x^2 + 68x + 289$

20. $(9a+8b)^2 = (9a)^2 + 2(9a)(8b) + (8b)^2$
$= 81a^2 + 144ab + 64b^2$

21. $(L^2-1)^2 = (L^2)^2 - 2 \cdot L^2 \cdot 1 + 1^2$
$= L^4 - 2L^2 + 1$

22. $(b^2-6)^2 = b^4 + 2(-6b^2) + 36 = b^4 - 12b^2 + 36$

23. $(4a+7xy)^2 = 16a^2 + 56axy + 49x^2y^2$

Ch. 6.2: Factoring: Common Factor and Difference of Squares

- To ***factor*** an expression we decompose that expression into its smallest factors.
- It involves reversing the process of finding a product.
- A polynomial or a factor is called ***prime*** if it contains no factors other than **+1** or **-1** and plus or minus itself.

Factoring


ALGEBRAIC EXPRESSION
Factoring
factor
factor
factor
and so on ...

$$a(x+y) = ax + ay$$
$$(x+y)(x-y) = x^2 - y^2$$
$$(x+y)^2 = x^2 + 2xy + y^2$$
$$(x-y)^2 = x^2 - 2xy + y^2$$
$$(x+a)(x+b) = x^2 + (a+b)x + ab$$
$$(ax+b)(cx+d) = acx^2 + (ad+bc)x + bd$$

Factoring

- The ability to factor algebraic expressions depends heavily on the proper **recognition** of the special products.
- ***Example***:

$$14x^2 + 21x$$

$$7x \quad (2x + 3)$$

Common Monomial Factors

- Given: $\mathbf{14x^2 + 21x = 7x(\,2x + 3)}$

$\mathbf{7x}$ → common monomial factor

Common Monomial Factors

- We check the result by multiplication:

$$7x(2x + 3) = 14x^2 + 21x$$

Factoring with '1'

- Factor: $15x + 45x^2$

  $= 15x(1 + 3x)$

- When factoring out $15x$, 1 is left behind to remind us that $15x$ exists in the original algebraic expression.
- Multiplying back through provides the check that we need.

Factoring the Difference of Two Squares

- The factors only differ by the middle sign.
- In general: $(x + y)(x - y) = x^2 - y^2$
- So when factoring, we know that we can decompose $x^2 - y^2$ into $(x + y)(x - y)$
- Also note:

$$(ax + by)(ax - by) = (ax)^2 - (by)^2$$

$$= a^2x^2 - b^2y^2$$

Example

$a(x+y) = ax + ay$
$(x+y)(x-y) = x^2 - y^2$
$(x+y)^2 = x^2 + 2xy + y^2$
$(x-y)^2 = x^2 - 2xy + y^2$
$(x+a)(x+b) = x^2 + (a+b)x + ab$
$(ax+b)(cx+d) = acx^2 + (ad+bc)x + bd$

- Factor: $4x^2 - 9$

$= (2x)^2 - 3^2$

$= (2x + 3)(2x - 3)$ ✓

a difference of squares

Complete Factoring

- Factoring an algebraic expression may require more than one step.
- ***Example***:

$$6x^3 - 24x$$

$$= 6x(x^2 - 4)$$

difference of squares

$$= 6x\,(x + 2)(x - 2)$$

https://www.khanacademy.org/math/algebra/multiplying-factoring-expression/factoring-special-products/v/factoring-difference-of-squares

https://www.khanacademy.org/math/algebra/multiplying-factoring-expression/factoring-special-products/v/factoring-to-produce-difference-of-squares

Factoring by Grouping

- In some polynomials, terms can be grouped together to help factor the algebraic expression.
- We look for a common binomial factor in these situations.

? 6.2

$a(x+y) = ax + ay$
$(x+y)(x-y) = x^2 - y^2$
$(x+y)^2 = x^2 + 2xy + y^2$
$(x-y)^2 = x^2 - 2xy + y^2$
$(x+a)(x+b) = x^2 + (a+b)x + ab$
$(ax+b)(cx+d) = acx^2 + (ad+bc)x + bd$

Factor the following

Look for the common multiplication factor

12. $5a^2 - 20ax$

14. $90p^3 - 15p^2$

16. $23a - 46b + 69c$

18. $4pq - 14q^2 - 16pq^2$

20. $27a^2b - 24ab - 9a$

22. $5a + 10ax - 5ay - 20az$

12. $5a^2 - 20ax = 5a(a-4x)$ ($5a$ is a c.m.f.)

A

12. $5a^2 - 20ax = 5a(a - 4x)$ ($5a$ is a c.m.f.)

13. $288n^2 + 24n = 24n(12n + 1)$ ($24n$ is a c.m.f.)

14. $90p^3 - 15p^2 = 15p^2(6p - 1)$ ($15p^2$ is a c.m.f.)

15. $2x + 4y - 8z = 2(x + 2y - 4z)$ (2 is a c.m.f.)

16. $23a - 46b + 69c = 23(a - 2b + 3c)$

17. $3ab^2 - 6ab + 12ab^3 = 3ab(b - 2 + 4b^2)$
($3ab$ is a c.m.f.)

18. $4pq - 14q^2 - 16pq^2 = 2q(2p - 7q - 8pq)$
($2q$ is a c.m.f.)

19. $12pq^2 - 8pq - 28pq^3 = 4pq(3q - 2 - 7q^2)$
($4pq$ is a c.m.f.)

20. $27a^2b - 24ab - 9a = 3a(9ab - 8b - 3)$
($3a$ is a c.m.f.)

21. $2a^2 - 2b^2 + 4c^2 - 6d^2 = 2(a^2 - b^2 + 2c^2 - 3d^2)$
(2 is a c.m.f.)

22. $5a + 10ax - 5ay + 20az = 5a(1 + 2x - y + 4z)$
($5a$ is a c.m.f.)

23. $x^2 - 4 = x^2 - 2^2 = (x - 2)(x + 2)$
(because $-2x + 2x = 0x = 0$)

24. $r^2 - 25 = r^2 - 5^2 = (r - 5)(r + 5)$
(because $-5r + 5r = 0r = 0$)

Ch. 6.3: Factoring Trinomials

- Recall:

$$(x+a)(x+b) = x^2 + \underbrace{(a+b)}_{\text{sum}}x + \underbrace{ab}_{\text{product}}$$

$$(x+a)(x+b)=x^2+\underbrace{(a+b)}_{\text{sum}}x+\underbrace{ab}_{\text{product}}$$

Observations:

- We are to find integers ***a*** and ***b***, and they are found by noting that:

1. The ***coefficient of x***^{2} is **1**.
2. The ***final constant*** is the ***product*** of the constants ***a*** and ***b*** in the factors, and,
3. The ***coefficient of x*** is the ***sum*** of ***a*** and ***b***.

Example

$$(x+a)(x+b) = x^2 + \underbrace{(a+b)}_{\text{sum}}x + \underbrace{ab}_{\text{product}}$$

- Factor:

$$x^2 \underbrace{- x}_{\text{sum}} \underbrace{- 6}_{\text{product}}$$

$$= x^2 + (-3 + 2)\, x + (-3 \times 2)$$

$$= (x - 3)\,(x + 2)$$

Factoring General Trinomials

- Recall:

$$acx^2 + (ad + bc)x + bd = (ax + b)(cx + d)$$

- ***Observations*:**

1. The coefficient of x^2 is the product of the coefficients a and c in the factors,
2. The final constant is the product of the constants b and d in the factors, and,
3. The coefficient of x is the sum of the inner and outer products.

Summary

- Be sure to factor an expression completely.
- We first look for the common monomial factors and then check each resulting factor to see if it can be factored when we complete each step.

? 6.3

16. $b^2 - 12bc + 36c^2$

18. $2n^2 - 13n - 7$

20. $25x^2 + 45x - 10$

22. $7y^2 - 12y + 5$

24. $5R^4 - 3R^2 - 2$

26. $3n^2 - 20n + 20$

28. $3x^2 + xy - 14y^2$

30. $2z^2 + 13z - 5$

$$(x+a)(x+b) = x^2 + \underbrace{(a+b)}_{\text{sum}}x + \underbrace{ab}_{\text{product}}$$

$$acx^2 + (ad + bc)x + bd = (ax + b)(cx + d)$$

A

16. $b^2 - 12bx + 36c^2 = (b-6c)(b-6c) = (b-6c)^2$

(because $-12bc = -6bc - 6bc$)

17. $3x^2 - 5x - 2 = (3x+1)(x-2)$

(because $-5x = -6x + x$)

18. $2n^2 - 13n - 7 = (2n+1)(n-7)$

(because $-13n = -14n + n$)

19. $12y^2 - 32y - 12 = 4(3y+1)(y-3), (-8y = y - 9y)$

20. $25x^2 + 45x - 10 = 5(5x-1)(x+2)$

(because $9x = 10x - x$)

21. $2s^2 + 13s + 11 = (2s+11)(s+1)$

(because $13s = 2s + 11s$)

22. $7y^2 - 12y + 5 = (7y-5)(y-1)$

(because $-12y = -7y - 5y$)

23. $3f^4 - 16f^2 + 5 = (3f^2 - 1)(f^2 - 5)$

24. $5R^4 - 3R^2 - 2 = (5R^2+2)(R^2-1)$

$= (5R^2+2)(R+1)(R-1)$

25. $2t^2 + 7t - 15 = (2t-3)(t+5)$

(because $7t = 10t - 3t$)

26. $3n^2 - 20n + 20 =$ prime (cannot be further factored)

27. $3t^2 - 7tu + 4u^2 = (3t-4u)(t-u)$

(because $-7tu = -4tu - 3tu$)

28. $3x^2 + xy - 14y^2 = (3x+7y)(x-2y)$

(because $xy = -6xy + 7xy$)

29. $4x^2 - 3x - 7 = (4x-7)(x+1)$

(because $-3x = 4x - 7x$)

30. $2z^2 + 13z - 5 =$ prime (cannot be further factored)

1 $a(x + y) = ax + ay$

2 $(x + y)(x - y) = x^2 - y^2$

3 $(x + y)^2 = x^2 + 2xy + y^2$

4 $(x - y)^2 = x^2 - 2xy + y^2$

5 $(x + a)(x + b) = x^2 + (a + b)x + ab$

6 $(ax + b)(cx + d) = acx^2 + (ad + bc)x + bd$

7 $(x+a)(x+b) = x^2 + \underbrace{(a+b)}_{\text{sum}}x + \underbrace{ab}_{\text{product}}$

M1 ch6

8 $acx^2 + (ad + bc)x + bd = (ax + b)(cx + d)$

Chapter 7

Quadratic Equations

Learning Outcomes

- At the end of this chapter the student will:
  - be able to identify the general form of the quadratic equation.
  - solve a quadratic equation by factoring, completing the square, using the quadratic formula and graphing.
  - describe characteristics of the graph of the quadratic equation.

Ch. 7.1: Quadratic Equations; Solution by Factoring

- The general form of the quadratic equation in x is:
  - Given that a, b and c are constants ($a \neq 0$), the equation:

$$ax^2 + bx + c = 0$$

Example 1

- Identify ***a***, ***b*** and ***c*** in the quadratic equation:

$$x^2 - 5x - 11 = 0$$

$a = 1$

$b = -5$

$c = -11$

Identifying Quadratic Equations

- Examples of equations which are not quadratic:

$$2x - 9 = 0$$

- No x^2 terms.

$$x^4 - 4x^3 - x^2 + 3 = 0$$

- No power of x should be higher than 2.

$$x^2 + 7x = x^2$$

- Simplifying produces a linear equation.

Solutions of a Quadratic Equation

- The solution of an equation consists of all numbers (**roots**) which, when substituted in the equation, give equality.
- There are 2 roots in a solution of a quadratic equation.

Examples of Solutions of a Quadratic Equation

① $x^2 - 5x + 6 = 0$

$x = 2, x = 3$

real roots

Examples of Solutions of a Quadratic Equation (*continued*)

② $x^2 - 4x + 4 = 0$

$$x = 2, x = 2$$

double roots

Examples of Solutions of a Quadratic Equation (*continued*)

③ $x^2 + 1 = 0$

$$x = \sqrt{-1},\ x = -\sqrt{-1}$$

imaginary roots

Procedure for Solving a Quadratic Equation by Factoring

1. Collect all terms on the left & simplify into the general quadratic equation form.
2. Factor the quadratic expression.
3. Set each factor equal to zero.
4. Solve the resulting linear equations. These numbers are the roots of the quadratic equation.
5. Check the solutions in the original equations.

Example 2

Can use special products

- Solve the quadratic equation by factoring.

$$x^2 + 2x - 15 = 0$$

factor

$$(x+5)(x-3) = 0$$

set each factor to 0

$\boldsymbol{x} + 5 = 0$ $\boldsymbol{x} - 3 = 0$

solve

$\boldsymbol{x} = -5$ $\boldsymbol{x} = 3$

Solving a Quadratic Equation by Factoring

- Remember to check the solutions by substituting into the original equation.
- The answer should be **‘0’**.
- It is essential for the quadratic expression on the left to be equal to zero on the right.
- If a product equals a nonzero number, it is probable that neither factor will give a correct root.

? Ex 7.1

In Exercises 3–8, determine whether or not the given equations are quadratic. If the resulting form is quadratic, identify a, b, and c, with $a > 0$. Otherwise, explain why the resulting form is not quadratic.

3. $x(x-2)=4$

4. $(3x-2)^2=2$

5. $x^2=(x+2)^2$

6. $x(2x^2+5)=7+2x^2$

7. $n(n^2+n-1)=n^3$

8. $(T-7)^2=(2T+3)^2$

In Exercises 9–44, solve the given quadratic equations by factoring.

9. $x^2-4=0$

10. $B^2-400=0$

11. $4x^2=9$

12. $x^2=0.16$

13. $x^2-8x-9=0$

14. $x^2+x-6=0$

A

3. $x(x-2)=4;\ x^2-2x-4=0;\ a=1,\ b=-2,$
$c=-4$

4. $(3x-2)^2=2;\ 9x^2-12x+4=2;\ 9x^2-12x+2=0$
$a=9,\ b=-12,\ c=2$

5. $x^2=(x+2)^2$
$x^2=x^2+4x+4$
$4x+4=0,$ no x^2 term, not quadratic

6. $x(2x^2+5)=7+2x^2;\ 2x^3+5x=7+2x^2$
Not quadratic; there is an x^3 term.

7. $n(n^2+n-1)=n^3;\ n^3+n^2-n=n^3;\ n^2-n=0$
$a=1,\ b=-1,\ c=0$

8. $(T-7)^2=(2T+3)^2$
$T^2-14t+49=4T^2+12T+9$
$-3T^2-26T+40=0;\ 3T^2+26T-40=0$
$a=3,\ b=26,\ c=-40$

9. $x^2-4=0$
$(x+2)(x-2)=0$
$x+2=0$ or $x-2=0$
$x=-2 \qquad x=2$

10. $B^2-400=0;\ (B-20)(B+20)=0$
$B-20=0,\ B=20$ or $B+20=0,\ B=-20$

11. $4x^2=9$
$4x^2-9=0;\ (2x+3)(2x-3)=0$
$2x+3=0;\ 2x=-3,\ x=-\frac{3}{2}$ or
$2x-3=0;\ 2x=3,\ x=\frac{3}{2}$

12. $x^2=0.16$
$x^2-0.16=0$
$(x-0.4)(x+0.4)=0$
$x-0.4=0;\ x=0.4$
$\ldots=0;\ x=-0.4$

13. $x^2-8x-9=0$
$(x-9)(x+1)=0$
$x-9=0$ or $x+1=0$
$x=9 \qquad x=-1$

14. $x^2+x-6=0;\ (x+3)(x-2)=0$
$x+3=0,\ x=-3$ or $x-2=0,\ x=2$

https://www.khanacademy.org/math/algebra/quadratics/factoring_quadratics/v/Example%201:%20Solving%20a%20quadratic%20equation%20by%20factoring

https://www.khanacademy.org/math/algebra/quadratics/quadratics-square-root/v/solving-quadratic-equations-by-square-roots

Ch. 7.2: Completing the Square

- Factoring is not the only way that quadratic equations can be solved.
- Nor is factoring the easiest way.
- The method known as ***Completing the Square*** can be used to solve any quadratic equation.

Solving a Quadratic Equation by Completing the Square

1. Divide each side by a (coefficient of x^2).
2. Rewrite the equation with the constant on the right side.
3. Complete the square: add the square of ½ of the coefficient of x to both sides.
4. Write the left side as a square & simplify the right side.
5. Equate the square root of the left side to the principal square root of the right side & to its negative.
6. Solve the 2 resulting linear equations.

1. Divide each side by a (coefficient of x^2).
2. Rewrite the equation with the constant on the right side.
3. Complete the square: add the square of ½ of the coefficient of x to both sides.
4. Write the left side as a square & simplify the right side.
5. Equate the square root of the left side to the principal square root of the right side & to its negative.
6. Solve the 2 resulting linear equations.

Example

- Solve the quadratic equation by completing the square.

$$3x^2 - 3x - 2 = 0$$

Solution

$$3x^2 - 3x - 2 = 0$$

1. Divide each side by $\boldsymbol{a}$ (coefficient of $\boldsymbol{x^2}$).

$$\frac{3x^2}{3} - \frac{3x}{3} - \frac{2}{3} = \frac{0}{3}$$

2. Rewrite the equation with the constant on the right side.

$$x^2 - x = \frac{2}{3}$$

1. Divide each side by a (coefficient of x^2).
2. Rewrite the equation with the constant on the right side.
3. Complete the square: add the square of ½ of the coefficient of x to both sides.
4. Write the left side as a square & simplify the right side.
5. Equate the square root of the left side to the principal square root of the right side & to its negative.
6. Solve the 2 resulting linear equations.

$$x^2 - x = \frac{2}{3}$$

3. Complete the square: add the square of ½ of the coefficient of x to both sides.

$$x^2 - x + \frac{1}{4} = \frac{2}{3} + \frac{1}{4}$$

4. Write the left side as a square & simplify the right side.

$$\left(x - \frac{1}{2}\right)\left(x - \frac{1}{2}\right) = \frac{11}{12}$$

Solution (*continued*)

$$\left(x-\frac{1}{2}\right)\left(x-\frac{1}{2}\right)=\frac{11}{12}$$

5. Equate the square root of the left side to the principal square root of the right side & to its negative.

$$x-\frac{1}{2}=\pm\sqrt{\frac{11}{12}}$$

6. Solve the 2 resulting linear equations.

$$x=1.46,\ x=-0.46$$

15 min

https://www.khanacademy.org/math/algebra/quadratics/completing_the_square/v/solving-quadratic-equations-by-completing-the-square

? Ex7.2 3-6, 11-14

In Exercises 3–10, solve the given quadratic equations by finding appropriate square roots as in Example 1.

3. $x^2 = 25$
4. $x^2 = 100$
5. $x^2 = 7$
6. $x^2 = 15$
7. $(x - 2)^2 = 25$
8. $(x + 2)^2 = 10$
9. $(x + 3)^2 = 7$
10. $\left(x - \frac{5}{2}\right)^2 = 100$

In Exercises 11–30, solve the given quadratic equations by completing the square. Exercises 11–14 and 17–20 may be checked by factoring.

11. $x^2 + 2x - 8 = 0$
12. $x^2 - 8x - 20 = 0$
13. $D^2 + 3D + 2 = 0$
14. $t^2 + 5t - 6 = 0$
15. $n^2 = 4n - 2$
16. $(R + 9)(R + 1) = 13$
17. $v(v + 2) = 15$
18. $Z^2 + 12 = 8Z$

A

3. $x^2 = 25$

$x = \pm\sqrt{25}$

$x = \pm 5$

4. $x^2 = 100$

$\sqrt{x^2} = \pm\sqrt{100}$

$x = -10$ or $x = 10$

5. $x^2 = 7$

$\sqrt{x^2} = \pm\sqrt{7}$

$x = \sqrt{7}$ or $x = -\sqrt{7}$

6. $x^2 = 15$

$\sqrt{x^2} = \pm\sqrt{15}$

$x = \sqrt{15}$ or $x = -\sqrt{15}$

11. $x^2 + 2x - 8 = 0$

$x^2 + 2x + 1 = 8 + 1$

$(x+1)^2 = 9$

$x + 1 = \pm\sqrt{9} = \pm 3$

$x + 1 = 3$ or $x + 1 = -3$

$x = 2 \qquad x = -4$

12. $x^2 - 8x - 20 = 0$

$x^2 - 8x = 20$

$x^2 - 8x + 16 = 20 + 16$

$(x-4)^2 = 36$

$x - 4 = 6$ or $x - 4 = -6$

$x = 10 \qquad x = -2$

13. $D^2 + 3D + 2 = 0$

$D^2 + 3D = -2$

$D^2 + 3D + \frac{9}{4} = -2 + \frac{9}{4}$

$\left(D + \frac{3}{2}\right)^2 = \frac{1}{4}$

$D + \frac{3}{2} = -\frac{1}{2}$ or $D + \frac{3}{2} = \frac{1}{2}$

$D = -\frac{4}{2} \qquad D = -\frac{2}{2}$

$= -2 \qquad = -1$

14. $t^2 + 5t - 6 = 0$

$t^2 + 5t = 6$

$t^2 + 5t + \frac{25}{4} = 6 + \frac{25}{4}$

$\left(t + \frac{5}{2}\right) = \frac{49}{4}$

$t + \frac{5}{2} = -\frac{7}{2}$ or $t + \frac{5}{2} = \frac{7}{2}$

$t = -\frac{12}{2} \qquad t = \frac{2}{2}$

$= -6 \qquad = 1$

Ch. 7.3: The Quadratic Formula

- The quadratic formula can be used to find solutions (roots) to a quadratic equation.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Info only > Following completing the square derives this (p220)

Show

https://www.khanacademy.org/math/algebra/quadratics/quadratic-formula/v/quadratic-formula-1

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example

- Solve the quadratic equation by the quadratic formula.

$$2x^2 + 5x - 3 = 0$$

Solution

- Identify ***a***, ***b*** and ***c*** in the quadratic equation:

$$2x^2 + 5x - 3 = 0$$

a = 2   ***b*** = 5   ***c*** = -3

$a = 2$ $b = 5$ $c = -3$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Solution (*continued*)

- Using the quadratic formula:

$$x = \frac{-5 \pm \sqrt{5^2 - 4(2)(-3)}}{2(2)}$$

- Answer: $x = -3, x = 0.5$

Characteristics of the Roots of a Quadratic Equation

1. If $b^2 - 4ac$ is positive & a perfect square, the roots are real, rational, & unequal.
2. If $b^2 - 4ac$ is positive but not a perfect square, the roots are real, irrational, & unequal.
3. If $b^2 - 4ac = 0$, the roots are equal, rational & equal.
4. If $b^2 - 4ac < 0$, the roots contain imaginary numbers & are unequal.

?Ex 7.3. 5-9

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

EXERCISES 7.3

In Exercises 1–4, make the given changes in the indicated examples of this section and then solve the resulting equations by the quadratic formula.

1. In Example 1, change the − sign before $5x$ to +.
2. In Example 2, change the coefficient of x^2 from 2 to 3.
3. In Example 3, change the + sign before $24x$ to −.
4. In Example 4, change 4 to 3.

In Exercises 5–36, solve the given quadratic equations, using the quadratic formula. Exercises 5–8 are the same as Exercises 11–14 of Section 7.2.

5. $x^2 + 2x - 8 = 0$
6. $x^2 - 8x - 20 = 0$
7. $D^2 + 3D + 2 = 0$
8. $t^2 + 5t - 6 = 0$
9. $x^2 - 4x + 2 = 0$
11. $v^2 = 15 - 2v$
13. $2s^2 + 5s = 3$
15. $3y^2 = 3y + 2$
17. $y + 2 = 2y^2$
19. $30y^2 + 23y - 40 = 0$
21. $8t^2 + 61t = -120$
23. $s^2 = 9 + s(1 - 2s)$
25. $25y^2 = 121$
27. $15 + 4z = 32z^2$
29. $x^2 - 0.20x - 0.40 = 0$
31. $0.29Z^2 - 0.18 = 0.63Z$

$ax^2 + bx + c = 0$

A

5. $x^2+2x-8=0;\ a=1,\ b=2,\ c=-8$

$$x=\frac{-2\pm\sqrt{2^2-4(1)(-8)}}{2(1)}=\frac{-2\pm\sqrt{36}}{2}$$

$$=\frac{-2\pm6}{2}$$

$x=2$ or $x=-4$

6. $x^2-8x-20=0;\ a=1,\ b=-8,\ c=-20$

$$x=\frac{8\pm\sqrt{64-4(1)(-20)}}{2(1)}=\frac{8\pm\sqrt{144}}{2}$$

$$=\frac{8\pm\sqrt{144}}{2}$$

$$=\frac{8\pm12}{2}=-2,\ 10$$

7. $D^2+3D+2=0;\ a=1,\ b=3,\ c=2$

$$D=\frac{-3\pm\sqrt{9-4(1)(2)}}{2(1)}=\frac{-3\pm\sqrt{1}}{2}$$

$$=\frac{-3\pm1}{2}=-2,\ -1$$

8. $t^2+5t-6=0;\ a=1,\ b=5,\ c=-6$

$$t=\frac{-5\pm\sqrt{25-4(1)(-4)}}{2(1)}=\frac{-5\pm\sqrt{49}}{2}$$

$$=\frac{-5\pm7}{2}=6,\ 1$$

9. $x^2-4x+2=0;\ a=1,\ b=-4,\ c=2$

$$x=\frac{-(-4)\pm\sqrt{(-4)^2-4(1)(2)}}{(2)}$$

$$=\frac{4\pm\sqrt{8}}{2}$$

$$=\frac{4\pm2\sqrt{2}}{2}$$

$$=2\pm\sqrt{2}$$

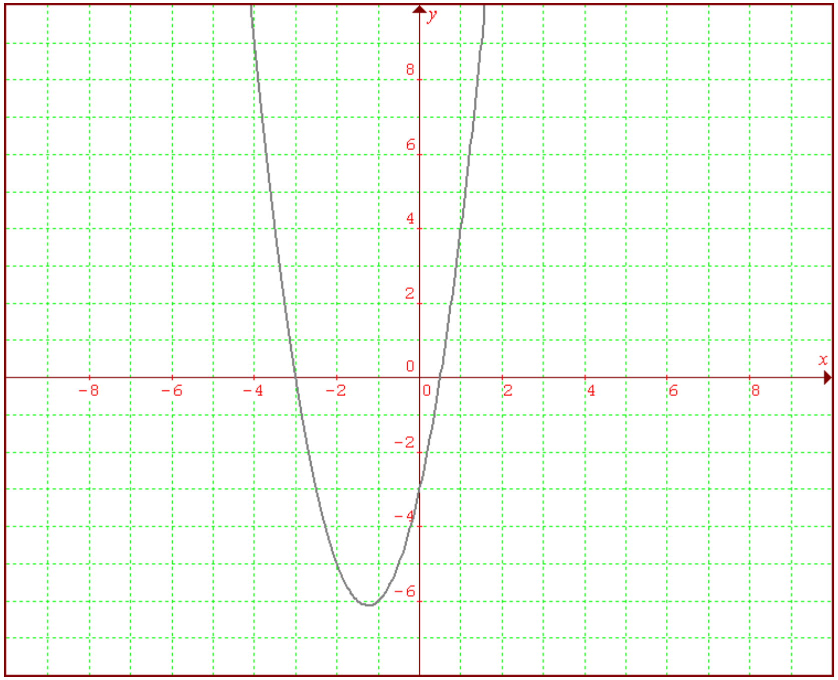
Ch. 7.4: The Graph of the Quadratic Equation

- By graphing the function $ax^2 + bx + c$, we can find its solution.
- We let: $y = ax^2 + bx + c$ to graph.

The Graph of the Quadratic Equation

- Using a graphing utility, graph:

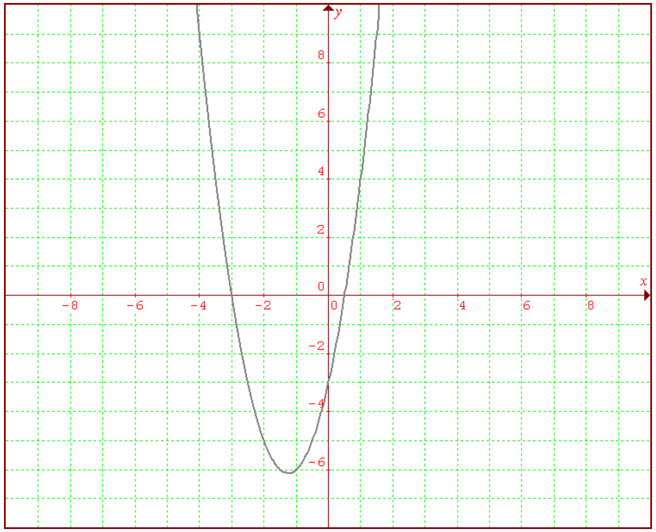
$$2x^2 + 5x - 3 = 0$$



https://www.khanacademy.org/math/algebra/quadratics/solving_graphing_quadratics/v/graphing-a-quadratic-function

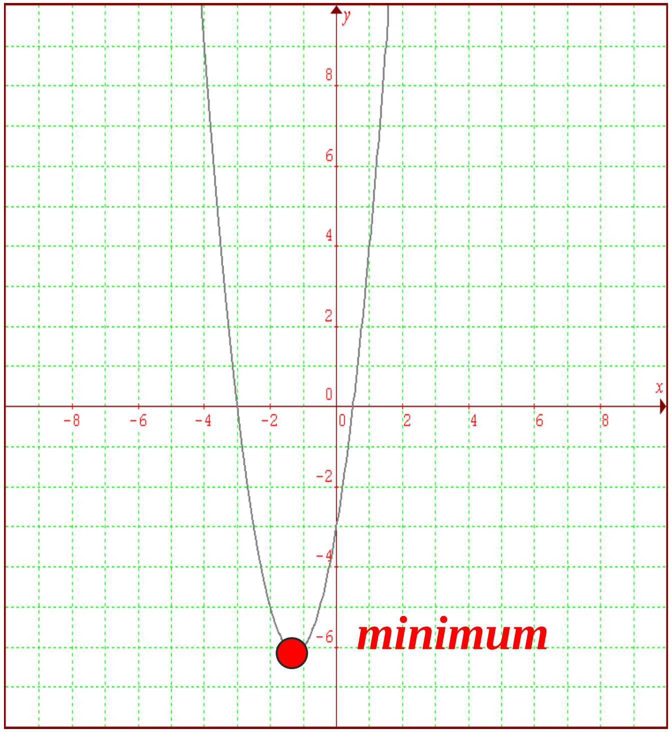
The Graph of the Quadratic Equation

- The graph of any quadratic function $y = ax^2 + bx + c$ will have the same basic shape, the shape of a parabola.



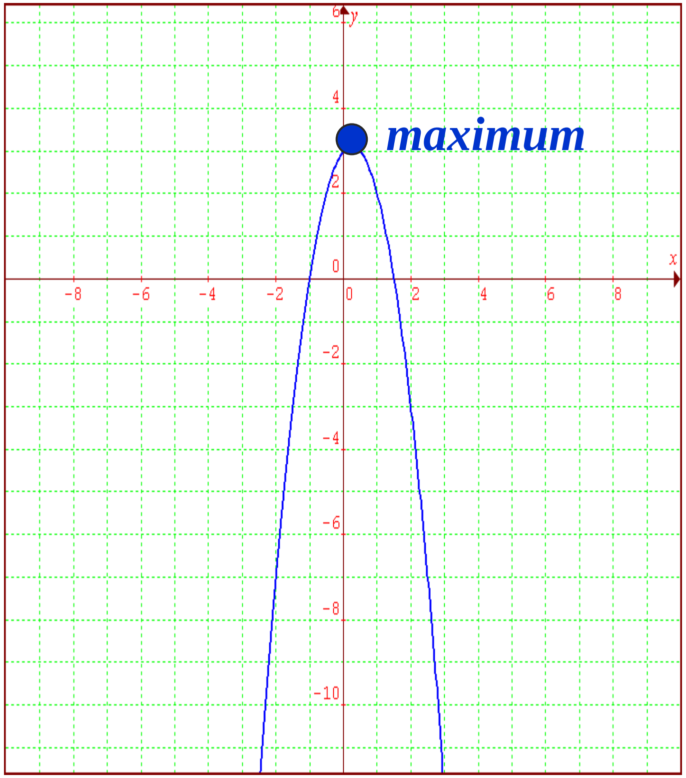
The Graph of the Quadratic Equation

- All parabolas have an ***extreme point***.
- This point is known as a ***vertex***.
- If $\boldsymbol{a > 0} \rightarrow$ ***minimum***.
- The graph opens upward.



The Graph of the Quadratic Equation

- All parabolas have an ***extreme point***.
- This point is known as a ***vertex***.
- If $\boldsymbol{a < 0} \rightarrow$ ***maximum***.
- The graph opens downward.



Finding the Vertex

- We can find the ***x***-coordinate of the vertex with:

$$x = \frac{-b}{2a}$$

- Substituting this value into the given equation, we can find the ***y***-coordinate of the vertex.

Solving Quadratic Equations Graphically

- When a quadratic equation is graphed, the roots of the equation are the $\boldsymbol{x}$-coordinates of the points for which $\boldsymbol{y} = \mathbf{0}$ (the $\boldsymbol{x}$-intercepts).
- Knowing when a quadratic curve is at a maximum or a minimum is a useful concept in maximization problems.

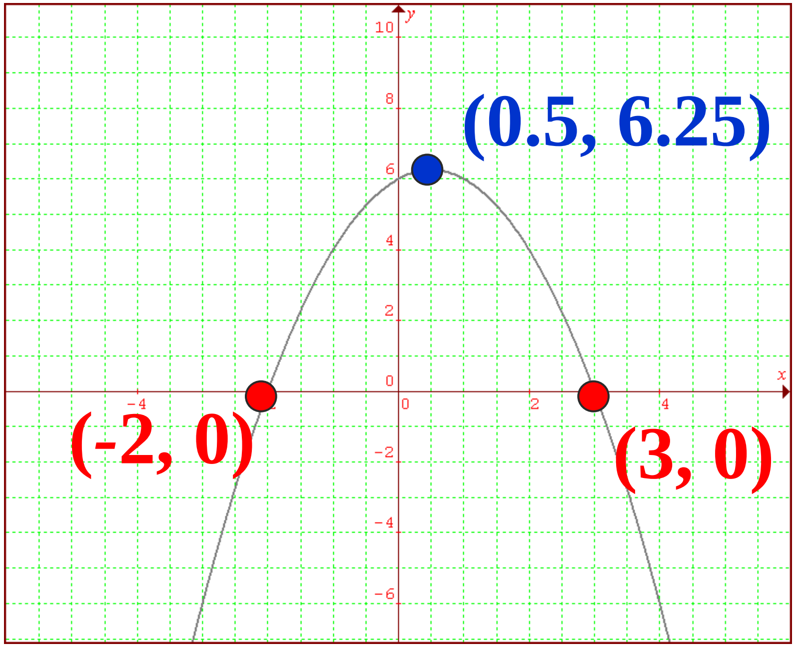
https://www.khanacademy.org/math/algebra/quadratics/solving_graphing_quadratics/v/quadratic-functions-2

Explanation of vertex etc > for information only

Example

- From the graph of $y = -x^2 + x + 6$, identify the roots of the equation.
- $x = -2$
- $x = 3$
- Vertex: **(0.5, 6.25)**
- Since $a < 0$, the vertex is at a maximum.

$$x = \frac{-b}{2a}$$



Summary

- We can sketch a quadratic equation when we know the vertex, the x-intercepts and the y-intercepts.
- We also note, the curve is symmetric to a vertical line through the vertex.
- Knowing when the vertex of the curve is a maximum or a minimum point is a useful concept in maximization problems.

https://www.desmos.com/calculator

? Ex 7.4 q 3&4

In Exercises 3–8, sketch the graph of each parabola by using only the vertex and the y-intercept. Check the graph using a graphing calculator.

3. $y = x^2 - 6x + 5$

4. $y = -x^2 - 4x - 3$

5. $y = -3x^2 + 10x - 4$

6. $s = 2t^2 + 8t - 5$

7. $R = v^2 - 4v$

8. $y = -2x^2 - 5x$

In Exercises 9–12, sketch the graph of each parabola by using the vertex, the y-intercept, and the x-intercepts. Check the graph using a graphing calculator.

9. $y = x^2 - 4$

10. $y = x^2 + 3x$

11. $y = -2x^2 - 6x + 8$

12. $u = -3v^2 + 12v - 5$

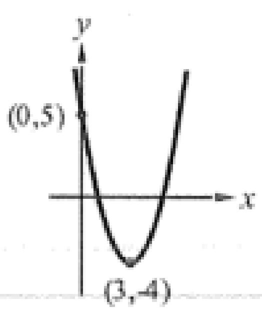
3. $y = x^2 - 6x + 5$; $a = 1$, $b = -6$, $c = 5$

The x-coordinate of the extreme point is

$\frac{-b}{2a} = \frac{-(-6)}{2(1)} = 3$, and the y-coordinate is

$y = 3^2 - 6(3) + 5 = -4$

The extreme point is $(3, -4)$. Since $a > 0$ it is a minimum point.



Since $c = 5$, the y-intercept is $(0, 5)$. Use the minimum point $(3, -4)$ and the y-intercept $(0, 5)$, and the fact that the graph is a parabola, to sketch the graph.

A

4. $y = -x^2 - 4x - 3$; $a = -1$, $b = -4$.

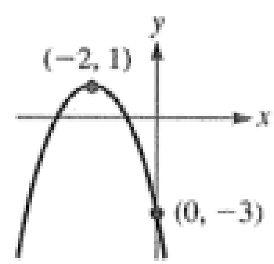
This means that the x-coordinate of the extreme is

$$\frac{-b}{2a} = \frac{-(-4)}{2(-1)} = -2,$$

and the y-coordinate is

$$y = -(-2)^2 - 4(-2) - 3 = 1.$$

Thus the extreme point is $(-2, 1)$. Since $a < 0$, it is a maximum point.

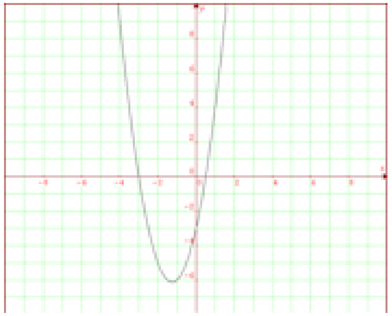


Since $c = -3$, the y-intercept is $(0, -3)$. Use the maximum point $(-2, 1)$ and the y-intercept $(0, -3)$, and the fact that the graph is a parabola, to sketch the graph.

M1 Ch 7 Quadratic

$$ax^2 + bx + c = 0$$

The graph of any quadratic function $y = ax^2 + bx + c$ will have the same basic shape, the shape of a parabola.



$x^2 + 2x - 15 = 0$ factor

$(x+5)(x-3) = 0$ set each factor to 0

$x + 5 = 0$ $x - 3 = 0$ solve

$x = -5$ $x = 3$

$$x = \frac{-b}{2a}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Chapter 11.1 and 11.2

Exponents and Radicals

Learning Outcomes

- At the end of this chapter the student will:
  - apply the laws of exponents to solve expressions containing exponents and fractional roots,
  - simplify radical expressions,
  - perform the basic arithmetic operations on expressions containing radicals.

Ch. 11.1: Simplifying Expressions with Integral Exponents

- In this section, we review the laws of exponents and the concept of the negative exponent.

Laws of Exponents

- Product Law: $\boxed{a^m \times a^n = a^{m+n}}$

- Quotient Law: $\boxed{\dfrac{a^m}{a^n} = a^{m-n}}$ $\quad$ ***m* > *n*, *a* ≠ 0**

$$\boxed{\frac{a^m}{a^n} = \frac{1}{a^{m-n}}} \qquad m < n,\ a \neq 0$$

Laws of Exponents (*continued*)

- Power Law:

$$\boxed{(a^m)^n = a^{mn}}$$

$$\boxed{(ab)^n = a^n b^n}$$

$$\boxed{\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}} \quad \boldsymbol{b \neq 0}$$

Zero & Negative Exponents

- A zero exponent is defined by:

$$\boxed{a^0 = 1}$$

- A negative exponent is defined by:

$$\boxed{a^{-n} = \frac{1}{a^n}}$$

The Negative Exponent

- Negative exponents are generally not used in the expression of a final result, unless specified otherwise.
- When a factor is moved from the denominator to the numerator of a fraction, or conversely, the sign of the exponent is changed.

Example

- Express the given expression in simplest form with only positive exponents.

$$\frac{x^{-1}-y}{x-y^{-1}} \Rightarrow = \frac{\frac{1}{x}-y}{x-\frac{1}{y}} \Rightarrow = \frac{\frac{1-xy}{x}}{\frac{xy-1}{y}} \Rightarrow = \left(\frac{1-xy}{x}\right)\left(\frac{y}{xy-1}\right)$$

$$\Rightarrow \boxed{= -\frac{y}{x}}$$

? Ex 11.1 q 10,12,23,25

In Exercises 5–52, express each of the given expressions in simplest form with only positive exponents.

5. x^7x^{-4} **6.** y^9y^{-2} **7.** $2a^2a^{-6}$

8. $5ss^{-5}$ **9.** $5^0 \times 5^{-3}$ ✓ **10.** $(3^2 \times 4^{-3})^3$

11. $(2\pi x^{-1})^2$ **12.** $(3xy^{-2})^3$ **13.** $2(5an^{-2})^{-1}$

14. $4(6s^2t^{-1})^{-2}$ ✓**15.** $(-4)^0$ ✓**16.** -4^0

17. $-7x^0$ **18.** $(-7x)^0$ **19.** $3x^{-2}$

20. $(3x)^{-2}$ **21.** $(7a^{-1}x)^{-3}$ ✓ **22.** $7a^{-1}x^{-3}$

23. $\left(\frac{2}{n^3}\right)^{-3}$ **24.** $\left(\frac{3}{x^3}\right)^{-2}$ **25.** $3\left(\frac{a}{b^{-2}}\right)^{-3}$

A

10. $\left(3^2\times 4^{-3}\right)^3=\left(\frac{3^2}{4^3}\right)^3=\frac{3^6}{4^9}$

11. $(2\pi x^{-1})^2=\left(\frac{2\pi}{x}\right)^2=\frac{4\pi^2}{x^2}$

12. $\left(3xy^{-2}\right)^3=3^3x^3y^{-6}=\frac{27x^3}{6}$

13. $2\left(5an^{-2}\right)^{-1}=2\times 5^{-1}a^{-1}n^{(-2)(-1)}=\frac{2n^2}{5a}$

14. $4\left(6s^2t^{-1}\right)^{-2}=4\left(\frac{6s^2}{t}\right)^2=\frac{4\times 6^{-2}s^{-4}}{t^{-2}}=\frac{4t^2}{6^2s^4}=\frac{t^2}{9s^4}$

15. $(-4)^0=1$

16. $-4^0=-(1)=-1$

17. $-7x^0=-7\times 1=-7$

18. $(-7x)^0=1$

19. $3x^{-2}=3\left(\frac{1}{x^2}\right)=\frac{3}{x^2}$

20. $(3x)^{-2}=3^{-2}x^{-2}=\frac{1}{9x^2}$

21. $\left(7a^{-1}x\right)^3=7^{-3}a^3x^{-3}=\frac{a^3}{7^3x^3}=\frac{a^3}{343x^3}$

22. $7a^{-1}x^{-3}=\frac{7}{ax^3}$

23. $\left(\frac{2}{n^3}\right)^{-3}=\frac{2^{-3}}{n^{-9}}=\frac{n^9}{8}$

24. $\left(\frac{3}{x^3}\right)^{-2}=\frac{3^{-2}}{x^{-6}}=\frac{x^6}{9}$

25. $\left(\frac{a}{b^{-2}}\right)^{-3}=\frac{3a^{-3}}{\left(b^{-2}\right)^{-3}}=\frac{\frac{3}{a^3}}{b^{(-2)(-3)}}=\frac{\frac{3}{a^3}}{b^6}=\frac{3}{a^3b^6}$

Ch. 11.2: Fractional Exponents

- Radical & Exponential Forms

$$\boxed{a^{1/n}} = \boxed{\sqrt[n]{a}}$$

Exponential form

Radical form

Examples

1. $\sqrt[3]{x} = x^{1/3}$

2. $\sqrt{x} = x^{1/2}$

Fractional Exponents

- If a fractional exponent is negative, it does not mean that the expression is negative.
- ***Example***:

$$y^{-1/2} = \frac{1}{\sqrt{y}}$$

Fractional Exponents

$$(a^m)^n = a^{mn}$$

- Applying the product law:

$$a^{m/n} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$$

Example 1

$$(a^m)^n = a^{mn}$$

- Simplify:

$$x^{2/3} = (x^{1/3})^2 = (\sqrt[3]{x})^2$$

or, $x^{2/3} = (x^2)^{1/3} = (\sqrt[3]{x})^2$

Example 2

$$(a^m)^n = a^{mn}$$

- Simplify:

$$27^{2/3} = (\sqrt[3]{27})^2$$

$$= (\sqrt[3]{3 \times 3 \times 3})^2$$

$$= 3^2$$

$$= 9$$

- Generally, it is easier to take the root first.

[? Ex 11.2,q 7,13,21,28]

In Exercises 5–28, evaluate the given expressions.

5. $25^{1/2}$ **6.** $27^{1/3}$ **7.** $81^{1/4}$ **8.** $125^{2/3}$

9. $100^{25/2}$ **10.** $-16^{5/4}$ **11.** $8^{-1/3}$ **12.** $16^{-1/4}$

13. $64^{-2/3}$ **14.** $-32^{-4/5}$ **15.** $5^{1/2}5^{3/2}$ **16.** $(4^4)^{3/2}$

17. $(3^6)^{2/3}$ **18.** $\dfrac{121^{-1/2}}{100^{1/2}}$ **19.** $\dfrac{1000^{1/3}}{-400^{-1/2}}$ **20.** $\dfrac{-7^{-1/2}}{6^{-1}7^{1/2}}$

21. $\dfrac{15^{2/3}}{5^2 15^{-1/3}}$ **22.** $\dfrac{(-27)^{1/3}}{6}$ **23.** $\dfrac{(-8)^{2/3}}{-2}$ **24.** $\dfrac{-4^{-1/2}}{(-64)^{-2/3}}$

25. $125^{-2/3} - 100^{-3/2}$ **26.** $32^{0.4} + 25^{-0.5}$

27. $\dfrac{16^{-0.25}}{5} + \dfrac{2^{-0.6}}{2^{0.4}}$ **28.** $\dfrac{4^{-1}}{36^{-1/2}} - \dfrac{5^{-1/2}}{5^{1/2}}$

A

7. $(81)^{1/4} = \sqrt[4]{81} = 3$

13. $64^{-2/3} = \dfrac{1}{\left(64^{1/3}\right)^2} = \dfrac{1}{\left(\sqrt[3]{64}\right)^2} = \dfrac{1}{4^2} = \dfrac{1}{16}$

21. $\dfrac{15^{2/3}}{5^2 \times 15^{-1/3}} = \dfrac{15^{2/3+1/3}}{5^2} = \dfrac{15^1}{25} = \dfrac{3}{5}$

28. $\dfrac{4^{-1}}{(36)^{-1/2}} - \dfrac{5^{-1/2}}{5^{1/2}} = \dfrac{(36)^{1/2}}{4} - \dfrac{1}{5^{1/2}5^{1/2}}$

$= \dfrac{\sqrt[2]{36}}{4} - \dfrac{1}{5}$

$= \dfrac{6}{4} - \dfrac{1}{5} = \dfrac{3}{2} - \dfrac{1}{5}$

$= \dfrac{3(5)-1(2)}{10}$

$= \dfrac{15-2}{10} = \dfrac{13}{10}$

$$a^m \times a^n = a^{m+n}$$

$$a^0 = 1$$

$$\frac{a^m}{a^n} = a^{m-n}$$

$$\frac{a^m}{a^n} = \frac{1}{a^{m-n}}$$

$$a^{-n} = \frac{1}{a^n}$$

$$(a^m)^n = a^{mn}$$

$$(ab)^n = a^n b^n$$

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

- Radical & Exponential Forms

$$\boxed{a^{1/n}} = \boxed{\sqrt[n]{a}}$$

Exponential form

Radical form

$$a^{m/n} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$$

M1 w9 ch11

Chapter 13

Exponential and Logarithmic Functions

9 10	Logarithms and logarithmic functions	Laws of indices Properties of logarithms Solving for any variable to any base Exponential growth and decay	11.1 Simplifying expressions with integral exponents 11.2 Fractional exponents 13.2 Logarithmic functions 13.3 Properties of logarithms 13.4 Logarithms to the base 10 13.5 Natural logarithms 13.6 Exponential and logarithmic equations Various applications (e.g. radioactivity, capacitor charging and discharging, etc.)

9. Solve logarithmic and exponential functions; represent them in graphs and solve applications. [PO2] [PO6]

Learning Outcomes

- At the end of this chapter the student will:
  - define exponential & logarithmic functions.
  - solve exponential & logarithmic equations using the properties of logarithms.
  - be able to work with natural & common logarithms.

Ch. 13.2: Logarithmic Functions

- How do we solve equations where the exponent is the unknown?
- For example:
  - The formula for the growth of bacteria is $n = 1500(2)^t$ where n is the number of bacteria in t hours. How long will it take for 50 000 bacteria to grow?
  - That is: $50\,000 = 1500(2)^t$
- We use logarithms to find the answer.

Logarithmic Functions

- Forms of a logarithm:

exponent

exponential form $b^x = y$ $x = \log_b y$ logarithmic form

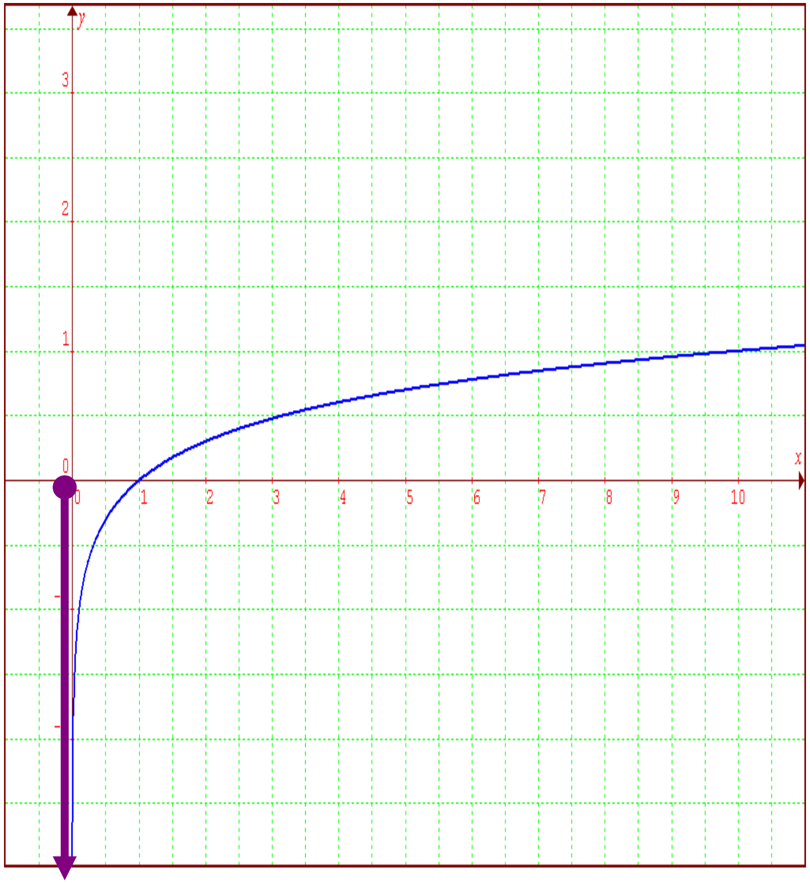
base

- Remember, exponents can be negative.

Graphing Logarithmic Functions

- Graphing: $y = \log_{10} x$

- Note the vertical ***asymptote*** along the negative *y*-axis where the graph ***never*** touches.



Basic Features of Logarithmic Functions (b > 1)

1. The domain is $x > 0$; the range is all values of y.
2. The negative y-axis is an asymptote of graph of $y = \log_b x$.
3. If $0 < x < 1$, $\log_b x < 0$; if $x = 1$, $\log_b x = 0$; if $x > 1$, $\log_b x > 0$.
4. If $x > 1$, x increases more rapidly than $\log_b x$.

? Ex 13.2 q 7, 14, 16, 25, 30

EXERCISES 13.2

In Exercises 1–4, perform the indicated operations if the given changes are made in the indicated examples of this section.

1. In Example 3(b), change the exponent to 4/5 and then make any other necessary changes.
2. In Example 4(b), change the 1/2 to 5/2 and then make any other necessary changes.
3. In Example 6, change the logarithm base to 4 and then make any other necessary changes.
4. In Example 7, change the logarithm base to 4 and then plot the graph.

In Exercises 5–16, express the given equations in logarithmic form.

5. $3^3 = 27$ **6.** $5^2 = 25$

7. $4^4 = 256$ **8.** $2^7 = 128$

9. $7^{-2} = \frac{1}{49}$ **10.** $3^{-2} = \frac{1}{9}$

11. $2^{-6} = \frac{1}{64}$ **12.** $(12)^0 = 1$

13. $8^{1/3} = 2$ **14.** $(81)^{3/4} = 27$

15. $\left(\frac{1}{4}\right)^2 = \frac{1}{16}$ **16.** $\left(\frac{1}{2}\right)^{-2} = 4$

In Exercises 17–28, express the given equations in exponential form.

17. $\log_3 81 = 4$ **18.** $\log_{11} 121 = 2$

19. $\log_9 9 = 1$ **20.** $\log_{15} 1 = 0$

21. $\log_{25} 5 = \frac{1}{2}$ **22.** $\log_8 16 = \frac{4}{3}$

23. $\log_{243} 3 = 0.2$ **24.** $\log_{32}\left(\frac{1}{8}\right) = -0.6$

25. $\log_{10} 0.1 = -1$ **26.** $\log_7\left(\frac{1}{49}\right) = -2$

27. $\log_{0.5} 16 = -4$ **28.** $\log_{1/3} 3 = -1$

In Exercises 29–44, determine the value of the unknown.

29. $\log_4 16 = x$ **30.** $\log_5 125 = x$

A

7. $4^1 = 256$ has base 4, exponent 4, and number 256.

$\log_4 256 = 4$

14. $(81)^{3/4} = 27$ has base 81, exponent $\dfrac{3}{4}$, and number 27.

$\log_{81} 27 = \dfrac{3}{4}$

16. $\left(\dfrac{1}{2}\right)^{-2} = 4$ has base $\dfrac{1}{2}$, exponent -2, and number 4.

$\log_{1/2} 4 = -2$

25. $\log_{10} 0.1 = -1$ has base 10, exponent -1, and number 0.1.

$0.1 = 10^{-1}$

30. $\log_5 125 = x$ has base 5, exponent x, and number 125.

$5^x = 125,\ x = 3$

Ch. 13.3: Properties of Logarithms

- Since a logarithm is an exponent, the properties of logarithms will be similar to those of exponents.
- We will compare the ***laws of exponents*** with the ***laws of logarithms***.

Logarithm of a Product

- Product Law of Exponents

$$b^{u} \times b^{v}$$

$$= b^{u+v}$$

Exponents are added

- The Logarithm of a Product

$$\log_b (u \times v)$$

$$= \log_b u + \log_b v$$

Logarithm of a Product

- **Example 1:**
  - Write as the sum or difference of 2 or more logarithms.

$$\log 5x = \log 5 + \log x$$

- **Example 2:**
  - Express as a single logarithm with a coefficient of 1.

$$\log 2 + \log 4 = \log (2 \times 4) = \log 8$$

Logarithm of a Quotient

- Quotient Law of Exponents

$$b^{u} \div b^{v}$$

$$= b^{\,u-v}$$

Exponents are subtracted

- Logarithm of a Quotient

$$\log_b (u \div v)$$

$$= \log_b u - \log_b v$$

Logarithm of a Quotient

Write as the sum or difference of 2 or more logarithms.

- **Example 3:** log (x/**5**) = log x - log **5**
- **Example 4:**

$$\log\left(\frac{11x}{6y}\right) = \log 11x - \log 6y$$

$$= \left(\log 11 + \log x\right) - \left(\log 6 + \log y\right)$$

Why the use of brackets?

Logarithm of a Quotient

- **Example 5:**

Express as a single logarithm with a coefficient of 1.

$\log 2 + \log 6 - \log 4$

$= \log (2 \times 6) - \log 4$

$= \log 12 - \log 4$

$$= \log \frac{12}{4} = \log 3$$

Logarithm of a Power

- Power Law of Exponents

$$(b^u)^n = b^{u \times n}$$

- Logarithm of a Power

$$\log_b u^n$$

$$= n \log_b u$$

Logarithm of a Power

- **Example 6:**

$\log 3^{12} = 12 \log 3$

- **Example 7:**

$\log 3^{y} = y \log 3$

Notice that we now have a product with the potential to divide and leave y by itself.

- **Example 8:**

Solve: $10 = \log x^{5}$

Properties of Logarithms

- **Example 9:**

Express as a single logarithm with a coefficient of 1.

$3\log x - 2\log y + 5\log z$

$= \log x^{3} - \log y^{2} + \log z^{5}$

$$= \log\left(\frac{x^3}{y^2}\right) + \log z^5 = \log\left(\frac{x^3 z^5}{y^2}\right)$$

Summary

- Remember the ***Order of Operations*** when working with the properties of logarithms.
- Avoid clearing your calculator screen after each calculation.

?Ex 13.3 q 14, 16, 25

In Exercises 9–20, express each as a sum, difference, or multiple of logarithms. See Example 2.

9. $\log_5 33$

10. $\log_3 14$

11. $\log_7\left(\frac{5}{3}\right)$

12. $\log_3\left(\frac{2}{11}\right)$

13. $\log_2(a^3)$

14. $2\log_8(n^5)$

15. $\log_6 abc$

16. $\log_2\left(\frac{xy}{z^2}\right)$

17. $8\log_5\sqrt[4]{y}$

18. $\log_4\sqrt[7]{x}$

19. $\log_2\left(\frac{\sqrt{x}}{a^2}\right)$

20. $\log_3\left(\frac{\sqrt[3]{y}}{7}\right)$

In Exercises 21–28, express each as the logarithm of a single quantity. See Example 3.

21. $\log_b a + \log_b c$

22. $\log_2 3 + \log_2 x$

23. $\log_5 9 - \log_5 3$

24. $-\log_8 R + \log_8 V$

25. $-\log_b\sqrt{x} + \log_b x^2$

26. $\log_4 3^3 + \log_4 9$

27. $2\log_e 2 + 3\log_e \pi$

28. $\frac{1}{2}\log_b a - 2\log_b 5$

A

14. $2\log_8\left(n^5\right) = 10\log_8 n$

16. $\log_2\left(\dfrac{xy}{x^2}\right) = \log_2 xy - \log_2 z^2$

$= \log_2 x + \log_2 y - 2\log_2 z$

25. $-\log_b\sqrt{x} + \log_b x^2 = \log_b \dfrac{x^2}{x^{1/2}} = \log_b x^{3/2}$

Ch. 13.4: Logarithms to the Base 10

- A common logarithm has a base of **10**.

$$\log_{10} N = \log N$$

- If there is no base identified in the logarithmic form, then assume it is to the base **10**.

Common Logarithms

Calculator Use:

Use the log key on your calculator to solve for N.

Common Logarithms

- Example:
  - Find **log N** if **N = 260**.
- Solution:
  - Rounding to 3 decimal places, **log 260 = 2.415**.

Common Logarithms

- Remember, when you are finding the logarithm of a number, you are finding the power to which **10** must be raised to give the answer.
- For example, $\mathbf{\log 260 = 2.415}$ means

$$10^{2.415} = 260$$

Finding Antilogarithms

- Given: $\log N = 2.415$
- What is N?
- Rearranging this into exponential form, $10^{2.415} = N$
- We use the 10^x key on the calculator to find the answer.
- $N = 260$ antilogarithm

Summary

- Common logarithms are logarithms to the base 10.
- They are readily calculated using a scientific calculator that has been preprogrammed for common logarithms.
- We use the properties of logarithms to solve equations using common logarithms

? Ex 13.4 q 9 , 15

In Exercises 3–12, find the common logarithm of each of the given numbers by using a calculator.

3. 567 **4.** 0.0640

5. 9.24×10^6 **6.** 3.19^3

7. 1.174^{-4} **8.** 8.043×10^{-8}

9. $\cos 12.5°$ **10.** $\tan 12.6$

11. $\sqrt{274}$ **12.** $\log_2 16$

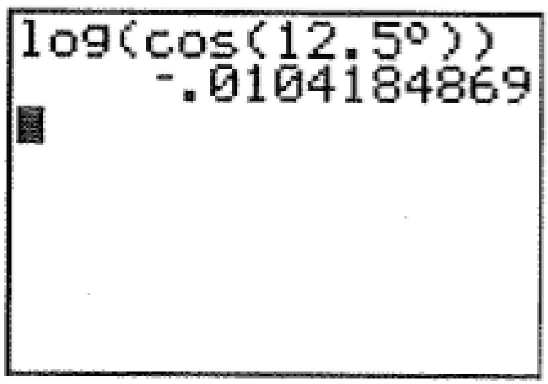
In Exercises 13–20, find the antilogarithm of each of the given logarithms by using a calculator.

13. 4.437 **14.** 0.929 **15.** −1.3045 **16.** −6.9788

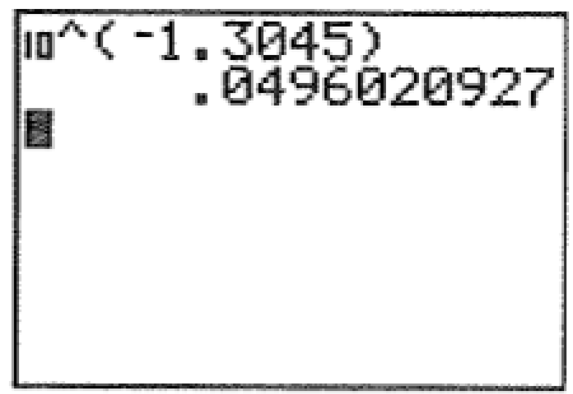
17. 3.30112 **18.** 8.82436 **19.** −2.23746 **20.** −10.336

A

9. $\log\left(\cos 12.5^{\circ}\right) = -0.0104$



15. $10^{-1.3045} = 0.04960$



Ch. 13.5: Natural Logarithms

- A natural logarithm has a base of e.

$$\log_e N = ln\, N$$

- Natural logarithms have widespread application in science and business.

Natural Logarithms

Calculator Use:

Use the *ln* key on your calculator to solve for N.

Natural Logarithms

- Example:
  - Find ***ln* N** if **N** = 260.

- Answer:
  - To three decimal places: ***ln* 260 = 5.561**

Natural Logarithms

- Remember, when you are finding the logarithm of a number, you are finding the power to which e must be raised to give the answer.
- For example, $\ln 260 = 5.561$ means $e^{5.560\,7} = 260$

Natural Logarithms

- Example:
  - Find **N** if ***ln* N** is: 0.367

- Solution:
  - We take the antilogarithm of ***ln* N**
  - This gives us: $e^{0.367}$
  - Therefore, ***ln* 0.367 = 1.443**

Converting Logarithms

- Scientific calculators are programmed for logarithms in bases **10** & ***e***.
- We can ***solve*** any logarithm to a different base using the equation:

$$\log_b x = \frac{\log_a x}{\log_a b}$$

a = 10

a = e

Converting Logarithms

- Example: Find $\log_2 75$.
- Solution: We will find the answer in base **10**.

$$\log_b x = \frac{\log_a x}{\log_a b} \Rightarrow \log_2 75 = \frac{\log 75}{\log 2}$$

$$\Rightarrow \log_2 75 \cong 6.23$$

Summary

- Natural logarithms are logarithms to the base ***e***.
- They are readily calculated using a scientific calculator that has been preprogrammed for natural logarithms.
- We use the properties of logarithms to solve equations using natural logarithms

? Ex 13.5 q 19, 27

In Exercises 15–22, find the natural logarithms of the given numbers.

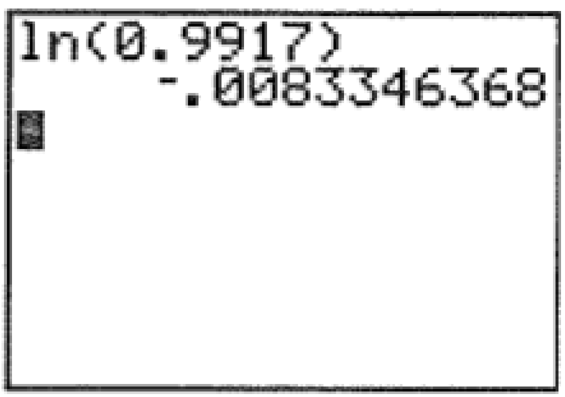
15. 51.4
16. 293
17. 1.394
18. 6552
19. 0.9917
20. 0.002086
21. $(0.012937)^4$
22. $\sqrt{0.000060808}$

In Exercises 27–34, find the natural antilogarithms of the given logarithms.

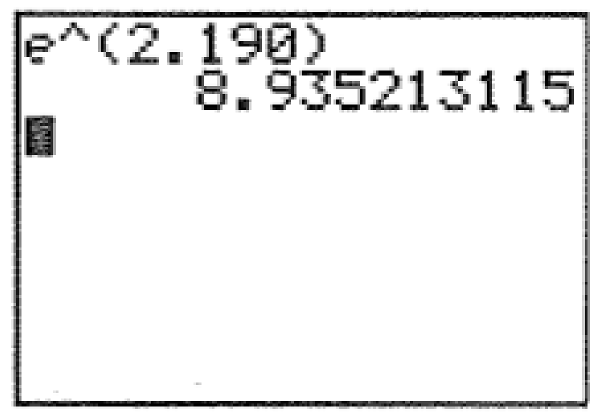
27. 2.190
28. 5.420
29. 0.0084210
30. 0.632
31. −0.7429
32. −2.94218
33. −23.504
34. −0.00804

A

19. $\ln 0.9917 = -0.008335$



27. $e^{2.190} = 8.935$



Ch. 13.6: Exponential and Logarithmic Equations

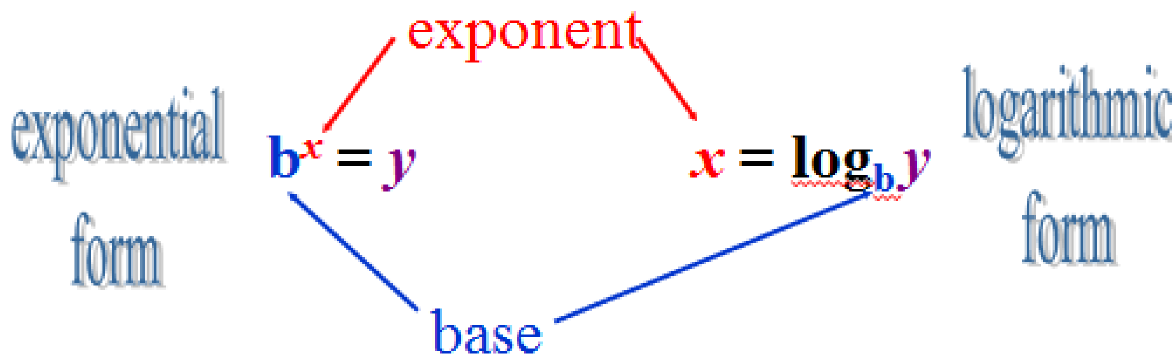
- In the fields of electronics and business, we are called upon to solve equations containing variable exponents or logarithms in some or all of the terms.

Solving Exponential & Logarithmic Equations

Our tools:

1. Converting between exponential form and logarithmic forms.
2. The Properties of Logarithms
3. The Identities in Logarithms.
4. Taking the logarithm of both sides.

It follows from



We get

$$ln(e^n)\text{=n}$$

$$log_{10}(10^n)\text{=n}$$

Like

$$\sqrt{x^2} = x$$

Radioactive decay

$$N = N_0 e^{\frac{-0.693\,t}{T_{1/2}}}$$

N= Number at time t
N_0 = Number at time t_0
$T_{1/2}$ = half life

If a source has a half life of 2000yrs. How many years will it take to decay to 10% of its original value?

$$\frac{1}{10} = e^{\frac{-0.693\,t}{T_{1/2}}}$$

$$\ln(0.1) = \frac{-0.693\,t}{2000}$$

t = 6,645 yrs

The Identities in Logarithms

- We use the identity: $\mathbf{log_b b = 1}$
- In common logarithms, this is $\mathbf{log\ 10 = 1}$.
- In natural logarithms, this is $\boldsymbol{ln\ e} = \mathbf{1}$.

Exponential Equations

- **Example 1:**
  - Solve for *x*. $2^{3x-1} = 6$
- **Solution:**
  - Take the logarithm of both sides.
    $\log(2^{3x-1}) = \log 6$
  - Apply the power law of logarithms.
    $(3x - 1)\log 2 = \log 6$
  - Solve. $x = 1.19$ (:to 2 decimal places)

Logarithmic Equations

1. We use algebra to isolate the logarithm with the unknown in it (x).
2. We convert the logarithmic equation into its exponential counterpart.

Logarithmic Equations

- **Example 2:**
  - $\log_x 64 = 3$
  - $x^3 = 64$
  - $\boldsymbol{x = 4}$
- **Example 3:**
  - $\log_{49} x = 1/2$
  - $49^{1/2} = x$
  - $\boldsymbol{x = 7}$

Logarithmic Equations

- **Example 4:**
  - $ln\ x - ln\ x^2 = ln\ 27$
  - $\boldsymbol{x = 1/27}$
- **Example 5:**
  - $\log (x^2 - 9) - 1 = \log (x + 3)$
  - $\boldsymbol{x = 3x90}$

Summary

- Exponential & logarithmic equations are readily solved when we:

1. convert between exponential form and logarithmic forms.
2. apply the Properties of Logarithms
3. apply the Identities in Logarithms.
4. in some instances, take the logarithm of both sides.

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6. $\pi^x = 15$

7. $3^{-x} = 0.525$

8. $e^{-x} = 17.54$

9. $6^{x+1} = 78$

10. $5^{x-1} = 0.07$

11. $3(14^x) = 400$

12. $0.8^x = 0.4$

13. $0.6^x = 2^{x^2}$

14. $15.6^{x+2} = 23^x$

15. $3\log_8 x = -2$

16. $5\log_{32} x = -3$

17. $\log x^2 = (\log x)^2$

18. $x^{\log x} = 1000x^2$

19. $\log_2 x + \log_2 7 = \log_2 21$

20. $2\log_2 3 - \log_2 x = \log_2 45$

21. $2\log(3 - x) = 1$

22. $3\log(2x - 1) = 1$

23. $\log 12x^2 - \log 3x = 3$

24. $\ln x - \ln(1/3) = 1$

25. $3\ln 2 + \ln(x - 1) = \ln 24$

26. $\log_2 x + \log_2(x + 2) = 3$

27. $\frac{1}{2}\log(x + 2) + \log 5 = 1$

28. $2\log_x 2 + \log_2 x = 3$

29. $\log(2x - 1) + \log(x + 4) = 1$

30. $\ln(2x - 1) - 2\ln 4 = 3\ln 2$

22. $3\log(2x-1)=1$

$$\log(2x-1)=\frac{1}{3}$$

$$2x-1=10^{1/3}=2.154$$

$$2x=2.154+1=3.154$$

$$x=\frac{3.154}{2}=1.58$$

29. $\log(2x-1)+\log(x+4)=1$

$$\log[(2x-1)(x+4)]=1$$

$$(2x-1)(x+4)=10$$

$$2x^2+7x-4=10$$

$$2x^2x^2 7x-14=0$$

Use the quadratic formula to solve for x:

$$x=\frac{-7\pm\sqrt{49-4(2)(-14)}}{2(2)}=\frac{-7\pm\sqrt{161}}{4}$$

$$=\frac{-7\pm 12.689}{4}=-4.92,\ 142$$

$x=1.42$ (Since logs are not defined on negatives.)

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exponent

exponential form $b^x = y$ $x = \log_b y$ logarithmic form

base

$\log_b (u \times v)$

$= \log_b u + \log_b v$

$\log_b u^n$

$= n \log_b u$

$$\log_b x = \frac{\log_a x}{\log_a b}$$

$\log_b (u \div v)$

$= \log_b u - \log_b v$

$\log_b b = 1$

$\log 10 = 1$.

$\ln e = 1$.

Exponential function	$y = b^x$
Logarithmic form	$x = \log_b y$
Logarithmic function	$y = \log_b x$
Laws of exponents	$b^u b^v = b^{u+v}$
	$\dfrac{b^u}{b^v} = b^{u-v}$
	$(b^u)^n = b^{nu}$
Properties of logarithms	$\log_b xy = \log_b x + \log_b y$
	$\log_b\left(\dfrac{x}{y}\right) = \log_b x - \log_b y$
	$\log_b(x^n) = n \log_b x$
	$\log_b 1 = 0 \qquad \log_b b = 1$
	$\log_b(b^n) = n$
Changing base of logarithms	$\log_b x = \dfrac{\log_a x}{\log_a b}$
	$\ln x = \dfrac{\log x}{\log e}$
	$\log x = \dfrac{\ln x}{\ln 10}$