Rotation

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(ref the cartoon guide to Physics, Gonick & Huffman)

Inertia and mass

An object's inertia depends only on its *mass*. The definition of mass is very difficult, and you will probably have met the idea that 'mass is a measure of the amount of matter in a body'. While this statement is not false, it is not the whole truth either. The most satisfactory definition of mass uses the idea of inertia. If two objects A and B have the same acceleration, but the resultant force on object A is $2F$ while that on object B is F, then object A must have twice the mass of object B. Figure 1.2.9 shows an application of this.

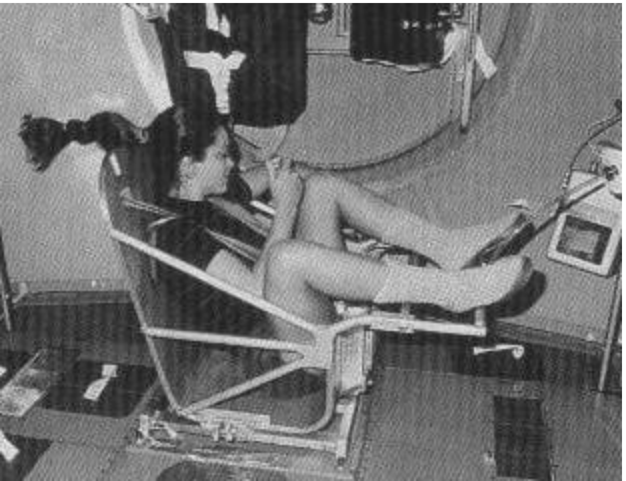


Figure 1.2.9 Newton's first law in action. It is important to know about changes in body mass of astronauts during long periods in orbit. Obviously bathroom scales are useless in this situation. This device uses the inertia of the astronaut's body to affect the way in which oscillations happen – the oscillations are then timed and used to calculate the mass of the astronaut.

$\text{Momentum} = mv$ (linear)

linear Inertia $\propto m$

$$\text{angular velocity} = \omega = \frac{\Delta\theta}{\Delta t}$$

$\Rightarrow$

Period

$$T = \text{time for 1 revolution} = \frac{\Delta\theta}{\omega} = \frac{2\pi}{\omega}$$

$$f = \frac{1}{T} = \frac{\omega}{2\pi}$$

$$\text{angular momentum} = m\,v$$
$$= m\,\omega r$$

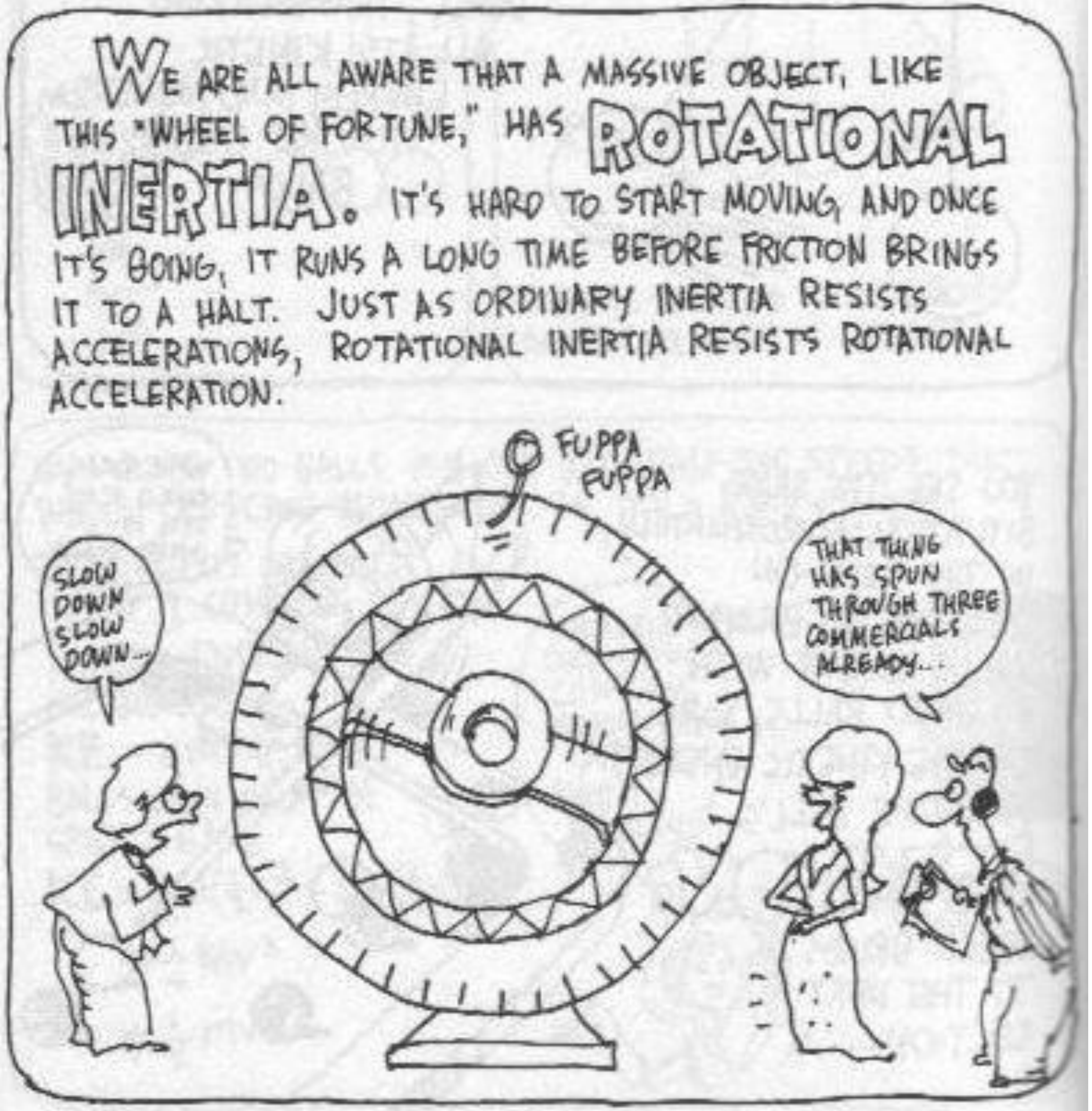
$v = \omega r$

~~a~~ rotational inertia $\propto m\,r$

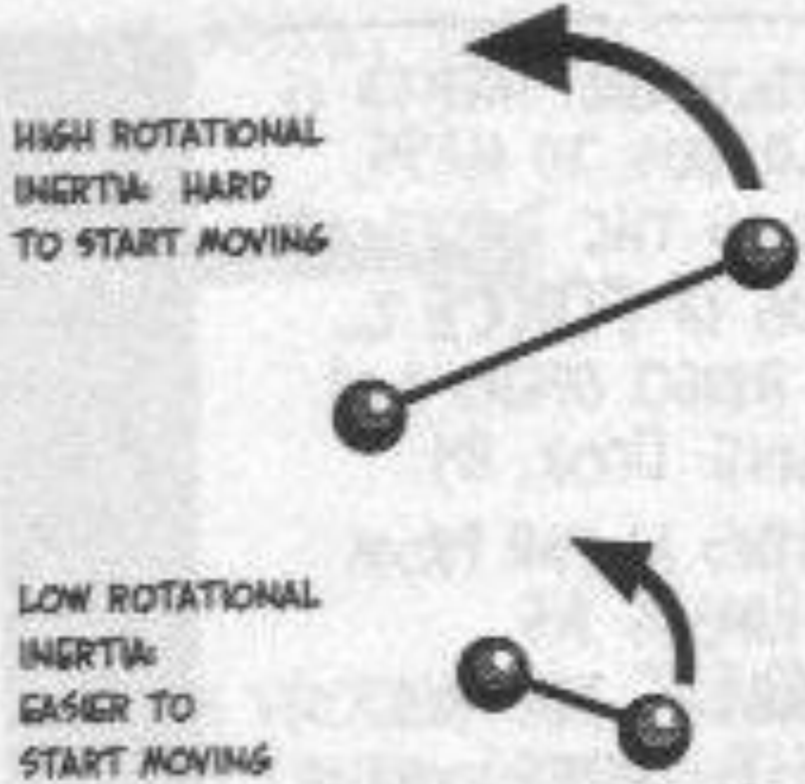
$$\text{angular momentum} = \text{rotational Inertia} \times \text{angular velocity}$$
$$= m\,r \times \omega$$

Momentum is conserved

Angular momentum is conserved $=$ $m\,r\,\omega$ is conserved

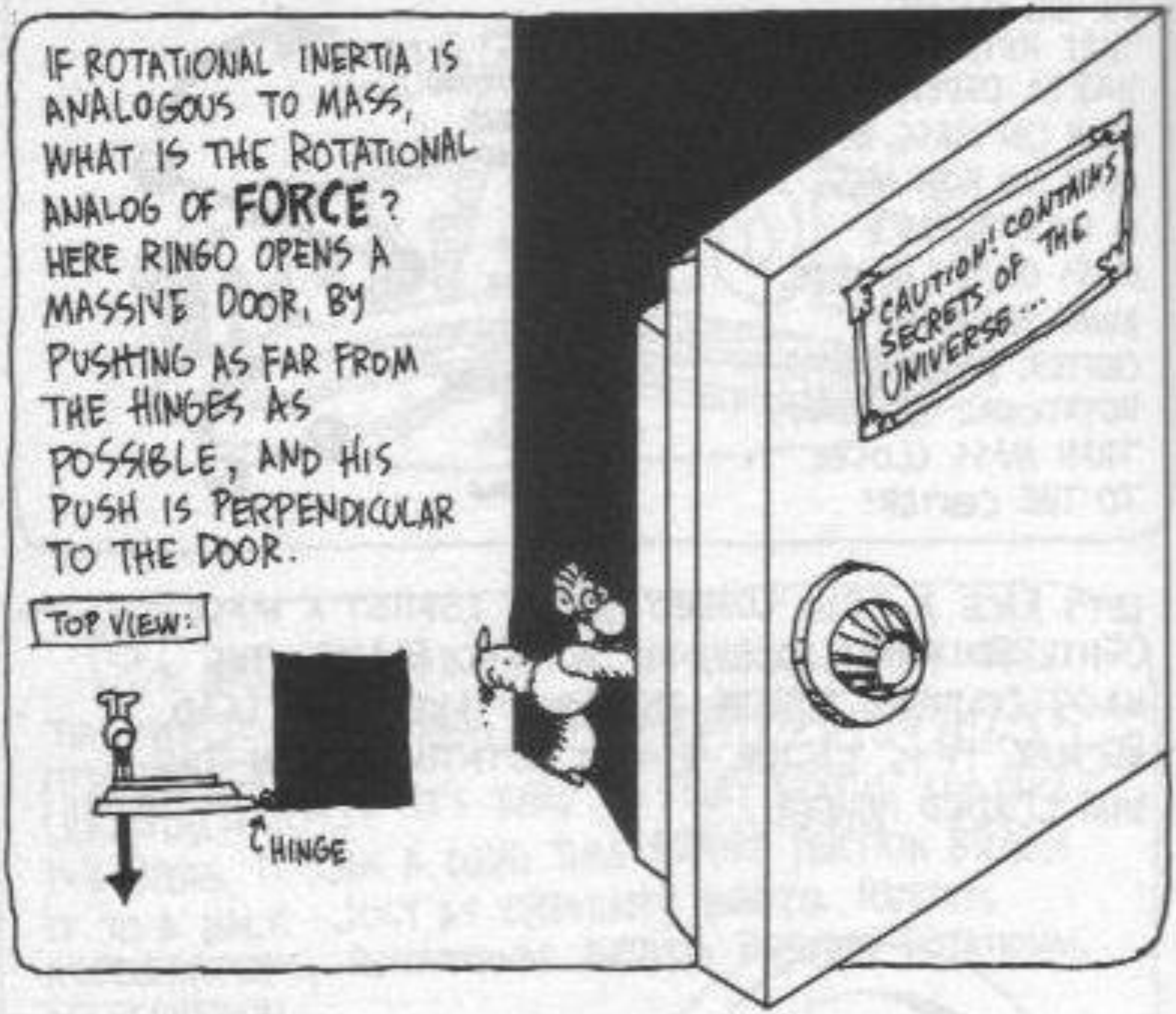

WE ARE ALL AWARE THAT A MASSIVE OBJECT, LIKE THIS "WHEEL OF FORTUNE," HAS ROTATIONAL INERTIA. IT'S HARD TO START MOVING, AND ONCE IT'S GOING, IT RUNS A LONG TIME BEFORE FRICTION BRINGS IT TO A HALT. JUST AS ORDINARY INERTIA RESISTS ACCELERATIONS, ROTATIONAL INERTIA RESISTS ROTATIONAL ACCELERATION.
FUPPA FUPPA
SLOW DOWN SLOW DOWN...
THAT THING HAS SPUN THROUGH THREE COMMERCIALS ALREADY...

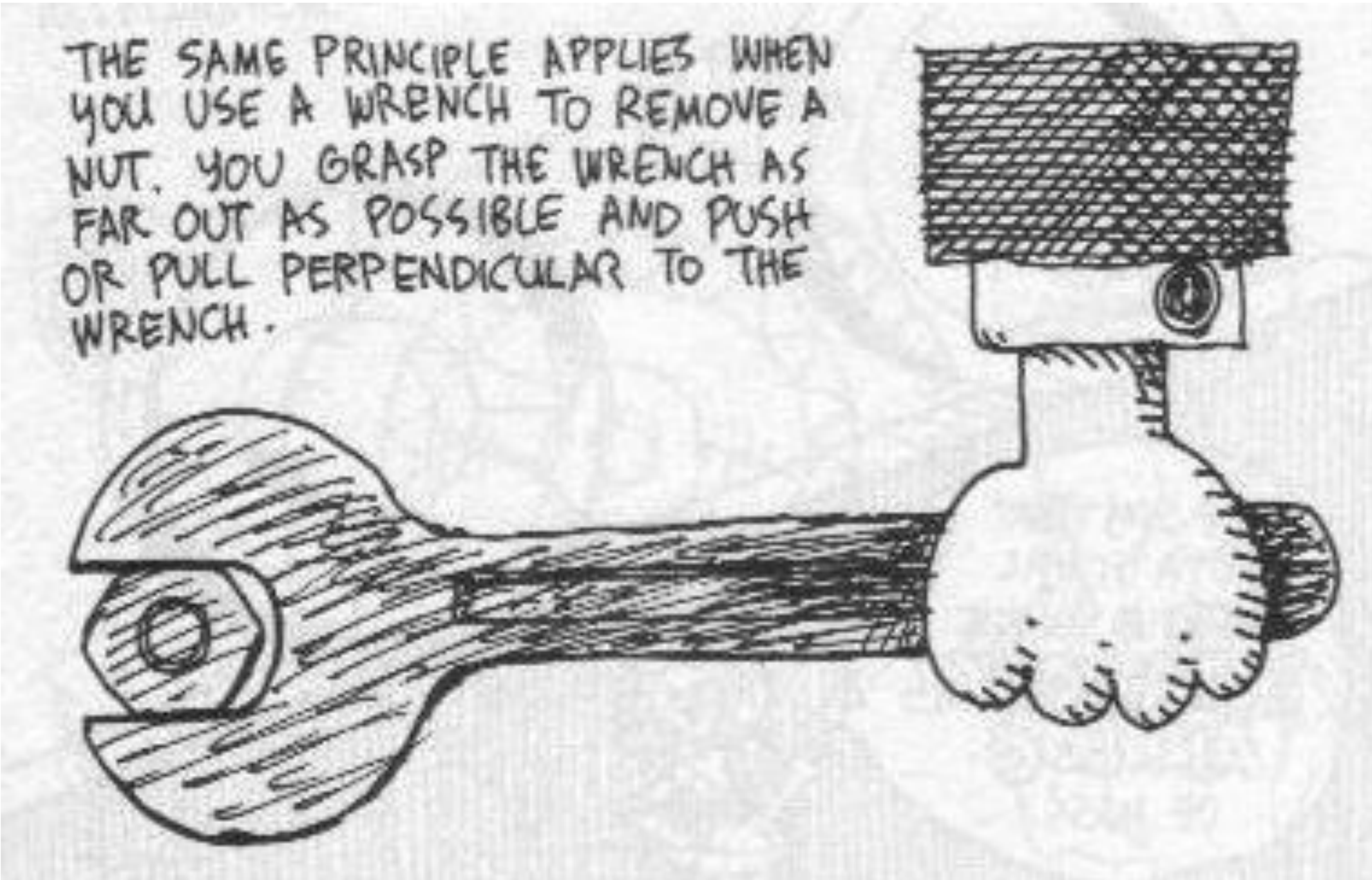
DID YOU REALIZE THAT ROTATIONAL INERTIA DEPENDS NOT ONLY ON MASS, BUT ALSO ON HOW MASS IS DISTRIBUTED? MASS ON THE OUTSIDE, AWAY FROM THE CENTER, HAS MORE ROTATIONAL INERTIA THAN MASS CLOSER TO THE CENTER!

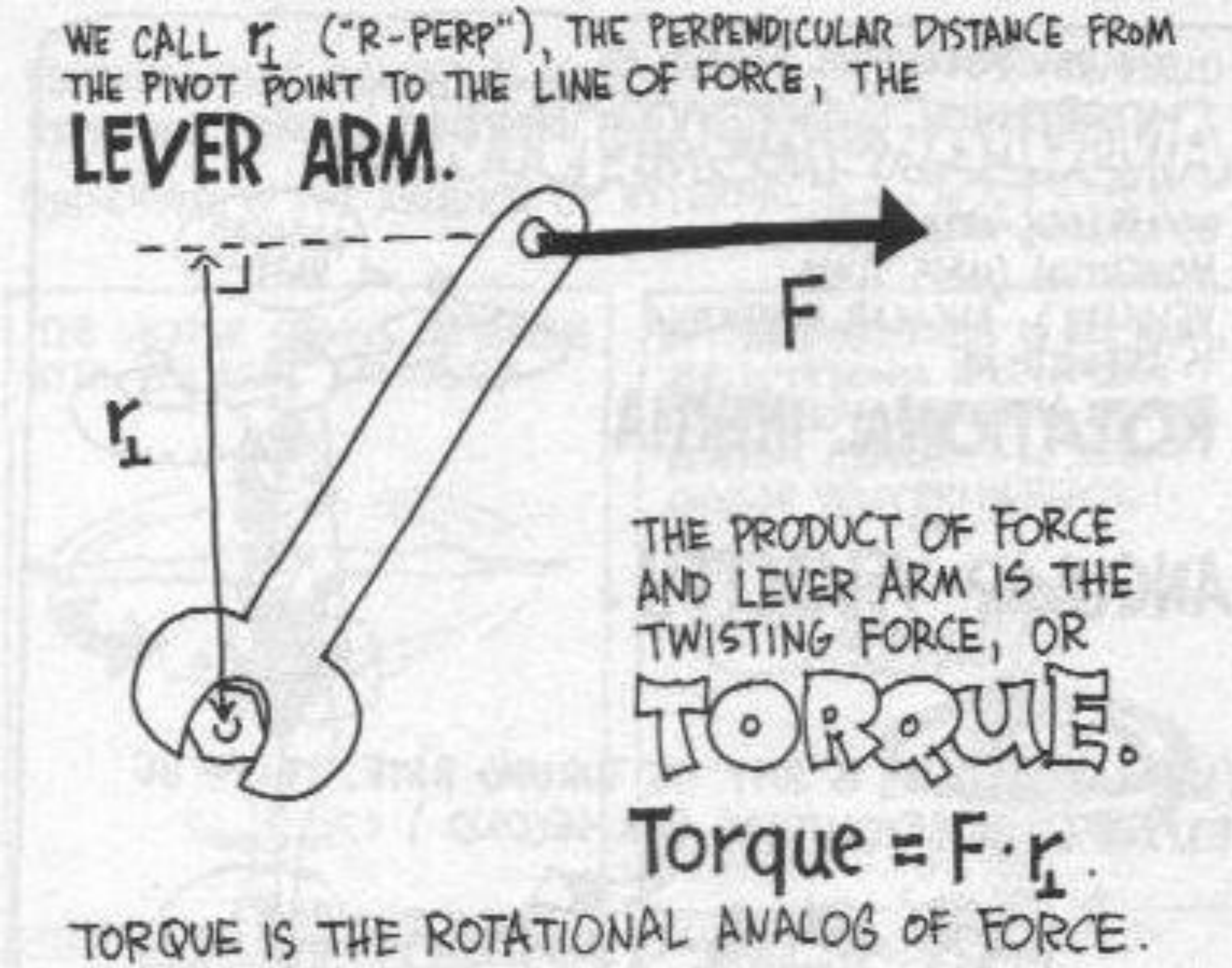


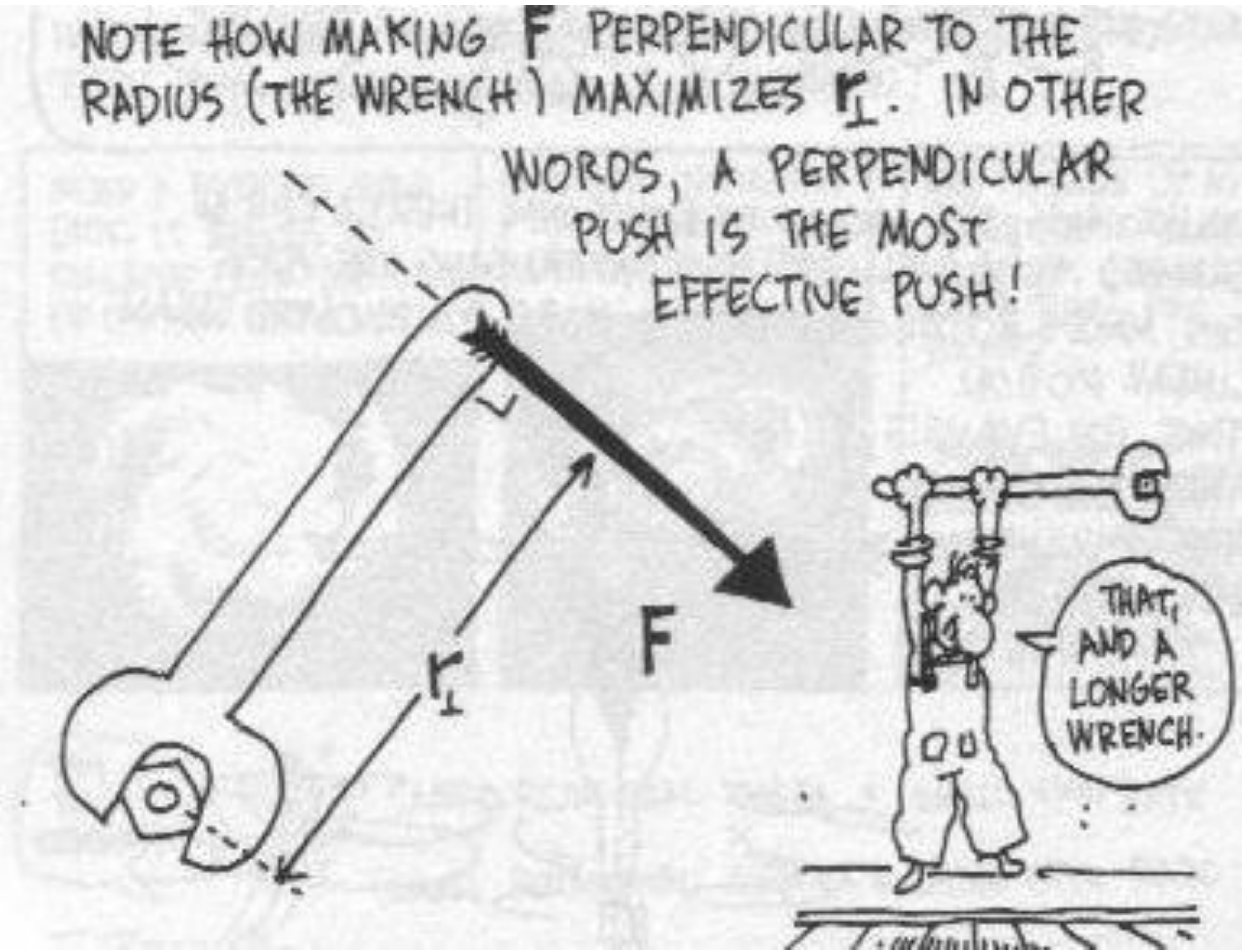
LET'S RACE A "RIM-LOADED" WHEEL AGAINST A MASS-CENTERED WHEEL DOWN AN INCLINED PLANE. THE MASS-CENTERED WHEEL QUICKLY TAKES THE LEAD, BECAUSE IT IS EASIER TO GET ROTATING THAN THE RIM-LOADED WHEEL.


WE SAY THAT ROTATIONAL INERTIA IS THE ROTATIONAL ANALOG OF MASS.


IF ROTATIONAL INERTIA IS ANALOGOUS TO MASS, WHAT IS THE ROTATIONAL ANALOG OF **FORCE**? HERE RINGO OPENS A MASSIVE DOOR, BY PUSHING AS FAR FROM THE HINGES AS POSSIBLE, AND HIS PUSH IS PERPENDICULAR TO THE DOOR.
TOP VIEW:
HINGE
CAUTION! CONTAINS SECRETS OF THE UNIVERSE...


THE SAME PRINCIPLE APPLIES WHEN YOU USE A WRENCH TO REMOVE A NUT. YOU GRASP THE WRENCH AS FAR OUT AS POSSIBLE AND PUSH OR PULL PERPENDICULAR TO THE WRENCH.


WE CALL $r_\perp$ ("R-PERP"), THE PERPENDICULAR DISTANCE FROM THE PIVOT POINT TO THE LINE OF FORCE, THE
LEVER ARM.
F
$r_\perp$
THE PRODUCT OF FORCE AND LEVER ARM IS THE TWISTING FORCE, OR
TORQUE.
Torque = $F \cdot r_\perp$.
TORQUE IS THE ROTATIONAL ANALOG OF FORCE.


NOTE HOW MAKING F PERPENDICULAR TO THE RADIUS (THE WRENCH) MAXIMIZES $r_\perp$. IN OTHER WORDS, A PERPENDICULAR PUSH IS THE MOST EFFECTIVE PUSH!
F
$r_\perp$
THAT, AND A LONGER WRENCH.


OUR FINAL ROTATIONAL ANALOG IS
ANGULAR MOMENTUM.
BY ANALOGY WITH LINEAR MOMENTUM (MASS TIMES VELOCITY), ANGULAR MOMENTUM IS DEFINED AS
ROTATIONAL INERTIA
×
ANGULAR VELOCITY.
ANGULAR BARF--
(ANGULAR VELOCITY IS JUST THE TURNING RATE. IT CAN BE EXPRESSED IN REVOLUTIONS PER SECOND.)
THAT'S WHAT WE HAVE IN MY COUNTRY: A REVOLUTION EVERY SECOND!

UNLIKE MASS, THE AMOUNT OF ROTATIONAL INERTIA CAN BE CHANGED "IN MID-FLIGHT" BY REARRANGING THE MASS. THIS MAKES ROTATIONAL MOTION MORE COMPLICATED THAN LINEAR MOTION.

TAKE, FOR EXAMPLE, THE CASE OF THE SPINNING ICE SKATER...

REMEMBER THAT MOMENTUM IS CONSERVED IN THE ABSENCE OF EXTERNAL FORCES. LIKEWISE, ANGULAR MOMENTUM IS CONSERVED IN THE ABSENCE OF EXTERNAL TORQUES.


IN THIS RESPECT, AN ICE SKATER RESEMBLES A COLLAPSING STAR. THEY BOTH CONSERVE ANGULAR MOMENTUM!
WHEN A ROTATING STAR DIES, IT BEGINS TO COLLAPSE FROM THE FORCE OF ITS OWN GRAVITY.
ITS SPIN INCREASES TO CONSERVE ANGULAR MOMENTUM.
AND IT ENDS UP AS A SUPER-DENSE BLOB OF STUFF, SPINNING MANY TIMES PER SECOND.
TRY TO REMEMBER...
LARGE ROTATIONAL INERTIA × SMALL SPIN RATE
=
SMALL ROTATIONAL INERTIA × LARGE SPIN RATE