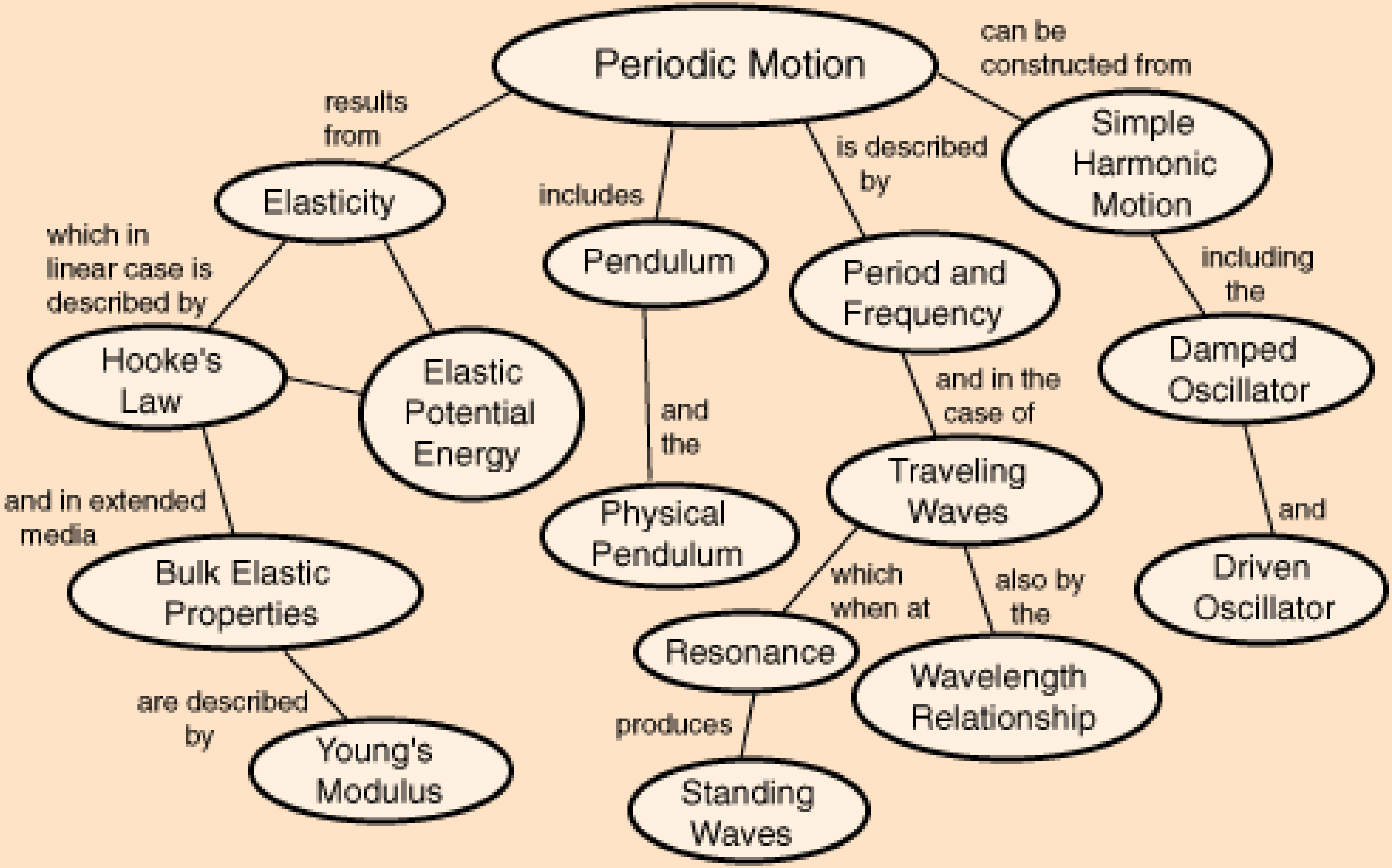
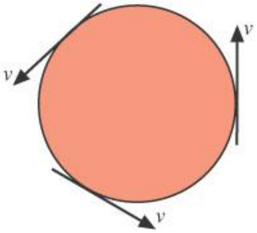
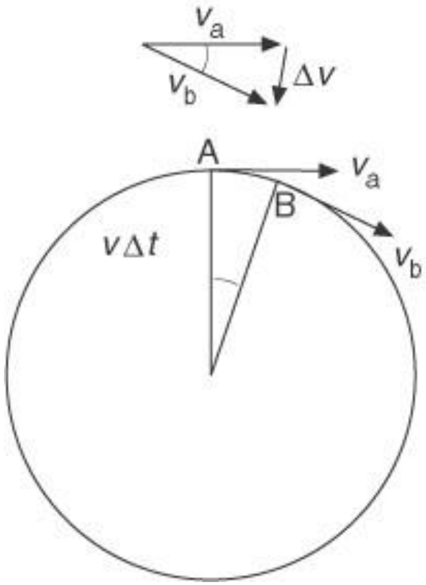
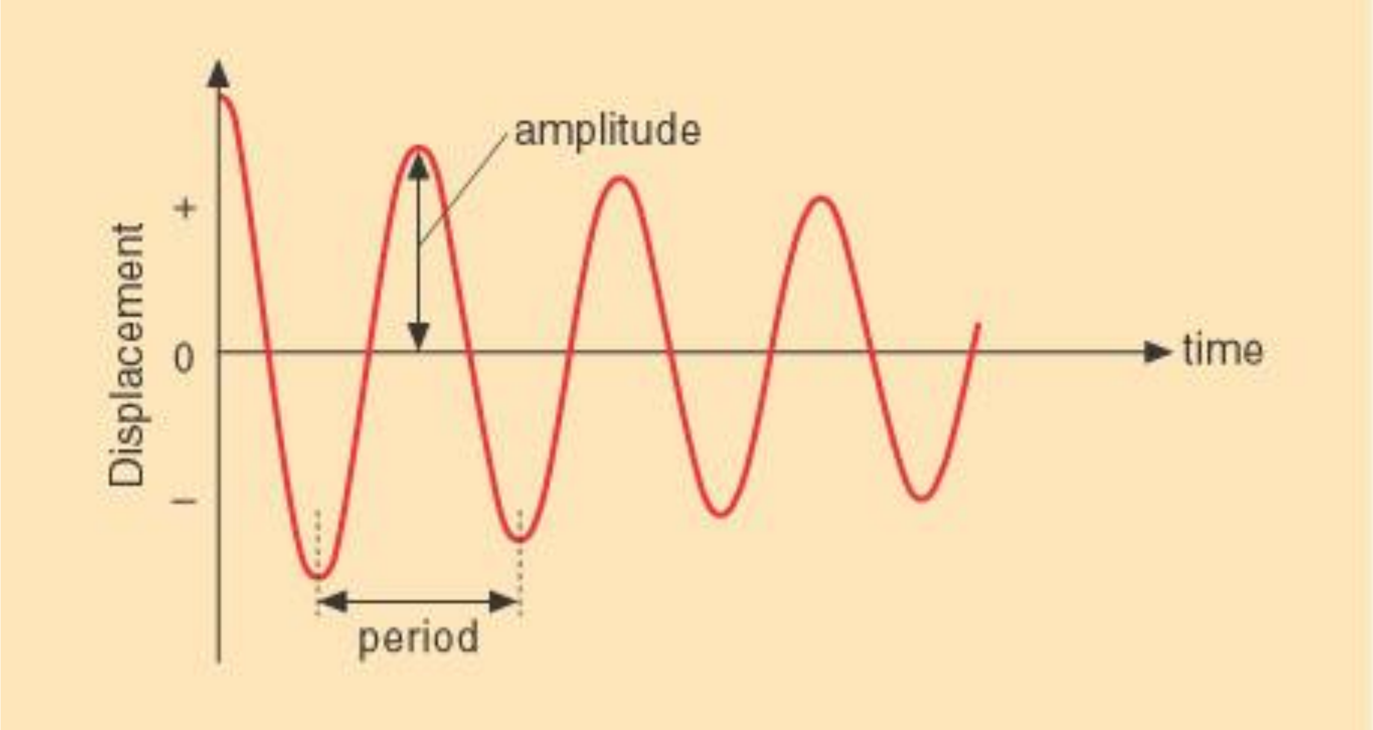
Waves and SHM

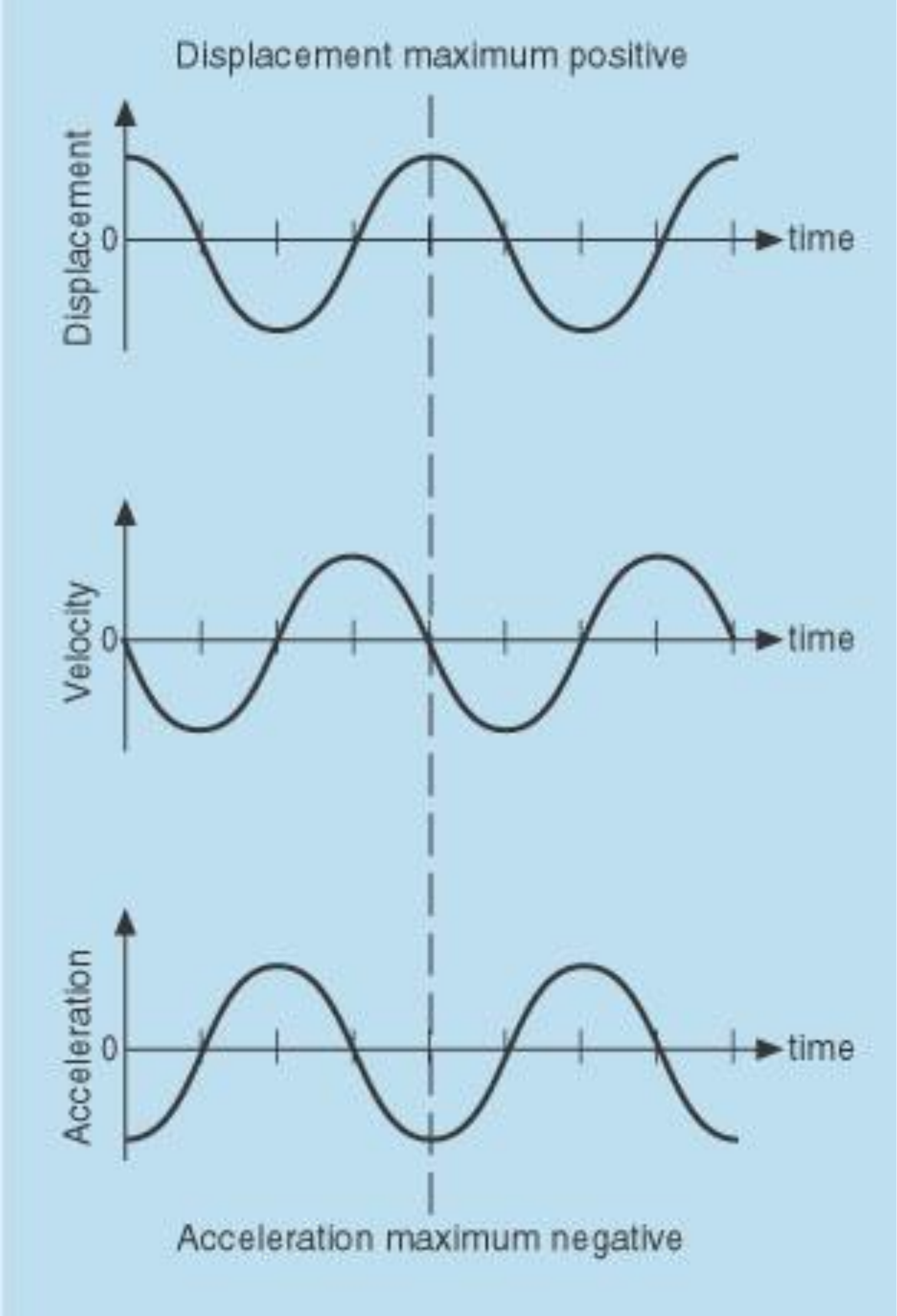
Dr P Lawson

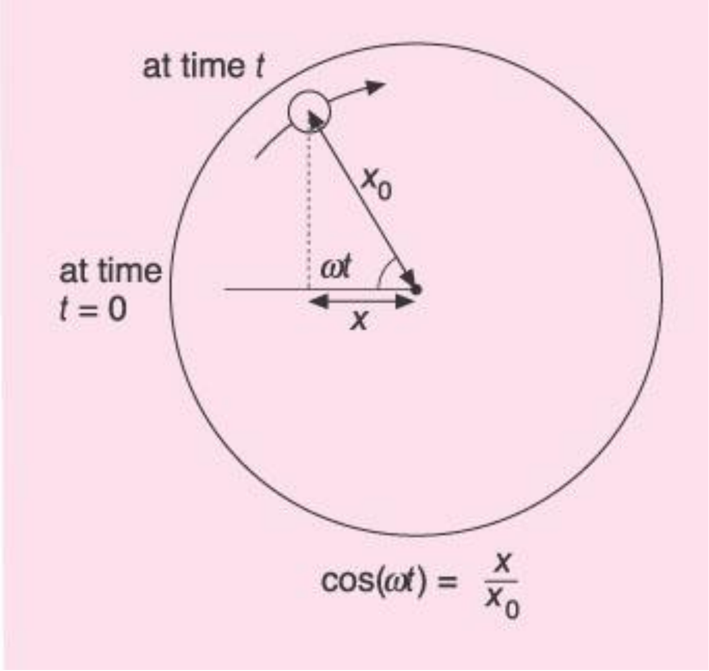
Periodic Motion
results from
Elasticity
which in linear case is described by
Hooke's Law
Elastic Potential Energy
and in extended media
Bulk Elastic Properties
are described by
Young's Modulus
includes
Pendulum
and the
Physical Pendulum
is described by
Period and Frequency
and in the case of
Traveling Waves
which when at
Resonance
produces
Standing Waves
also by the
Wavelength Relationship
can be constructed from
Simple Harmonic Motion
including the
Damped Oscillator
and
Driven Oscillator


v
v
v

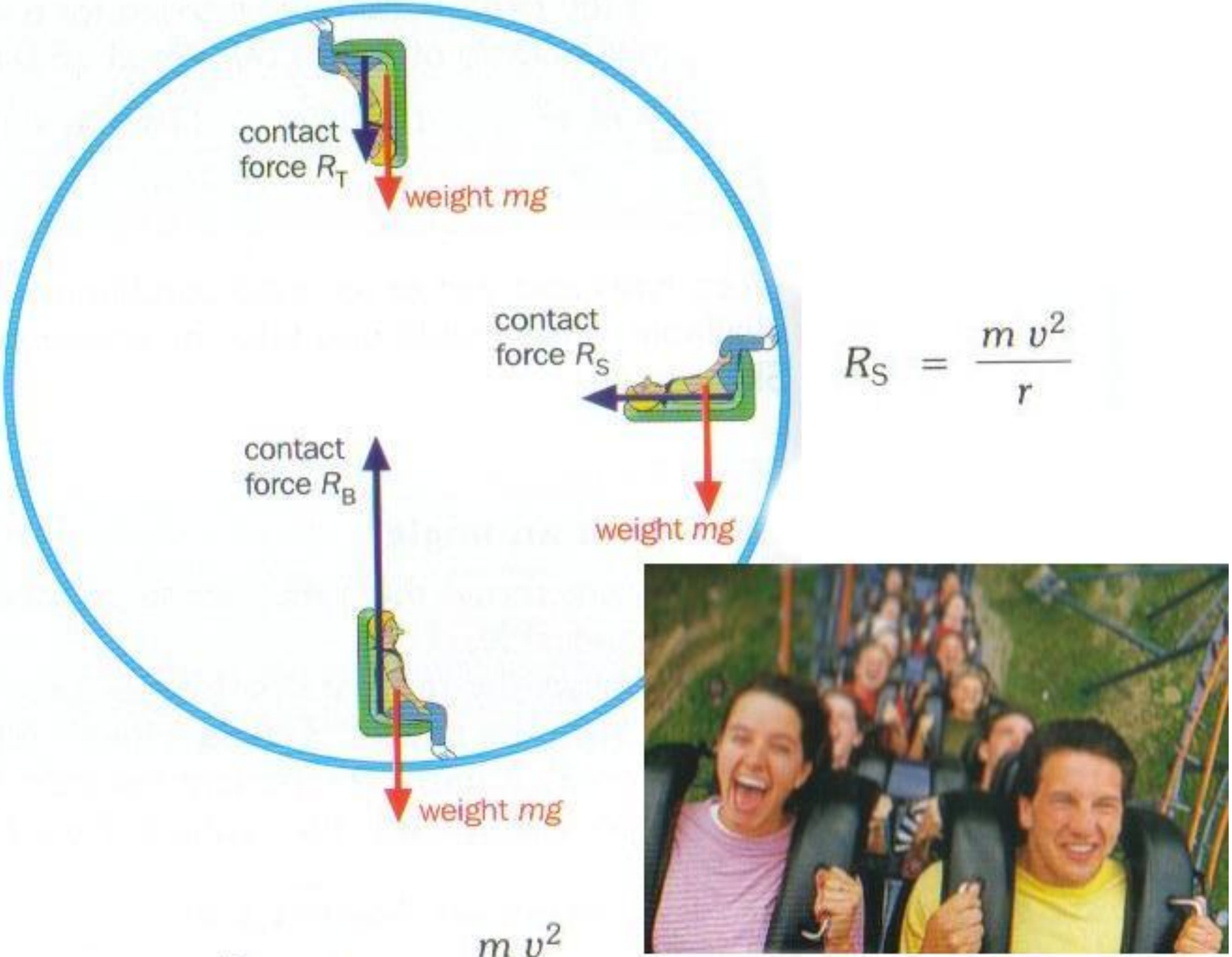

v_a
v_b
Δv
A
v_a
B
v_b
$v\Delta t$


amplitude
Displacement
+
0
–
time
period


Displacement maximum positive
Displacement
0
time
Velocity
0
time
Acceleration
0
time
Acceleration maximum negative


at time t
x_0
ωt
at time $t = 0$
x
$\cos(\omega t) = \frac{x}{x_0}$

$$R_T + mg = \frac{m v^2}{r}$$



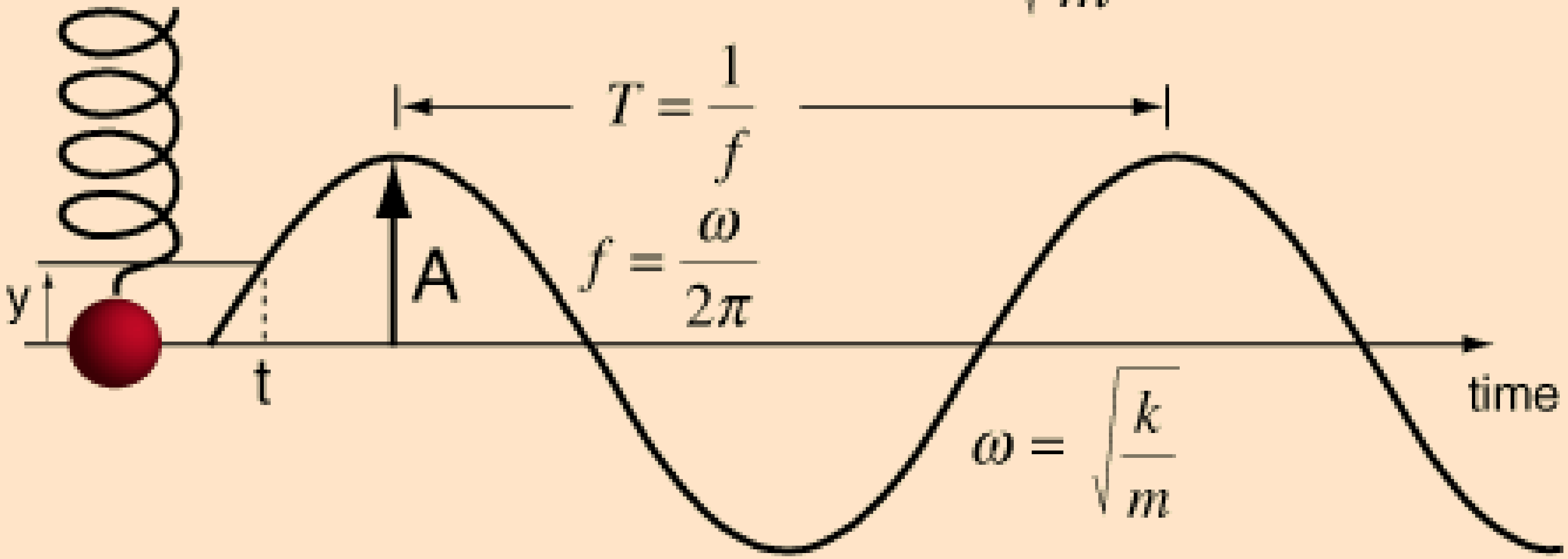
$$R_S = \frac{m v^2}{r}$$

$$R_B - mg = \frac{m v^2}{r}$$

Simple Harmonic Motion

Simple harmonic motion is typified by the motion of a mass on a spring when it is subject to the linear elastic restoring force given by Hooke's Law. The motion is sinusoidal in time and demonstrates a single resonant frequency.

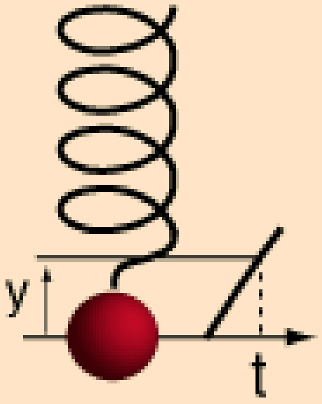
$$y = A\sin\omega t = A\sin\sqrt{\frac{k}{m}}\,t$$



Simple Harmonic Motion Equations

The motion equation for simple harmonic motion contains a complete description of he motion, and other parameters of the motion can be calculated from it.

$$y = A\sin\omega t = A\sin\sqrt{\frac{k}{m}}t$$



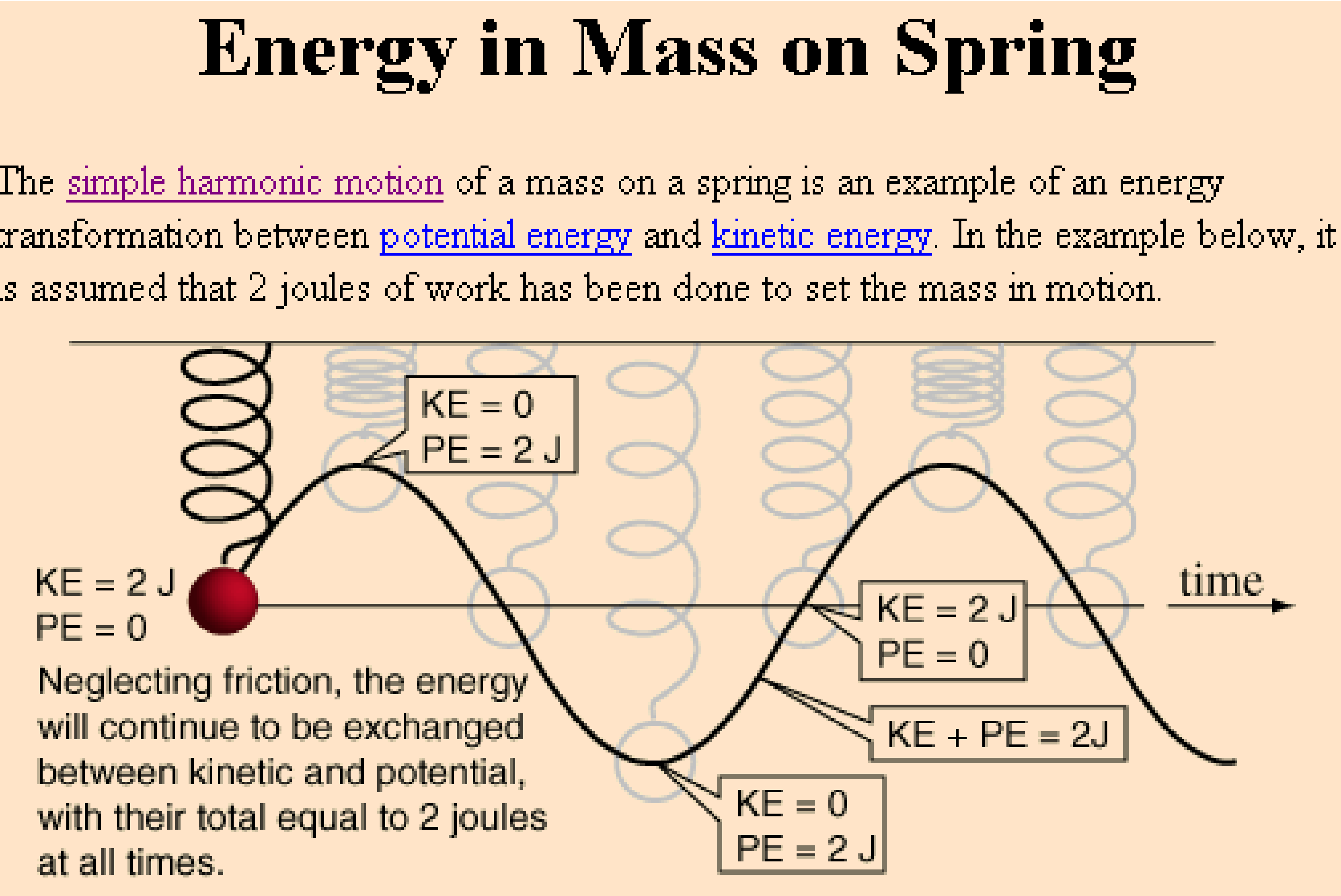
The velocity and acceleration are given by

$$v = \omega A\cos\omega t$$

$$a = -\omega^2 A\sin\omega t = -\omega^2 y$$

The total energy for an undamped oscillator is the sum of its kinetic energy and potential energy, which is constant at

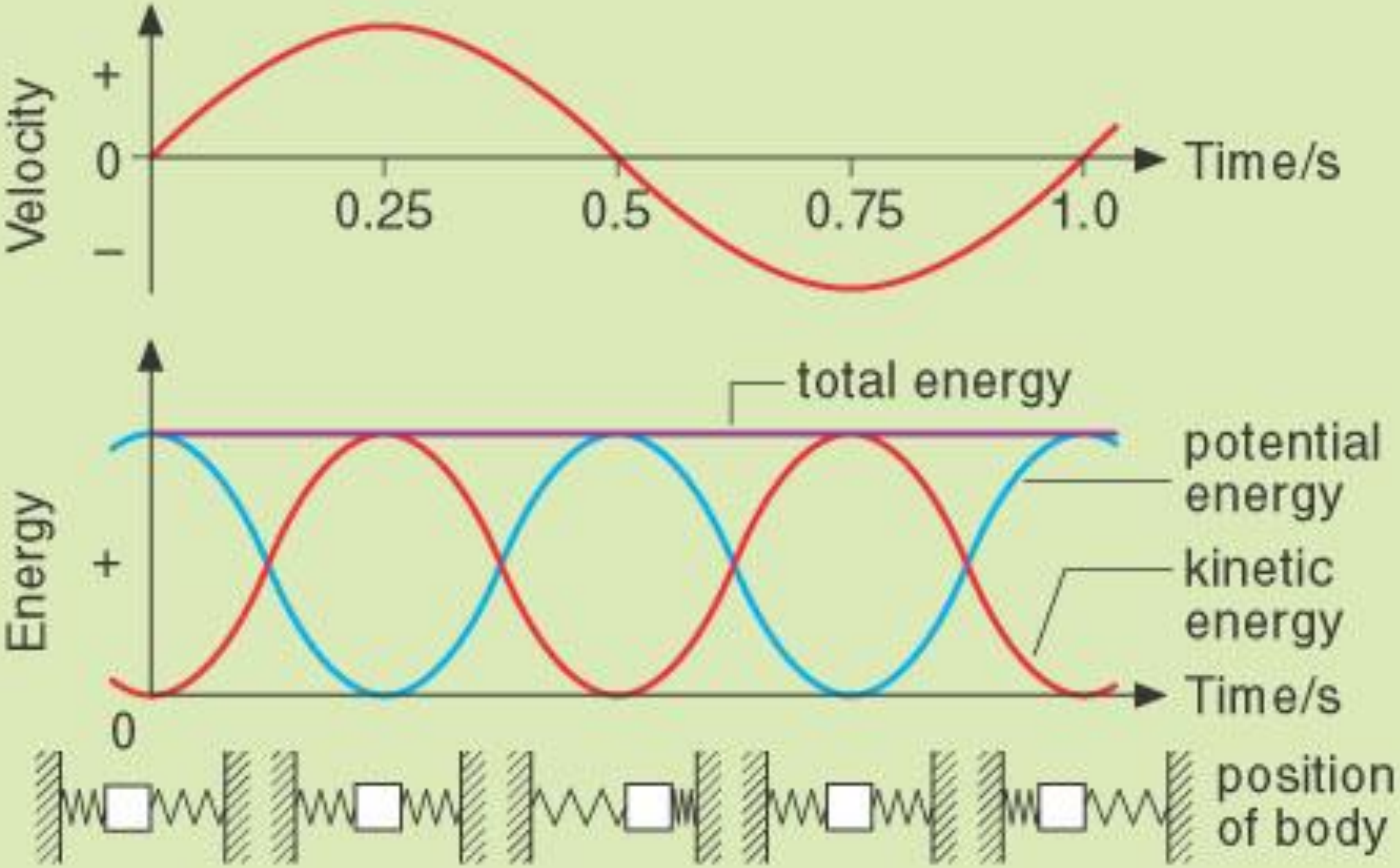
$$E = KE + PE = \frac{1}{2}kA^2$$

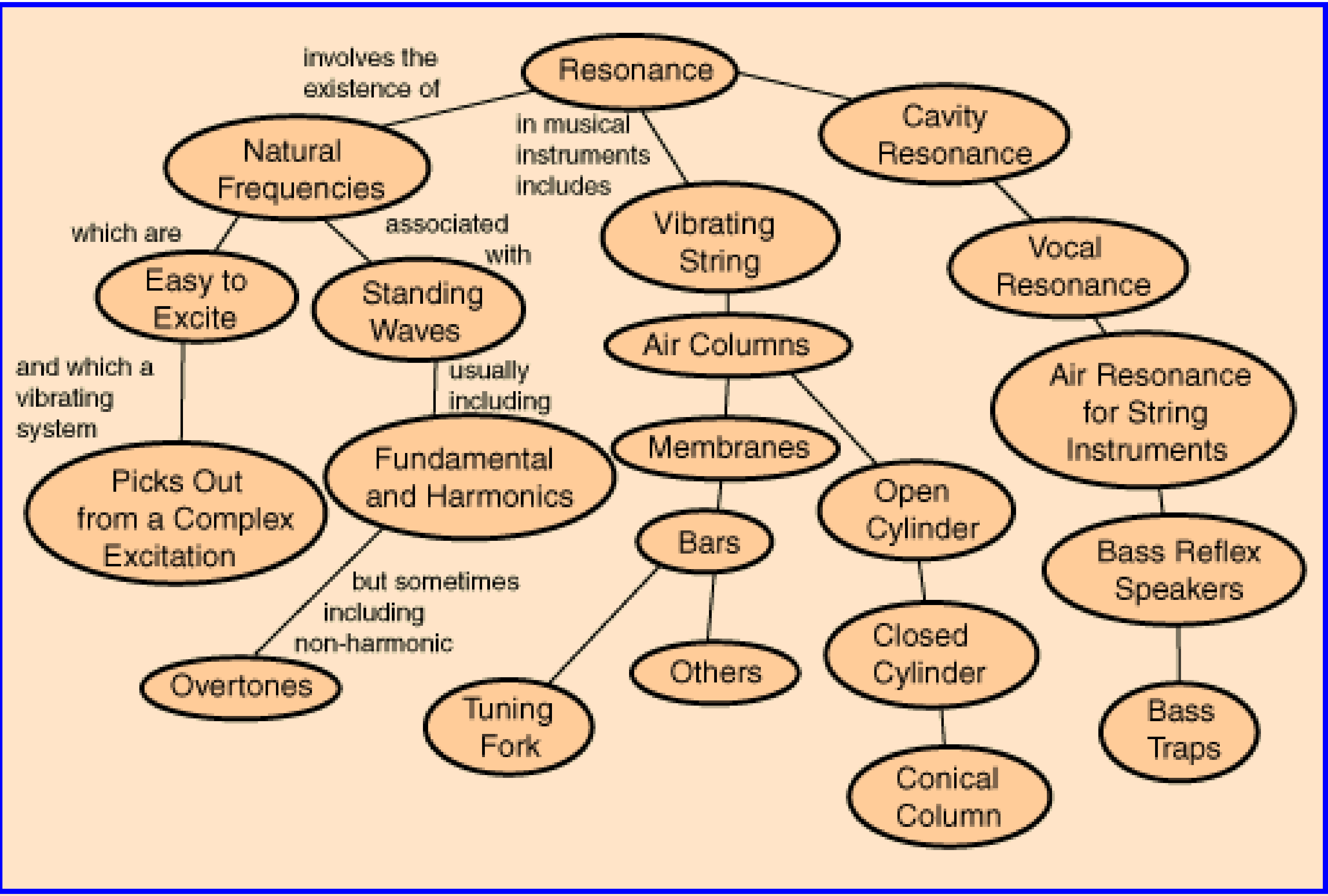


Energy in Mass on Spring

The simple harmonic motion of a mass on a spring is an example of an energy ransformation between potential energy and kinetic energy. In the example below, it s assumed that 2 joules of work has been done to set the mass in motion.

Neglecting friction, the energy will continue to be exchanged between kinetic and potential, with their total equal to 2 joules at all times.


Velocity
+
0
−
Time/s
0.25
0.5
0.75
1.0
Energy
+
total energy
potential energy
kinetic energy
Time/s
0
position of body



Resonance

involves the existence of

Natural Frequencies

which are

Easy to Excite

and which a vibrating system

Picks Out from a Complex Excitation

associated with

Standing Waves

usually including

Fundamental and Harmonics

but sometimes including non-harmonic

Overtones

in musical instruments includes

Vibrating String

Air Columns

Open Cylinder

Closed Cylinder

Conical Column

Membranes

Bars

Tuning Fork

Others

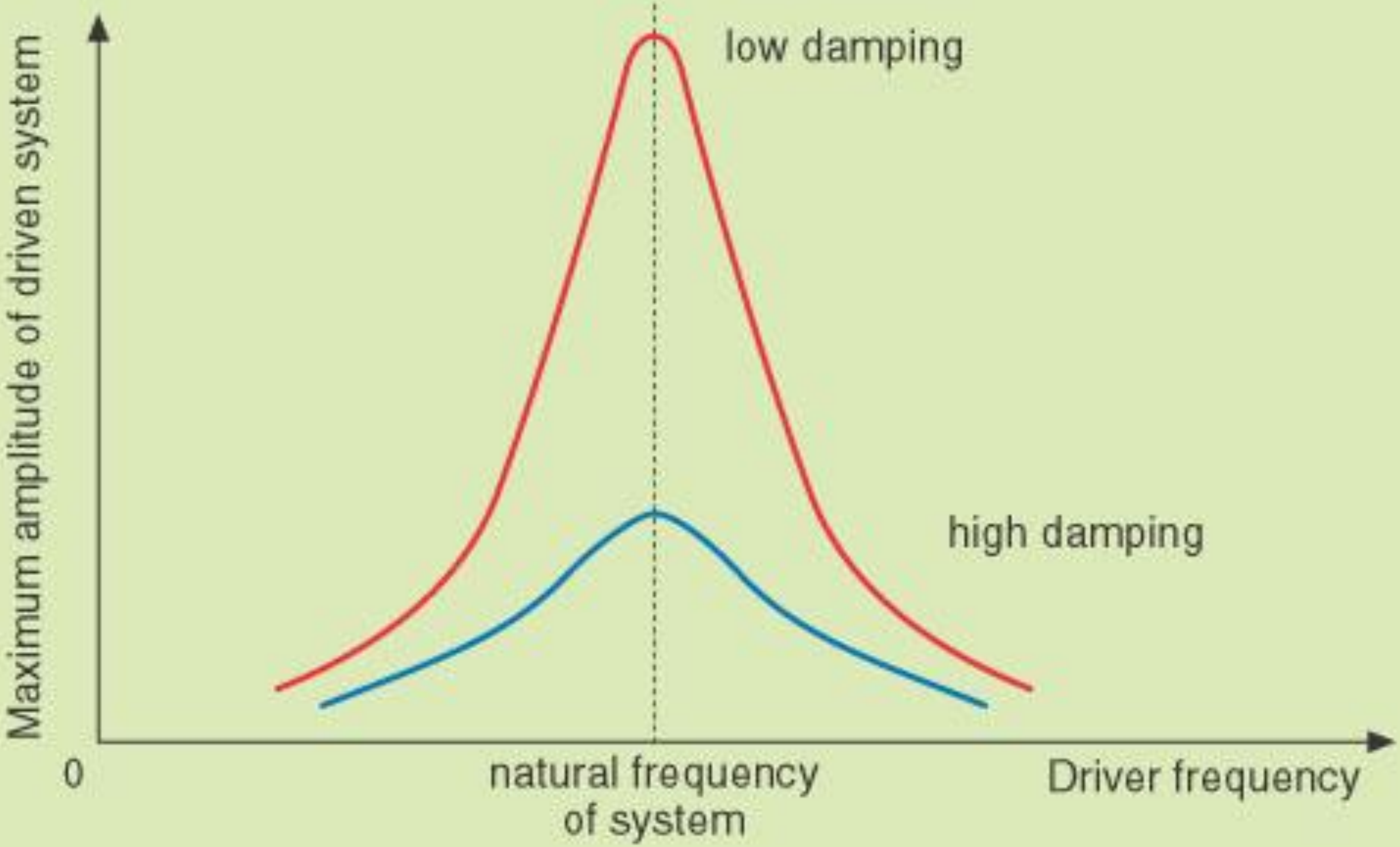
Cavity Resonance

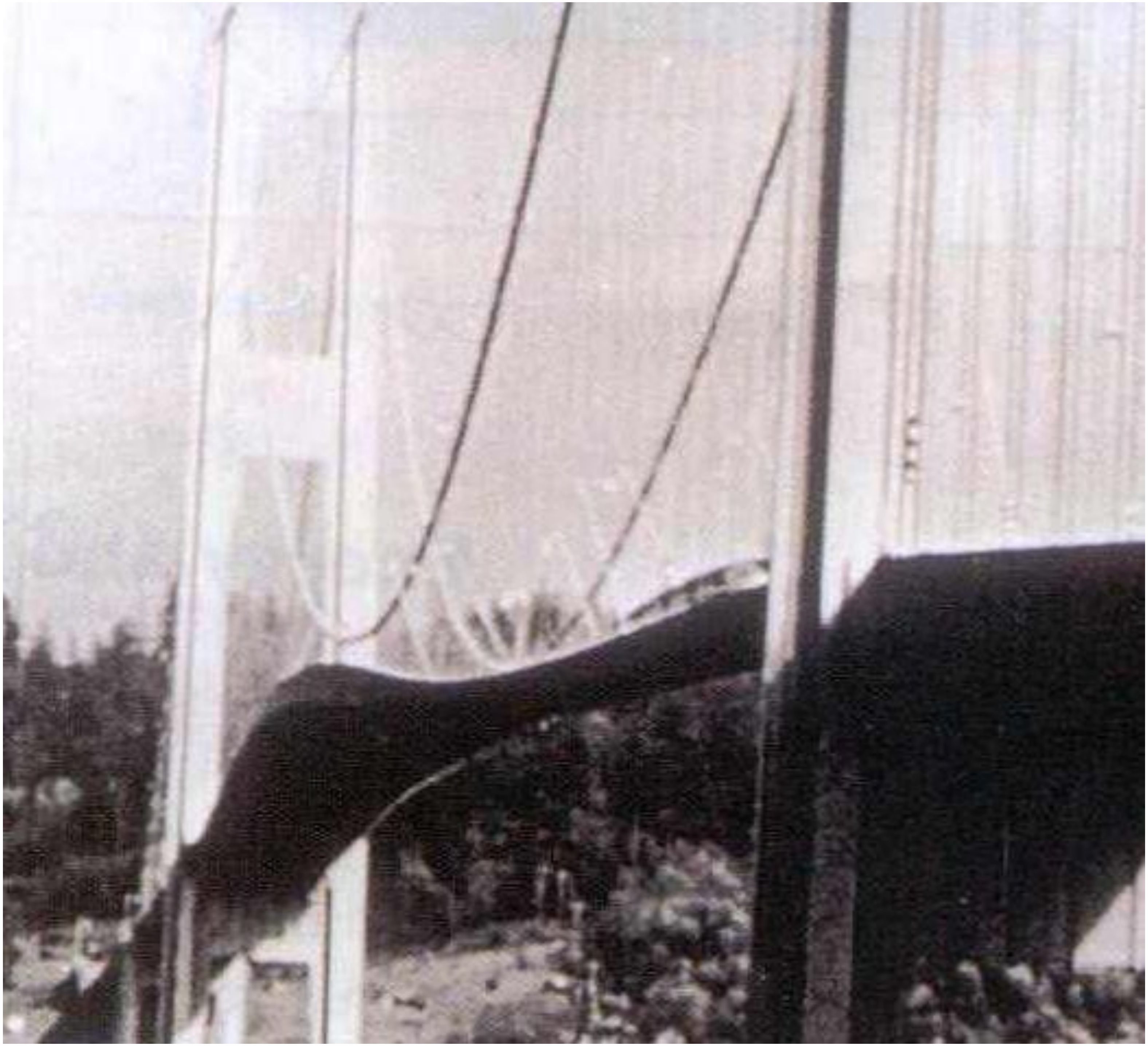
Vocal Resonance

Air Resonance for String Instruments

Bass Reflex Speakers

Bass Traps


Maximum amplitude of driven system
low damping
high damping
0
natural frequency
of system
Driver frequency



Forced oscillations

- Hang 200 g from a spring and take sufficient readings to find the natural frequency of this system.
- Attach the spring and mass to the vibration generator as shown in Figure 9.1. Set the signal generator to produce sinusoidal waves and use a meter to measure its output frequency.
- Observe the mass's motion as you slowly increase the output frequency from well below the natural frequency of the mass–spring system to well above it. What do you expect to happen to the oscillations?
- Repeat the experiment for a different oscillating mass.

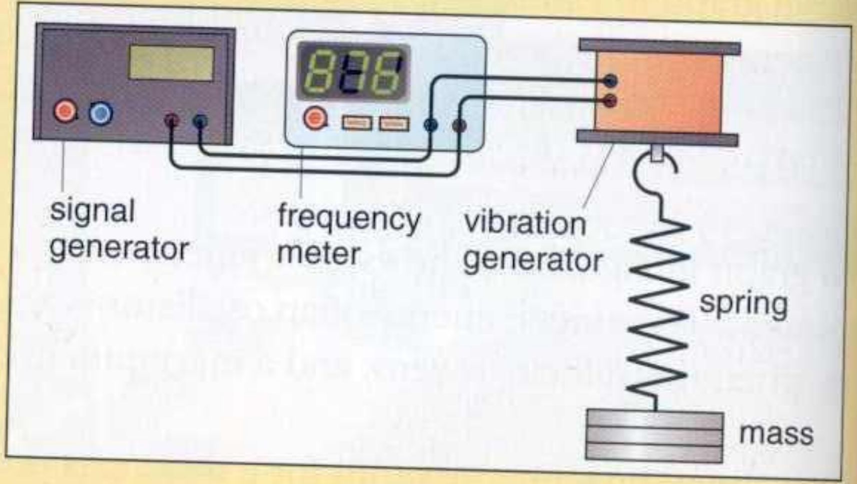
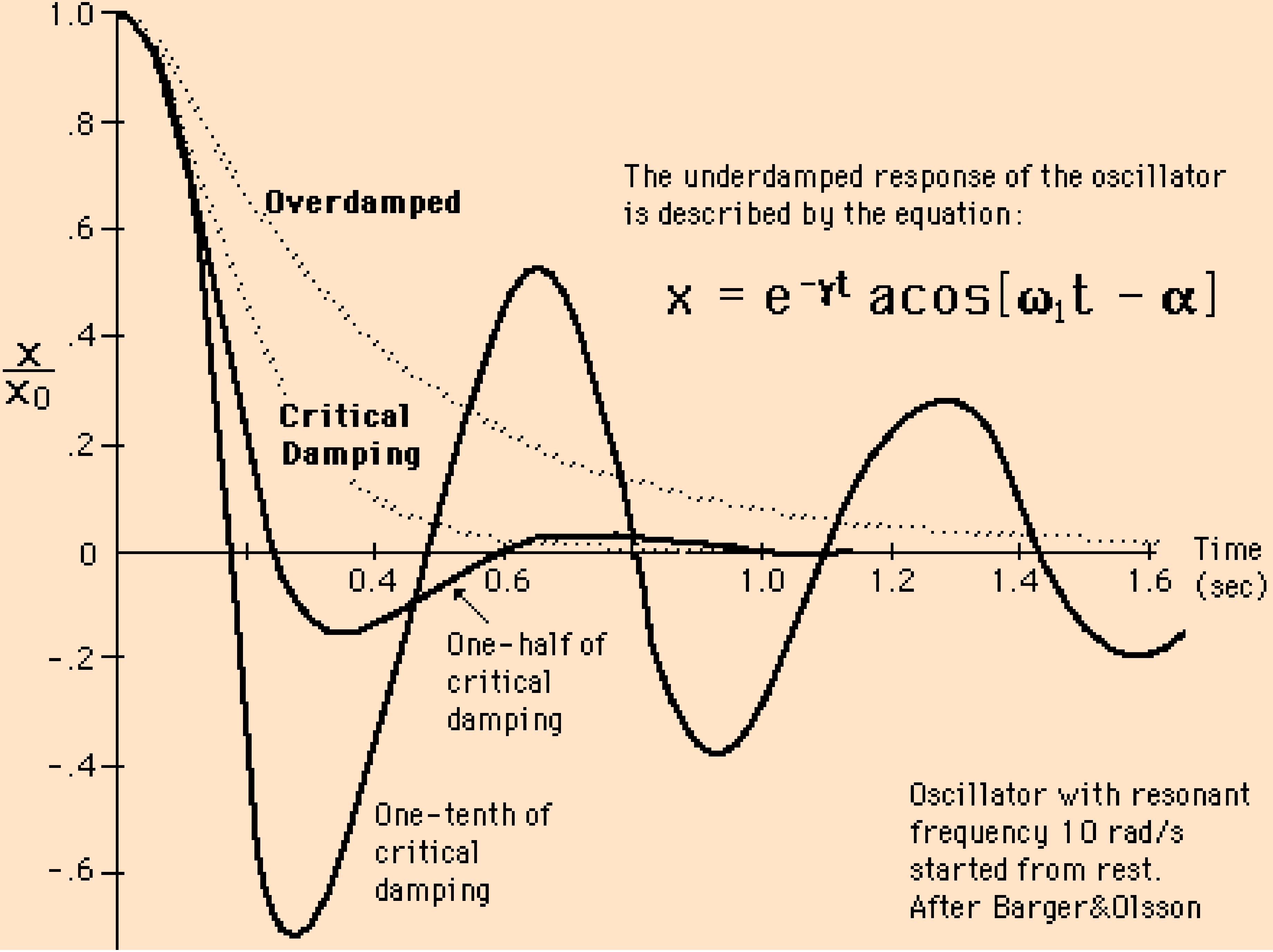


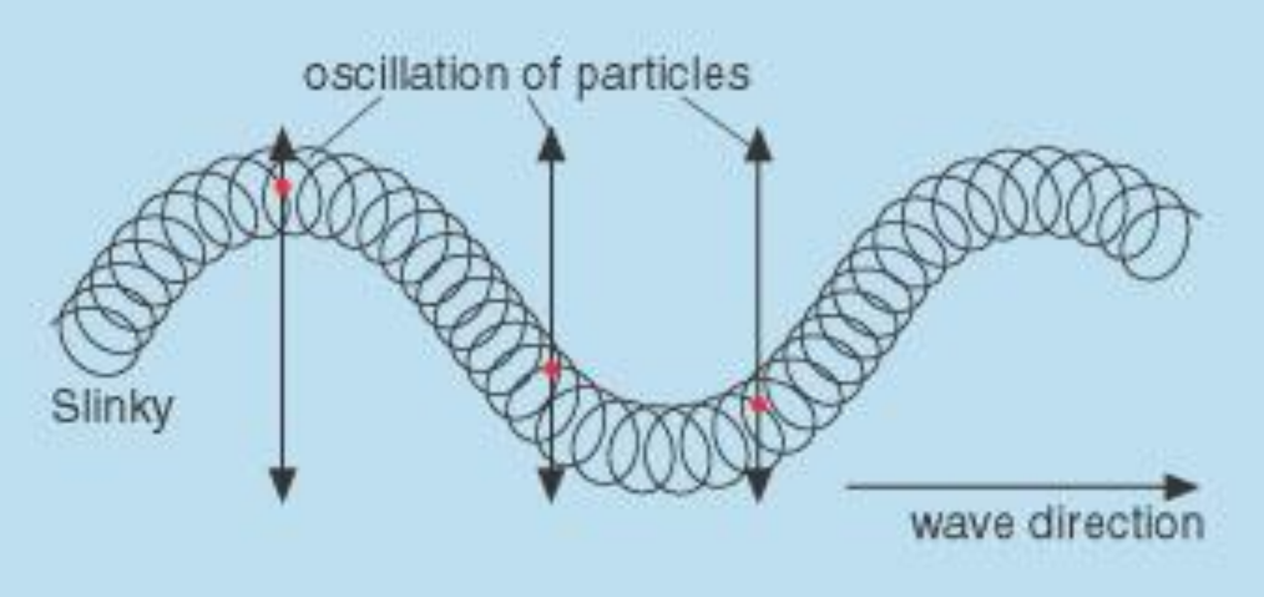
Figure 9.1 The vibrator causes the mass to oscillate

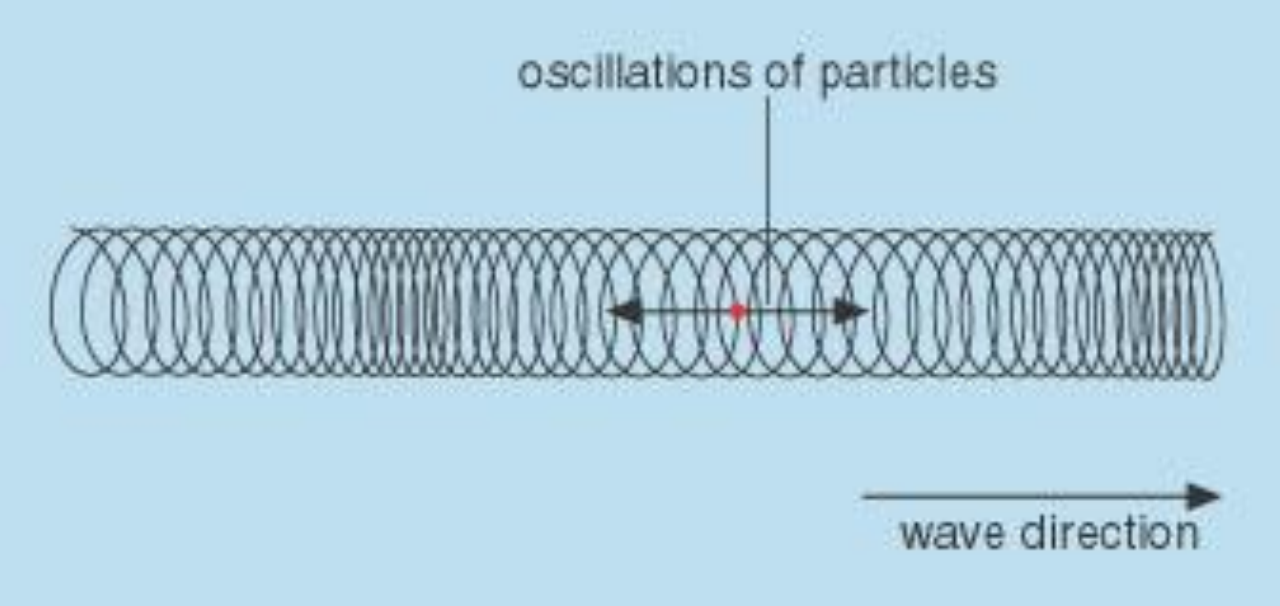


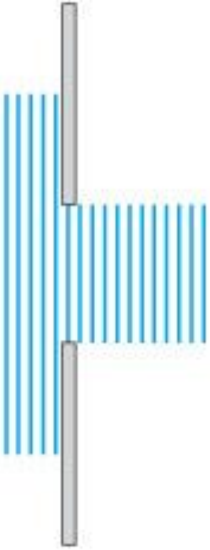
The underdamped response of the oscillator is described by the equation:

$$x = e^{-\gamma t} a\cos[\omega_1 t - \alpha]$$

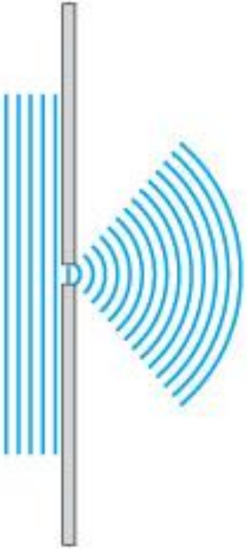
Oscillator with resonant frequency 10 rad/s started from rest.
After Barger&Olsson


oscillation of particles
Slinky
wave direction

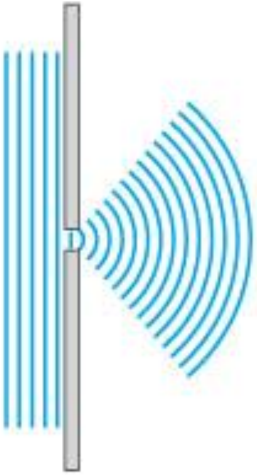

oscillations of particles
wave direction



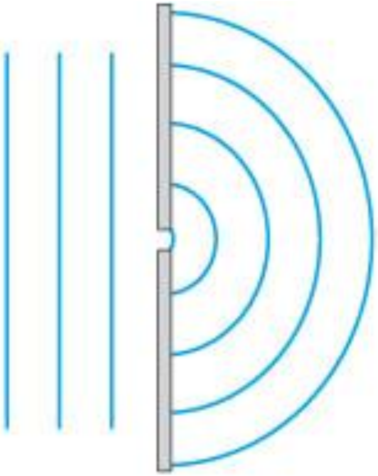
small
wavelength,
large gap



small
wavelength,
small gap



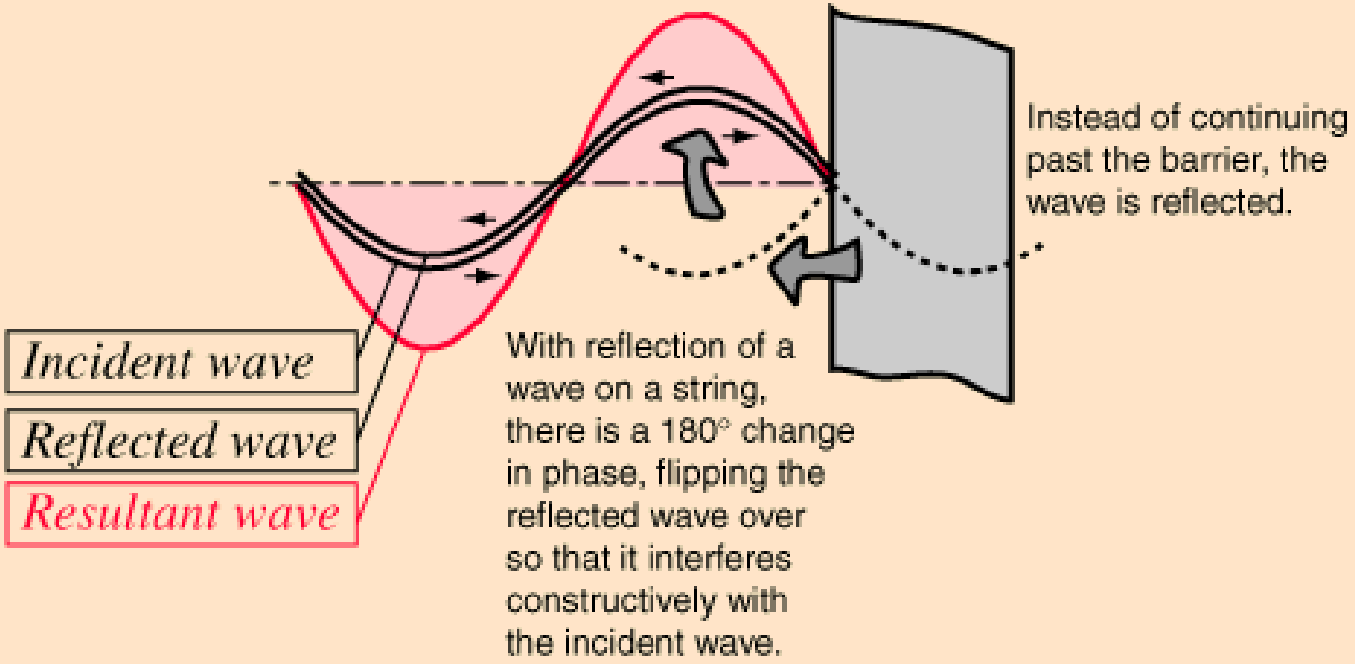
small wavelength,
small gap



large wavelength,
small gap

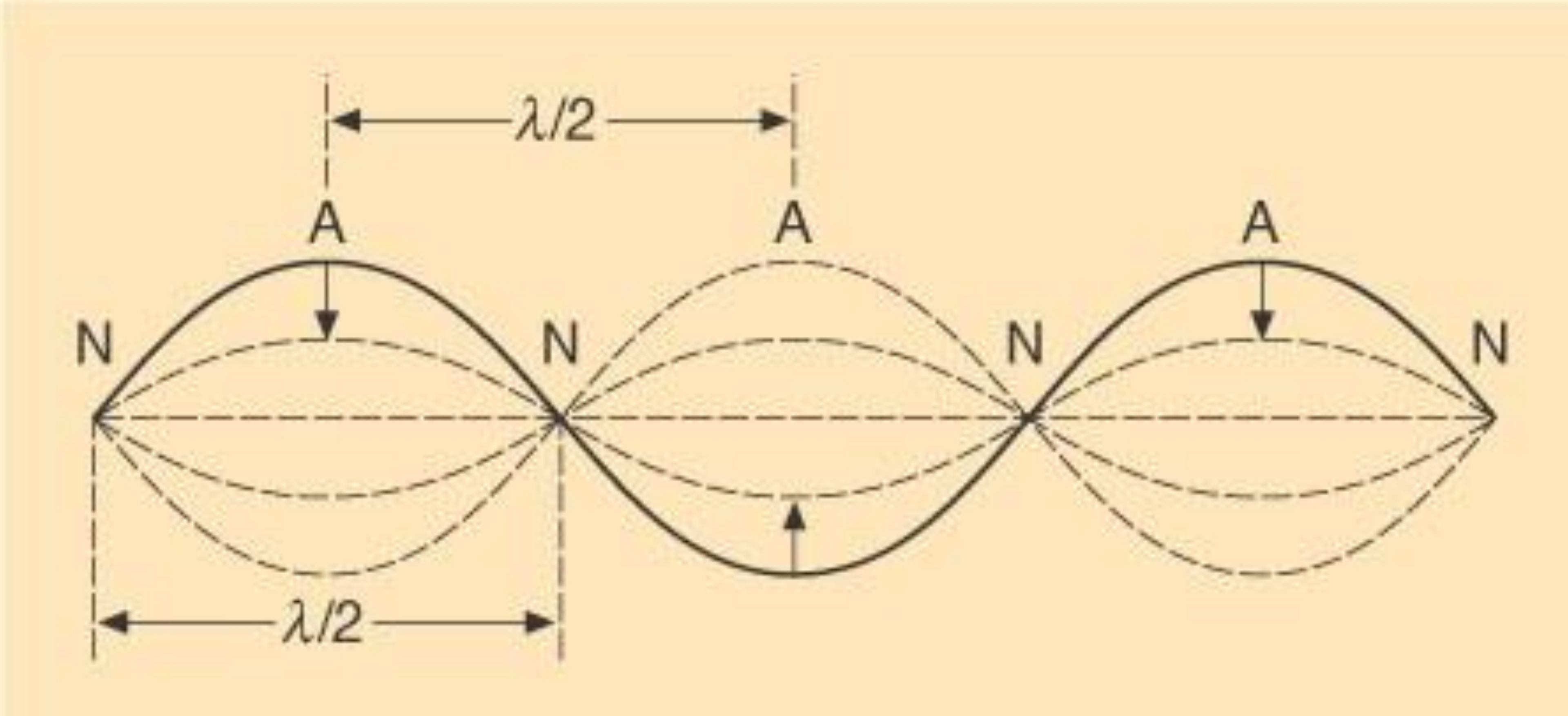
Standing Waves

The modes of vibration associated with resonance in extended objects like strings and air columns have characteristic patterns called standing waves. These standing wave modes arise from the combination of reflection and interference such that the reflected waves interfere constructively with the incident waves. An important part of the condition for this constructive interference for stretched strings is the fact that the waves change phase upon reflection from a fixed end. Under these conditions, the medium appears to vibrate in segments or regions and the fact that these vibrations are made up of traveling waves is not apparent - hence the term "standing wave".



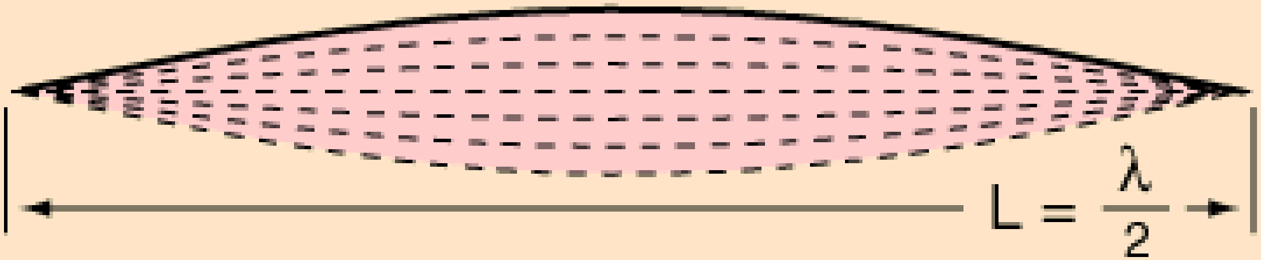
Instead of continuing past the barrier, the wave is reflected.

With reflection of a wave on a string, there is a 180° change in phase, flipping the reflected wave over so that it interferes constructively with the incident wave.


λ/2
A
A
A
N
N
N
N
λ/2

Vibrating String

The fundamental vibrational mode of a stretched string is such that the wavelength is twice the length of the string.



Applying the basic wave relationship gives an expression for the fundamental frequency:

$$f_1 = \frac{v_{\text{wave on string}}}{2L}$$

Calculation

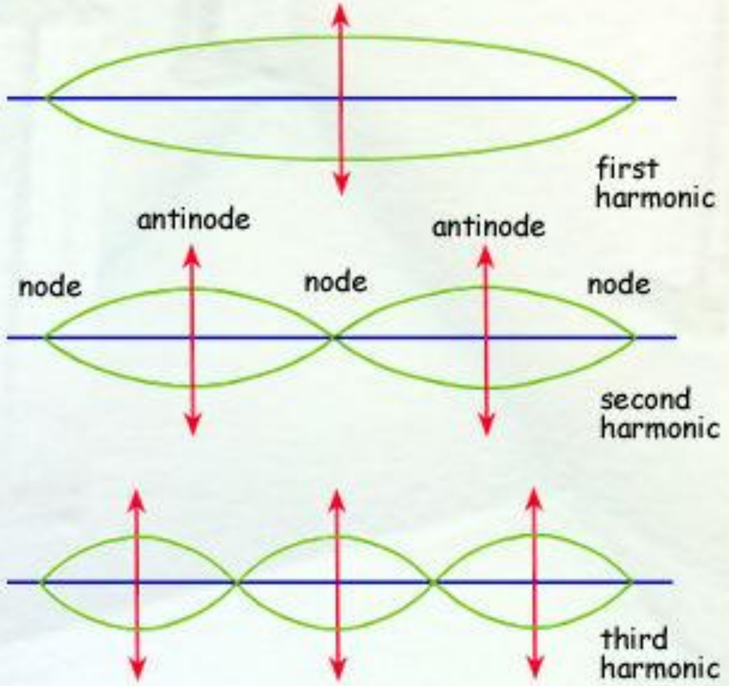
Since the wave velocity is given by $v = \sqrt{\frac{T}{m / L}}$, the frequency expression

The string will also vibrate at all harmonics of the fundamental. Each of these harmonics will form a standing wave on the string.

Standing Waves

Standing or stationary waves are produced when two waves with the same frequency and amplitude move towards each other and become superimposed. The result is the creation of fixed nodal points of zero displacement which alternate with fixed antinodal points of maximum displacement. These nodes and antinodes are half a wavelength apart. Standing waves do not transfer energy along their length, as do progressive waves.

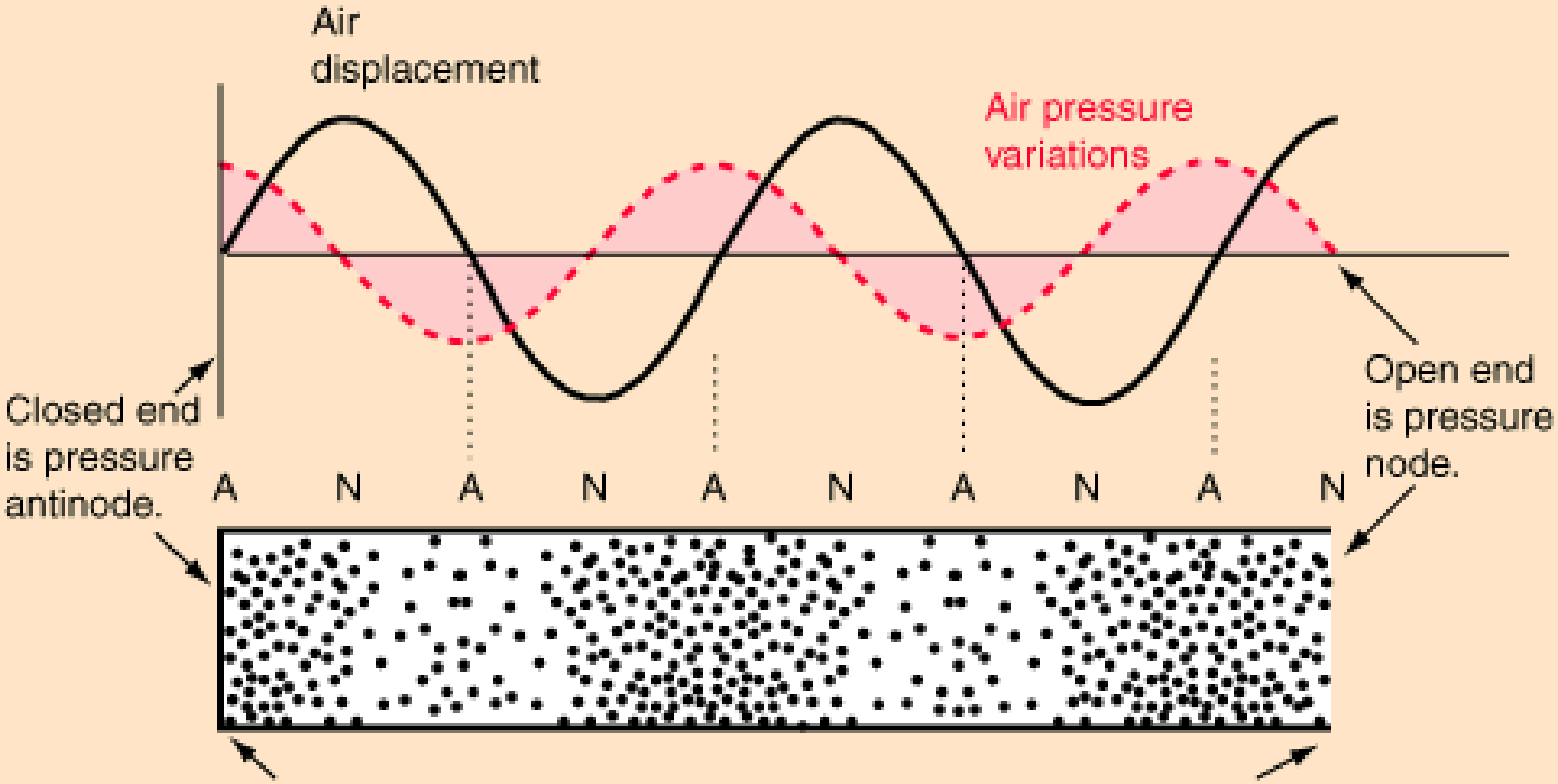
Plucking a stretched string fixed at both ends, such as one on a stringed instrument, produces a standing wave with two fixed nodes at each end. The three simplest standing waves or modes are the first, second and third harmonics.



The first harmonic (or fundamental) has a node separation of $\frac{\lambda}{2} = L$
The string length is half a wavelength.
The second harmonic (or first overtone) has a node separation of $\frac{\lambda}{2} = \frac{L}{2}$
The string length is one wavelength.
The third harmonic (or second overtone) has a node separation of $\frac{\lambda}{2} = \frac{L}{3}$
The string length is one and a half wavelengths.

Standing waves such as the vibrations (overtones) of a stringed instrument can generate progressive waves. The progressive sound waves passing through air have the same frequency as the plucked string which generates them. The frequencies of each harmonic are whole number multiples of the lowest fundamental frequency. For a stringed instrument, when plucked, the string vibrates with all these frequencies.

Production of a standing wave in an air column involves reflections from both the closed end and the open end of the column.



Closed end is pressure antinode.

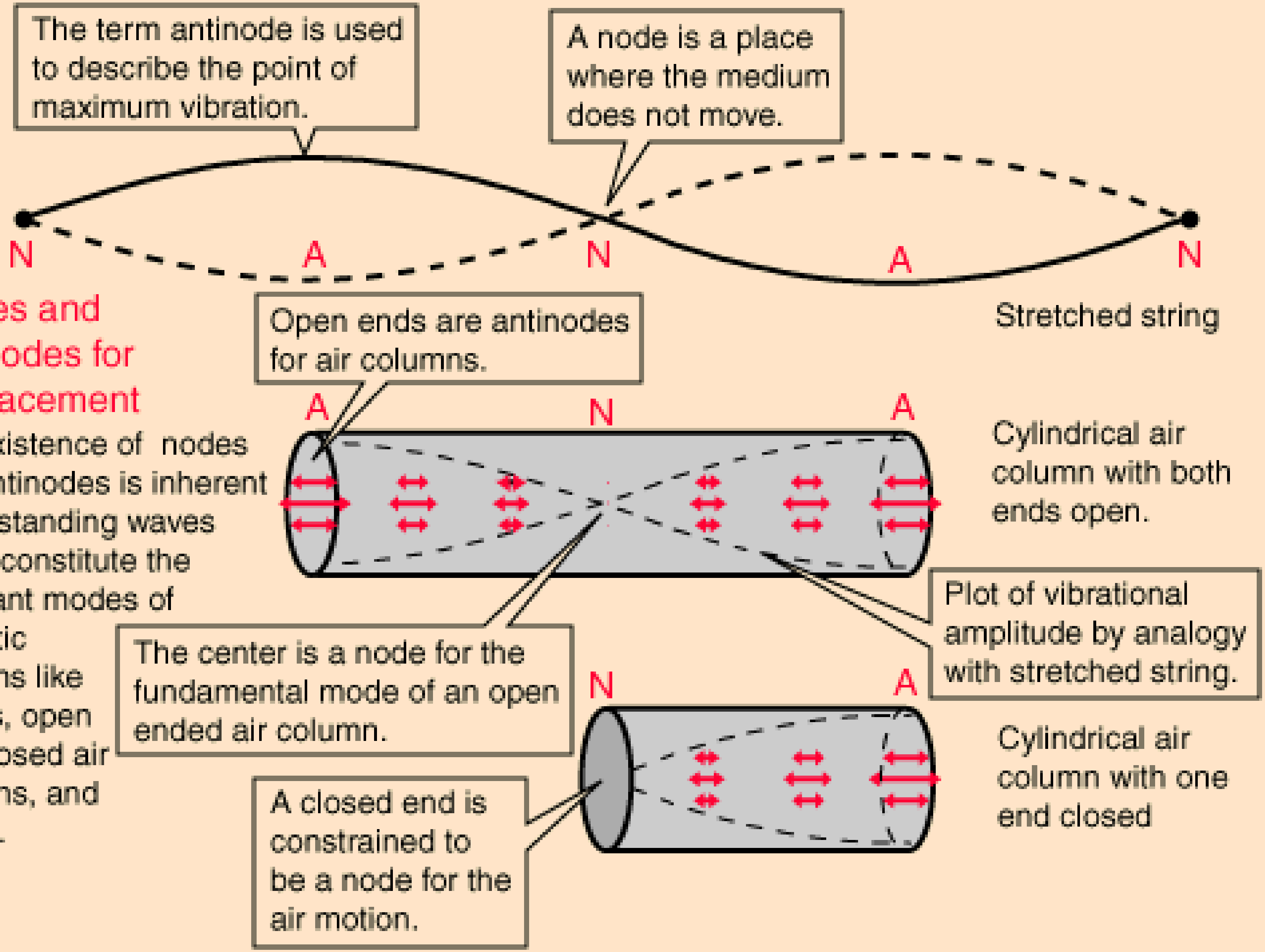
Open end is pressure node.

No phase change upon reflection since the wave encounters a greater acoustic impedance.

180° phase change upon reflection since the wave encounters a lesser acoustic impedance. This phase change is associated with energy loss as sound energy is transmitted to the outside air.

Nodes and antinodes for displacement

The existence of nodes and antinodes is inherent in the standing waves which constitute the resonant modes of acoustic systems like strings, open and closed air columns, and others.



The term antinode is used to describe the point of maximum vibration.

A node is a place where the medium does not move.

Open ends are antinodes for air columns.

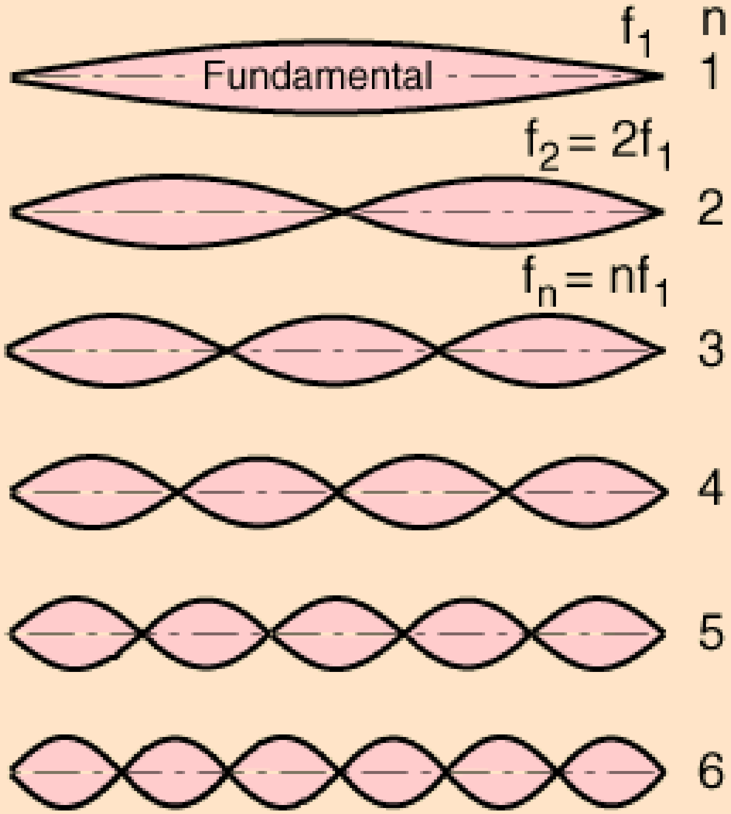
The center is a node for the fundamental mode of an open ended air column.

Plot of vibrational amplitude by analogy with stretched string.

A closed end is constrained to be a node for the air motion.

Harmonics

An ideal vibrating string will vibrate with its fundamental frequency and all harmonics of that frequency. The position of nodes and antinodes is just the opposite of those for an open air column.



The fundamental frequency can be calculated from

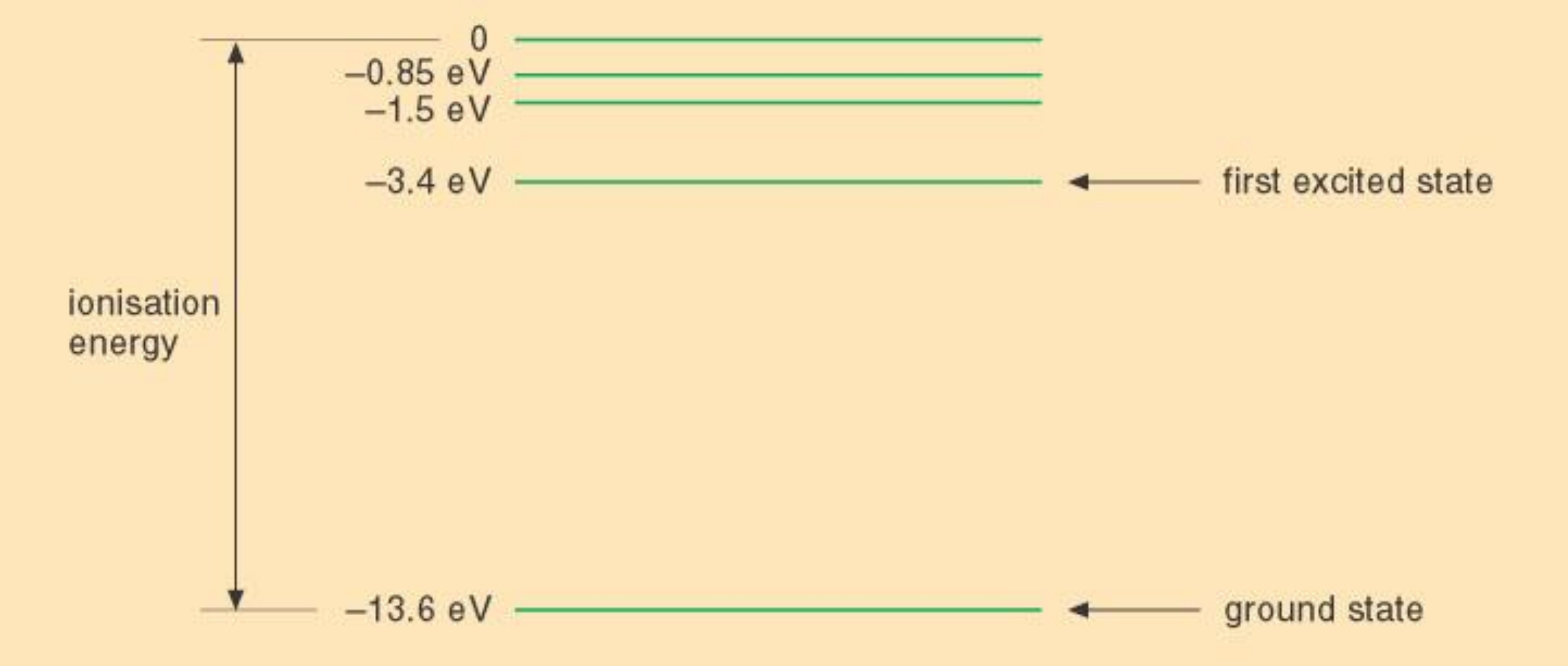
$$f_1 = \frac{v_{wave\ on\ string}}{2L}$$

where

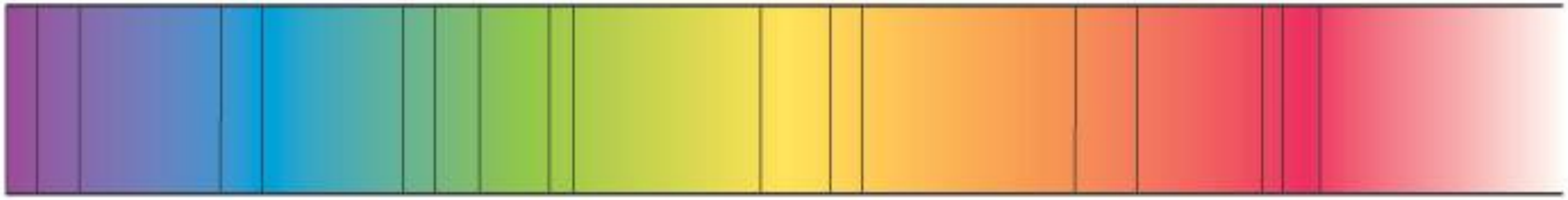
$$v_{\text{wave in string}} = \sqrt{\frac{T}{m/L}}$$

and the harmonics are integer multiples.

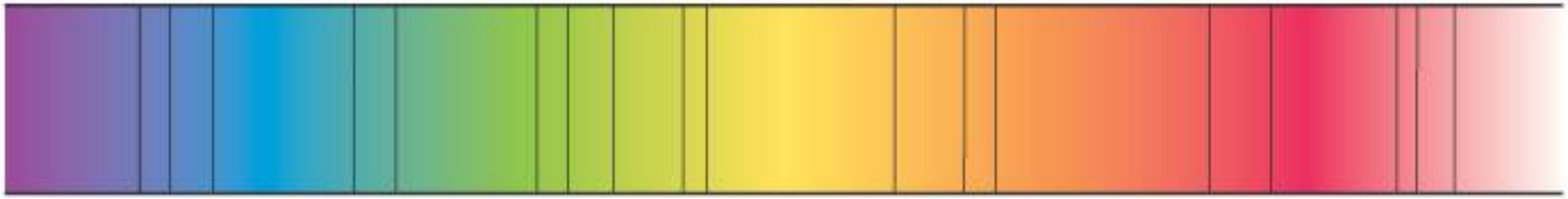
T = string tension
m = string mass
L = string length



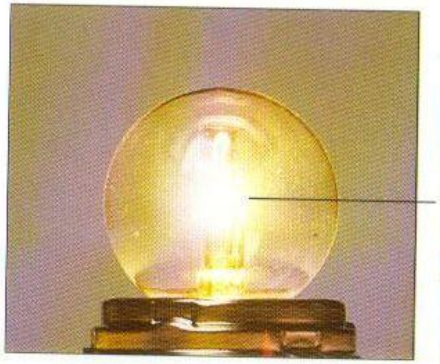
(a) Absorption bands in light from nearby galaxies



(b) Absorption bands in light from distant galaxies



SOURCES OF LIGHT

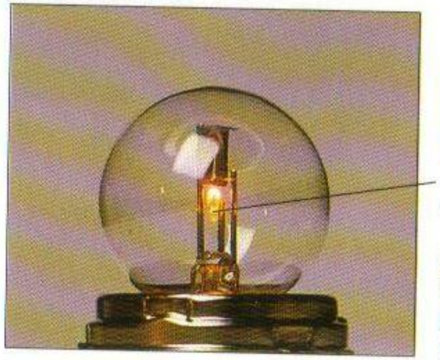


BRIGHT FILAMENT LAMP

This spectrum shows which colours are produced

All colours of light together combine to produce white

BRIGHT FILAMENT LAMP
With a high **electric current**, the whole spectrum of visible light is produced (see p. 39).



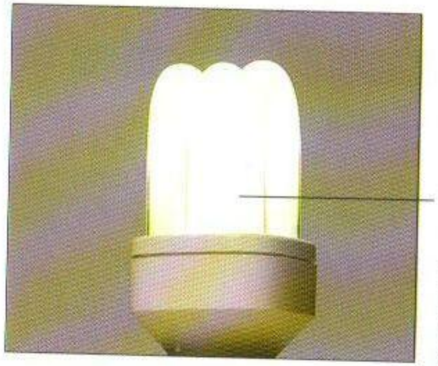
DIM FILAMENT LAMP

Red, yellow, and green light combine to produce orange

Lamp appears orange

No blue light produced

DIM FILAMENT LAMP
With a smaller current, the **temperature** of the **filament** (see pp. 32-33) is low.



FLUORESCENT LAMP

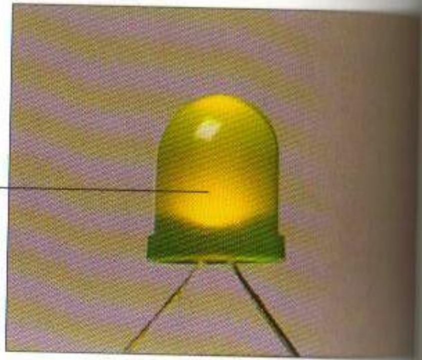
Lamp produces certain colours in each part of the spectrum

All three types of cone are stimulated and lamp appears white

FLUORESCENT LAMP
In a **fluorescent** lamp, chemicals called **phosphors** produce colours in many parts of the spectrum.

LED produces colours in the green part of the spectrum

LED appears green

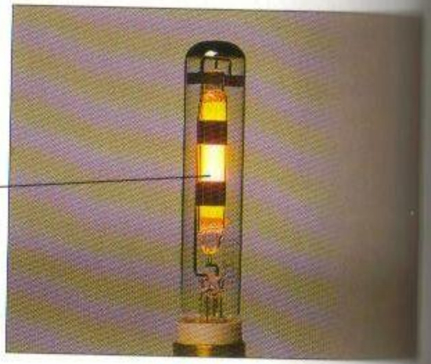


GREEN LED

GREEN LED
An LED (light-emitting diode) is made of a **semiconductor**, and produces certain colours of light.

Two colours of light very close together in the orange part of the spectrum are produced

Lamp appears orange

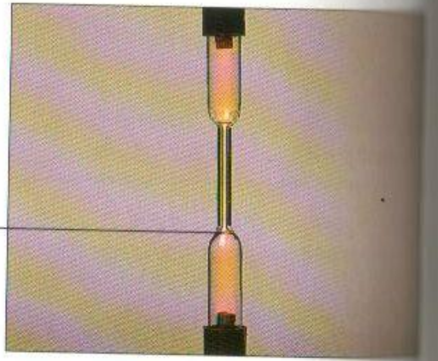


SODIUM LAMP

SODIUM LAMP
In a sodium lamp, an electric current excites electrons in sodium vapour, giving them extra energy. The electrons give the energy out as light.

Only certain colours characteristic of neon are produced

Lamp appears orange



NEON TUBE

NEON TUBE
In a similar way to a sodium lamp, a neon discharge lamp produces a characteristic orange glow.

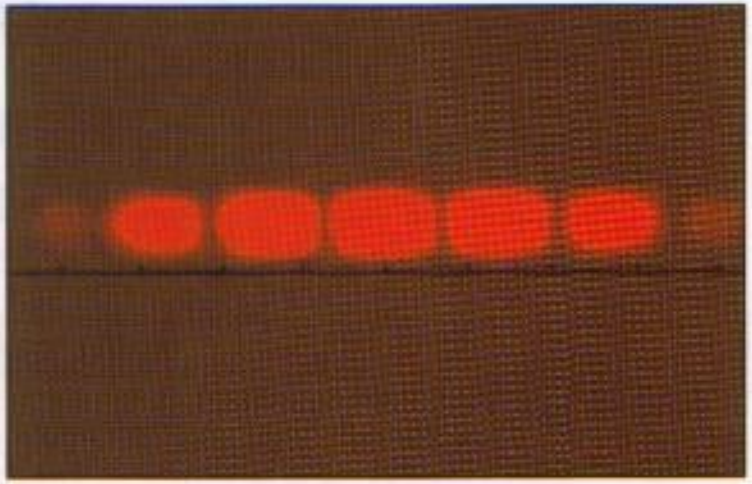


Figure 16.4 Superposition pattern for laser light through a double slit

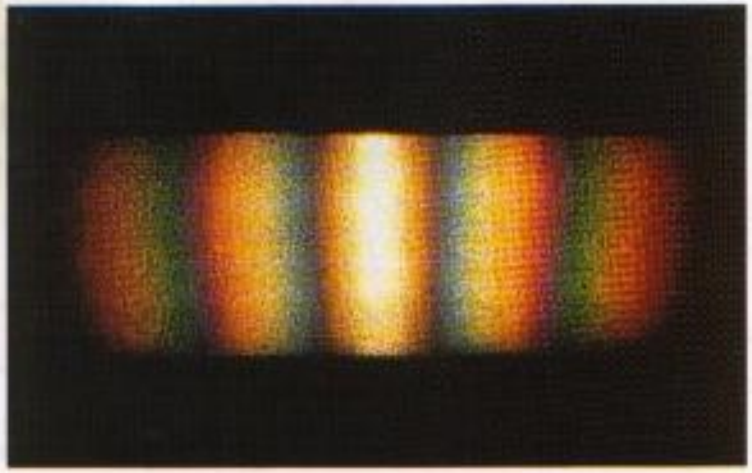
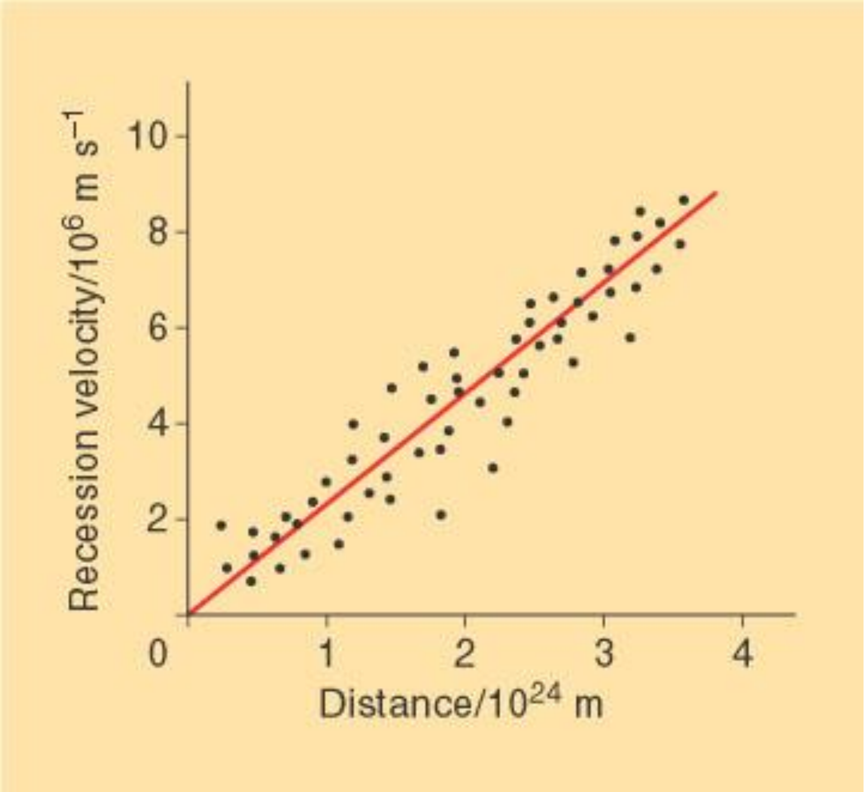


Figure 16.5 Superposition pattern caused by white light through a double slit


Recession velocity/10^6 m s^{-1}
10
8
6
4
2
0
1
2
3
4
Distance/10^{24} m

STANDARD CANDLES

Cepheid variable

Pulse $\propto$ size

Pulse rate

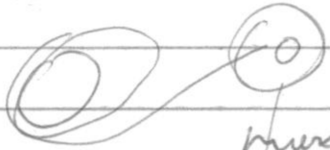
↳ size of ~~star~~ star

Look at size it appears on earth

↓

Determine distance

Type 1 supernova



Draws matter from nearby source.

mass $\approx 1\frac{1}{2}$ sun

↓

explodes

constant flash intens

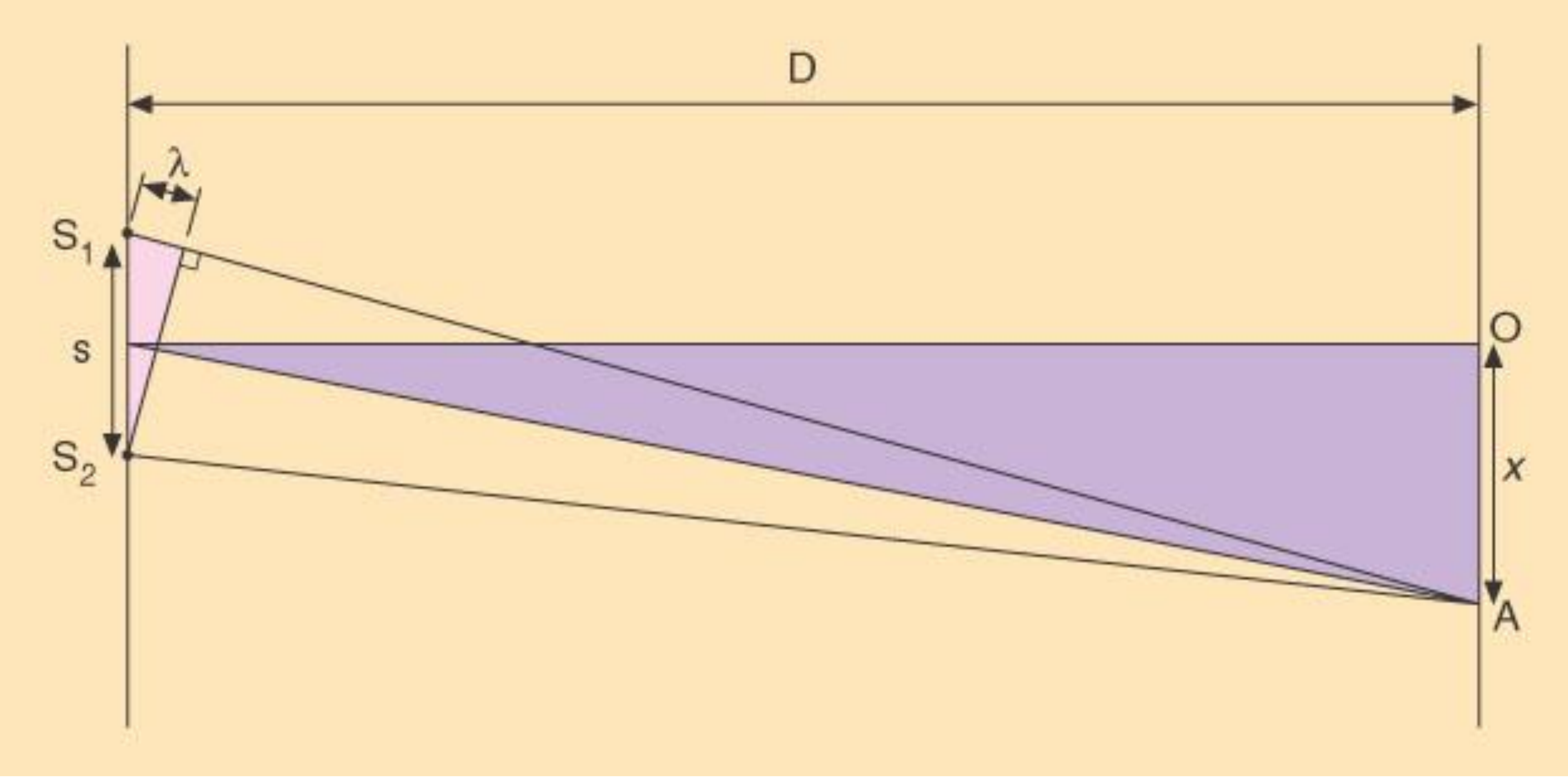
↓

Determine distance

Closer objects → Parralax

Table 22.1 *The electromagnetic spectrum*

Type	Frequency	Wavelength	How made	Uses	Photon energy
long-wave radio	~250 kHz	~1200 m	oscillating currents in aerials	radio	~10^{-28} J
medium-wave radio	~1000 kHz (1 MHz)	~300 m	oscillating currents in aerials	radio	~10^{-27} J
short-wave radio	~10 MHz	~30 m	oscillating currents in aerials	radio	~10^{-26} J
VHF	~100 MHz	~3 m	oscillating currents in aerials	radio	~10^{-25} J
UHF	~400 MHz	~1 m	oscillating currents in aerials	television	~10^{-25} J
microwaves	~2.5 GHz	~10 cm	directly produced in waveguides	radar, cooking, communicating	~10^{-24} J
infra-red	~10^{14} Hz	~1 μm (> 700 nm)	hot bodies, LEDs	night-sights, heating, short-distance communication	~10^{-19} J ~1 eV
visible	~5 × 10^{14} Hz	700 – 400 nm	very hot bodies, LEDs	seeing, etc	2 – 3 eV
ultra-violet	>7.5 × 10^{14} Hz	<400 nm	extremely hot bodies, sparks, discharge tubes	sun-tanning, detecting invisible marking, sterilising	> 3 eV
X-rays	~10^{18} Hz	~10^{-10} m	stopping fast electrons	X-raying people and materials	~10 000 eV
gamma rays (overlap with X-rays)	~10^{20} Hz	~10^{-12} m	nuclear decay	X-raying thick objects, killing cancerous cells, sterilising	~1 MeV ~10^{-13} J
cosmic rays	very high	very short	from distant parts of the Universe	just cause a hazard	up to tens of joules


D
λ
S₁
s
S₂
O
x
A

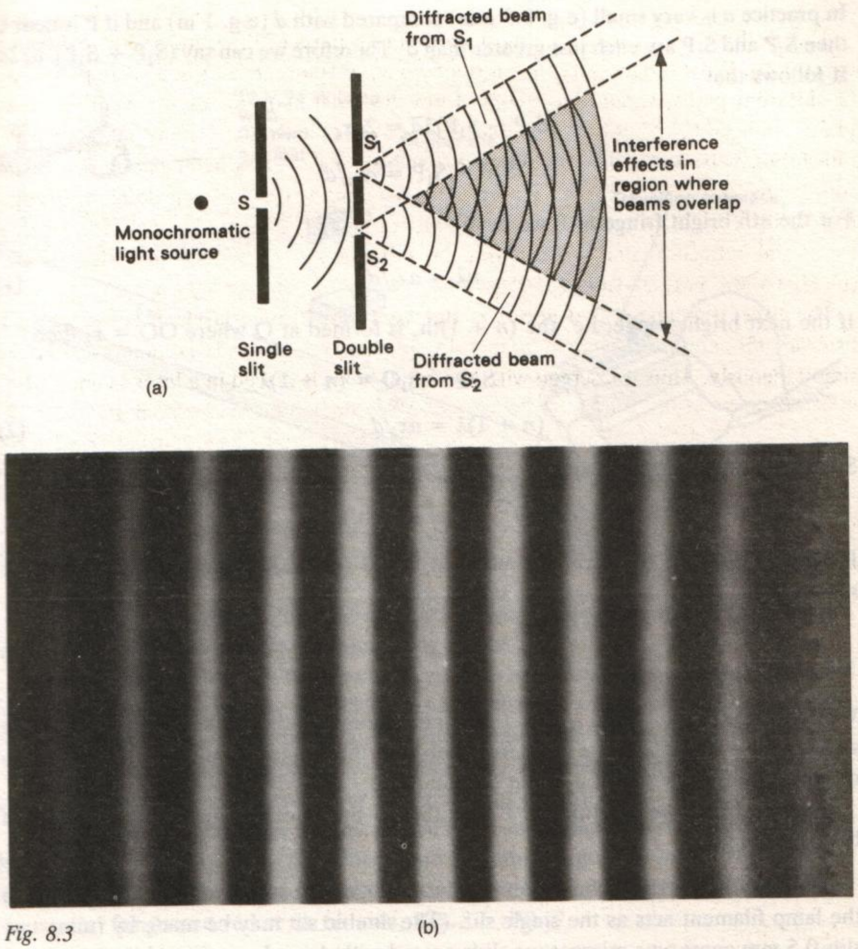

Diffracted beam from S1
Interference effects in region where beams overlap
S1
S
Monochromatic light source
S2
Single slit
Double slit
Diffracted beam from S2
(a)
(b)

Fig. 8.3

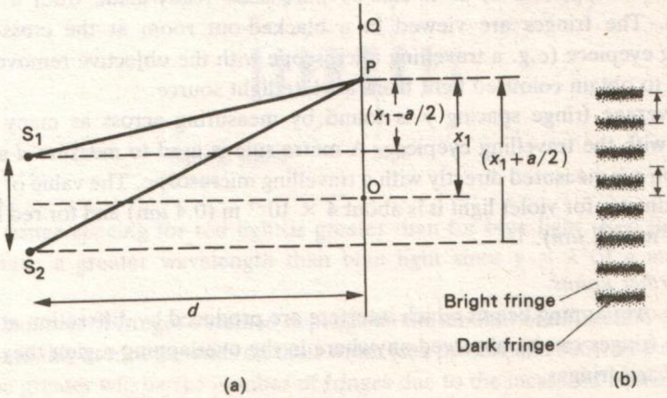
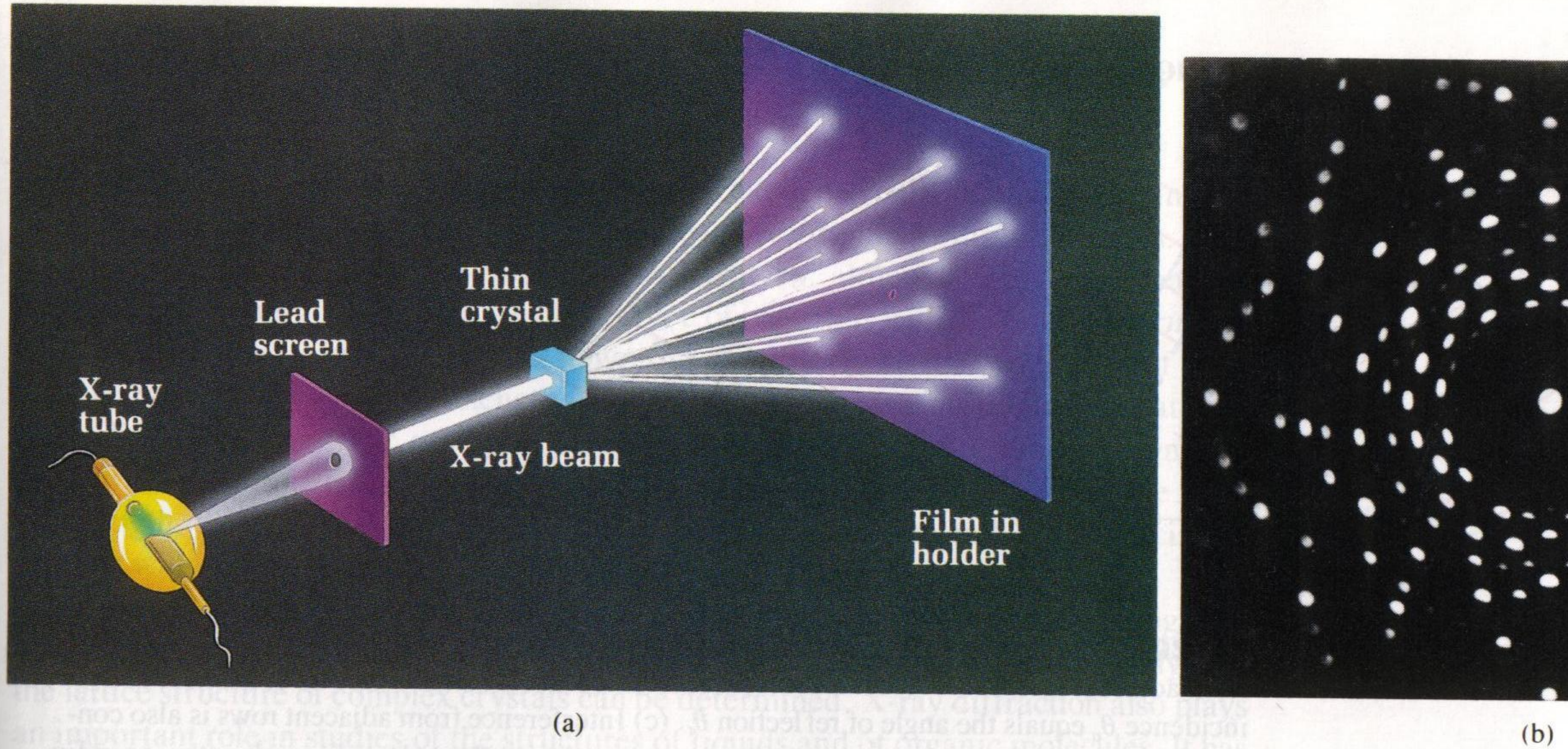
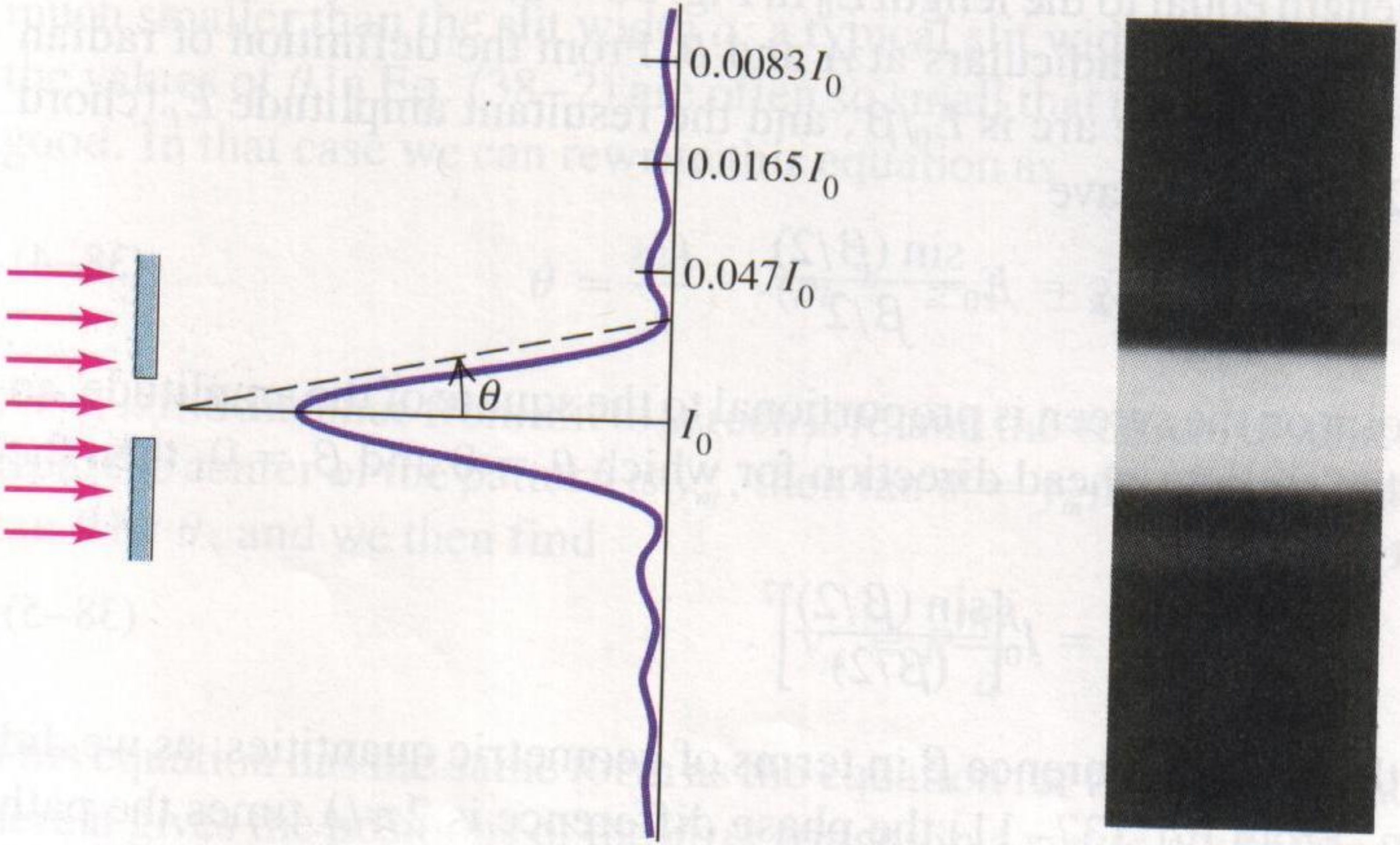

Q
P
S1
(x1 − a/2)
x1
(x1 + a/2)
O
a
S2
d
y
y
Bright fringe
Dark fringe
(a)
(b)

Fig. 8.4

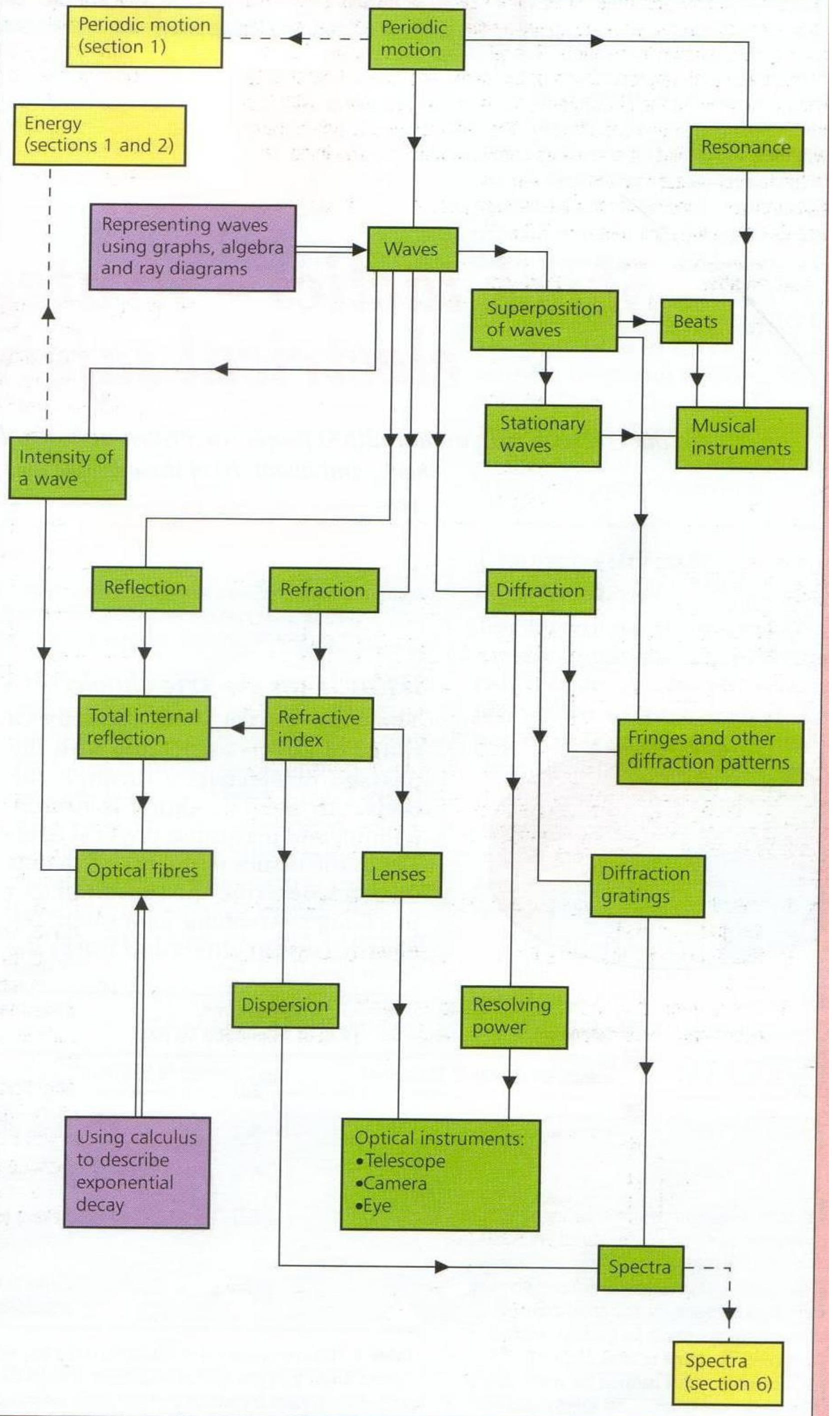


38–18 (a) In an x-ray diffraction experiment, most x rays pass straight through the crystal, but some are scattered, forming an interference pattern that exposes the film in a pattern related to the atomic arrangement in the crystal. (b) Laue diffraction pattern formed by directing a beam of x rays at a thin section of quartz crystal.



(a)

(b)


Periodic motion (section 1)
Periodic motion
Energy (sections 1 and 2)
Resonance
Representing waves using graphs, algebra and ray diagrams
Waves
Superposition of waves
Beats
Stationary waves
Musical instruments
Intensity of a wave
Reflection
Refraction
Diffraction
Total internal reflection
Refractive index
Fringes and other diffraction patterns
Optical fibres
Lenses
Diffraction gratings
Dispersion
Resolving power
Using calculus to describe exponential decay
Optical instruments:
•Telescope
•Camera
•Eye
Spectra
Spectra (section 6)